A Derivation of the Standard Model

Beatriz Gato-Rivera, Bert Schellekens

CSIC

NIKHEF

Nucl.Phys. B883 (2014) 529-580

Nijmegen, 9-3-2015

Towards

(not including the number of families)

discrete structure of the

A Derivation of the Standard Model

(in a class of intersecting brane models)

Our goal:

Derive the discrete structure of the Standard Model:
The gauge group and representations.

The standard approach is to use Grand Unification.

But this does not really work.

Grand Unification

The simplicity is undeniable:

$$SU(3)\times SU(2)\times U(1) \quad \subset \quad SU(5) \quad \subset \quad SO(10)$$

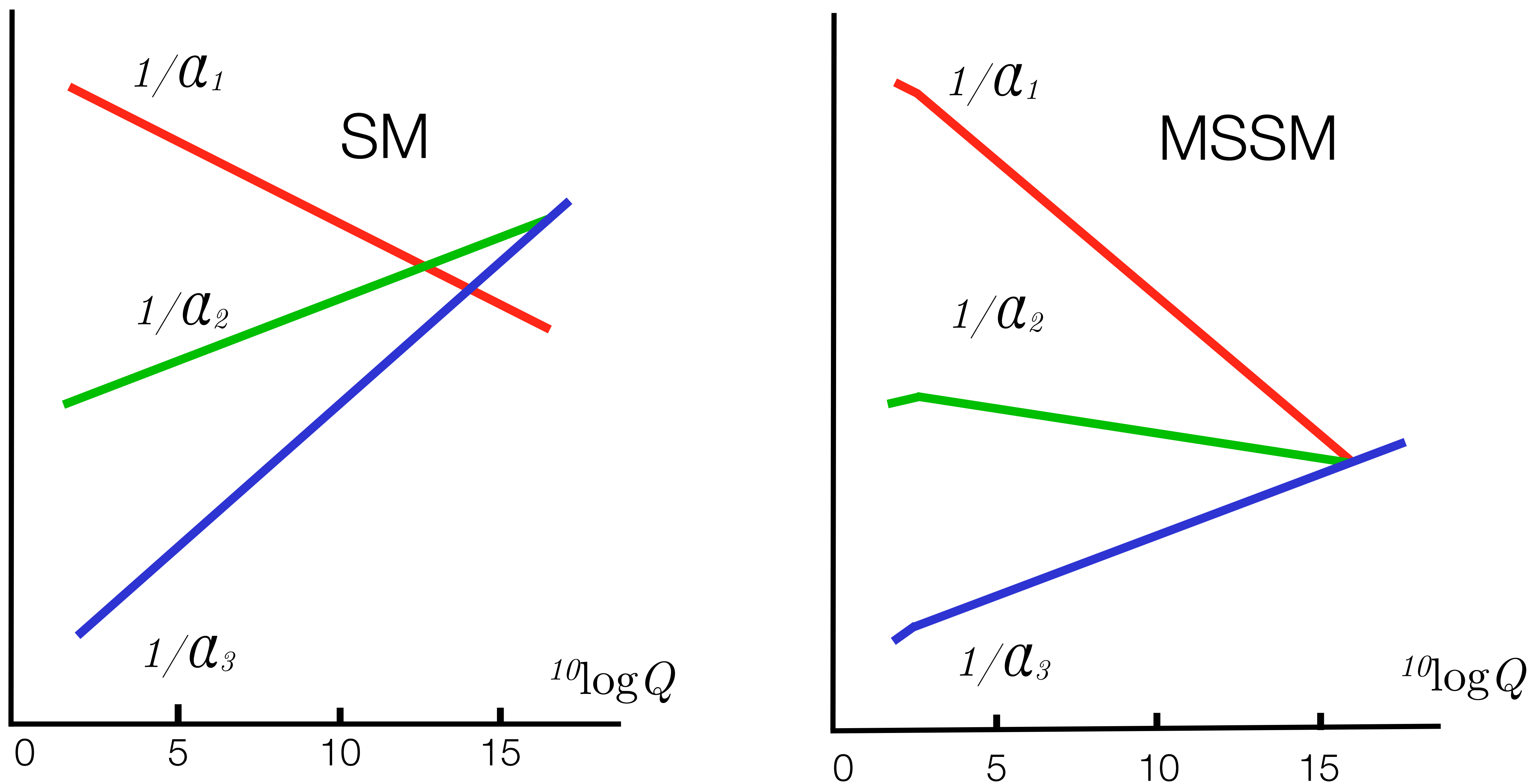
One family matter representation (left-handed)

$$(3,2,\frac{1}{6}) + (3^*,1,\frac{1}{3}) + (3^*,1,-\frac{2}{3}) + (1,2,-\frac{1}{2}) + (1,1,1) \; + (1,0,0)$$

Fits beautifully in the $\mathbf{(16)}$ of $SO(10)$

And the coupling constants meet each other if there is low energy supersymmetry.

So how could this be wrong?



SM
$1/\alpha_1$
$1/\alpha_2$
$1/\alpha_3$
$^{10}\log Q$
0
5
10
15
MSSM
$1/\alpha_1$
$1/\alpha_2$
$1/\alpha_3$
$^{10}\log Q$
0
5
10
15

Grand Unification

Even if correct, GUTs do not lead to a derivation of the SM structure:

- Even the smallest group, $SU(5)$, can break in two ways, to $SU(3)\times SU(2)\times U(1)$ or $SU(4)\times U(1)$.
- The Standard Model Higgs is not determined, and does not fit in an $SU(5)$ multiplet.
- In QFT the representations are determined if one assumes some kind of minimality, but what is the motivation for that?
- No top-down arguments selecting $SU(5)$ or $SO(10)$.

We will show that in a certain minimal string setting where GUT realizations are available, anthropic arguments work far better:

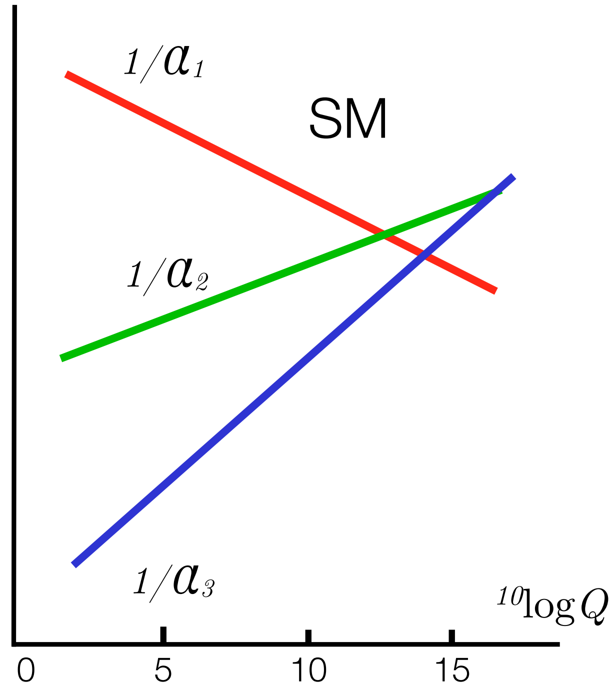
- Gauge group determined to be $SU(3)\times SU(2)\times U(1)$.
- Matter determined to be a number of standard families.
- Correct charge quantization without GUTs.
- Standard Model Higgs determined.

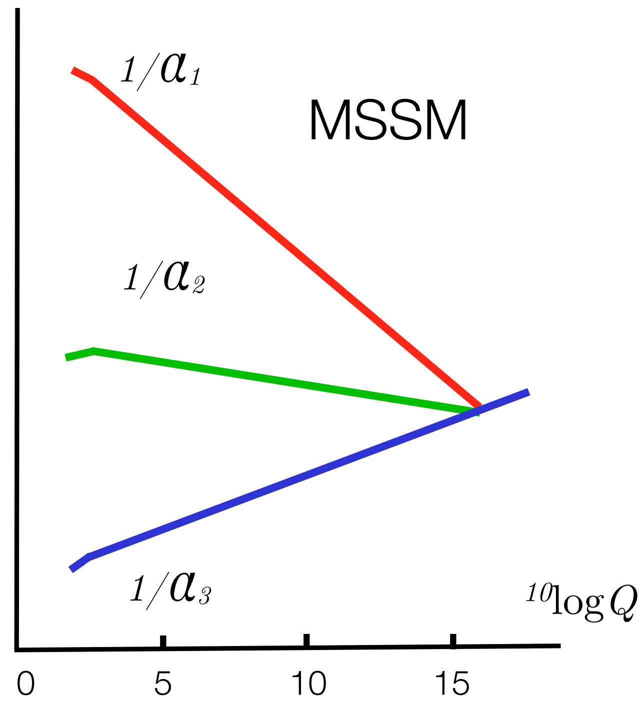
Assuming at least one unbroken non-abelian and at least one unbroken electromagnetic interaction

GUTs, Anomalies and Charge Quantization

If there is no low-energy supersymmetry, the three gauge coupling constants do not converge.

This removes one of the arguments in favor of GUTs.





But the arguments based on family structure and charge quantization remain valid.

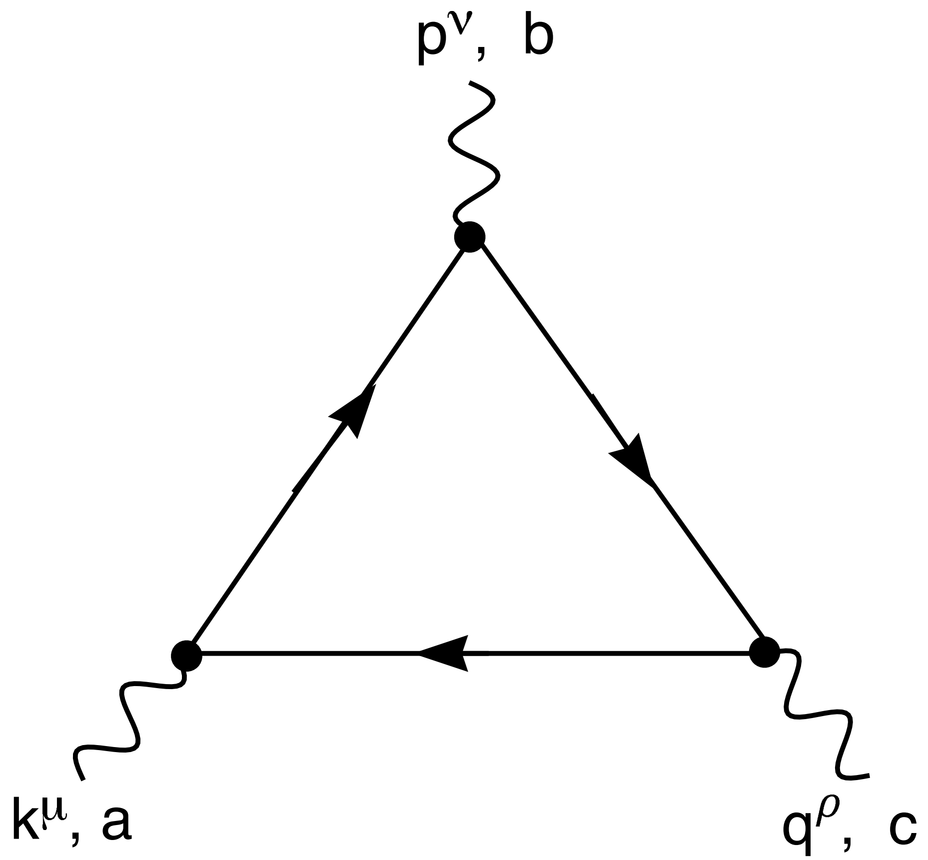
GUTs, Anomalies and Charge Quantization

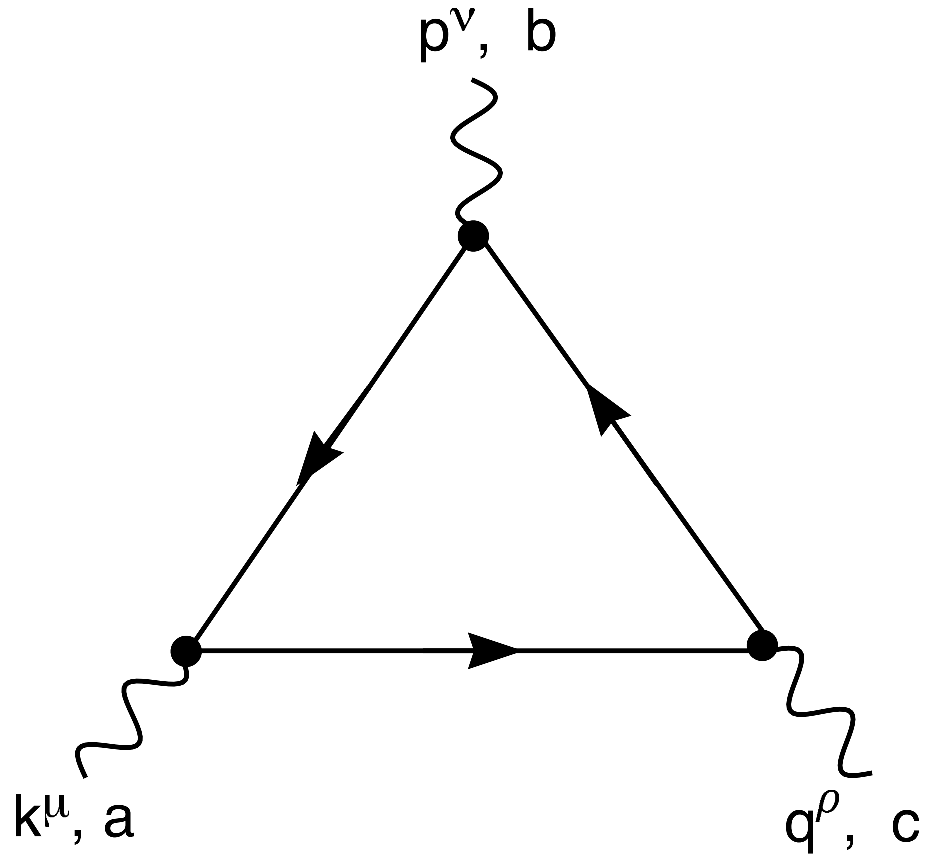
The observed charged quantization is excellent evidence for BSM physics.

Imagine we end up with a consistent theory of quantum gravity that imposes no constraints on QFT. Then this would allow particles with arbitrary real charges. It is hard to accept that we just happen to live in a universe with quantized charges.

One often hears the arguments that anomaly cancellation imposes charge quantization.

Triangle anomalies





		SU(3)	SU(2)	$SU(3)^2$ x U(1)	$SU(2)^2$ x U(1)	$U(1)^3$	(Grav) x U(1)
Q	(3,2,1/6)	2	0	1/3	1/2	1/36	1
U*	(3*,1,-2/3)	-1	0	-2/3	0	-8/9	-2
D*	(3*,1,1/3)	-1	0	1/3	0	1/9	1
L	(1,2,-1/2)	0	0	0	-1/2	-1/4	-1
E*	(1,1,1)	0	0	0	0	1	1
	Sum	0	0	0	0	0	0

Old QFT arguments

Geng and Marshak (1989)

A single SM family (without right-handed neutrino) is the smallest non-trivial chiral anomaly-free representation of $SU(3) \times SU(2) \times U(1)$.

OK, but:

- There are **three** families.
- There probably are right-handed neutrinos.
- Why is the smallest representation preferred anyway?

See also:

Minahan, Ramond, Warner (1990), *Geng and Marshak (1990)*

GUTs, anomalies and Charge Quantization

Anomaly cancellation does *not* impose charge quantization:

One can add scalars or Dirac fermions of arbitrary real charge.

But even for chiral matter anomaly cancellation is not enough:
one could add an entire family with rescaled charges.

Such rescalings are not possible if one wishes to couple the
extra family to the SM Higgs.

GUTs, anomalies and Charge Quantization

One can try to impose one-family charge quantization on all three families by requiring that they all couple to the same Higgs.

But even that does not work:
One can have chiral fermions with irrational charges (in SM units) that get their mass from the SM Higgs

$$(3, 2, \frac{1}{6} - \frac{x}{3}) + (\bar{3}, 1, -\frac{2}{3} + \frac{x}{3}) + (\bar{3}, 1, \frac{1}{3} + \frac{x}{3})$$
$$+(1, 2, -\frac{1}{2} + x) + (1, 1, 1 - x) + (1, 1, -x)$$

Charge Quantization

We need some kind of BSM physics to explain charge quantization.

Our working hypothesis is there exists at least some BSM physics related to quantum gravity: a fundamental theory that imposes restrictions on the allowed QFT's.

In other words, we are not going to end up with a consistent theory of quantum gravity that couples to ***any*** QFT.

The most promising, perhaps only candidate for such a theory is string theory.

String theory is likely to quantize the charges.
(although not necessarily in the right way)

If we already have string theory, do we also need GUTs?

The String Theory Landscape

String theory certainly does not predict the Standard Model uniquely.
As far as we know it leads to a huge ensemble ("landscape") of possibilities, realized in a multiverse. All of this is still in its infancy, but non-uniqueness of the QFT choice has been clear from the very beginning.

At this point, people tend to get nervous and start asking: but how do you ever falsify that statement? Those people should understand that the opposite point of view has the same problem. If you believe derive the Standard Model can be derived (the standard "Einstein" paradigm), you must have reasons to believe that it is unique.

But the only thing unique about it is that it is the only QFT we can observe, in principle.

Carl Sagan once said: "extraordinary claims requires extraordinary evidence".

But what is the most extraordinary claim, that there might (theoretically at least) exist other universes with other realizations of QFT, or that what we can see is all there is?

Either one of these claims can ultimately only be established by determining the fundamental theory and counting how many alternatives to the Standard Model it contains.

The String Theory Landscape

String theory certainly not predict the Standard Model uniquely. As far as we know it leads to a huge ensemble ("landscape") of possibilities, realized in a multiverse.

So then how can we hope to derive the Standard Model?

We still have two clues, that are inevitable in a large landscape:

- Anthropic arguments
- Landscape distributions

The String Theory Landscape

The anthropic argument we will use is that the spectrum must be sufficiently complicated. In our universe this is achieved by quarks binding into protons and neutrons, which bind into nuclei, which together with electrons form atoms.

We cannot really derive this from $SU(3)\times SU(2)\times U(1)$, and hence we can certainly not expect to be able to derive this from any QFT that is more complicated.

But in some simpler theories the existence of a complicated set of bound states can be plausibly ruled out.

The String Theory Landscape

More complicated QFT's that *cannot* be anthropically ruled out certainly exist, for example

$$SU(5)\times SU(2)\times U(1)$$

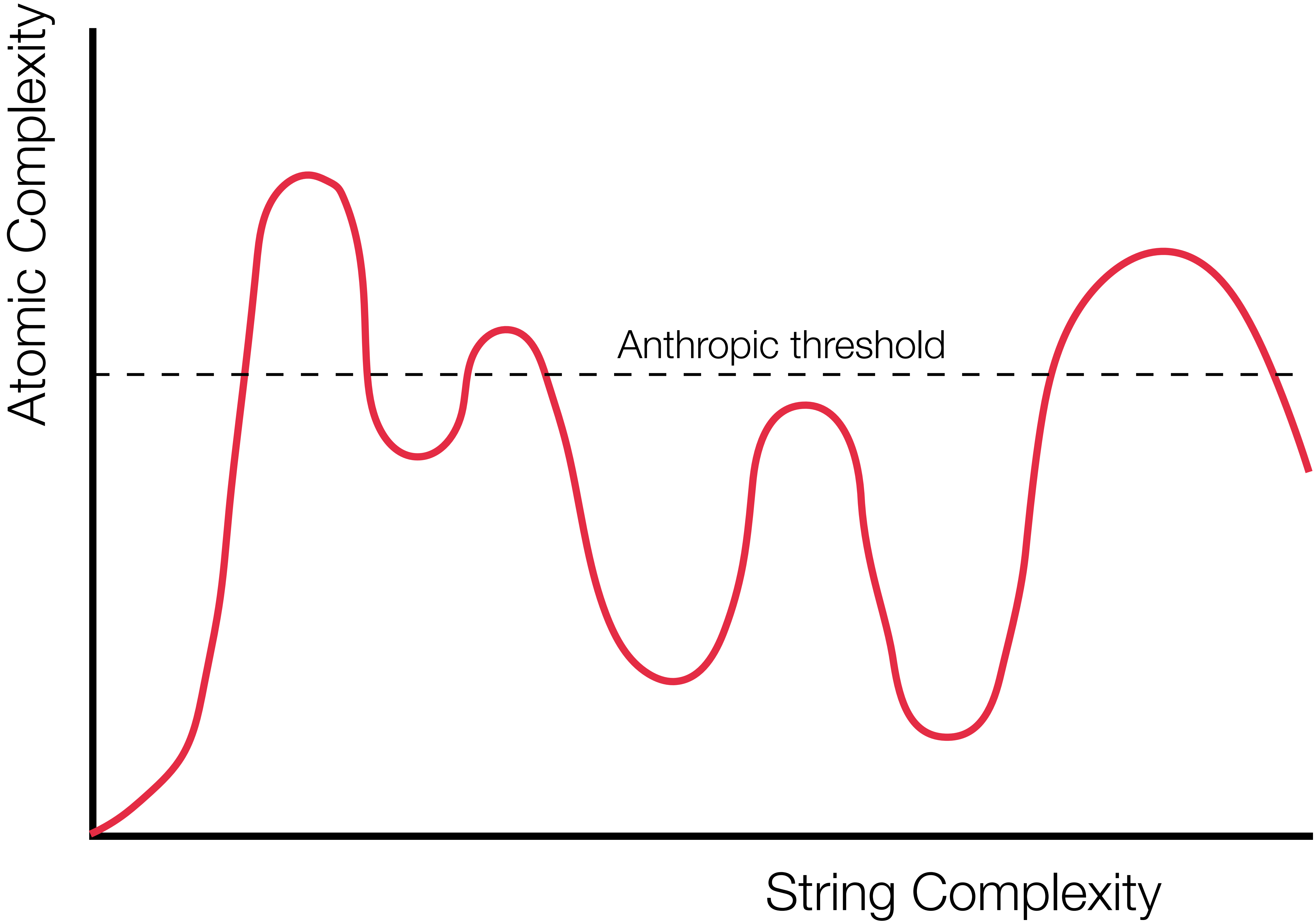
With fifth-integer fractionally charged quarks.

So anthropic arguments alone will not do, given our current knowledge about strongly interacting gauge theories.

The String Theory Landscape

The hope is then that we can establish that the Standard Model is the simplest one with a complicated spectrum.

Then one may also hope that landscape statistics prefers simpler QFT's over more complicated ones.


Atomic Complexity
Anthropic threshold
String Complexity

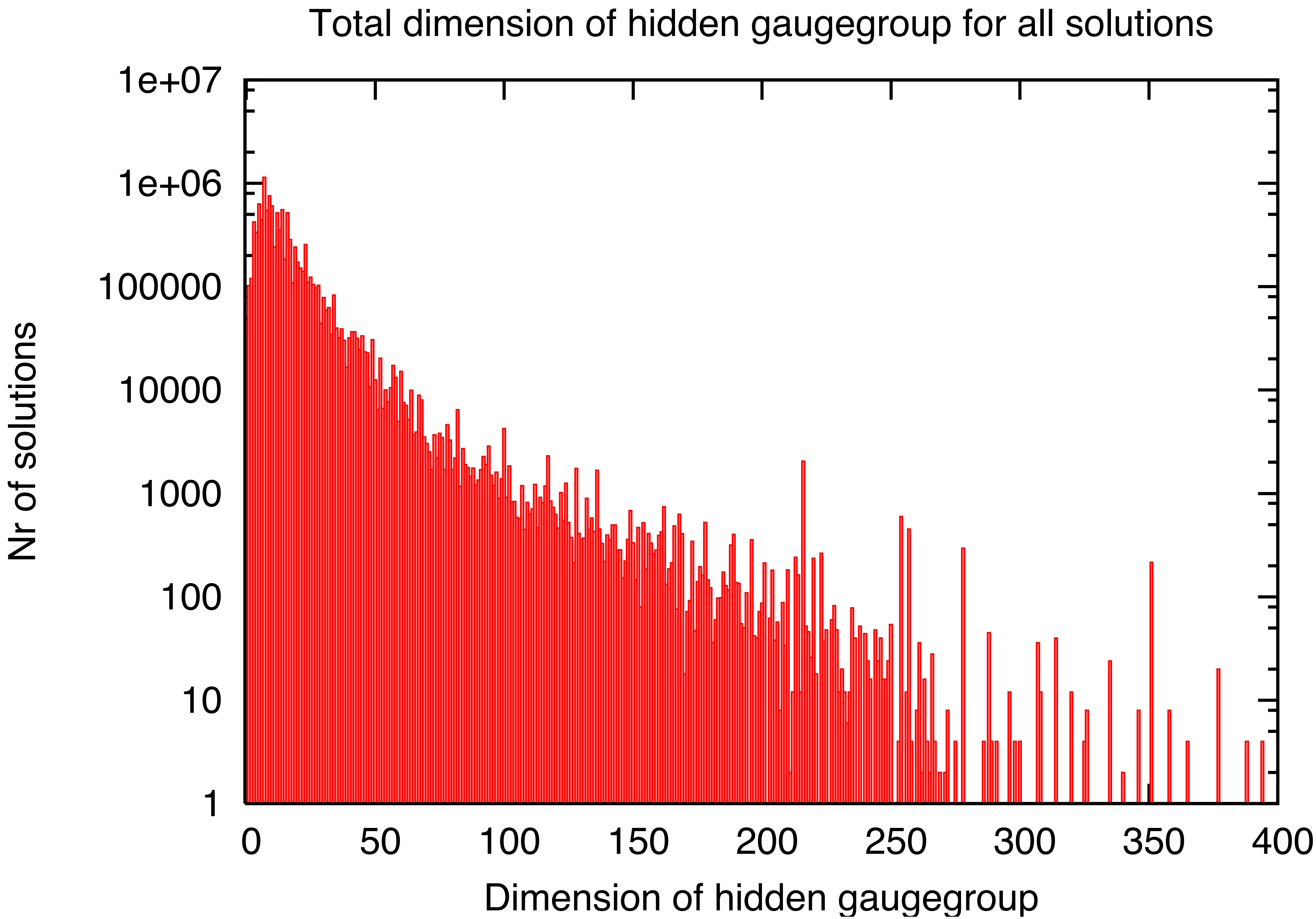
The String Theory Landscape

The hope is then that we can establish that the Standard Model is the simplest one with a complicated spectrum.

Then one may also hope that landscape statistics prefers simpler QFT's over more complicated ones.

Here "simpler" means smaller gauge groups, smaller representations, fewer participating building blocks (e.g. membranes).

In string theory all these quantities are indeed fundamental limited, and hence their distribution will approach zero for large values.



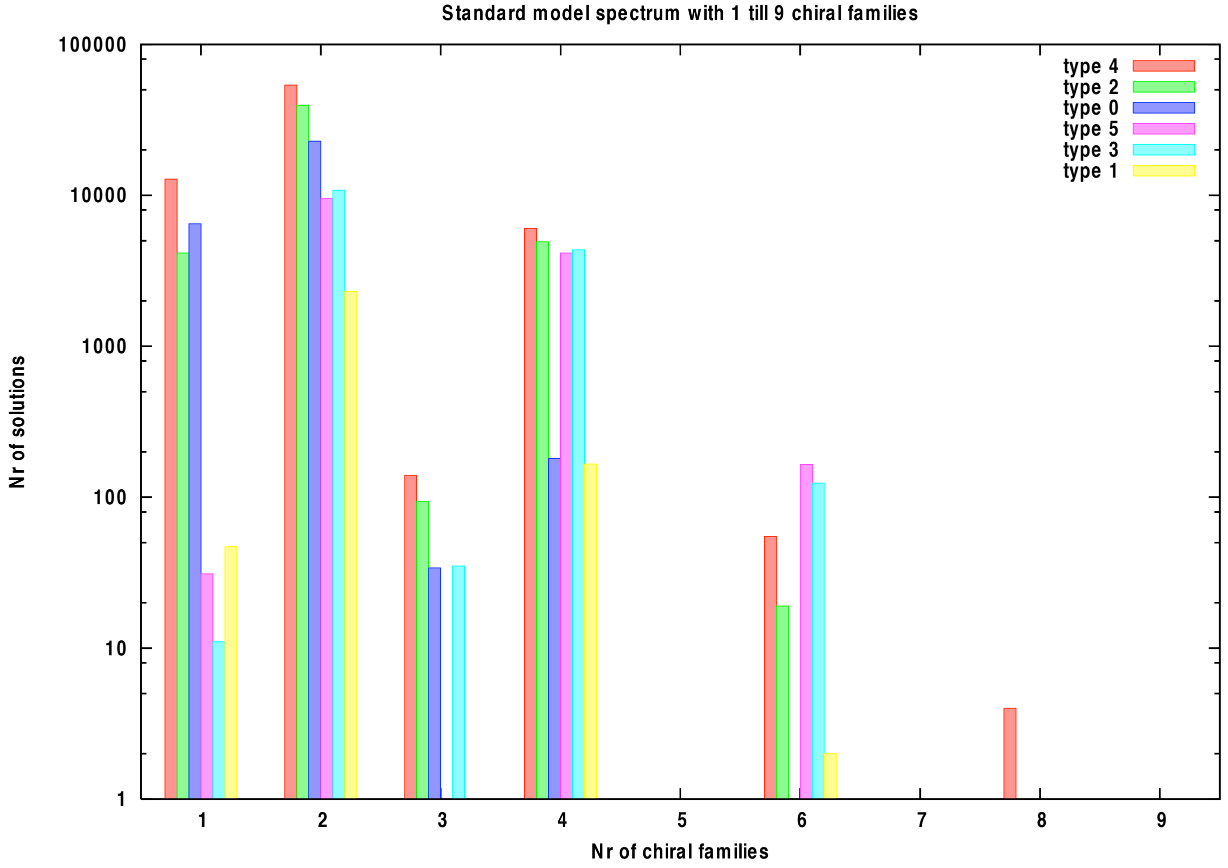
Dijkstra, Huiszoon, Schellekens (2004)

Type-II RCFT orientifolds

The String Theory Landscape

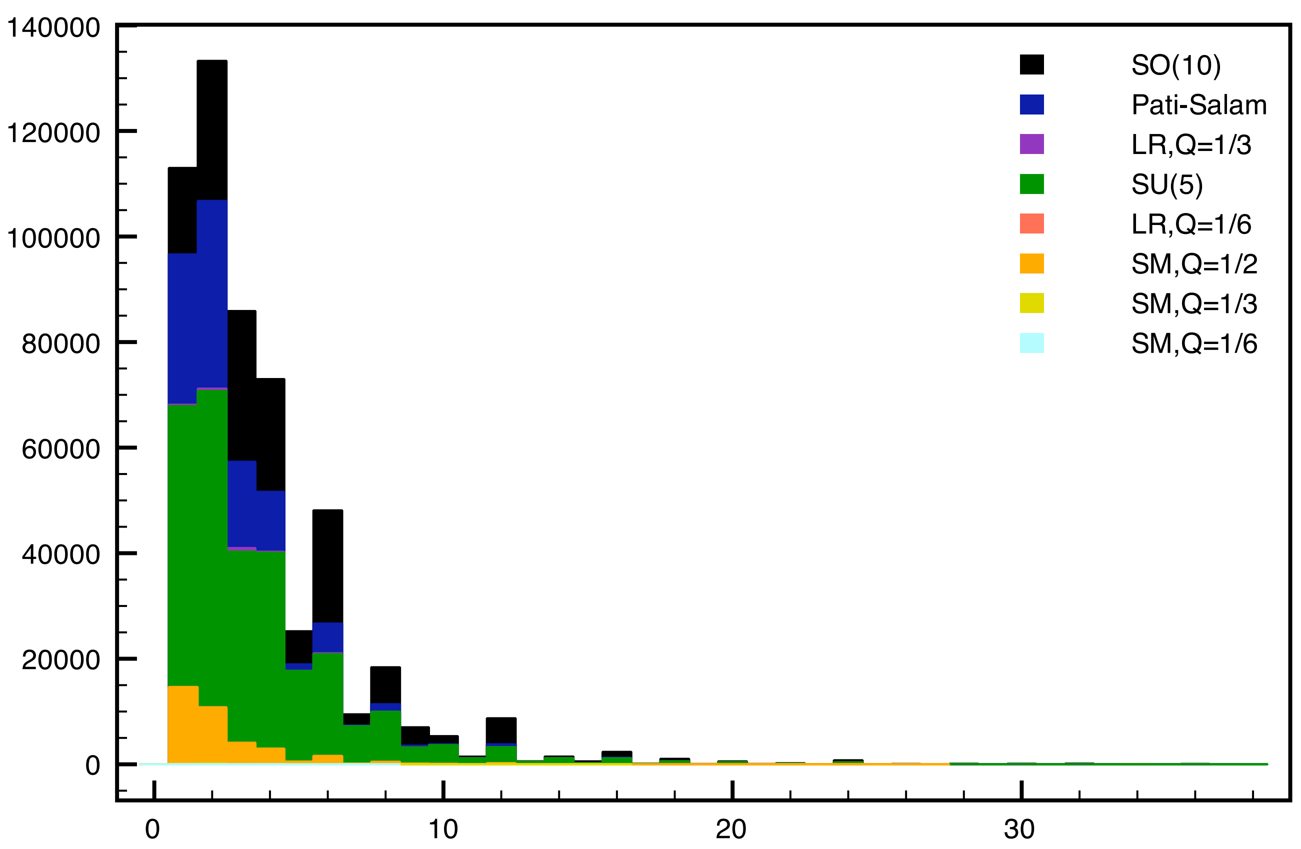
Unfortunately, the fact that we observe three families rather than one is counter evidence…

Type-II RCFT orientifolds



Dijkstra, Huiszoon, Schellekens (2004)

Heterotic Strings

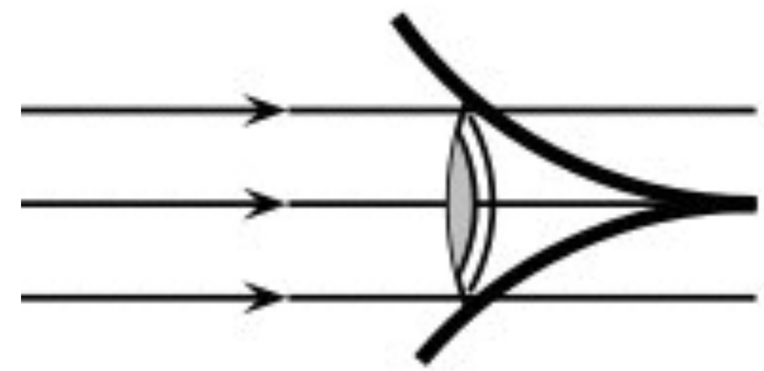


Gato-Rivera, Schellekens (2010)

Towards a derivation of the Standard Model

Main anthropic assumption:

To have observers we will need electromagnetism and a handful of particles with various charges.



We are not asking for a particular quantization, and we are *not* requiring particles of charge 6 (Carbon) to exist, but too simple sets will not do (e.g. charges $-1,1,2$: just Hydrogen and Helium)

So perhaps one could just "emulate" atomic physics with some fundamental particles with charges $-1,1,2,\ldots,N$ for sufficiently large N: fundamental "electrons" and "nuclei".

Towards a derivation of the Standard Model

Pure QED with a set of charged particles has some problems:
No fusion-fueled stars, no stellar nucleosynthesis, baryogenesis difficult, ….

But we focus on another problem, namely that there has to be a hierarchy between the Planck scale and the masses of the building blocks of life.

Maximal number of building blocks with mass m_p of an object that does not collapse into a black hole

$$\left(\frac{m_{\text{Planck}}}{m_p}\right)^3$$

Brain with 10^{27} building blocks requires a hierarchy of 10^{-9}

Towards a derivation of the Standard Model

So to get a substantial number of light atoms, we have to solve a hierarchy problem for each of the constituents.

In the Standard Model this is solved by getting the particle masses from a single Higgs.

There may be landscape distribution arguments to justify this.

Is having N light fermions* statistically more costly than having a single light boson? (The N fermions can be either elementary nuclei or the two light quarks and the electron; then $N=3$)

(*) Our nuclei can be bosons and fermions, but that is not essential

The Hierarchy Problem

Renormalization of scalar masses

$$\mu^2_{\text{phys}} = \mu^2_{\text{bare}} + \sum_i a_i \Lambda^2$$

Computable statistical cost of about 10^{-34} for the observed hierarchy. This is the "hierarchy problem".

Renormalization of fermion masses

$$\lambda_{\text{phys}} = \lambda_{\text{bare}} \left(\sum_i b_i \log(\Lambda/Q) \right)$$

Statistical cost determined by landscape distribution of λ_{bare}

The Hierarchy Problem

It is certainly possible that one fundamental scalar wins against N fermions for moderate N (even for $N \geq 3$).

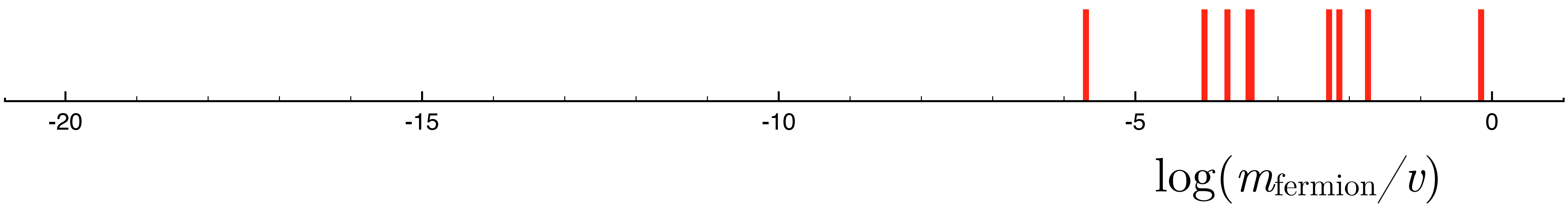
This depends on the landscape distribution of Yukawa couplings and Dirac masses of vector-like particle.

There is circumstantial experimental evidence that these distributions do not favor small values

cf. Harnik, Kribs, Perez, "A universe without weak interactions"

The Hierarchy Problem

- The charged quark and lepton Yukawa coupling distributions may be flat on a log scale*, but not over a large range.



- String theory has a large number of massless vector-like particles, but none of them have been seen, suggesting that they acquire masses, with a distribution that suppresses small masses.

(*) Donoghue, 1997

The Hierarchy Problem

One would also have to show that one fundamental scalar wins against dynamical Higgs mechanism or low energy supersymmetry.

Not enough is known theoretically to decide this, so we take experiment as our guiding principle.

Currently it seems we have a single Higgs + nothing.

This suggests that in a landscape the Higgs is not the *origin* but the *solution* of the Hierarchy problem: it could be the optimal way to create the anthropically required large hierarchy.

This would immediately imply that there is only a single Higgs.

No Higgs?

Statistically, no Higgs is better than one.
If there is a credible alternative to the SM with only dynamical symmetry breaking, that would be a serious competitor.

But generically these theories will have a number of problems.

Consider the SM without a Higgs. It is well-known that in that case the QCD chiral condensate will act like a composite Higgs and give mass to the quarks. The photon survives as a massless particle.

But the quark masses are not tuneable, and the leptons do not acquire a mass.

Massless charged leptons turn the entire universe into an opaque particle-antiparticle plasma.
(C. Quigg, R. Shrock, Phys.Rev. D79 (2009) 096002)

Lessons:

1. Dynamical Symmetry Breaking can play the role of the Higgs mechanism
2. Dynamical Symmetry Breaking should not make the photon massive
3. There should not be any massless charged leptons

String Theory Input

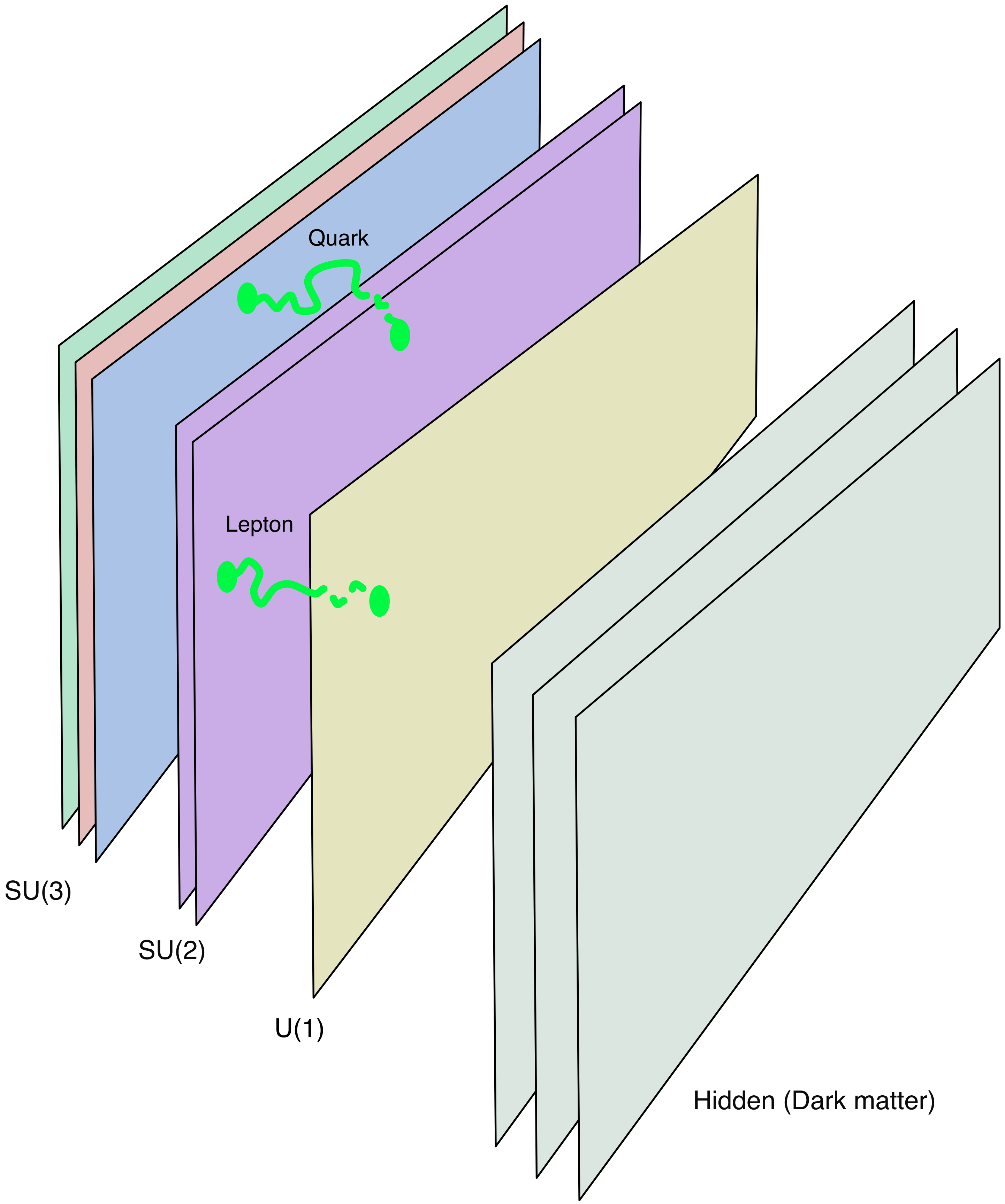
We would like to enumerate all QFT's with a gauge group and chiral matter. All non-chiral matter is assumed to be heavy, with the exception of at most one scalar field, the Higgs. We demand that after the Higgs gets a vev, and that when all possible dynamical symmetry breakings have been taken into account, at least one massless photon survives, and all charged leptons* are massive.

This condition is very restrictive, but still has an infinite number of solutions in QFT.

So at this point we invoke string theory. Its main rôle is to restrict the representations. It also provides a more fundamental rationale for anomaly cancellation.

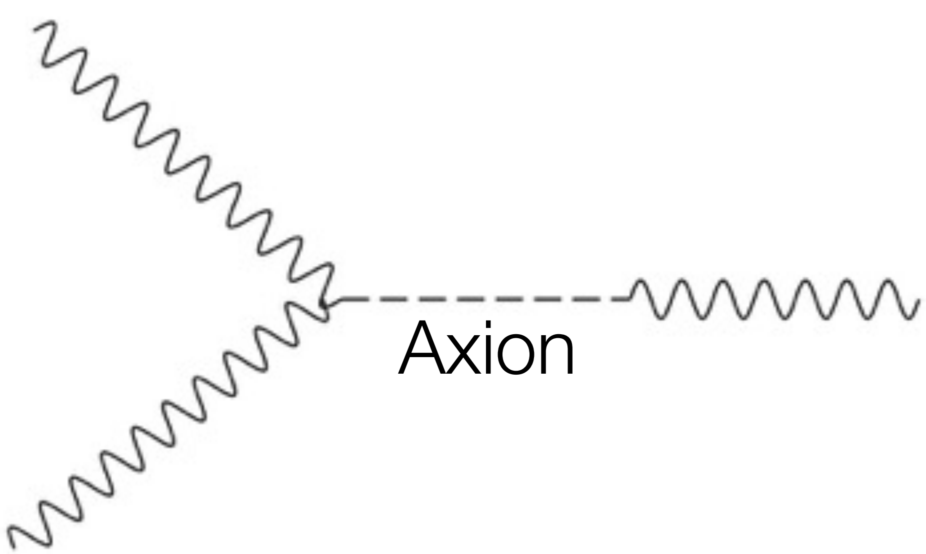
*lepton: a fermion not coupling to any non-abelian vector boson

Intersecting Brane Models



Intersection brane models

- Intersections of branes in extra dimensions determine the massless spectrum.
- Brane multiplicities are subject to a constraint: tadpole cancellation (automatically implies absence of triangle anomalies in QFT).
- Massless photons may mix with axions and acquire a mass.



(Green-Schwarz mechanism)

Intersecting Brane Models

We will assume that all matter and the Higgs bosons are massless particles in intersecting brane models. Then the low-energy gauge groups is a product of $U(N)$, $O(N)$ and $Sp(N)$ factors.

The low energy gauge group is assumed to come from S stacks of branes.
There can be additional branes that do not give rise to massless gauge bosons: $O(1)$ or $U(1)$ with a massive vector boson due to axion mixing.

All matter (fermions as well a the Higgs) are bi-fundamentals, symmetric or anti-symmetric tensors, adjoints or vectors (open strings with one end on a neutral brane)

We start with $S = 1$, and increase S until we find a solution.

Intersecting Brane Models: $S=1$

Chan-Paton group can be $U(N)$, $O(N)$ or $Sp(N)$, but only $U(N)$ can be chiral.

Matter can be symmetric or anti-symmetric tensors or vectors.
Chiral multiplicities $S,\ A,\ K$; charges $2q,\ 2q,\ q$.

Anomaly cancellation:

$$\begin{aligned} KNq^3 + \tfrac{1}{2}N(N+1)S(2q)^3 + \tfrac{1}{2}N(N-1)A(2q)^3 &= 0 \\ KNq + \tfrac{1}{2}N(N+1)S(2q) + \tfrac{1}{2}N(N-1)A(2q) &= 0 \\ Kq + (N+2)S(2q) + (N-2)A(2q) &= 0 \end{aligned}$$

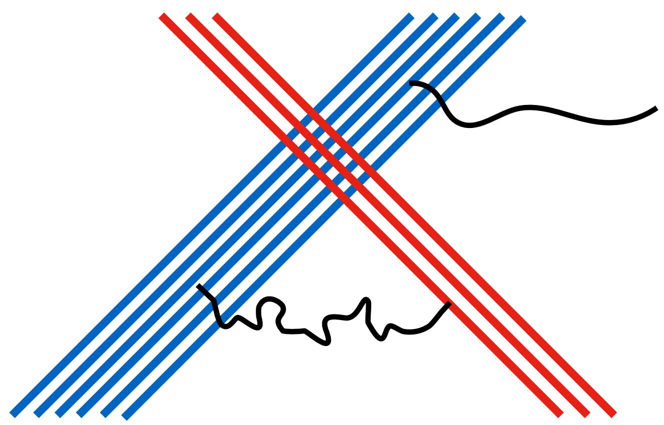
Solutions: $K=S=A=0$ or $q=0$. In the former case, there is no chiral spectrum, in the latter case no electromagnetism.

Higgs symmetry breaking could still produce a $U(1)$, but the choice of Higgses is limited to vectors and rank-2 (anti)-symmetric tensors or adjoints.

Adjoints never turn a chiral spectrum into a non-chiral one.

The others produces a $U(1)$ only for a symmetric tensor breaking $SU(2)$ to $SO(2)$. But $SU(2)$ is not chiral, so by assumption all matter then has Planck scale masses.

Two stack models



$$SU(M) \times SU(N) \times U(1)$$

(We have only considered unitary branes so far)

$$Y = q_a Q_a + q_b Q_b$$

$q_a, \; q_b$ determined by axion couplings

Q	$(M, N, q_a + q_b)$
U	$(A, 1, 2q_a)$
D	$(\overline{M}, 1, -q_a)$
S	$(S, 1, 2q_a)$
X	$(M, \overline{N}, q_a - q_b)$
L	$(1, \overline{N}, -q_b)$
T	$(1, S, 2q_b)$
E	$(1, A, 2q_b)$

Anomalies

$SU(M) \times SU(N) \times U(1)$

S W Y

There are six kinds of anomalies:

SSS
WWW } From tadpole cancellation: also for $M,\ N < 3$
YYY
SSY
WWY
GGY Mixed gauge-gravity

At most one linear combination of the $U(1)$'s is anomaly-free

Anomalies

$$\begin{aligned}(S+U)\tilde{q}_a &= C_1\\(T+E)\tilde{q}_b &= -C_2\\(D+8U)\tilde{q}_a &= (4+M)C_1+NC_2\\L\tilde{q}_b+D\tilde{q}_a &= 0\\2E\tilde{q}_b+2U\tilde{q}_a &= C_1-C_2\end{aligned}$$

$$\tilde{q}_a \equiv Mq_a,\ \tilde{q}_b \equiv Nq_b$$

$$C_1 = -(Q-X)\tilde{q}_b$$

$$C_2 = (Q+X)\tilde{q}_a$$

Only five independent ones. In most cases of interest,the stringy $SU(2)^3$ anomaly is not an independent constraint.

Cubic charge dependence can be linearized.

($q_a = 0$ and/or $q_b = 0$ must be treated separately)

Abelian theories

Single $U(1)$: Higgs must break it, no electromagnetism left

$U(1) \times U(1)$: No solution to anomaly cancellation for two stacks

So in two-stack models we need at least one non-abelian factor in the high-energy theory.

Strong Interactions

It is useful to have a non-abelian factor in the low-energy theory as well, since the elementary particle charge spectrum is otherwise too poor. We need some additional interaction to bind these particles into bound states with larger charges (hadrons and nuclei in our universe).

For this to work there has to be an approximately conserved baryon number.
This means that we need an $SU(M)$ factor with $M \geq 3$, and that this $SU(M)$ factor does not become part of a larger group at the "weak" scale.

Note that $SU(2)$ does not have baryon number, and the weak scale is near the constituent mass scale. We cannot allow baryon number to be broken at that scale.

But let's just call this an additional assumption.

Higgs Choice

This implies that at least one non-abelian factor is not broken by the Higgs. We take this factor to be $U(M)$.

Therefore we do not consider bi-fundamental Higgses breaking both $U(M)$ and $U(N)$. We assume that $U(N)$ is the broken gauge factor. Then the only Higgs choices are **L**,**T** and **E**.

We will assume that $U(M)$ it is strongly coupled in the IR-regime and stronger than $U(N)$.

$SU(M) \times U(1)$ $(i.e.\ N{=}1)$

Higgs can only break $U(1)$, but then there is no electromagnetism.

Hence there will be a second non-abelian factor, broken by the Higgs.

$M = 3,\ N = 2$

Higgs = L

Decompose L, E, T: chiral charged leptons avoided only if

$$L = E,\ T = 0$$

Substitute in anomaly equation:

$$S\tilde{q}_a = \left(\frac{5 - N - M}{2M}\right) C_1$$

For $M = 3,\ N = 2$: $S = 0$

Therefore we get standard QCD without symmetric tensors.

$$M = 3,\ N = 2$$

Quark sector

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$$Q + X - D = 0$$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$M = 3,\ N = 2$

Hence either $Q = 0$ ***or*** $X = 0$; the choice is irrelevant.

Take $X = 0$.
Then $D = Q = U,\ T = 0,\ L = E$
Remaining anomaly conditions: $L = Q$

Hence the only solution is a standard model family, occurring Q times.

The branes **a** and **b** are in principle unrelated, and can generally not be combined to a $U(5)$ stack

$M = 3,\ N = 2$

Higgs = T

The symmetric tensor can break $SU(2)\times U(1)$ in two ways, either to $U(1)$, in the same way as **L**, or to $SO(2)$.

Breaking to $U(1)$ (same subgroup as **L**)

No allowed Higgs couplings to give mass to the charged components of L, E and T, so we must require $E = L = T = 0$. Then there is no solution.

Breaking to $SO(2)$

Then $SO(2)$ must be electromagnetism. Y-charges forbid cubic T couplings, so $T = 0$ to avoid massless charged leptons. Quark charge pairing (to avoid chiral QED, broken by QCD) requires $Q = -X$. **If we also require $S = 0$, everything vanishes.**

Note: stronger dynamical assumption: $S = 0$

$M > 3$ and/or $N > 2$: lepton pairing

Lepton charge pairing: $-L + (N-1)E + (N+1)T = 0$

Combined with the five anomaly constraints this gives the following solution

$$
\begin{aligned}
U\tilde{q}_a &= \tfrac{3+M}{6} C_1 \\
S\tilde{q}_a &= \tfrac{3-M}{6} C_1 \\
D\tilde{q}_a &= N C_2 - \tfrac{M}{3} C_1 \\
L\tilde{q}_b &= -N C_2 + \tfrac{M}{3} C_1 \\
E\tilde{q}_b &= -\tfrac{1}{2} C_2 + \tfrac{M}{6} C_1 \\
T\tilde{q}_b &= -\tfrac{1}{2} C_2 - \tfrac{M}{6} C_1
\end{aligned}
$$

$$C_1 = -(Q - X)\tilde{q}_b$$

$$C_2 = (Q + X)\tilde{q}_a$$

For $M = 3,\ S = 0$ automatically!

$M > 3$ and/or $N > 2$: quark pairing

$Q \neq -X$: Left-handed and righthanded quark representations have different dimensions. Then no subgroup of $SU(N)$ is non-chiral.
Hence dynamical symmetry breaking breaks $SU(N)$ completely.

But $SU(N) \times U(1)$ ***does*** contain a current that is non-chiral.
Note that now U and D participate, which are neutral under $SU(N)$, but carry a $U(1)$ charge. The surviving $U(1)$ symmetry must be a linear combination

$$Q_{\mathrm{em}} = \Lambda + Y,$$

where $\Lambda \in SU(N)$. There can be at most one such $U(1)$ factor.
This is the only symmetry that can survive DSB+Higgs breaking.

($Q = -X$: see paper)

$M > 3$ and/or $N > 2$

$\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$ (surviving Higgs + any DSB)

Charges of Q: $q_a + q_b + \lambda_i$

Charges of X: $q_a - q_b - \lambda_i$

Charges of D: $-q_a$

Charges of U, S: $2q_a$

Lepton Charges: $q_b + \lambda_i;\ 2q_b + \lambda_i + \lambda_j$

Define $q_b + \lambda_i = \alpha q_a$

Quark charge pairing is possible only for $\alpha = 0,\ \pm 3$

All solutions satisfy Standard Model charge quantization!

$M > 3$ and/or $N > 2$

We can obtain a solution for any Q and X

$$\Lambda : n \times \{-q_b\} + n_+ \times \{-q_b + 3q_a\} + n_- \times \{-q_b - 3q_a\}$$

$$n_+ = \frac{Q}{R}$$

$$n_- = -\frac{X}{R}$$

$$N = n + n_+ + n_-$$

$$R = -(Q + X)\frac{\tilde{q}_a}{\tilde{q}_b} \in \mathbb{Z}$$

The trace of Λ must vanish

$$\mathrm{Tr}\ \Lambda = \tilde{q}_b \left(\frac{3}{M} - 1 \right)$$

Hence $M = 3$!

$M > 3$ and/or $N > 2$

The spectrum can be computed

$$D = n(Q + X)$$

$$U = (N - n)(Q + X)$$

$$L = nR$$

$$E = \frac{1}{2}(N - n + 1)\,R$$

$$T = -\frac{1}{2}(N - n - 1)\,R$$

Absence of massless charged leptons only for $N = 2!$

Conclusions

- The Standard Model is the only anthropic solution within the set of two-stack models.
- Family structure, charge quantization, the weak interactions and the Higgs choice are all derived.
- Standard Model charge quantization works the same way, for any value of N, even if $N+3 \neq 5$.
- The GUT extension offers no advantages, only problems (doublet-triplet splitting)
- Only if all couplings converge (requires susy), GUTs offer an advantage.
- The general class is like a GUT with its intestines removed, keeping only the good parts: **GUTs without guts**.

Couplings

The $U(3)\times U(2)$ structure of this class of models implies one relation among the SM couplings, instead of the two of $SU(5)$

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}$$

see also:
Ibañez, Munos, Rigolin, 1998;
Blumenhagen, Kors, Lüst, Stieberger, 2007

Extrapolation this to higher energies we see that this is satisfied at $5.7\times 10^{13}\ \mathrm{GeV}$ $(1.4\times 10^{16}\ \mathrm{GeV}$ for susy).

Proton decay by SU(5) vector bosons would be far too large, but generically we do not have such bosons in the spectrum. There is no SU(5) in any limit.

But what happens at that scale?

If it is the string scale, one would still expect quantum-gravity related proton decay, which would be much too large.

But there are many ways out.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
1	1	2	2	-3	**L**	3	6	3	3	0	1	1	0
2	1	2	4	-1	**L**	2	1	1	0	0	2	3	1
3a	1	2	2	-1	**L**	3	4	1	3	-4	1	0	-1
3b	1	2	2	-1	**L**	2	2	1	1	-1	1	1	0
3c	1	2	2	-1	**L**	4	5	0	3	-4	0	1	-1
4	1	3	3	-2	**L**	2	3	2	1	0	1	1	0
5	1	3	3	-1	**E**	0	0	-2	-1	1	-2	1	0
6	1	4	4	-1	**L**	1	1	1	0	0	1	1	0
7	M	2	1	ρ	**T**	1	$-\rho$	$2M\rho$	$-\rho$	-1	$2M$	0	0
8	2	3	3	-2	**L**	1	1	1	0	0	1	1	0
9	3	2	2	-3	**L**	1	1	1	0	0	1	1	0

All chiral spectra without massless charged free leptons that can be obtained for all M and N with $q_a \neq 0$ and $q_b \neq 0$. Here $M = 1,2$ and ρ is a free integer parameter.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
6	1	4	4	-1	**L**	1	1	1	0	0	1	1	0

This realizes the $SU(4)\times U(1)$ subgroup of SU(5).

The Higgs boson breaks this to $SU(3)\times U(1)$, QCD × QED.

But this implies $SU(5)$-type proton decay at the weak scale.

A family constitutes a single, complete $SU(4)$ Higgs multiplet.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
8	2	3	3	-2	**L**	1	1	1	0	0	1	1	0

This is the same $SU(3)\times SU(2)\times U(1)$ subgroup of $SU(5)$ that gives rise to the Standard Model, but with a triplet Higgs instead of a doublet Higgs.

At low energies, there is a non-abelian $SO(4) \approx SU(2)\times SU(2)$ gauge group without conserved Baryon number.

The special case $q_a = 0$ (all M,N)

Anomaly cancellation:

$SU(M) \times SU(N) \times U(1)_Y$

$Q[(V, V, 1) + (V, \overline{V}, -1)] + \text{flavor-neutral } \mathbf{U}, \mathbf{D}, \mathbf{S} \text{ matter}$

For $M = 1{,}2$ this is vectorlike (hence massive)
For $M > 3$ there is no $U(1)$ in the flavor group that is non-chiral with respect to $SU(M)$, hence no electromagnetism.

Note: we treat Higgs and dynamical breaking on equal footing

The special case $q_b = 0$ (all M,N)

Anomaly cancellation:

$SU(M) \times SU(N) \times U(1)_Y$

$Q[(V, V, 1) + (\overline{V}, V, -1)] + Y\text{-neutral } \mathbf{L},\ \mathbf{E},\ \mathbf{T} \text{ matter}$

For $N = 1{,}2$ this is vector-like, and hence massive

For $N \geq 3$ the candidate Higgses do not break $U(1)_{\mathrm{Y}}$

Hence the Higgs just has to break $SU(N)$ to a real group, and this is indeed possible, for example Higgs = **T**, breaking $SU(N)$ to $SO(N)$

$Q[(V, V, 1) + (\overline{V}, V, -1) + 2M(1, V, 0)]$

No charged leptons; Baryon number is gauged, so baryogenesis would be problematic.