Parameter estimation of spinning binaries using MCMC

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1. Introduction

Stellar-mass compact binaries are amongst the most promising gravitational-wave sources for ground-based laser interferometers. If such a binary contains a black hole, it is believed to be spinning moderately [1]. A spinning black hole causes the binary orbit to precess, introducing phase and amplitude modulations in the waveform. This should be taken into account in the analysis of the signal. We have extended a non-spinning binary inspiral parameter-estimation code [2], to extract the source parameters of spinning binary inspirals. The code is based on a Markov-chain Monte-Carlo (MCMC) technique [3] to compute the posterior probability-density functions (PDFs) of the source parameters.

2. The waveform

In this first stage of our study, we model the gravitational-wave signal at the restricted 1.5PN approximation [4] and the effect of spins is included in the limit of *simple precession* [5]. Here we present results for a fiducial binary system of a $10\,M_\odot$ spinning black hole and a $1.4\,M_\odot$ neutron star. The waveform is described by twelve parameters: chirp mass M_c, symmetric mass ratio η, spin magnitude a_{spin}, the angle between the axes of spin and orbit θ_{SL}, time (t_c), phase (ϕ_c) and precession phase (α_c) at coalescence, distance d_L, position in the sky (R.A., Dec.), and orientation of the binary (θ_{J0}, ϕ_{J0}).

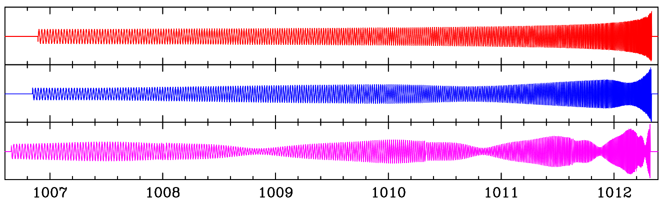

Fig. 1. Inspiral signals for $\theta_{SL} = 20°$ and a_{spin}= 0.0, 0.1 and 0.5.

3. Two detectors and moderate spin

We are carrying out a thorough exploration of the parameter space for spinning binaries. Here we present results for a fiducial source characterised by $a_{spin} = 0.5$, $\theta_{SL} = 20°$ and $d_L = 13\,\mathrm{Mpc}$ in a simulated observation with the two 4-km LIGO detectors. Ten serial chains of length $\sim 5 \times 10^6$ were computed to obtain these results. We show the marginalised PDFs of the twelve parameters in Fig. 2.

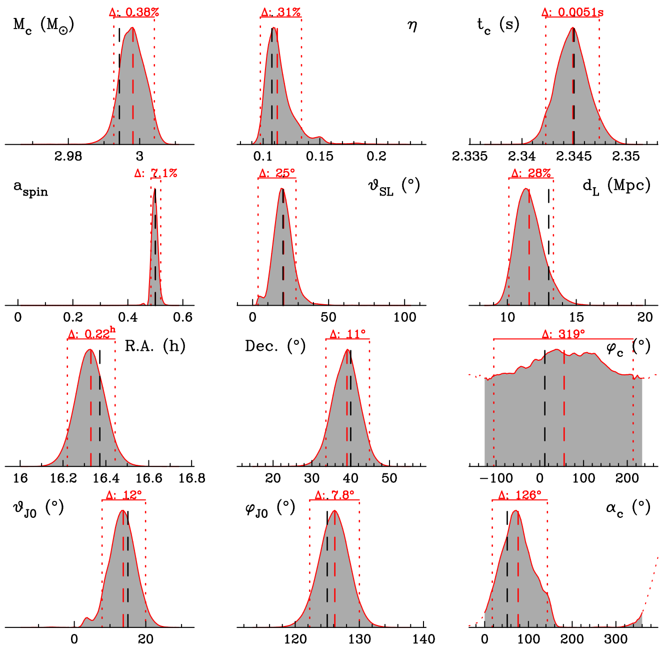

Fig. 2. Posterior PDFs for the twelve source parameters, as observed by the two 4-km LIGO detectors. For each PDF, the **black dashed line** indicates the true value, the **red dashed line** is the median and the **red dotted lines** show the 90%-probability interval, the width of which is indicated by Δ.

4. The effect of spin on the accuracy

The magnitude of the spin has a large influence on the accuracy with which the source parameters can be determined. A larger spin introduces more modulation in the waveform (Fig. 1), increases the SNR and can break degeneracies between parameters (Sect. 6). Figure 3 compares PDFs obtained from simulated detections with two LIGO interferometers of signals with different spin magnitudes, while all other parameters were kept constant.

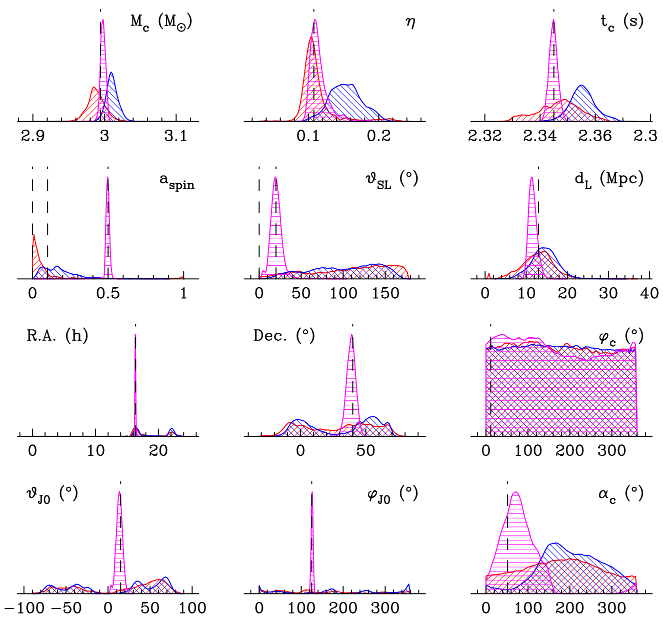

Fig. 3. Comparisons of posterior PDFs obtained with the two 4-km LIGO interferometers for signals with a spin magnitude a_{spin} = 0.0, 0.1 and 0.5, and $\theta_{SL} = 20°$, with SNRs between ~ 12 and ~ 14 per detector. The magenta PDFs are the same as in Fig. 2, the **black dashed lines** show the true parameter values (multiple values for a_{spin} and θ_{SL}). The PDFs are normalised to surface area.

5. One versus two detectors

For parameter estimation of an inspiralling binary with a spin of 0.5, there exists a degeneracy when observing with one detector. This degeneracy is lifted when using two instruments (see Sect. 6).

The accuracy with which the astrophysical binary parameters (masses and spin) are determined, increases when a second detector is added to the network. This happens firstly because more data is available with two detectors. The second reason for the increase in accuracy is that in the one-detector case, the mass and spin parameters are affected by the degeneracy, even if there is no degeneracy in these parameters themselves. Figure 4 compares the posterior PDFs of simulated detections with one and with two 4-km LIGO interferometers.

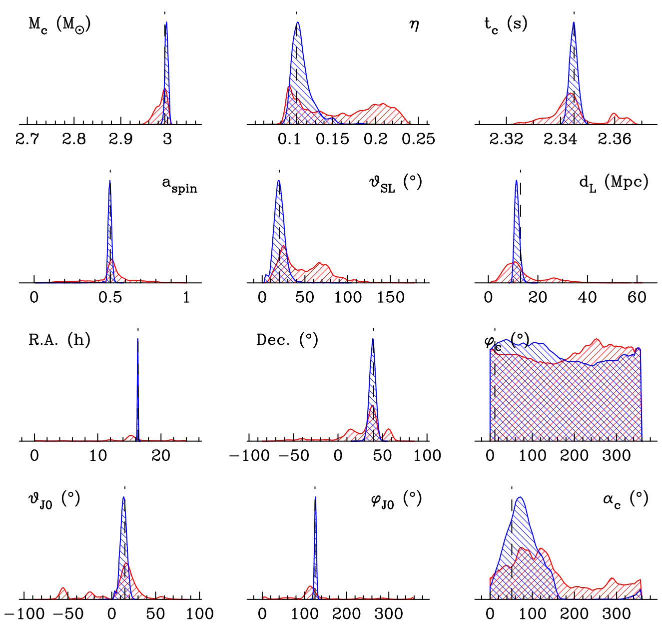

Fig. 4. PDFs obtained with the 4-km Hanford detector only and the 4-km Hanford and Livingston detectors, for $a_{spin} = 0.5$ and $\theta_{SL} = 20°$. The **black dashed lines** show the true values.

6. Degeneracy in the sky position

For the detection of a spinning inspiral with one detector, there exists a degeneracy between the position in the sky and the orientation of the binary. When there is no or low spin present in the binary, the degeneracy is complete, *i.e.* the posterior PDFs for the sky position are more or less flat and span most or all of the parameter space.

If the black hole in the binary spins moderately, the character of the degeneracy changes, and we find several confined regions in the sky that give an equally high likelihood. Figure 5a shows that the two-dimensional posterior PDF in that case changes from flat to multimodal.

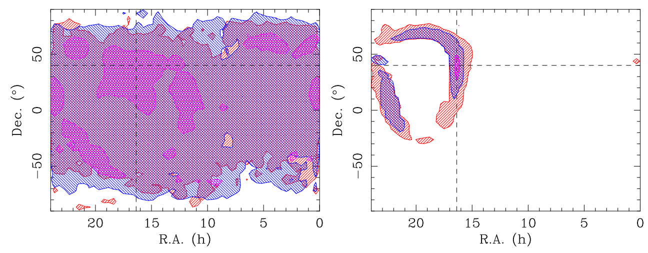

Fig. 5. Two-dimensional PDFs of the position in the sky, for the case of one detector (*left*, panel **a**) and two detectors (*right*, panel **b**). The two panels have the same scale. The dashed lines show the true position. The different colours show spin magnitudes of a_{spin} = 0.0, 0.1 and 0.5.

For a detection with two interferometers, the posterior PDFs are multimodal for zero or low spin, but the degeneracy is completely broken when the black hole has moderate spin. This results in a unique solution for the sky position (Fig. 5b).

7. Conclusions and future work

- Degeneracies in the sky position and binary orientation make it difficult to determine the astrophysical parameters (masses and spin) accurately with one detector.
- Observations with two (or more) detectors are necessary to fully resolve all parameters of interest, including the position in the sky. We show that our MCMC code can do this.
- In such a case, the combination of sky position, distance and time of the event could lead to association with an electromagnetic observation.
- The quality of astronomy improves (as expected) with the number of detectors, and we are already exploring the effect of including VIRGO into the network along with LIGO.
- We plan to include more realistic, higher-order waveforms, including the spin of both components.
- We are currently testing the code on injections into real interferometer data and on candidate events.
- We have detailed plans to include our MCMC code in the LIGO pipeline.

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