Electric Charge Quantization in String Theory

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Experimental Results

- All known particles have electric charges that are an integer multiple of the electron charge.
- Fractional charges have been looked for in bulk matter, particles accelerators and cosmic rays. Nothing was found.
- Relative abundance on earth is less that 10^{-20}
- The lightest fractionally charged particle must be stable.
- Unsuitable as a dark matter candidate.
- Unlikely to appear at the LHC.

Recent review:

Searches for fractionally charged particles.
Martin L. Perl (SLAC), Eric R. Lee, Dinesh Loomba (New Mexico U.). 2009. 19 pp.
Published in **Ann.Rev.Nucl.Part.Sci. 59**

Theoretical understanding?

Standard Model Lie Algebra $su(3) \oplus su(2) \oplus u(1)$

Observed representations (r_3, r_2, y)

$$(3, 2, \frac{1}{6})_L$$

$$(\bar{3}, 1, \frac{2}{3})_R$$

$$Q_u = \frac{2}{3}, Q_d = -\frac{1}{3}$$

$$(\bar{3}, 1, -\frac{1}{3})_R$$

$$(1, 2, -\frac{1}{2})_L$$

$$Q_e = -1, Q_\nu = 0$$

$$(1, 1, -1)_R$$

$$\frac{1}{3}t_3 + \frac{1}{2}t_2 + y = 0 \quad \text{mod } 1$$

Theoretical understanding?

Particle physics experiments only probe the Lie-algebra structure of the Standard Model.
But a given Lie algebra may correspond to several Lie-groups

$$SU(3) \times SU(2) \times U(1)$$

No restriction on y

$$su(3) \oplus su(2) \oplus u(1)$$

$$S(U(3) \times U(2))$$

$$\frac{1}{3}t_3 + \frac{1}{2}t_2 + y = 0 \mod 1$$

Theoretical understanding?

Dirac (1931)

Coexistence of electric and magnetic charges leads to a quantization condition (because a system of an electric and a magnetic charge has angular momentum, which must be quantized).

$$2\frac{q_e q_m}{\hbar c} \in \mathbb{Z}$$

If there exists even a single monopole, this would force q_e to be quantized.

But also the search for monopoles has been negative so far.
In most theories they are expected to be “inflated away”.

Grand Unification

$$S(U(3) \times U(2)) \subset SU(5)$$

$$\begin{pmatrix} \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} & \\ & \begin{pmatrix} * & * \\ * & * \end{pmatrix} \end{pmatrix}$$

Grand Unification

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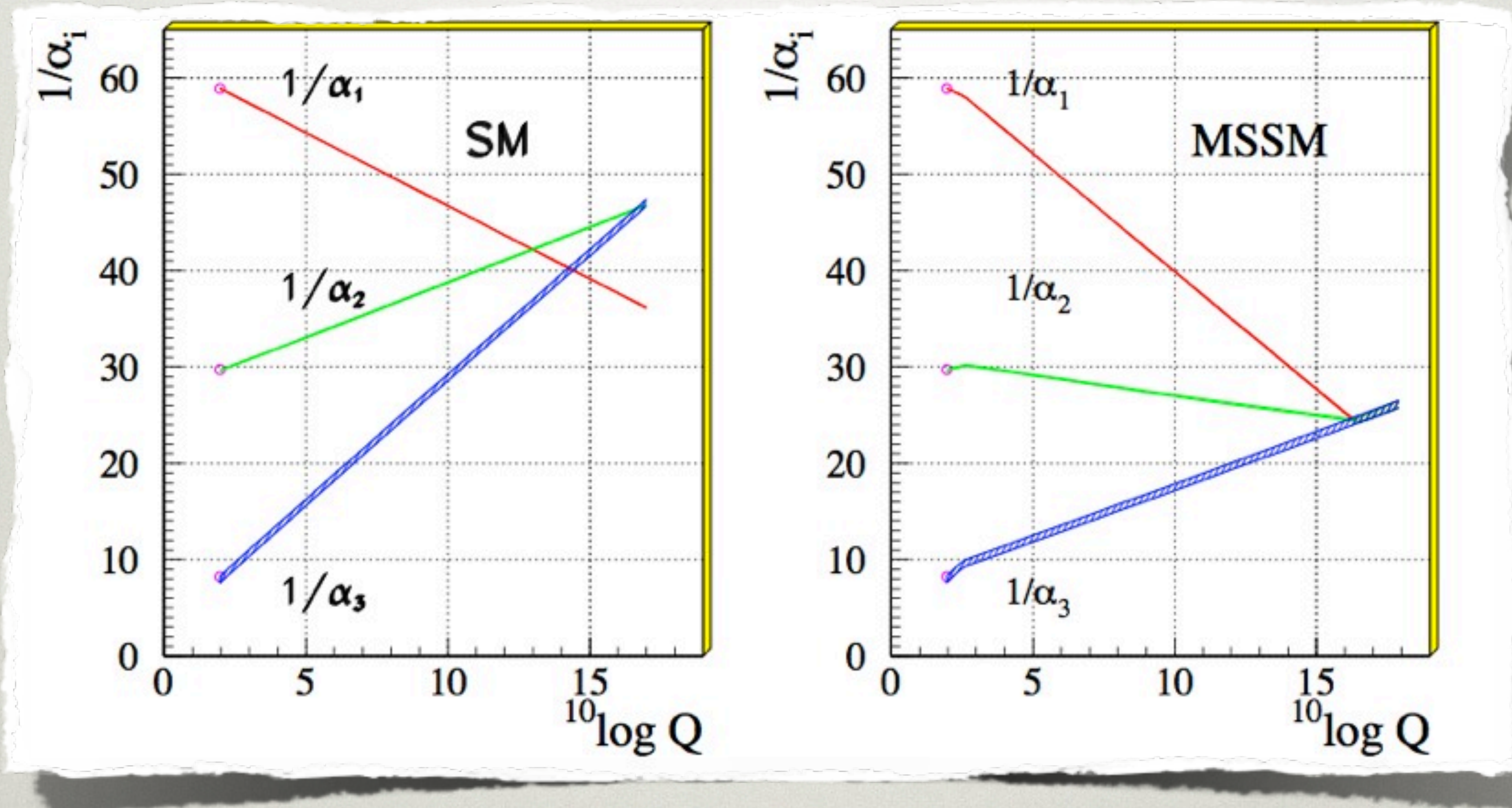
Grand Unification



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- Predicts proton decay (rules out simplest non-susy GUT).
- Has monopole solutions *('t Hooft, Polyakov (1974)).*
- Beautiful further extension to SO(10).

One family(*) fits exactly in an *SO(10)* spinor

u_1	:	$\vert\uparrow\downarrow\uparrow\uparrow\downarrow>$
u_2	:	$\vert\uparrow\downarrow\uparrow\downarrow\uparrow>$
u_3	:	$\vert\uparrow\downarrow\downarrow\uparrow\uparrow>$
d_1	:	$\vert\downarrow\uparrow\uparrow\uparrow\downarrow>$
d_2	:	$\vert\downarrow\uparrow\uparrow\downarrow\uparrow>$
d_3	:	$\vert\downarrow\uparrow\downarrow\uparrow\uparrow>$
u_1^c	:	$\vert\downarrow\downarrow\uparrow\downarrow\downarrow>$
u_2^c	:	$\vert\downarrow\downarrow\downarrow\uparrow\downarrow>$
u_3^c	:	$\vert\downarrow\downarrow\downarrow\downarrow\uparrow>$
d_1^c	:	$\vert\uparrow\uparrow\uparrow\downarrow\downarrow>$
d_2^c	:	$\vert\uparrow\uparrow\downarrow\uparrow\downarrow>$
d_3^c	:	$\vert\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν	:	$\vert\uparrow\downarrow\downarrow\downarrow\downarrow>$
e	:	$\vert\downarrow\uparrow\downarrow\downarrow\downarrow>$
e^c	:	$\vert\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν^c	:	$\vert\uparrow\uparrow\uparrow\uparrow\uparrow>$

(*) plus one right-handed neutrino

String Theory

- If the GUT idea is correct, one may hope to find an underlying principle leading to GUT unification.
- String theory was not invented for that purpose (1969: strong interactions; 1975: quantum gravity), but in 1984 it seemed to give Grand Unification automatically.

The Heterotic String

- Gauge group $E_8 \times E_8$, but in ten dimensions. (also: *SO(32)*)
  (Gross, Harvey, Martinec, Rohm (1984))
- Calabi-Yau compactification gives E_6 ($\times$ E_8) in four dimensions.
  E_6 contains *SO(10)* as a subgroup.
  (Candelas, Horowitz, Strominger, Witten (1984))
- But this is only a special case.
  More generally one finds *SO(10)* with *N* families in spinor representations.
  (Lerche, Lüst, Schellekens (1986)) "bosonic string map"
  (Gepner, (1987))
  (Schellekens, Yankielowicz (1989))

This seems to be exactly what one hoped for...

The heterotic string

... but we still have to break *SO(10)* to the Standard Model.

However, the usual Higgs bosons cannot appear in a massless string spectrum.

The heterotic string

The Heterotic string is a theory of closed strings.

Closed strings have left- and right-moving modes that can be treated independently for free strings.

In particular, the left-moving modes can be a bosonic string and the right-moving ones a fermionic string.

$$S = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau \partial^a X^\mu \partial_a X_\mu$$

$$S = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau \left[\partial_a X^\mu \partial^a X_\mu - i\bar{\psi}^\mu \rho^a \partial_a \psi_\mu\right]$$

The Bosonic String Map

This is fine for the free string, but there is an important constraint if interactions are allowed (splitting and joining of strings).

The first and most important one occurs for the one-loop diagram without external lines, the torus.
It is called "modular invariance".

It can be satisfied by mapping the fermionic string to a bosonic string ("bosonic string map"), and combining left and right symmetrically.

Roughly speaking, the left-moving sector gives rise to gauge groups and the right-moving ones to Lorentz representations.

$$\psi^\mu \quad \leftrightarrow \quad \text{12 free bosons on the } SO(10) \times E_8 \text{ torus}$$

$$\text{Space-time spinors} \quad \leftrightarrow \quad SO(10) \text{ spinors}$$

Space-time Supersymmetry

Left and right Hamiltonians: H_L, H_R

Physical states $|a\rangle \otimes |b\rangle \leftrightarrow \phi_a|0\rangle \otimes \bar{\phi}_b|0\rangle$

Subject to the condition $M_L^a = M_R^b$

$H_L|a\rangle = M_L^a|a\rangle \qquad H_R|b\rangle = M_R^b|b\rangle$

Space-time supersymmetry:
Requires operator ϕ_a = *S* in the fermionic (right-moving) sector.
(*S* is called the "spin field").

But we cannot have an operator in the right-moving sector without having an isomorphic (for the modular group) one in the left-moving (bosonic) sector.

The easiest solution is to take the image of *S* under the bosonic string map.
This yields an operator that extends *SO(10)* to E_6.

The state created by *S* on the right vacuum has M_R=0;
The state created by its bosonic image has M_L=0.

Asymmetric solutions

There are other solutions with $M_L > 0$.
These only yield massive states, and do not extend $SO(10)$ to E_6.

Using the same idea, we can start with a subgroup of $SO(10)$ and extend it by states with $M_L > 0$ that are isomorphic to the $SO(10)$ roots.

So we can start with $SU(3) \times SU(2) \times U(1)_Y \times U(1)_{B\text{-}L}$.

In string theory, a $U(1)$ corresponds to a boson compactified on a circle.
The radius of the circle is fixed if we require gauge coupling unification.

For $U(1)_Y$ the radius is such that the smallest allowed Y-charge is 1/6,
the minimal Y-charge occurring in the Standard Model.

This charge occurs in the quark doublets, (3, 2, 1/6).

A no-go theorem

But we must also satisfy

$$\frac{1}{3}t_3 + \frac{1}{2}t_2 + y = 0 \mod 1$$

Theorem:
In heterotic string theory this implies that the *SU(5)* roots must be in the massless spectrum. *(Schellekens, (1989)) (see also: Wen and Witten, 1985)*

This is a direct consequence of modular invariance.

One must either have unbroken *SU(5)* or fractional charges.

This result applies to all perturbative heterotic strings that respect coupling unification.

Possible ways out: the fractional charges may be massive, or confined by some additional gauge group.

Avoiding the no-go theorem

In randomly chosen heterotic string spectra, if we only require the Standard Model gauge group, we expect not even to find the usual family structure. In general there will be "chiral exotics": fractionally charged particles that cannot be paired up into massive Dirac fermions.

There may be spectra with only "vector-like" fractionally charged particles. They can get a mass from small perturbations of the theory.

Finally there may be some spectra with only massive fractionally charged particles.

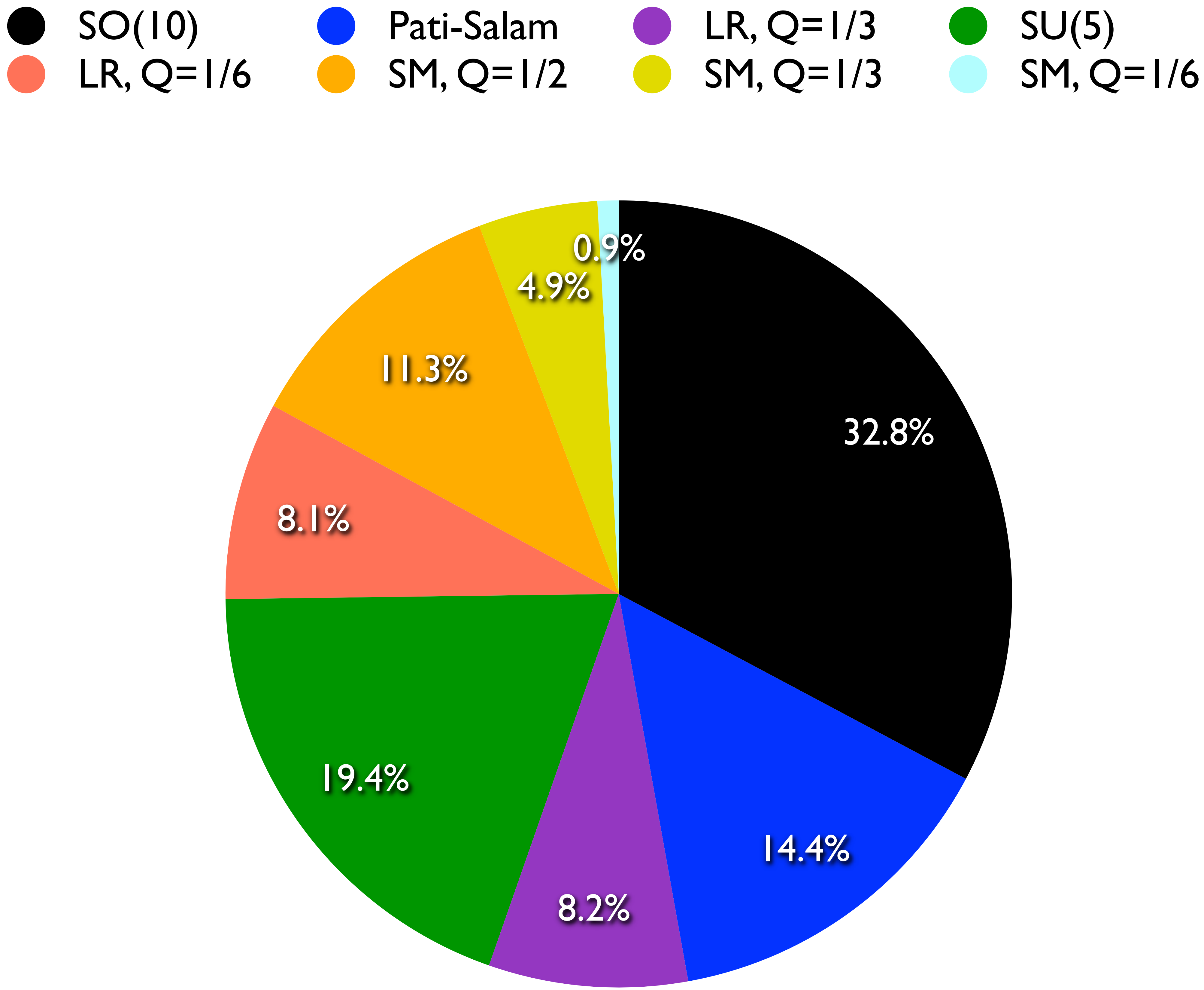
How often do each of these options occur?

To find out, we examined a large sample of heterotic spectra based on "Gepner models".

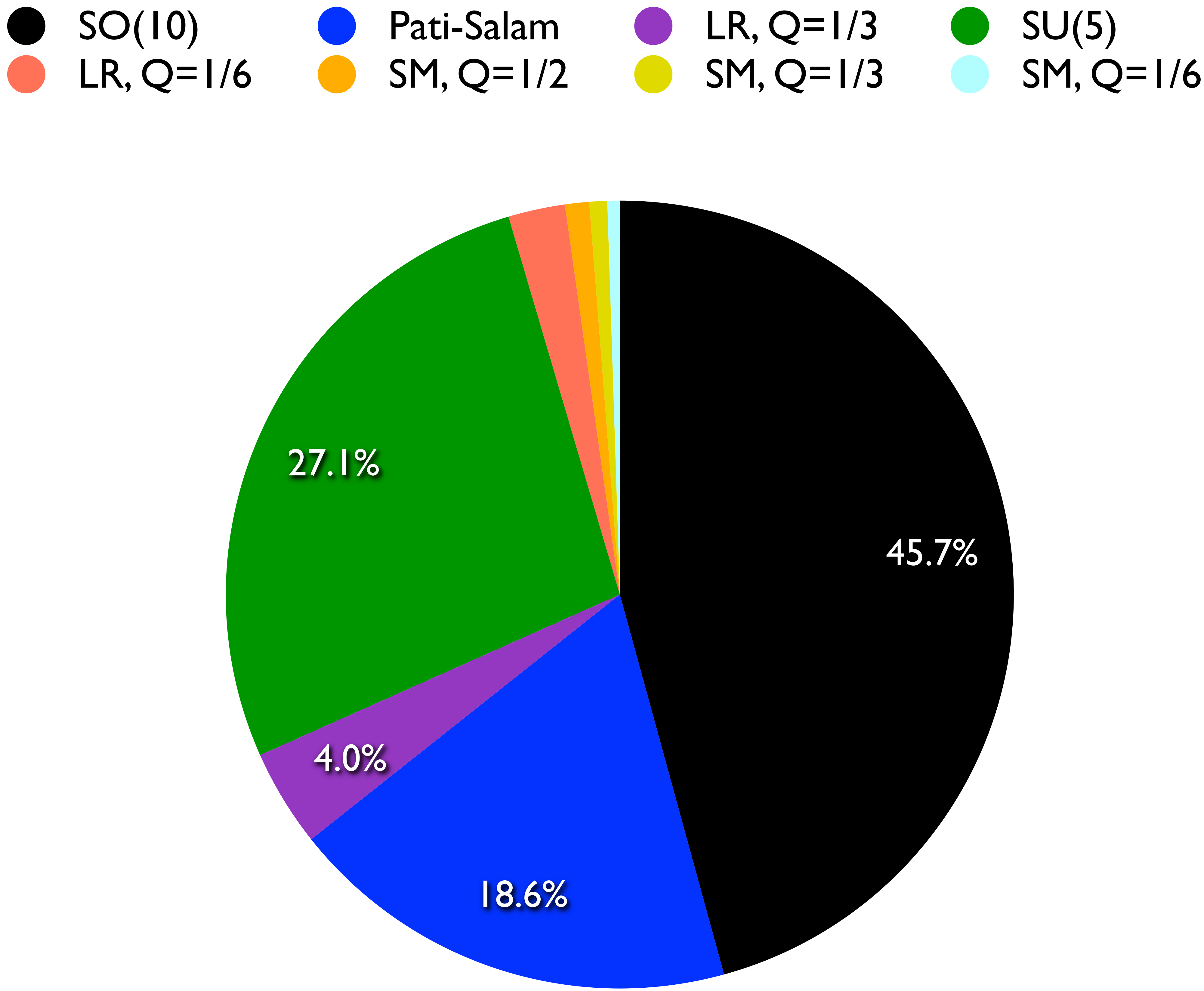
With B. Gato-Rivera (2010)

SO(10) CFT sub-algebras

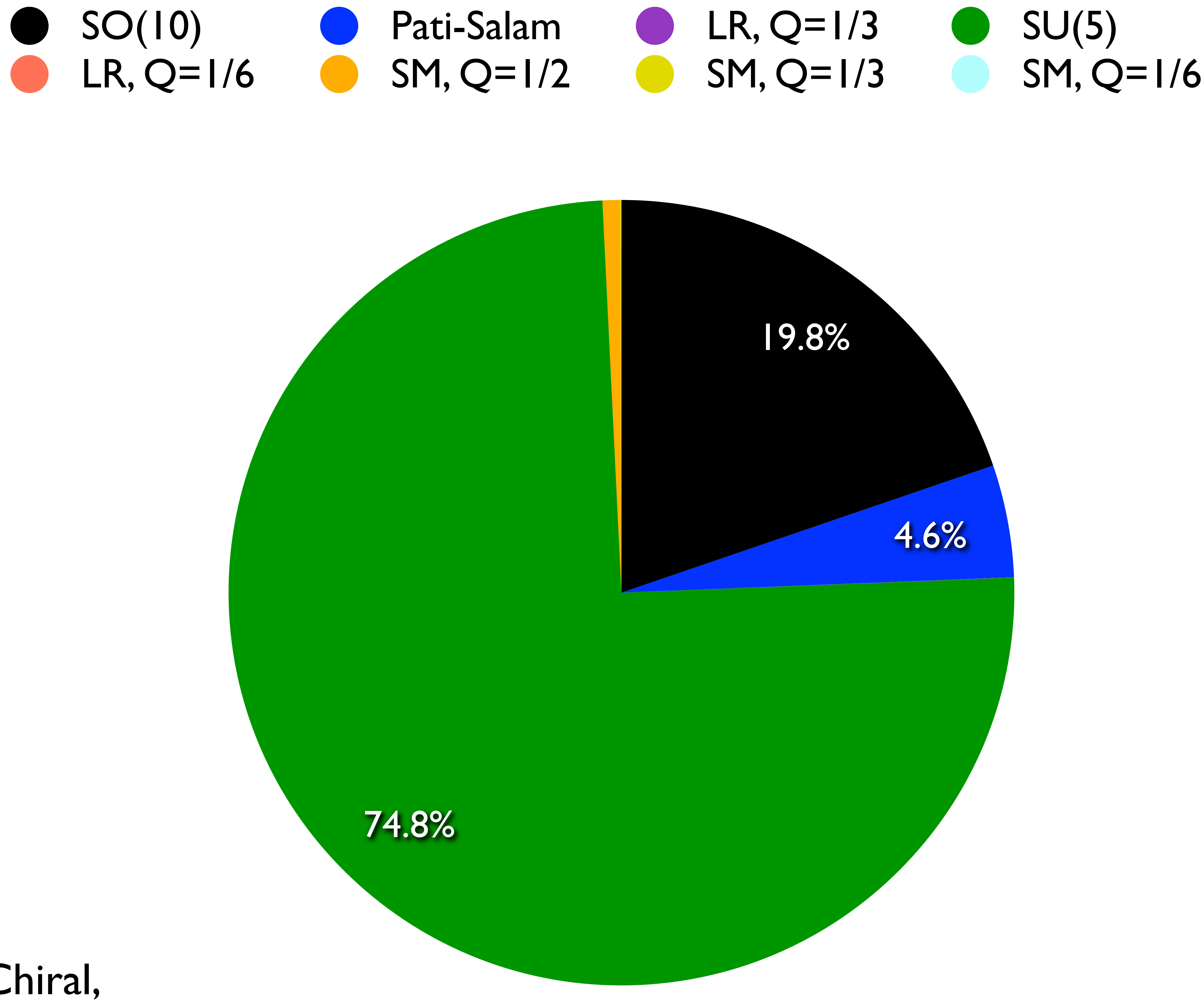
	Name	Current	Order	Gauge group	Grp.	CFT
	SM, Q=1/6	$(1,1,0,0)$	1	$SU(3)\times SU(2)\times U(1)\times U(1)$	$\frac{1}{6}$	$\frac{1}{6}$
	SM, Q=1/3	$(1,2,15,0)$	2	$SU(3)\times SU(2)\times U(1)\times U(1)$	$\frac{1}{6}$	$\frac{1}{3}$
	SM, Q=1/2	$(3,1,10,0)$	3	$SU(3)\times SU(2)\times U(1)\times U(1)$	$\frac{1}{6}$	$\frac{1}{2}$
	LR, Q=1/6	$(1,1,6,4)$	5	$SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$	$\frac{1}{6}$	$\frac{1}{6}$
	SU(5) GUT	$(\bar{3},2,5,0)$	6	$SU(5)\times U(1)$	1	1
	LR, Q=1/3	$(1,2,3,-8)$	10	$SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$	$\frac{1}{6}$	$\frac{1}{3}$
	Pati-Salam	$(\bar{3},0,2,8)$	15	$SU(4)\times SU(2)_L\times SU(2)_R$	$\frac{1}{2}$	$\frac{1}{2}$
	SO(10) GUT	$(3,2,1,4)$	30	$SO(10)$	1	1



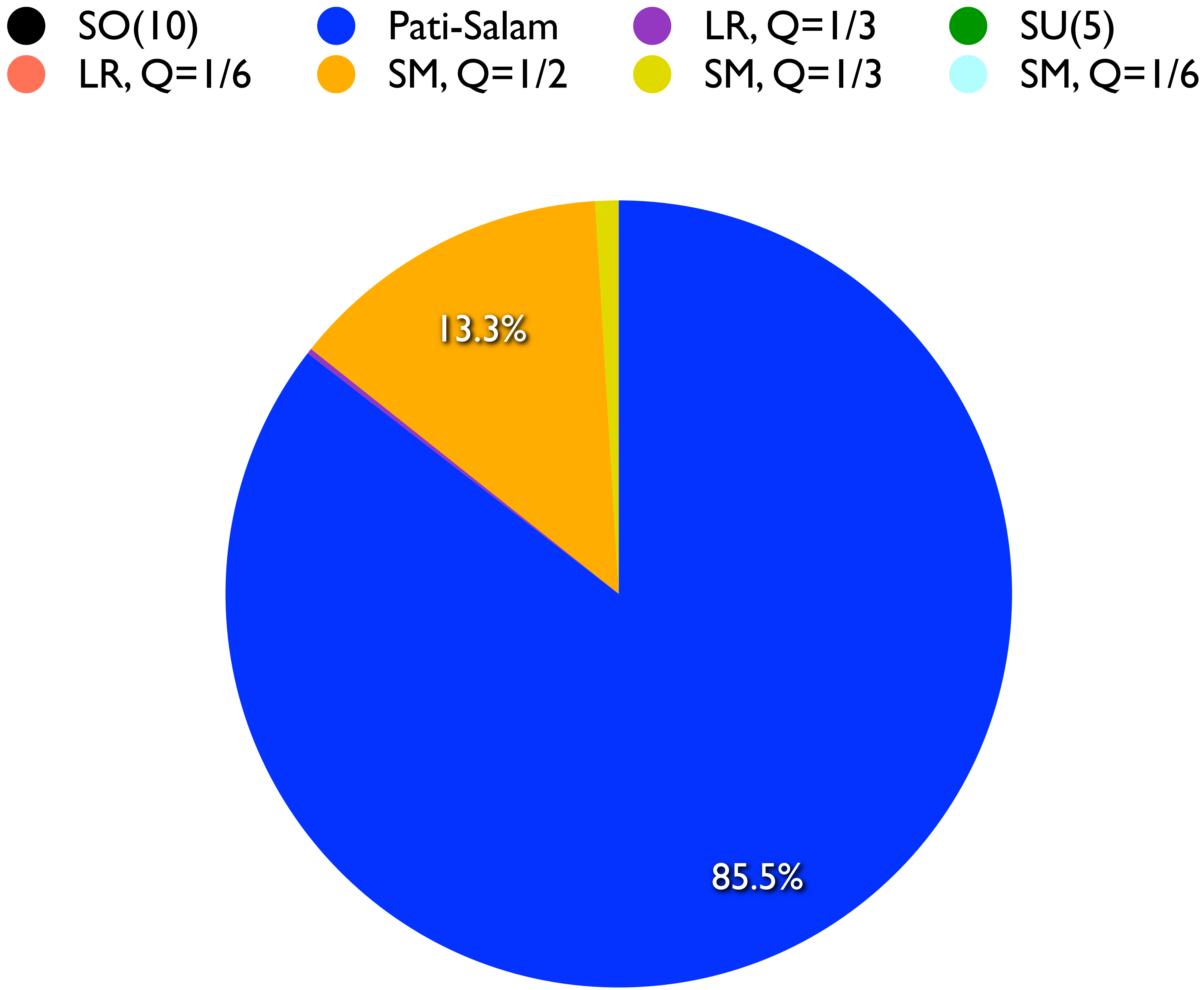
Group Type Distribution



No chiral exotics



Chiral,
N-family
N>0



Chiral, N-family,
No GUTs

Summary (all constructions)

Type	Chiral Exotics	GUT	Non-chiral	$N > 0$ fam.	No frac.
Standard*	37.4%	32.7%	20.5 %	9.3%	0
Standard, perm.	29.7%	33.4 %	27.9 %	8.9%	0
Free fermionic	1.5%	2.9%	94.4%	1.1%	0.072%
Lifted	28.3%	18.7%	51.9%	1.1%	0.00051%
Lifted, perm.	26.8%	8.9%	62.7 %	1.6%	0.00078%
$(\text{B-L})^*_{\text{Type-A}}$	21.3%	28.0%	50.4 %	0.3%	0.00017%
$(\text{B-L})_{\text{Type-A}}$, perm.	22.8%	8.1 %	69.1 %	0.03%	0
$(\text{B-L})^*_{\text{Type-B}}$	38.5%	8.7%	52.1%	0.6%	0
$(\text{B-L})_{\text{Type-B}}$, perm.	27.6%	7.3 %	65.0 %	0.1%	0

Vector-like Exotics

No Exotics

No-exotics models have an even number of families
For three-family examples see

Assel, Christodoulides, Faraggi, Kounnas and Rizos (2010) *[Free fermions]*

Blaszczyk, Nibbelink, Ratz, Ruehle, Trapletti, Vaudrevange (2010) *[Freely acting symmetries]*

Open Strings

Intersecting branes

The ends of open strings give rise to *U(N)*, *O(N)* or *Sp(2N)* gauge groups. (with unrelated gauge couplings)

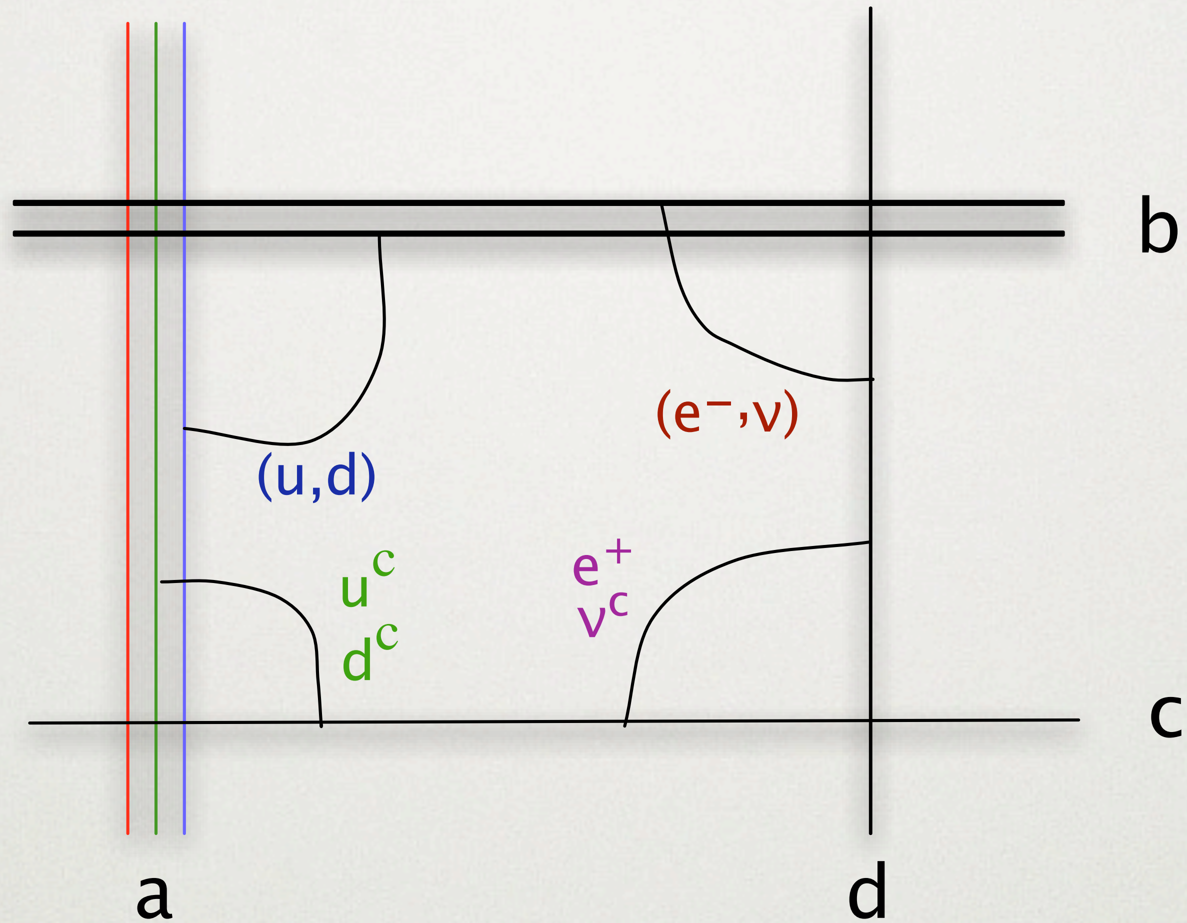
Since each open string has two ends, matter must be in bi-fundamentals (or rank-two tensors).

One may think of the endpoints as open strings ending on a membrane or a stack of *N* membranes.

By considering suitable combinations of stacks of intersecting branes one may obtain the standard model.

(hundreds of papers since ~ 2000)

The Madrid Model*



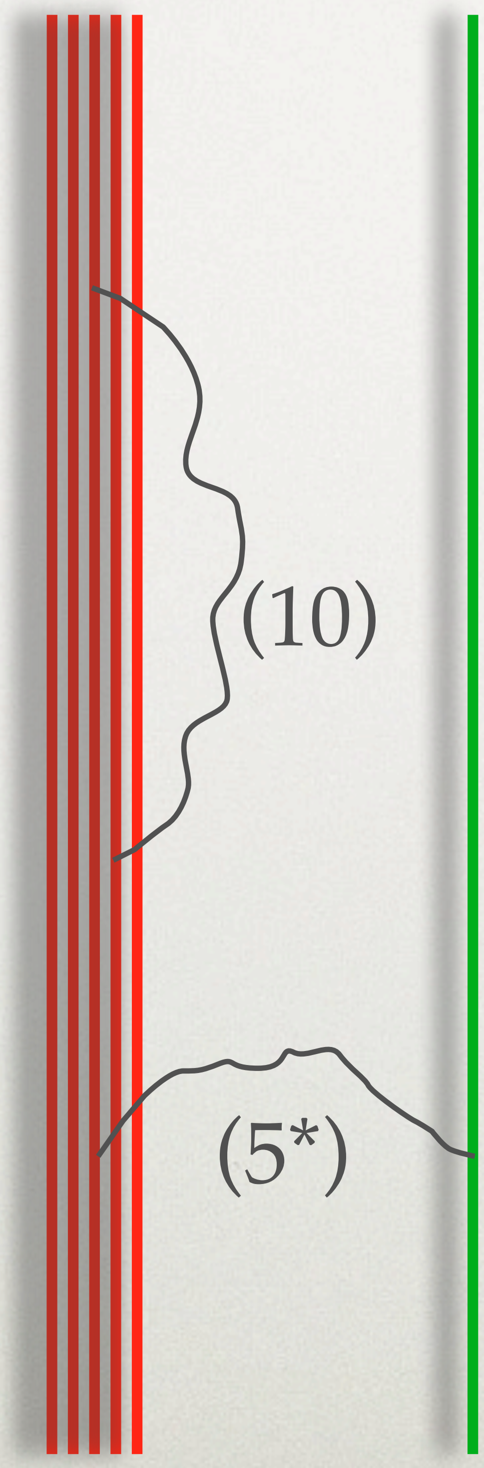
U(2), Sp(2)

U(1), O(2), Sp(2)

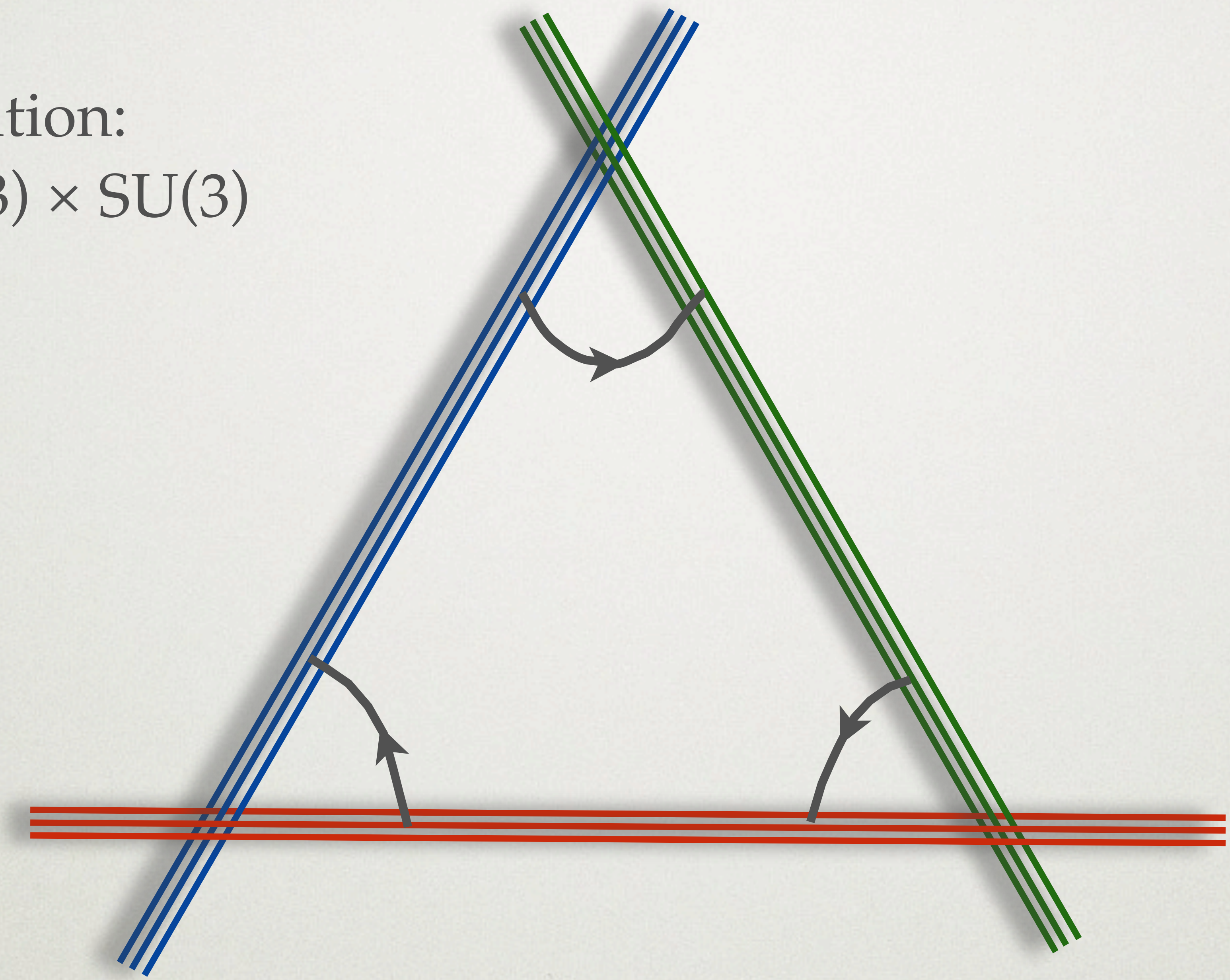
$$Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Qd$$

(*) Ibanez, Marchesano, Rabadan (2000)

SU(5)



Trinification:
SU(3) × SU(3) × SU(3)



$$(3, 3^*, 1) + (3^*, 1, 3) + (1, 3, 3^*)$$

Classes of open String Models

(assumption: Standard Model with at most 4 branes

$\Delta Q=½$ $\Delta Q=½$

Non-orientable

"x=½"

$\Delta Q=0$ $\Delta Q=0$

Non-orientable

"x=0"

$\Delta Q=x$ $\Delta Q=-x$

Orientable

Anastasopoulos, Dijkstra, Kiritsis, Schellekens (2006)

Open Strings: Fractional charges

All matter from intersections of standard model branes has integral charge.

But often there are additional "hidden sector" branes, intersecting the SM. These may have fractional charges x:

- x=½ class: Half-integer fractional charges.
- x=0 class: Only integer charges:
- orientable class: Fractional charge x.

  (e.g. third-integer for trinification)

Note: fractionally charged matter **must** couple to the hidden sector, and may be confined by it.

Input: SM particles from three of four branes.

Gauge group: Exactly SU(3) × SU(2) × U(1)!

	U(3)	Sp(2)	U(1)	U(1)		
3 x	V	V	0	0	chirality 3	Q
3 x	V	0	V	0	chirality -3	U*
3 x	V	0	V*	0	chirality -3	D*
3 x	0	V	0	V	chirality 3	L
5 x	0	0	V	V	chirality -3	E*+(E+E*)
3 x	0	0	V	V*	chirality 3	N*
18 x	0	V	V	0		Higgs
2 x	V	0	0	V		
2 x	Ad	0	0	0		
2 x	A	0	0	0		
6 x	S	0	0	0		
14 x	0	A	0	0		
6 x	0	S	0	0		
9 x	0	0	Ad	0		
6 x	0	0	A	0		
14 x	0	0	S	0		
3 x	0	0	0	Ad		
4 x	0	0	0	A		
6 x	0	0	0	S		

Dijkstra, Huiszoon, Schellekens (2004)

Gauge group: Exactly SU(3) × SU(2) × U(1)!

	U(3)	Sp(2)	U(1)	U(1)		
3 x	(V	,V	,0	,0)	chirality 3	Q
3 x	(V	,0	,V	,0)	chirality -3	U*
3 x	(V	,0	,V*	,0)	chirality -3	D*
3 x	(0	,V	,0	,V)	chirality 3	L
$\boxed{5}$ x	(0	,0	,V	,V)	chirality -3	E*+(E+E*)
3 x	(0	,0	,V	,V*)	chirality 3	N*
18 x	(0	,V	,V	,0)		Higgs
2 x	(V	,0	,0	,V)		
2 x	(Ad	,0	,0	,0)		
2 x	(A	,0	,0	,0)		
6 x	(S	,0	,0	,0)		
14 x	(0	,A	,0	,0)		
6 x	(0	,S	,0	,0)		
9 x	(0	,0	,Ad	,0)		
6 x	(0	,0	,A	,0)		
14 x	(0	,0	,S	,0)		
3 x	(0	,0	,0	,Ad)		
4 x	(0	,0	,0	,A)		
6 x	(0	,0	,0	,S)		

Vector-like matter

V=vector

A=Anti-symm. tensor

S=Symmetric tensor

Ad=Adjoint

Dijkstra, Huiszoon, Schellekens (2004)

Conclusions

- So far, no convincing explanation for the observed absence of fractional charges has been given in string theory.
- Examples without light fractional charges can be found, both with heterotic strings and in open strings, but they are constructed for that purpose, and not “generic”.
- From this point of view the x=0 open string models look most promising (related to “F-theory GUTs”), but there is no reason why that class would be preferred.