Part 3

tracking detectors

material effects

track models

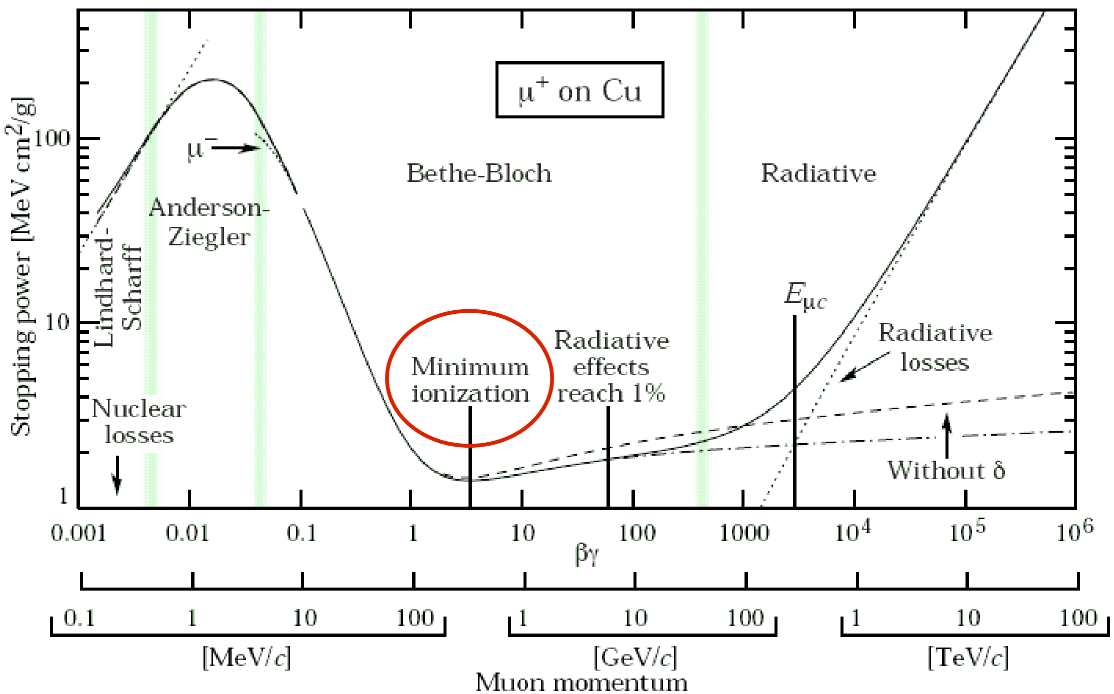
fitting with momentum

passage of particles through matter

- particles traversing through a medium interact with that medium
  - they loose energy
    - ionization: dominating effect for particles other than electrons/photons
    - bremsstrahlung: dominating effect for electrons
    - pair-production: dominating effect for photons
  - they scatter in (quasi-)elastic collisions: multiple coulomb scattering
- we'll briefly discuss these two effect because
  - important for understanding how tracking detectors work
  - important ingredient to track fitting
- for a real overview, look at relevant section in PDG

energy loss by ionization

- energy loss depends on particle momentum and properties of medium



from PDG:

energy loss of muons in Cu

expressed in MeV cm^2/g

- in the region 1-100 GeV energy, momentum dependence is mild
- mean specific energy loss described by Bethe-Bloch formula

$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

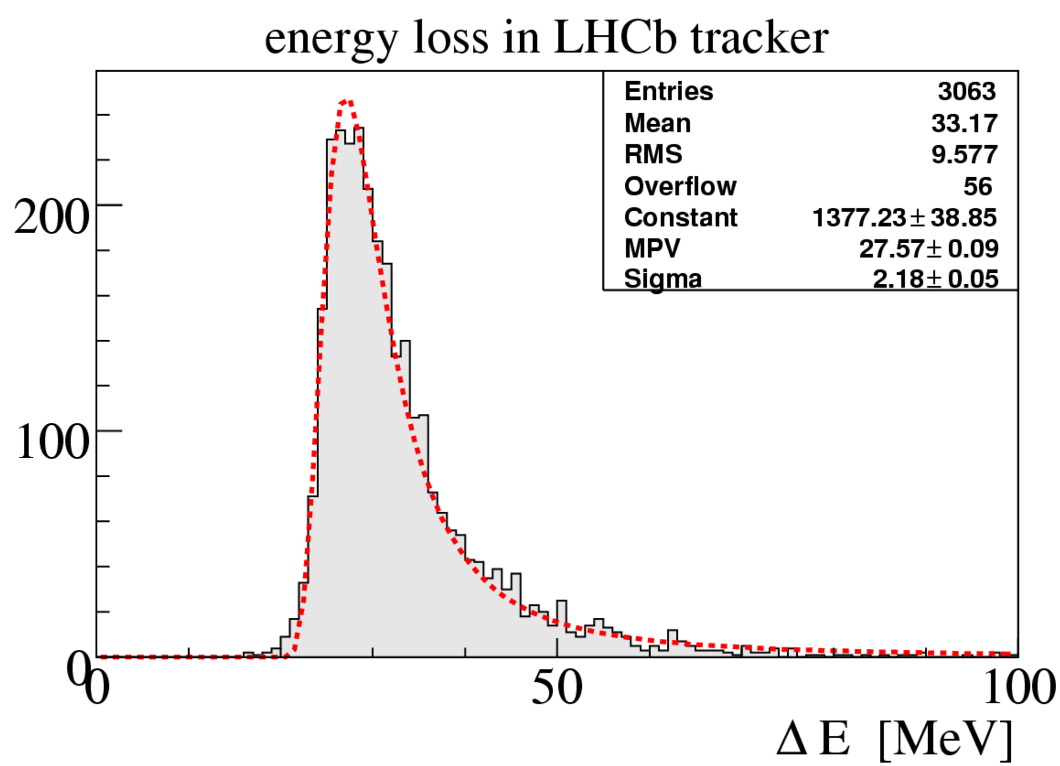
number density of electrons

ionization energy

energy loss by ionization (II)

- examples of typical energy loss, at 'minimum ionizing'
  - 1 meter air: 0.22 MeV
  - 0.3 mm Si: 0.12 MeV
  - 1 mm iron: 1.1 MeV
- energy loss is a stochastic process, approximately described by Landau distribution

- variations in energy loss can be large
- in particular, there is an 'infinitely' long tail

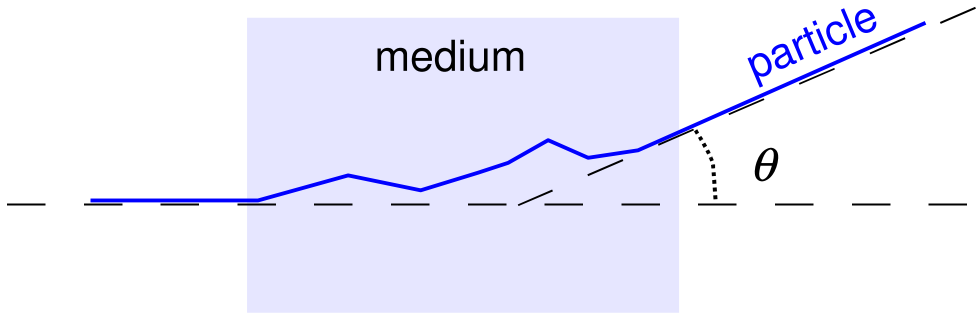


radiation length

- high energy electrons loose energy predominantly by bremsstrahlung
- radiation length X_0: mean distance over which electron looses all but 1/e of its energy by bremsstrahlung
- also characteristic length for pair production: survival probability for photon is 1/e over a length $7X_0/9$
- thickness of detector often expressed in 'fraction-of-radiation-length' x/X_0
- examples
  - 1 meter air: $x/X_0 = 0.003$
  - 300 micron silicon: $x/X_0 = 0.003$
  - 1 mm iron: $x/X_0 = 0.06$
- tracking detectors should be thin, calorimeters should be thick

multiple scattering

- charge particle traversing a medium is deflected by many small-angle scatters, mostly due to Coulomb scattering from nuclei



- scattering distribution described by theory of Molière
- roughly gaussian for small angles, *RMS* for central 98% given by

$$\theta_0 = \frac{13.6\mathrm{MeV}}{\beta c p} z \sqrt{x/X_0} \left(1 + 0.038 \ln(x/X_0)\right)$$

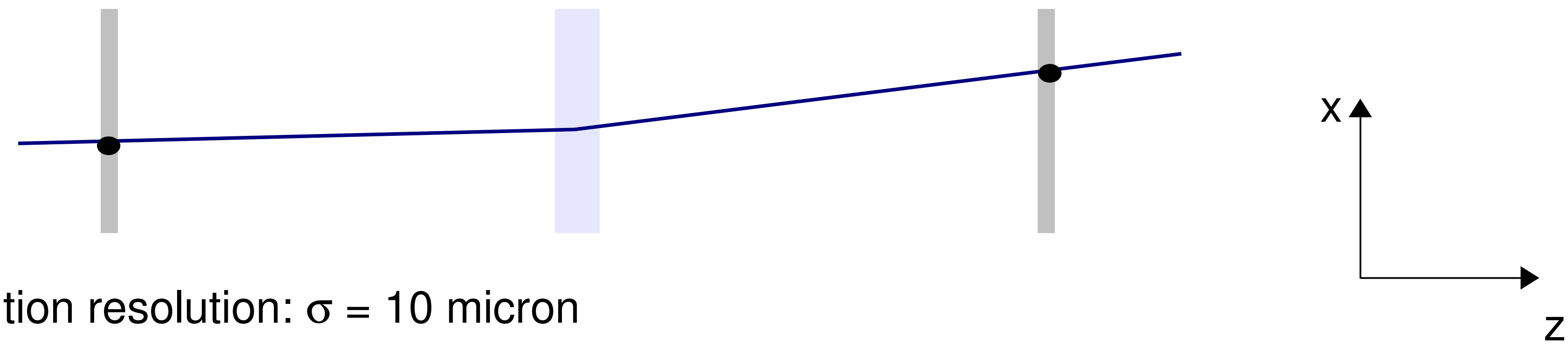
square-root of radiation thickness

~ 1/momentum

charge of incoming particle

example of multiple scattering effect

- take this two layer detector that measures position and angle of track



position resolution: σ = 10 micron

distance between planes: L = 10cm

thickness of scatterer: x/X0 = 0.003 --> typical thickness of Si wafer

- resolution on slope without scattering medium

$$\sigma(t_0) = \frac{\sigma}{\sqrt{N(\langle z^2\rangle - \langle z\rangle^2)}} = \frac{\sqrt{2}\sigma}{L} \approx 1.4\,\mathrm{mrad}$$

- filling in the Moliere scattering formula: θ_0(1GeV) = 0.7 mrad

- thickness of detectors itself can degrade resolution considerably

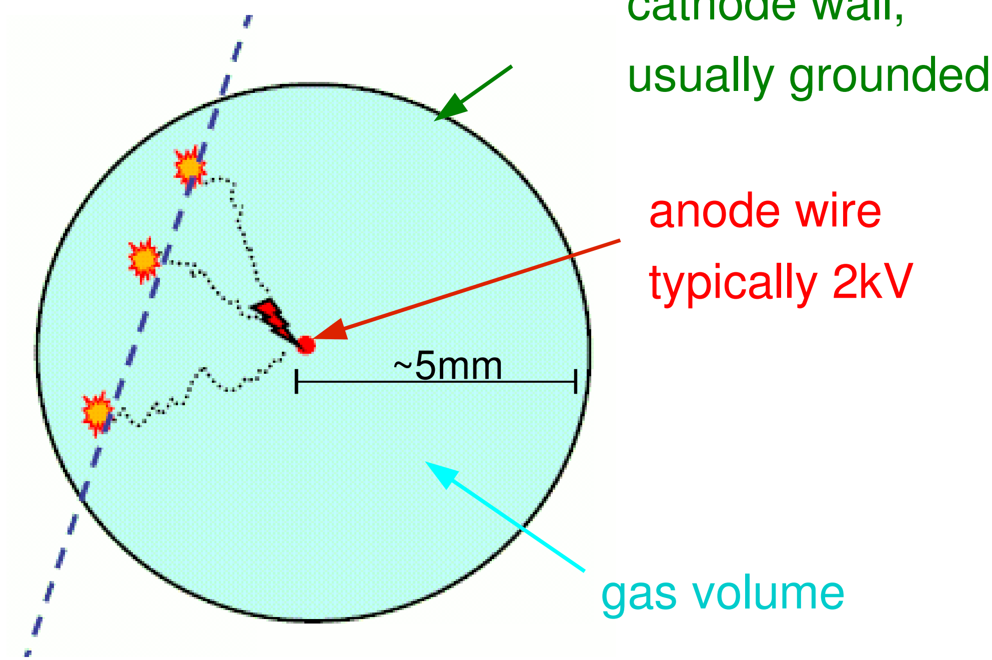
- precision of slope measurement in Si detector usually only determined by first few hits

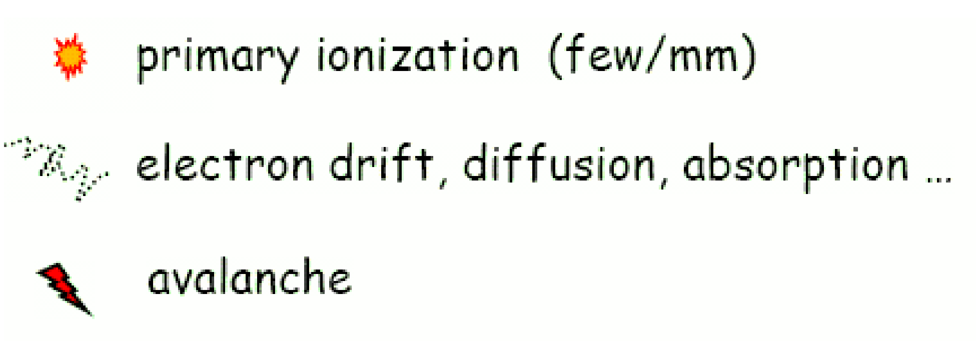
tracking detectors

- tracking detectors measure the position of a particle at several well-defined points along its trajectory
- in most multi-purpose detectors, tracking system consist of
  - a 'vertex' detector: good position resolution close to interaction point
  - a 'spectrometer': measure momentum from curvature in magnetic field
- some things to worry about when designing a tracking detector
  - impact parameter resolution (lifetimes)
  - momentum and direction resolution (invariant mass)
  - weight: multiple scattering, loss of particles due to interactions
  - radiation hardness, granularity, efficiency
- the most popular tracking devices are *wire drift chambers* and *silicon strip detectors*

wire drift chambers

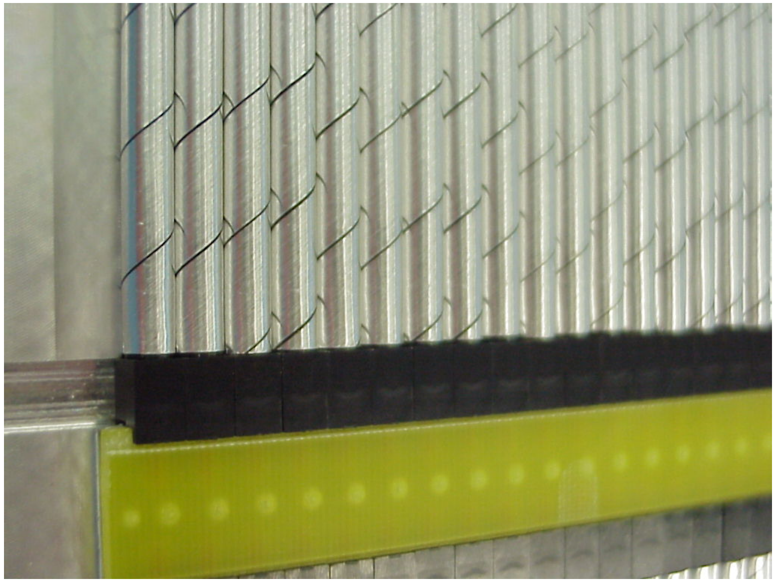
sketch of a 'drift cell'



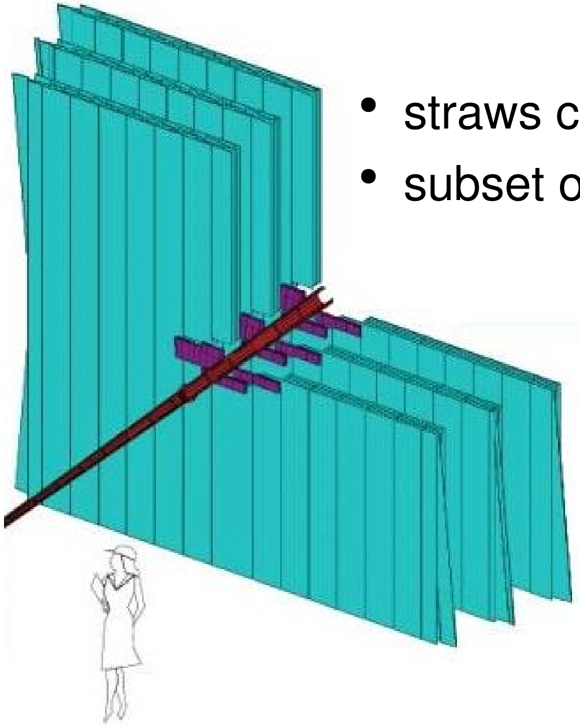


- ionization of molecules along particle trajectory
- electrons/ions separated by large field
- electrons collected at anode sense wire
- drift time is measure for distance between wire and trajectory
- typical spatial resolution: 200 μm
- typical spacing between cells: 1cm
- relatively low cost/covered surface
- relatively light

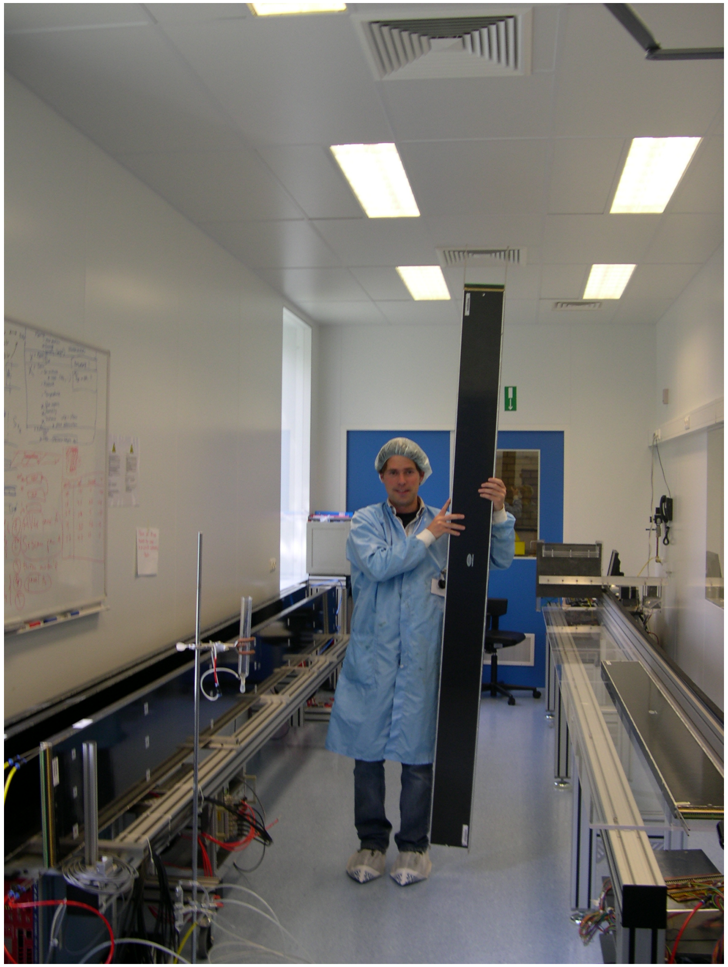
example: LHCb drift chamber



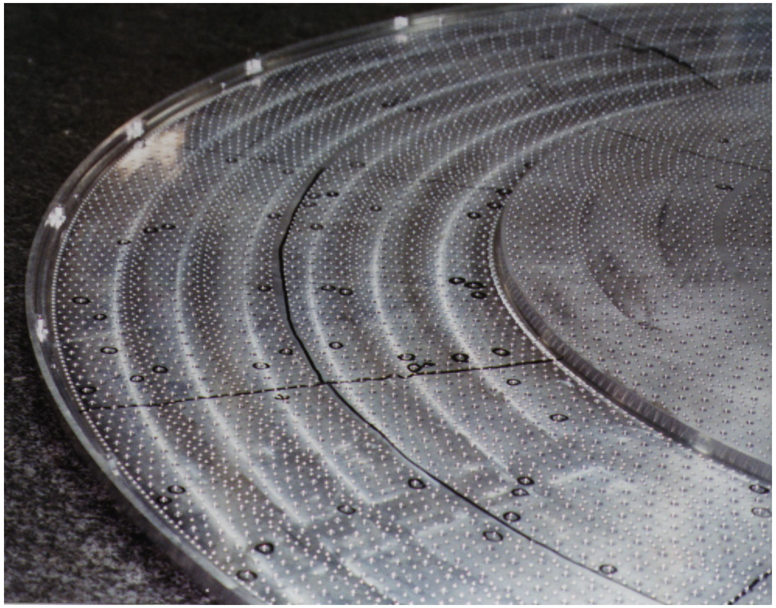
- 'straw tube' chamber
- cathode straws made of kapton with conductive condensate



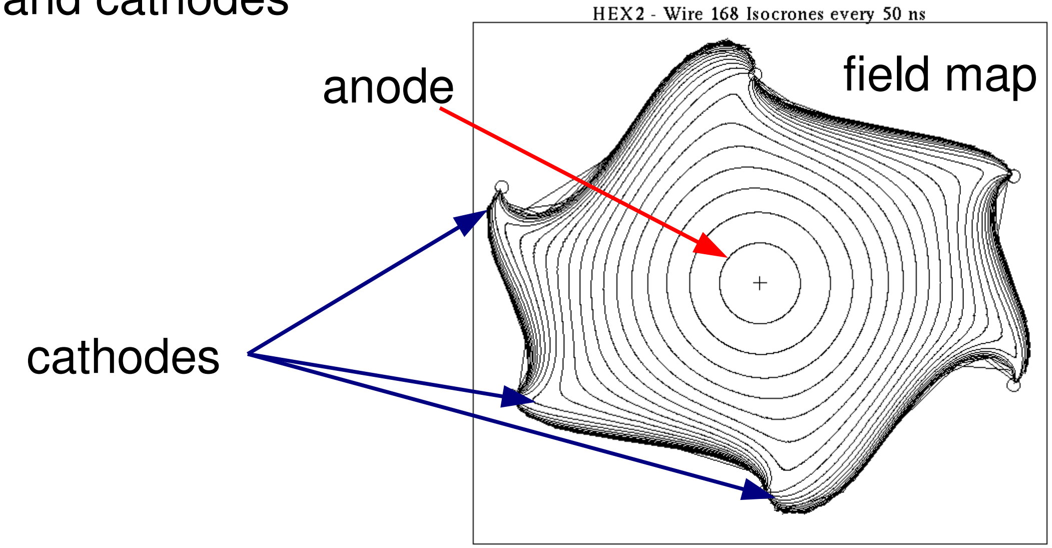
- straws combined in long (4m) modules
- subset of layers rotated by +-5^{o}



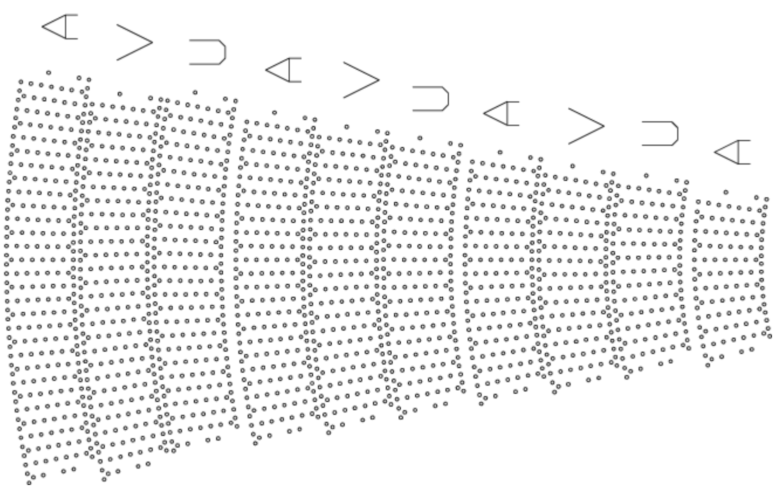
example: babar drift chamber

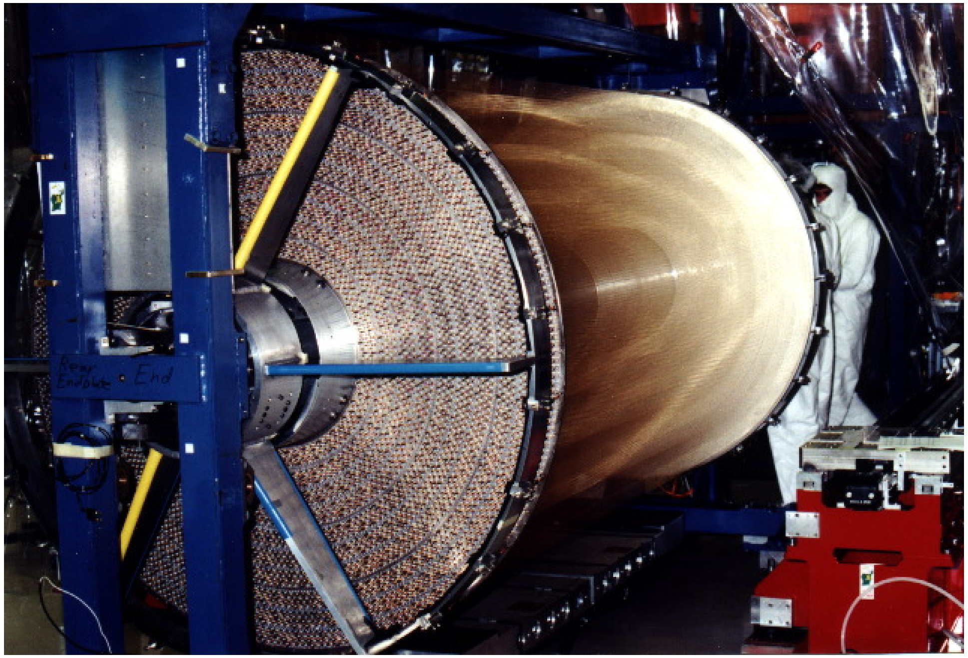


- cylindrical, 'open' geometry: wires for both anodes and cathodes

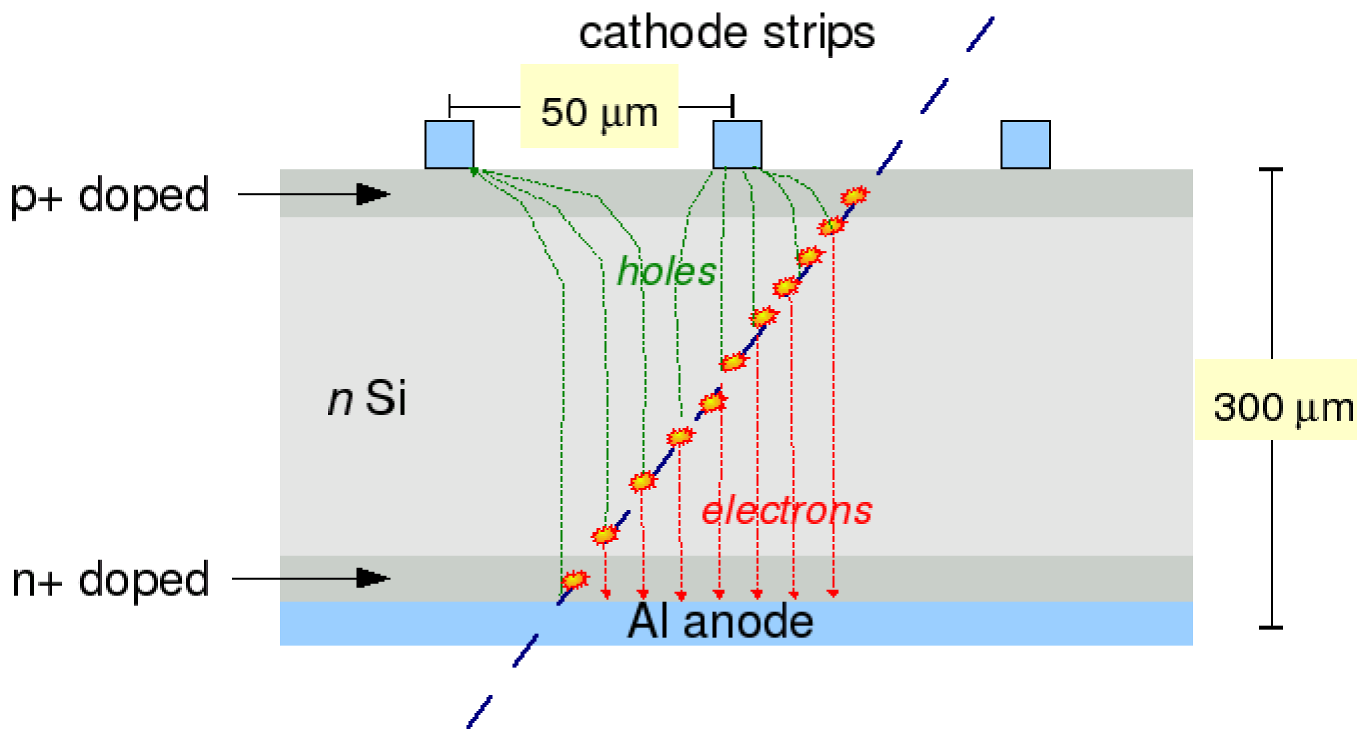


- in the 'U” and 'V' layers, the wires are not parallel to the z-axis: stereo angles





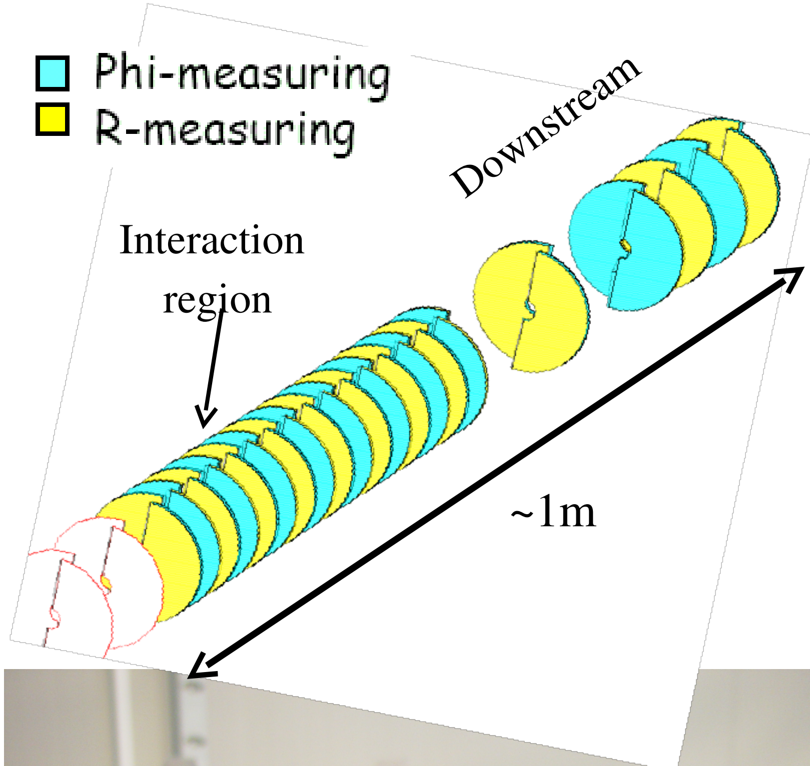
silicon strip detector



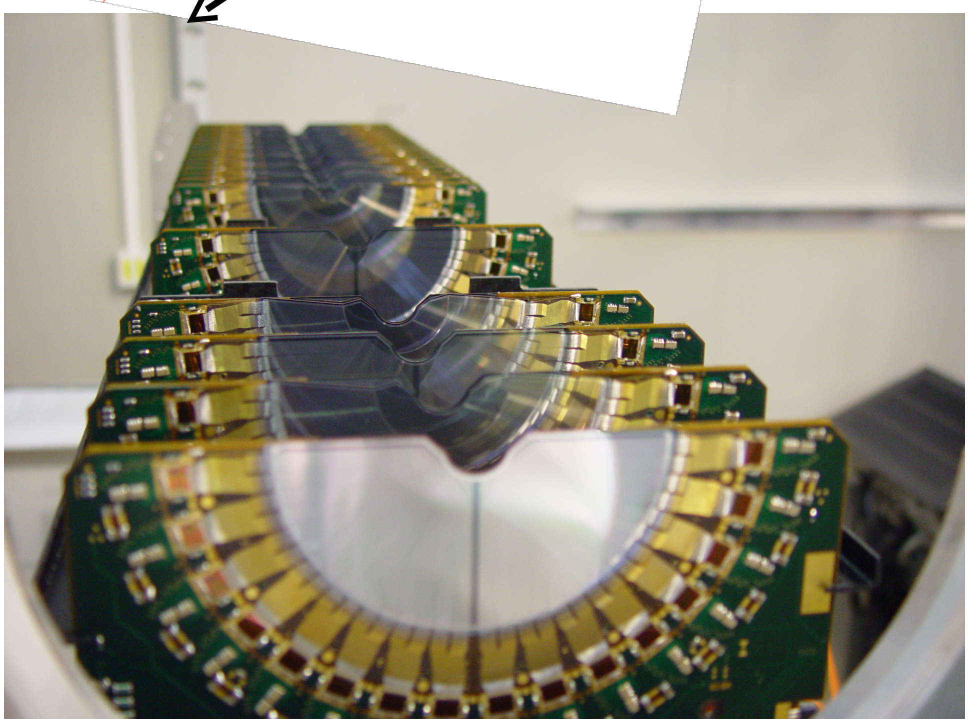
- measures ionization in doped silicon substrate
- typical strip pitch: 50-100 μm
- resolution: better than pitch/sqrt(12), down to ~10 μm

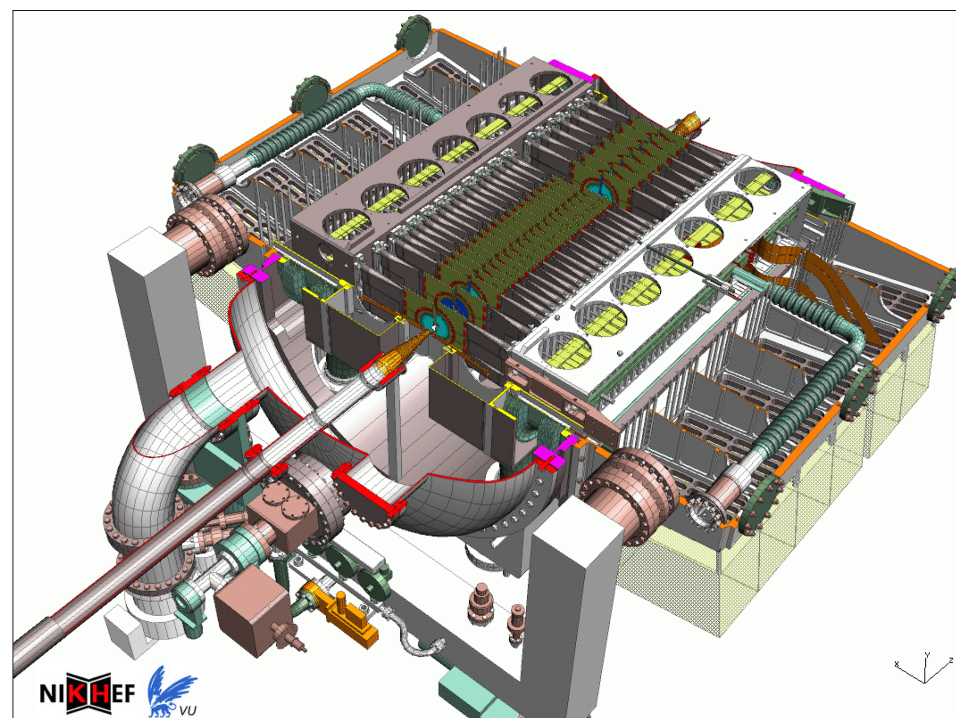
  (make use of division of charge between strips)
- relatively high cost per covered surface
- relatively heavy, radiation hard

example: LHCb silicon strip detector



- 21 tracking layers
- R-phi geometry: some planes measure distance to beam pipe, others measure angle phi





stereo views

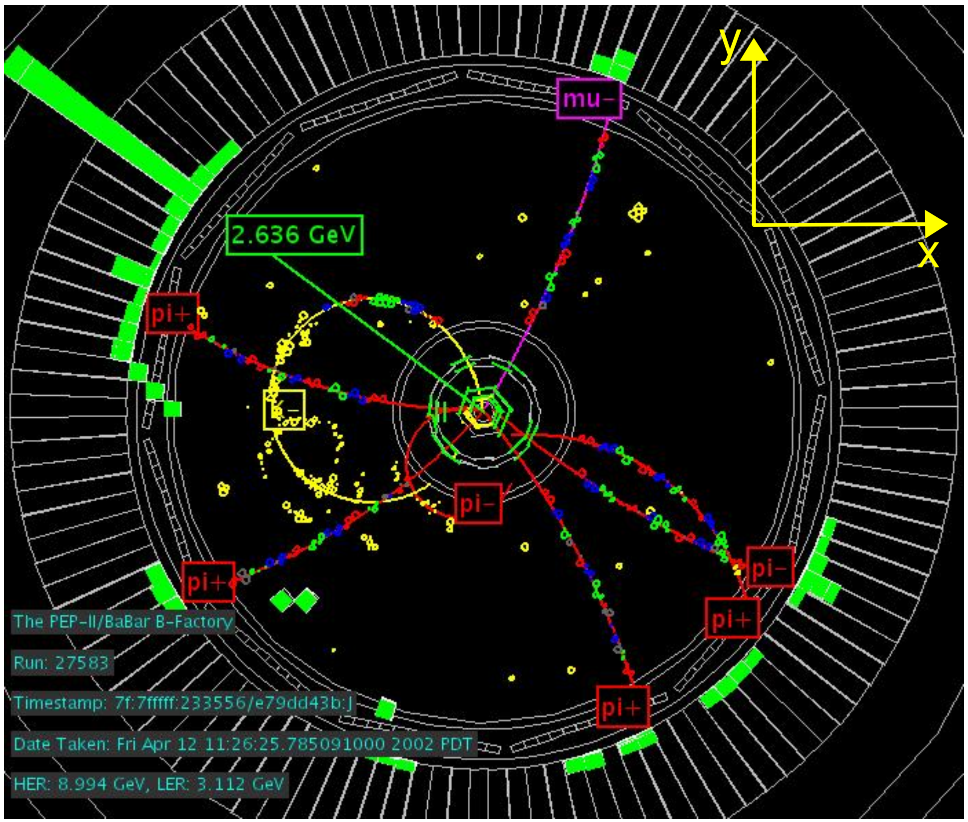
- wire chambers and silicon strip detectors measure distance of charged particle to a 'line' in space: *single coordinate measurements*
- if all strips are parallel, can only reconstruct trajectory in 2 dimensions, namely in plane perpendicular to the strips
  - this is why most tracking devices use so-called 'stereo views'
  - strip orientations different in different planes
- different scenerios for rotation angles
  - large angle, perpendicular strips
    - position resolution equal for both axis in plane
  - small angle, typically 5^{o}:
    - measure one coordinate more accurate than the other
    - easier for track finding (tomorrow, part 5)

tracking detector geometry

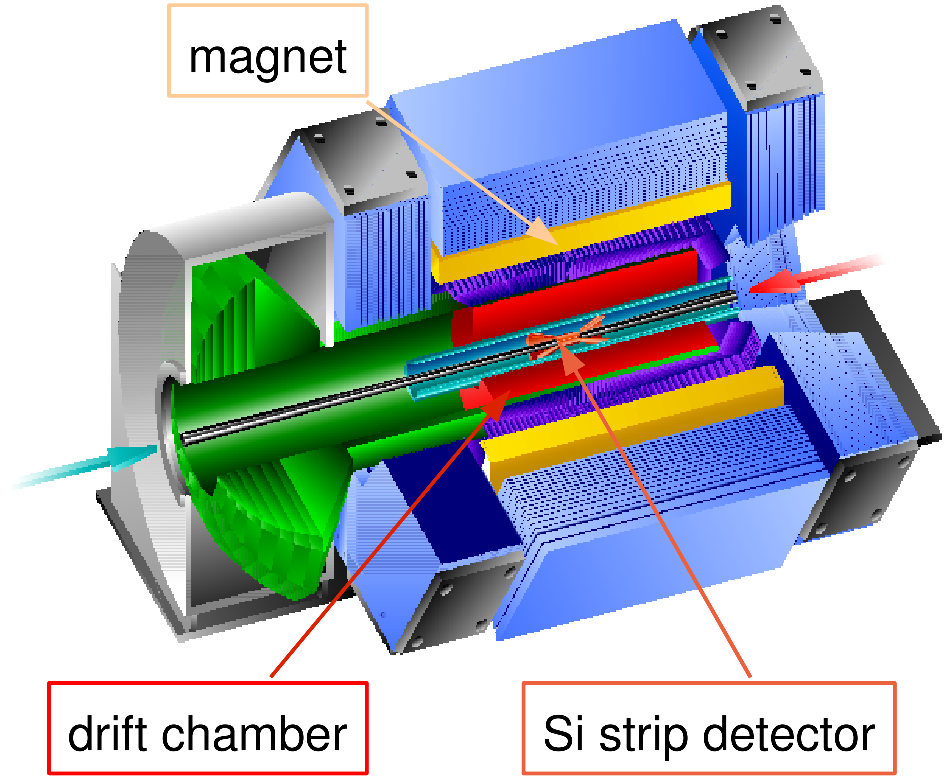
- two typical geometries
  - forward geometry (also 'forward spectrometer')
    - covers only small part of total acceptance
    - usually at fixed target, but sometimes at collider (LHCb)
  - cylindrical geometry (also 'central detector')
    - covers as much acceptance as possible
    - most detectors at colliders
- important for our discussion: coordinate systems and tracking parameterizations differ for these two setups

central detector

- example: Babar



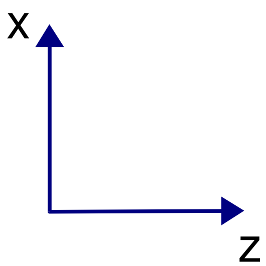
not typical: Babar is asymmetric

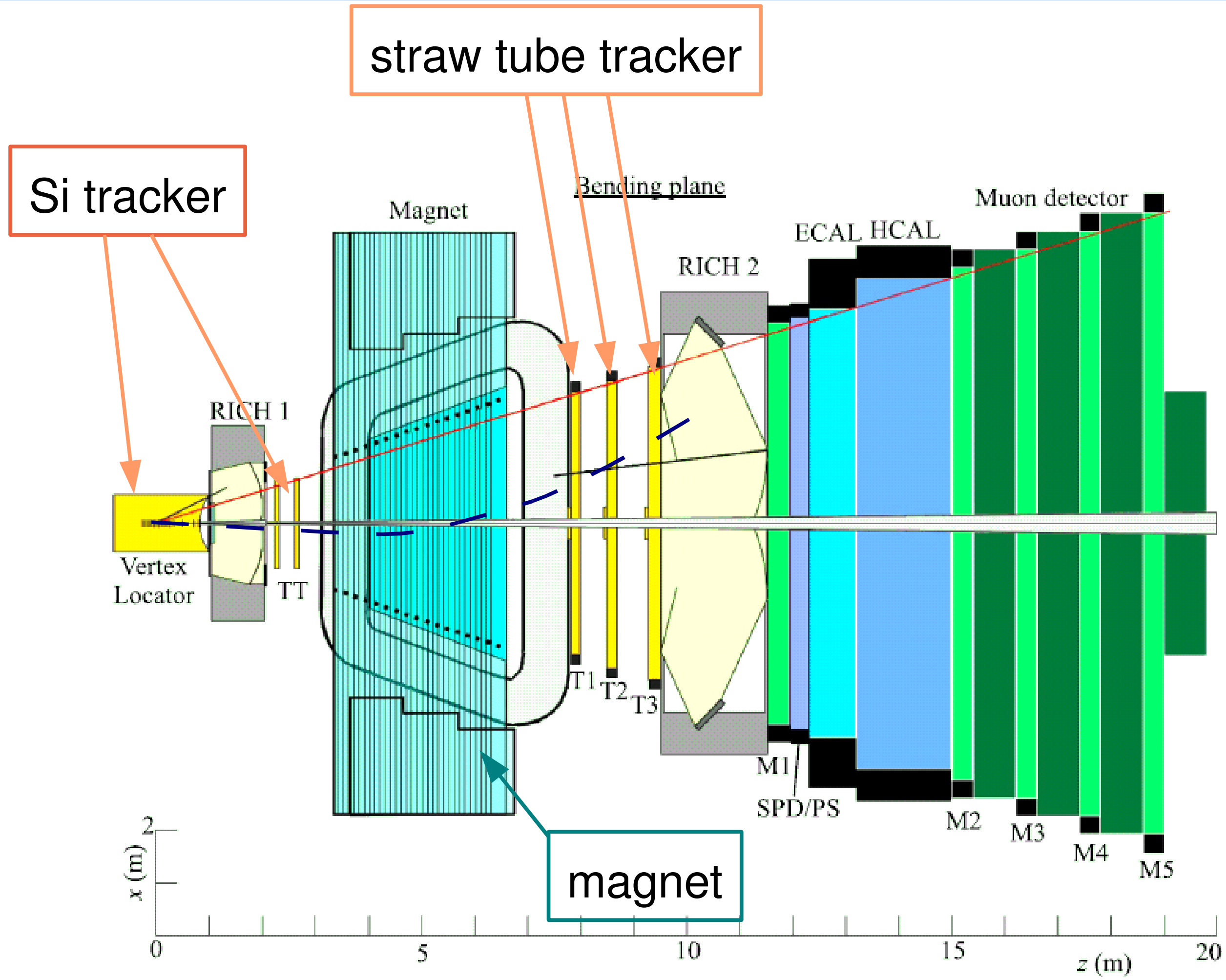


- B-field parallel to z-axis (~ parallel to beam)
- bending plane is xy-plane
- magnetic field approximately constant throughout tracker volume

forward detector

- example: LHCb





- B-field along y-axis (vertical)
- 'bending plane' is xz-plane
- magnetic field non-zero only in part of tracking volume

track models

- **track model:** parameterization of charged particle trajectory
- in stationary field in vacuum, motion of stable particle completely described by initial position and momentum --> 6 quantities
- however, tracking detectors don't measure where particle started
  - initial position can be chosen anywhere along trajectory
  - track models have **5** parameters
- if multiple scattering kinks are included in the track model, the situation becomes more complicated: many more parameters
- because magnetic field configurations differ, the actual model depends on the detector geometry

particle trajectory in constant magnetic field

- equation of motion in magnetic field

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = q\,\vec{v}\times\vec{B}$$

- velocity is constant: coordinate along trajectory $s = vt$

$$\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}s^2} = \frac{q}{p}\,\frac{\mathrm{d}\vec{r}}{\mathrm{d}s}\times\vec{B}$$

- in homogeneous field, particle trajectory follows **helix** with curvature radius

$$\rho = \frac{p_\perp}{q|B|}$$

- example: in Babar field of 1.5T

--> radius for particle with p_T = 1GeV/c is 2.2m

- helpful formula: electric charge in 'hybrid units'

$$e = 0.2998\left(\frac{\mathrm{GeV}}{c}\right)T^{-1}m^{-1}$$

helix parameterization

- if field along z-axis, helix trajectory can be parameterized as

$$\begin{pmatrix} d_0 \\ \phi_0 \\ q/p_\perp \\ z_0 \\ \theta \end{pmatrix} = \left\{ \begin{array}{l} \text{(signed) distance to } z \text{ axis at poca} \\ \text{azimuthal angle at poca} \\ \text{charge over transverse momentum} \\ z \text{ coordinate of poca} \\ \text{polar angle} \end{array} \right.$$

$\left.\begin{array}{l}d_0\\ \phi_0\\ q/p_\perp\end{array}\right\}$ motion in xy-plane

- 'poca': point of closest approach to z-axis
- position and momentum along trajectory in terms of these parameters

$$\begin{aligned} x &= \rho\sin(\phi) - (\rho + d_0)\sin(\phi_0) \\ y &= -\rho\cos(\phi) + (\rho + d_0)\cos(\phi_0) \\ z &= z_0 + s\cos\theta \\ p_x &= p_t\cos(\phi) \\ p_y &= p_t\sin(\phi) \\ p_z &= p_t\cot\theta \end{aligned}$$

where

$$\rho = \frac{p_\perp}{qB_z}$$

$$\phi = \phi_0 + \frac{s\sin\theta}{\rho}$$

- of course, no need to remember this, but useful to have seen it once

non-homogeneous magnetic field

- in real detectors magnetic field is not homogeneous
  - can still use helix parameterization, but parameters are now defined locally: they depend on coordinate along trajectory
  - changing this coordinate is called 'propagation'. we'll discuss it in more detail tomorrow

- usually, the magnetic field is measured on a grid (the 'field map')
  - numerical methods are used to interpolate on the grid and to integrate the field along the trajectory
  - there is always a compromise between the accuracy of the integration and speed
  - it is a not uncommon source of momentum bias

parameterization in forward detectors

- helix parameterization useful when detector fully embedded in approximately constant field
- in forward detector like LHCb, no field in large part of tracking system
- more suitable parameterization in terms of straight lines

$$\begin{pmatrix} x_0 \\ y_0 \\ t_{x0} \\ t_{y0} \\ q/p \end{pmatrix} = \left\{ \begin{array}{l} x\text{-position at } z = 0 \\ y\text{-position at } z = 0 \\ \mathrm{d}x/\mathrm{d}z \text{ at } z = 0 \\ \mathrm{d}y/\mathrm{d}z \text{ at } z = 0 \\ \text{charge over momentum} \end{array} \right.$$

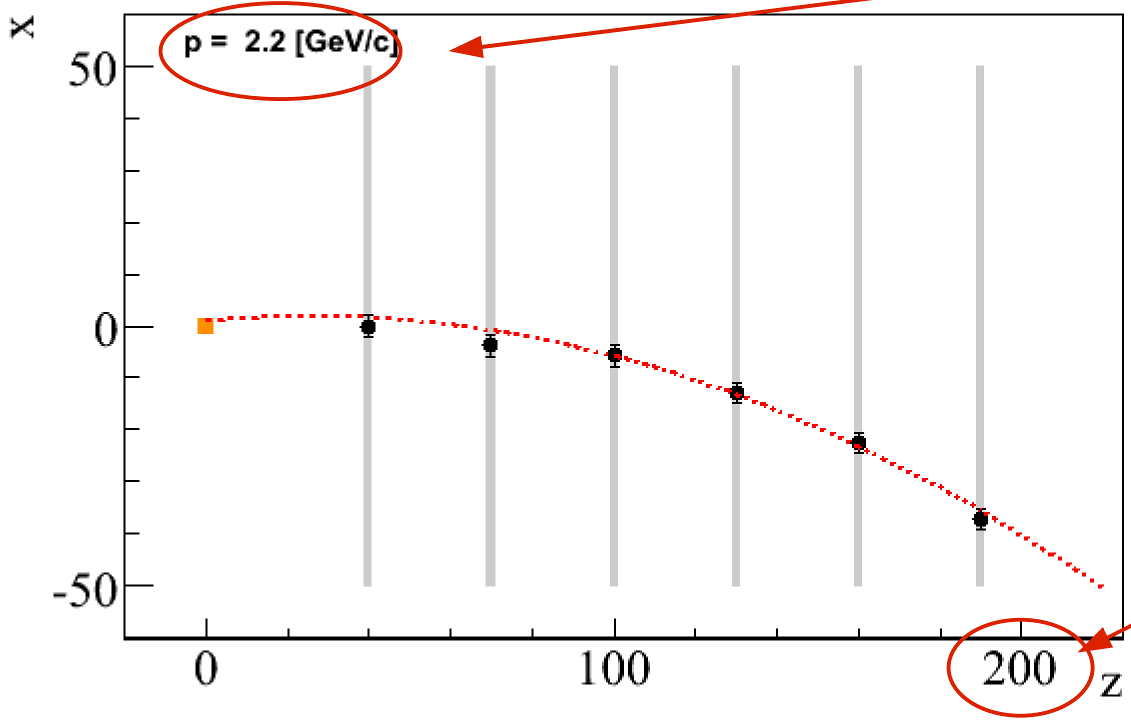
now express coordinates as function of z along track using (numerical) integration of B field between origin and z

- these coordinates are again defined locally: use B field integration to express them as function of z
- if field along y, 'kick' in magnetic field $\Delta t_x \approx \frac{q}{p_{xz}} \int B_y \mathrm{d}z$
- for example, LHCb: $\int B\,\mathrm{d}z = 4.2$ Tm --> $\Delta t_x \approx 0.13$ for 10 GeV particle

extension of toy track fit model

- introduce a 1 Tesla magnetic field

momentum of generated particle



from now on units are [cm]

- to keep things simple, use parabolic trajectory (rather than circle)

$$x(z) = x_0 + t_0\, z + \omega\, z^2$$

position at z=0

slope at z=0

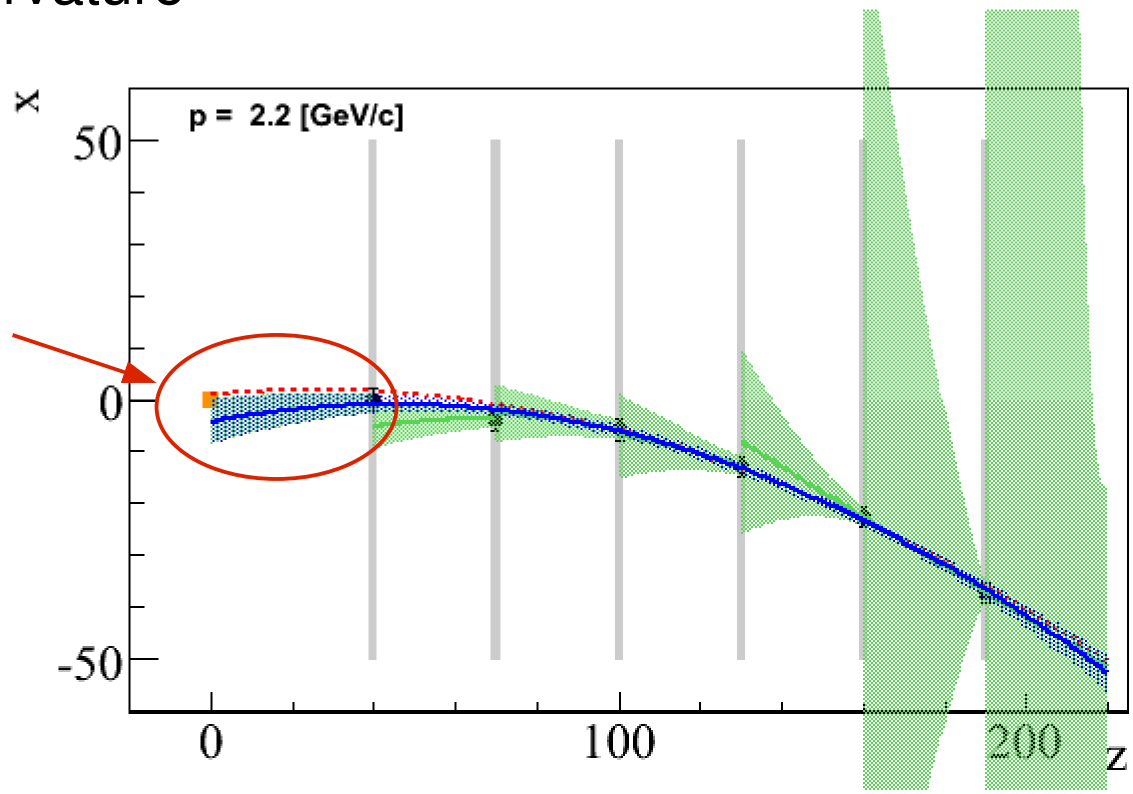
curvature

$$\omega \equiv \frac{1}{\rho} = \frac{qB}{p}$$

fitting with momentum

- fit for position, slope and curvature

kalman fit and global fit
still give identical results



- note that to fit for curvature, we need at least 3 points on the track

momentum resolution

- using what we discussed this morning, I tried to write down full expression

$$\mathrm{var}\left(\omega\right) \;=\; \frac{\sigma_{\mathrm{hit}}^2}{N} \, \frac{\langle z^2\rangle - \langle z\rangle^2}{\langle z^4\rangle\left(\langle z^2\rangle - \langle z\rangle^2\right) - \langle z^2\rangle^3 - \langle z^3\rangle^2 + 2\,\langle z\rangle\,\langle z^2\rangle\,\langle z^3\rangle}$$

- inverting a 3x3 matrix isn't that easy, so I cannot guarantee that it is right!
- it is also not particularly illuminating, so let's try something simpler:
  - note that measurement model is still linear in all track parameters
  - therefore, resolution in curvature does not depend on curvature

$$\sigma(\omega) \;=\; \text{cnst.} \qquad \Longrightarrow \qquad \frac{\sigma(p)}{p} \;\propto\; p$$

- in the absense of multiple scattering, the *relative momentum resolution is proportional to the momentum*

momentum resolution (II)

- we can go a little bit further: curvature resolution must be
  - proportional to hit resolution
  - inversely proportional to another length-squared (dimensions ...)
- translating that into the momentum, gives

$$\frac{\sigma(p)}{p} \propto \frac{\sigma_{\mathrm{hit}}}{|q|\, B\, L^2}\, p$$

- think of 'BL' is the field integral: the more field on the trajectory, the more bending and therefore more momentum resolution per hit-resolution
- think of the remaining 'L' the arm of the spectrometer: the longer the arm, the more precise you can measure direction, therefore curvature, therefore momentum
- the full expression contains moments of z-coordinates of tracking planes