Constanten

c	2.99792458×10^{8} m s^{-1}
G	$6.67428(67) \times 10^{-11}$ m^3 kg^{-1} s^{-2}
h	$6.62606896(33) \times 10^{-34}$ J s
e	$1.60217733(49) \times 10^{-19}$ C
k	$1.3806504(24) \times 10^{-23}$ J K^{-1}
σ	$5.670400(40) \times 10^{-8}$ W m^{-2} K^{-4}
amu	$1.660538782(83) \times 10^{-27}$ kg
$m_{\rm e}$	$9.10938215(45) \times 10^{-31}$ kg
	$\simeq 0.00054858$ amu
$m_{\rm p}$	$1.672621637(83) \times 10^{-27}$ kg
	$\simeq 1.00727647$ amu
$m_{\rm n}$	$1.674927211(84) \times 10^{-27}$ kg
	$\simeq 1.00866492$ amu
eV	$1.602176487(40) \times 10^{-19}$ J
$GM_\odot$	$1.32712441 \times 10^{20}$ m^3 s^{-2}
$M_\odot$	1.9884×10^{30} kg
$R_\odot$	6.96×10^{8} m
$L_\odot$	$\simeq 3.85 \times 10^{26}$ J s^{-1}
$L_{V,\odot}$	$\simeq 5.6 \times 10^{23}$ J s^{-1} nm^{-1}
$T_{\rm eff,\odot}$	$\simeq 5780$ K
$M_{\rm V,\odot}$	4.83
$M_{\rm bol,\odot}$	4.74
yr	365.25×86400 s
A.U.	$1.495978707 \times 10^{11}$ m
	$\simeq 499.00478384$ s $\times\ c$
pc	$\simeq 3.086 \times 10^{16}$ m
H_0	67.4(5) km s^{-1} Mpc^{-1}
t_0	13.787(20) Gyr
$\Omega_{\rm b}$	0.0486(10)
$\Omega_{\rm CDM}$	0.2589(57)
Ω_Λ	0.6911(62)

Formules

Afstand, tijd en licht

$d({\rm pc}) \equiv \frac{1}{p('')}$

$m_1 - m_2 = -2.5\,{}^{10}\!\log \frac{f_1}{f_2}$

$L = 4\pi R^2 F = 4\pi d^2 f$

$B_\lambda d\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT}-1} d\lambda$

$\lambda_{\rm max} T_{\rm eff} = {\rm constant} \simeq 0.29\,{\rm cm\,K}$

$F = B = \sigma {T_{\rm eff}}^4$

$\frac{R_{\rm ms}}{r_\odot} \approx \left(\frac{M_{\rm ms}}{M_\odot}\right)^n$; $M_{\rm ms} \lesssim M_\odot$: $n \approx 1$; $M_{\rm ms} \gtrsim M_\odot$: $n \approx 0.6$

$\frac{L_{\rm ms}}{L_\odot} \approx \left(\frac{M_{\rm ms}}{M_\odot}\right)^{3.8}$

Gassen, straling en sterren

$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$

$\frac{dP(r)}{dr} = -\frac{GM(r)}{r^2}\ \rho(r)$

$u = \frac{4\sigma}{c}\ {T_{\rm eff}}^4$

$F = -\frac{cl}{3}\nabla u = -\frac{c}{3\kappa\rho}\nabla u$

$\left(\frac{dT(r)}{dr}\right)_{\rm rad} = -\frac{L(r)}{4\pi r^2} \frac{3\kappa(r)\rho(r)}{4\sigma} \frac{1}{4T(r)^3}$

$\left(\frac{dT(r)}{dr}\right)_{\rm ad} = (1-\frac{1}{\gamma}) \frac{T(r)}{P(r)} \frac{dP(r)}{dr}$

$P = \frac{1}{3} nvp = \frac{1}{3} nmv^2 \equiv nkT$

$\frac{1}{2}mv^2 \equiv \frac{3}{2}kT$

$P_{\rm gas} = \frac{k}{\mu m_{\rm H}} \rho T$

$P_{\rm rad} = \frac{u}{3} = \frac{4\sigma T^4}{3c}$

$P_{\rm d} \equiv \frac{K_{\rm d}}{m_{\rm e}} \left(\frac{\rho}{\mu_{\rm e} m_{\rm H}}\right)^{5/3}$; $\quad K_{\rm d} \approx 2.13 \times 10^{-68}\,{\rm N^2\,m^2\,s^2}$

$P_{\rm d,r} \equiv K_{\rm d,r} \left(\frac{\rho}{\mu_{\rm e} m_{\rm H}}\right)^{4/3}$; $\quad K_{\rm d,r} \approx 2.45 \times 10^{-26}\,{\rm N\,m^2}$

$P_{\rm c} \propto \frac{GM}{R} \bar\rho \propto \frac{3GM^2}{4\pi R^4}$

$T_{\rm c} \propto \frac{\mu m_{\rm H}}{k} \frac{\bar\rho}{\rho_{\rm c}} \frac{GM}{R}$

$\tau_{\rm n} \sim 10\,{\rm Gyr}\,\frac{M}{M_\odot} \left(\frac{L}{L_\odot}\right)^{-1}$

$\tau_{\rm th} \equiv \frac{E_{\rm th}}{L} \sim \frac{GM^2}{RL} \sim 31\,{\rm Myr} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R}{R_\odot}\right)^{-1} \left(\frac{L}{L_\odot}\right)^{-1}$

$\tau_{\rm dyn} = \sqrt{\frac{R^3}{GM}} \sim 27\,{\rm min}\ \left(\frac{M}{M_\odot}\right)^{-1/2} \left(\frac{R}{r_\odot}\right)^{3/2}$

$R_{\rm wd} \approx 0.01125\,R_\odot \left[\left(\frac{M_{\rm wd}}{1.454\,M_\odot}\right)^{-2/3} - \left(\frac{M_{\rm wd}}{1.454\,M_\odot}\right)^{2/3}\right]^{1/2}$

$R_{\rm s} = \frac{2GM}{c^2}$

Dubbelsterren

$\frac{GM_{\rm T}}{a_{\rm b}^3} = \left(\frac{2\pi}{P}\right)^2 = \omega_{\rm b}^2$

$a_1 = \frac{M_2}{M_{\rm T}} a_{\rm b}; \qquad q_1 \equiv \frac{M_1}{M_2} = \frac{a_2}{a_1}$

$\mu \equiv \frac{M_1 M_2}{M_1+M_2}; \qquad \mathcal{M}_{\rm c} \equiv \left(\frac{M_1 M_2}{M_{\rm T}^{1/3}}\right)^{3/5}$

$R_{\rm Rl,i} \approx \frac{2}{3^{4/3}}\,a_{\rm b} \left(\frac{M_i}{M_{\rm T}}\right)^{1/3} \approx\ 0.46\,a_{\rm b} \left(\frac{M_i}{M_{\rm T}}\right)^{1/3}$

$E_{\rm b} = E_{\rm pot} + E_{\rm kin,1} + E_{\rm kin,2} = -\frac{GM_1M_2}{2\,a_{\rm b}}$

$J_{\rm b} = J_1 + J_2 = M_1 M_2 \left(\frac{G\,a_{\rm b}}{M_{\rm T}}\right)^{1/2}$

$\left(\frac{\dot J}{J}\right)_{\rm GW} = -\frac{32}{5} \frac{G^3}{c^5} \frac{M_1 M_2 M_{\rm T}}{a^4}$

$\tau_{\rm merge,gw} = \frac{5}{256} \frac{c^5}{G^3} \frac{a_{\rm b}^4}{M_1 M_2 M_{\rm T}}$
$\approx 150\,{\rm Myr} \left(\frac{a_{\rm b}}{R_\odot}\right)^4 \left(\frac{M_1}{M_\odot}\right)^{-1} \left(\frac{M_2}{M_\odot}\right)^{-1} \left(\frac{M_{\rm T}}{M_\odot}\right)^{-1}$

$\frac{\dot a}{a} = 2\dot M_{\rm d} \frac{M_{\rm d}-M_{\rm a}}{M_{\rm d} M_{\rm a}}$

$|E_{\rm bind,mntl}| = \alpha_{\rm CE}\,|\Delta E_{\rm b}| = \alpha_{\rm CE} \left[\frac{GM_{\rm d,e} M_{\rm a}}{2a_{\rm e}} - \frac{GM_{\rm d,b} M_{\rm a}}{2a_{\rm b}}\right]$

Zwaartekrachtgolven

$$\mathcal{M}_\mathrm{c} = \left(\frac{M_1 M_2}{M_\mathrm{T}^{1/3}}\right)^{3/5}$$

$$h_+(t) = \frac{4}{d}\frac{G}{c^4}\mu a_\mathrm{b}^2\omega_\mathrm{b}^2 \;\cos(2\omega_\mathrm{b} t)$$

$$h_\times(t) = \frac{4}{d}\frac{G}{c^4}\mu a_\mathrm{b}^2\omega_\mathrm{b}^2 \;\sin(2\omega_\mathrm{b} t)$$

$$|h_{+,\times}| = h_\mathrm{opt} = \frac{4}{d}\frac{G}{c^4}\mu a_\mathrm{b}^2\omega_\mathrm{b}^2 \;=\; \frac{4}{d}\frac{G^2}{c^4}\frac{M_1 M_2}{a_\mathrm{b}}$$

$$= \;\frac{4}{d}\frac{G^{5/3}}{c^4}\frac{M_1 M_2}{M_\mathrm{T}^{1/3}}\left(\pi f_\mathrm{gw}\right)^{2/3} \;=\; \frac{4}{d}\left(\frac{G\mathcal{M}_\mathrm{c}}{c^2}\right)^{5/3}\left(\frac{\pi f_\mathrm{gw}}{c}\right)^{2/3}$$

$$\langle h(t)\rangle = \frac{2}{5}h_\mathrm{opt}$$

$$P_\mathrm{gw} = \frac{32}{5}\frac{c^5}{G}\left(\frac{G\mathcal{M}_\mathrm{c}\omega_\mathrm{gw}}{2c^3}\right)^{10/3} = \frac{32}{5}\frac{G^4}{c^5}\frac{M_1^2 M_2^2 M_\mathrm{T}}{a_\mathrm{b}^5}$$

$$\dot{f}_\mathrm{gw} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_\mathrm{c}}{c^3}\right)^{5/3} f_\mathrm{gw}^{11/3}$$

$$f_\mathrm{gw} = \frac{1}{\pi}\left(\frac{5}{256}\frac{1}{t_\mathrm{coal}-t}\right)^{3/8}\left(\frac{G\mathcal{M}_\mathrm{c}}{c^3}\right)^{-5/8}$$

$$\Delta t = \frac{5}{256}\pi^{-8/3}\left(\frac{G\mathcal{M}_\mathrm{c}}{c^3}\right)^{-5/3}\left[f_\mathrm{min}^{-8/3} - f_\mathrm{max}^{-8/3}\right]$$

$$\approx 2.16\,\mathrm{s}\left(\frac{\mathcal{M}_\mathrm{c}}{1.22\,M_\odot}\right)^{-5/3}\left[\left(\frac{f_\mathrm{min}}{100\,\mathrm{Hz}}\right)^{-8/3} - \left(\frac{f_\mathrm{max}}{100\,\mathrm{Hz}}\right)^{-8/3}\right]$$

$$\left(\frac{da_\mathrm{b}}{dt}\right)_\mathrm{gw} = -\frac{64}{5}\frac{G^3}{c^5}\frac{M_1 M_2 M_\mathrm{T}}{a_\mathrm{b}^3}$$

$$\tau_\mathrm{merge,gw} = \frac{5}{256}\frac{c^5}{G^3}\frac{a_\mathrm{b}^4}{M_1 M_2 M_\mathrm{T}}$$

$$\approx 150\,\mathrm{Myr}\left(\frac{a_\mathrm{b}}{R_\odot}\right)^4\left(\frac{M_1}{M_\odot}\right)^{-1}\left(\frac{M_2}{M_\odot}\right)^{-1}\left(\frac{M_\mathrm{T}}{M_\odot}\right)^{-1}$$

$$R_\mathrm{isco} = 3R_\mathrm{s} = \frac{6GM_\mathrm{bh}}{c^2}$$

$$f_\mathrm{gw,isco} = \frac{1}{6^{3/2}\pi}\frac{c^3}{GM_\mathrm{T}} \;\approx\; 4.4\,\mathrm{kHz}\left(\frac{M_\mathrm{T}}{M_\odot}\right)^{-1}$$

$$f_\mathrm{gw,max} \sim \frac{1}{\pi}\sqrt{\frac{GM_\mathrm{T}}{a_\mathrm{min}^3}}; \qquad a_\mathrm{min} \sim 1.5\,(R_1 + R_2)$$

$$h(t) = \frac{\Delta L(t)}{L}$$

$$\Delta\varphi = 2h_0\,\frac{2\pi L}{\lambda_\mathrm{L}}\,\mathrm{sinc}\left(\frac{2\pi f_\mathrm{gw} L}{c}\right)\,\cos\left(2\pi f_\mathrm{gw} t\right)$$

$$L_\mathrm{opt} = \frac{c}{4f_\mathrm{gw}}$$

$$S_\mathrm{n}^{1/2}(f)\bigg|_\mathrm{shot} = h_\mathrm{min}(f)\bigg|_\mathrm{shot} = \frac{\Delta L_\mathrm{min}(f)}{L}\bigg|_\mathrm{shot}$$

$$= \frac{1}{2\pi L}\sqrt{\frac{hc\lambda_\mathrm{L}}{\eta_\mathrm{PD} P_\mathrm{L}}} \approx \frac{7\times10^{-20}}{\sqrt{\eta_\mathrm{PD}}}\left(\frac{L}{1\,\mathrm{km}}\right)^{-1}\left(\frac{\lambda_\mathrm{L}}{\mu\mathrm{m}}\right)^{1/2}\left(\frac{P_\mathrm{L}}{\mathrm{W}}\right)^{-1/2}$$

$$\tilde{h}(f) = \frac{2}{5}\left(\frac{5}{6}\right)^{1/2}\frac{1}{2\pi^{2/3}}\frac{c}{d}\left(\frac{G\mathcal{M}_\mathrm{c}}{c^3}\right)^{5/6} f_\mathrm{gw}^{-7/6}$$

$$\rho^2 \equiv 4\int_{f_\mathrm{min}}^{f_\mathrm{max}}\frac{|\tilde{h}(f)|^2}{S_\mathrm{n}(f)}df$$

$$\rho_\mathrm{tot} = \sqrt{\sum_i^{N_\mathrm{det}}\rho_i^2}$$

$$\rho \sim \frac{h_0}{n_0} \gtrsim \rho_\mathrm{min}\sqrt{\frac{\tau_0}{T_\mathrm{obs}}}$$

Sterrenstelsels en kosmologie

$$v_\mathrm{rot} = \sqrt{\frac{GM(r)}{r}}$$

$$L_\mathrm{V} \approx 50.2\,L_\odot\left(\frac{P}{\mathrm{day}}\right)^{0.972}$$

$$v = H_0\cdot d$$

$$r(t) = r_\mathrm{s}\,a(t); \qquad v(t) = r_\mathrm{s}\,\dot{a}(t)$$

$$H(t) = \frac{v(t)}{r(t)} = \frac{\dot{a}(t)}{a(t)}$$

$$v^2 - \frac{8\pi G}{3}r^2\rho = -kc^2 r_\mathrm{s}^2 = H^2 r^2 - \frac{8\pi G}{3}r^2\rho$$

$$H^2 - \frac{8\pi G}{3}\rho = \frac{-kc^2}{a^2}; \qquad H_0^2 - \frac{8\pi G}{3}\rho_0 = -kc^2$$

$$\rho_\mathrm{crit} = \frac{3H^2}{8\pi G}$$

$$\Omega \equiv \frac{\rho}{\rho_\mathrm{crit}}; \quad \Omega_0 = \frac{\rho_0}{\rho_\mathrm{crit,0}}; \quad \Omega_\mathrm{b,0} \approx 0.049\left(\frac{H_0}{67.4\,\mathrm{km/s/Mpc}}\right)^2$$

$$a^3\rho = a_0^3\rho_0 \equiv \rho_0$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3} = \frac{H_0^2}{a^3}$$

$$a(t) = \left(\frac{3}{2}H_0 t\right)^{2/3} \equiv \left(\frac{3}{2}\frac{t}{t_\mathrm{H}}\right)^{2/3}$$

$$t_0 = \frac{2}{3}t_\mathrm{H}; \quad t_\mathrm{H} \equiv 1/H_0$$

$$z \equiv \frac{\Delta\lambda}{\lambda} = \frac{\lambda_\mathrm{obs}-\lambda_\mathrm{bron}}{\lambda_\mathrm{bron}} = \frac{a_\mathrm{obs}-a_\mathrm{bron}}{a_\mathrm{bron}} \equiv \frac{1}{a_\mathrm{bron}} - 1$$

$$z_\mathrm{Doppler} \equiv \frac{\Delta\lambda}{\lambda} = \frac{\lambda_\mathrm{obs}-\lambda_\mathrm{bron}}{\lambda_\mathrm{bron}} = \sqrt{\frac{1+v_\mathrm{rad}/c}{1-v_\mathrm{rad}/c}} - 1 \approx \frac{v_\mathrm{rad}}{c}$$

$$1 + z = \frac{1}{a(z)}$$

$$d \approx \frac{cz}{H_0} \quad (z \ll 1)$$

$$t(z) = t_0\,\frac{1}{(1+z)^{3/2}}$$

$$t_\mathrm{L} \equiv t_0 - t(z) \equiv t_0\left(1 - t_\mathrm{rel}(z)\right)$$

$$d_\mathrm{L} = c\cdot t_\mathrm{L} = ct_0\left(1 - \frac{1}{(1+z)^{3/2}}\right)$$

$$u_\lambda d\lambda = \frac{4}{c}B_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}d\lambda$$

$$T(a) = \frac{T_0}{a}$$

$$\rho_\mathrm{rad} = \frac{u}{c^2} = \frac{4\sigma}{c^3}T(a)^4 = \frac{4\sigma T_0^4}{c^3 a^4}$$

$$\frac{\dot{a}}{a} = \left(\frac{32\pi G}{3c^3}\,\sigma T^4\right)^{1/2}$$

$$T(t) = \left(\frac{3c^3}{128\pi\,G\sigma}\right)^{1/4} t^{-1/2}$$

$$a(t) = \left(\frac{128\pi G}{3c^3}\,\sigma T_0^4\right)^{1/4} t^{1/2}$$