Interacting binaries and binary evolution

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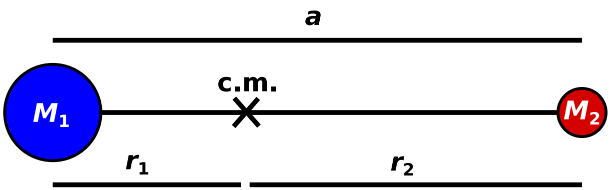
Outline

Binary geometry



$i = 1, 2$ = "star";
$(3 - i)$ = "other star".

Masses:

$$M_T = M_1 + M_2 \tag{1}$$

$$q_i = \frac{M_i}{M_{(3-i)}} \tag{2}$$

$$\mu = \frac{M_1 M_2}{M_T} \tag{3}$$

Centre of mass (rot. axis):

$$r_i \equiv \frac{M_{(3-i)}}{M_T} a \tag{4}$$

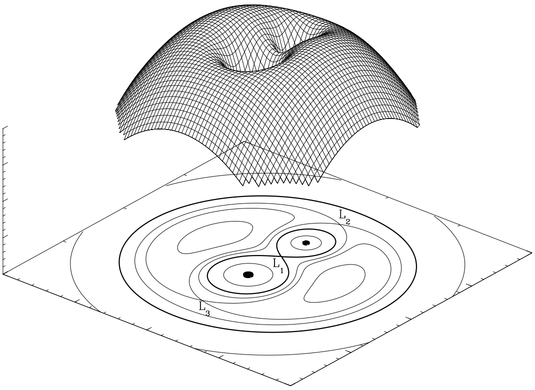
$$\frac{r_1}{M_2} = \frac{r_2}{M_1} = \frac{a}{M_T} \tag{5}$$

Kepler's law:

$$\omega^2 = \left(\frac{2\pi}{P}\right)^2 = \frac{GM_T}{a^3}, \tag{6}$$

where $\vec{\omega} = \frac{\vec{r} \times \vec{v}}{r^2}$ is the *angular frequency*.

Roche lobes



$$\Phi_{\rm Roche}(\vec{r}) = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r}_2|} - \frac{1}{2}\left(\vec{\omega}\times\vec{r}\right)^2 \quad (7)$$

(in a corotating frame). L_i: Lagrangian points

Roche-lobe radius (at which a spherical star has the same volume as its Roche lobe):

$$\frac{R_{\rm Rl,i}}{a} \approx \frac{2}{3^{4/3}}\left(\frac{M_i}{M_{\rm T}}\right)^{1/3}, \quad (8)$$

accurate within 2% for $q_i < 0.8$ (Paczyński, 1971).

$$\frac{R_{\rm Rl,i}}{a} \approx \frac{0.49\, q_i^{2/3}}{0.6\, q_i^{2/3} + \ln\left(1+q_i^{1/3}\right)} \approx \frac{0.44\, q_i^{0.33}}{(1+q_i)^{0.2}}. \quad (9)$$

The first part is accurate within 1% for $0 < q_i < \infty$ (Eggleton, 1983), the second is more convenient.

If $R_* > R_{\rm Rl}$, Roche-lobe overflow (RLOF) occurs and mass transfer can ensue through L_1.

Note that L_1 does generally *not* coincide with the centre of mass (rotation axis).

Minimum and maximum periods

From Eq. 8, minimum period w/o RLOF:

$$P_{\rm min} \propto \rho^{-1/2} \tag{10}$$

$$P_{\rm min} \sim 0.35 \sqrt{\left(\frac{R}{R_\odot}\right)^3 \left(\frac{M}{M_\odot}\right)^{-1}} \left(\frac{2}{1+q}\right)^{0.2} \,\mathrm{days} \tag{11}$$

For two Suns ($M = 1\,M_\odot$; $q = 1$):

$$P_{\rm min} \sim 0.35\,\mathrm{days} \;\sim 8.4\,\mathrm{h}.$$

Lowest-mass MS stars ($M = 0.1\,M_\odot$; $q = 1$):

$$P_{\rm min} \sim 8.4\,\mathrm{h}\,\sqrt{0.1^2} \sim 1\,\mathrm{h}.$$

Hence, binaries with $P \lesssim 1$ h cannot contain two MS stars!

From Eq. 8, maximum period for RLOF:

$$P_{\rm max} \sim 0.35 \sqrt{\left(\frac{R_{\rm max}}{R_\odot}\right)^3 \left(\frac{M}{M_\odot}\right)^{-1}} \left(\frac{2}{1+q}\right)^{0.2} \,\mathrm{days} \tag{12}$$

For two Suns ($M = 1\,M_\odot$; $q = 1$; $R_{\rm max} \sim 200\,R_\odot$):

$$P_{\rm max} \sim 990\,\mathrm{days} \;\; (\mathrm{really:}\ 1300\,\mathrm{days} \;-\; \mathrm{why?})$$

$M/M_\odot$	$P_{\rm min}$/day	$P_{\rm max}$/day
1	0.35	1300
8	0.85	1300
16	1.1	3200

Roche-lobe overflow $\rightarrow$ mass transfer

- If $P_{\rm min} < P_{\rm orb} < P_{\rm max}$, then $R_* \geq R_{\rm Rl}$ at some point and Roche-lobe overflow will occur.
- Mass transfer through the first Lagrangian point can strongly decrease the mass of the **donor star** and increase that of the **accretor**.
- Since the mass of a star is the most important parameter that determines its evolution, the future **evolution** of the two stars is **strongly influenced**.
- In addition, the **orbit can change dramatically**.
- Mass transfer can stop when the donor star shrinks, or when the orbit widens.

Key points that determine how mass transfer proceeds:

- how does $R_{\rm d}$ change? — ζ_*
- how does $R_{\rm Rl}$ change? — $\zeta_{\rm Rl}$
- what happens to the internal structure of the donor?
- what happens to the transferred mass (accreted, expelled)?
- what happens to the accretor ($M \rightarrow$ structure $\rightarrow R$)?

Mass transfer and angular momentum

Orbital angular momentum for binary component *i* in a circular orbit:

$$J_i = |\vec{J}_i| = M_i \left|\vec{r}_i \times \vec{v}_i\right| = M_i r_i v_i = M_i r_i^2 \omega \tag{13}$$

$$J_{\rm orb} = J_1 + J_2 = \mu a^2 \omega \;=\; M_1 M_2 \left(\frac{Ga}{M_{\rm T}}\right)^{1/2} \;=\; G^{2/3} \frac{M_1 M_2}{M_{\rm T}^{1/3}} \left(\frac{P}{2\pi}\right)^{1/3} \tag{14}$$

$$\frac{J_i}{J_{\rm orb}} \;=\; \frac{M_i\, r_i^2\, \omega}{\mu\, a^2\, \omega} \;=\; \frac{M_{(3-i)}}{M_{\rm T}} \tag{15}$$

$$\frac{\dot{J}}{J} \;=\; \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} - \frac{1}{2}\frac{\dot{M}_{\rm T}}{M_{\rm T}} + \frac{1}{2}\frac{\dot{a}}{a} \tag{16}$$

$$\;=\; \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} - \frac{1}{3}\frac{\dot{M}_{\rm T}}{M_{\rm T}} + \frac{1}{3}\frac{\dot{P}}{P} \tag{17}$$

Logarithmic derivative:

$$x \equiv a\, y^b \;\rightarrow\; \frac{\dot{x}}{x} = b\, \frac{\dot{y}}{y} \tag{18}$$

Conservative mass transfer

Simplest case: conservative mass transfer:

Eq. 16 with $\dot{J} = 0$; $\dot{M}_\mathrm{T} = 0 \rightarrow \dot{M}_\mathrm{a} = -\dot{M}_\mathrm{d}$:

$$\frac{\dot{a}}{a} = 2\,\dot{M}_\mathrm{d}\frac{M_\mathrm{d} - M_\mathrm{a}}{M_\mathrm{d}M_\mathrm{a}} = 2\frac{\dot{M}_\mathrm{d}}{M_\mathrm{d}}\,(q_\mathrm{d} - 1) \tag{19}$$

Note:

- Since J_orb is constant:

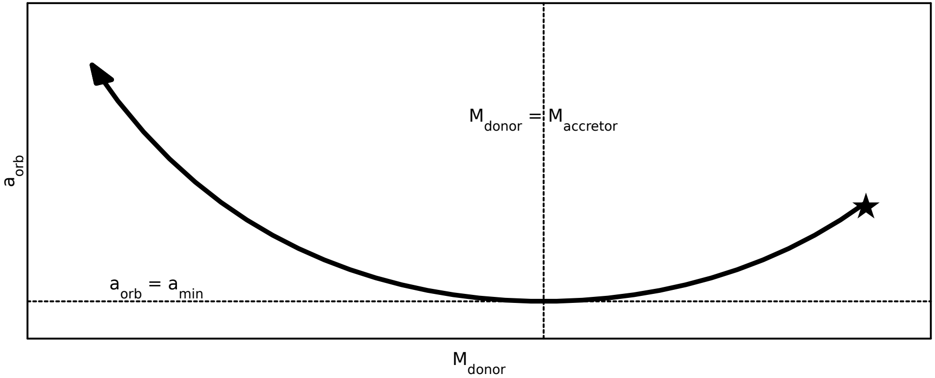
$$J_\mathrm{orb}(a_\mathrm{min}) = J_\mathrm{orb}(M_\mathrm{d} = M_\mathrm{a}) = \frac{M_\mathrm{T}^2}{4}\left(\frac{Ga_\mathrm{min}}{M_\mathrm{T}}\right)^2 \rightarrow \frac{a}{a_\mathrm{min}} = \left(\frac{M_\mathrm{T}^2}{4M_\mathrm{d}M_\mathrm{a}}\right)^2 \tag{20}$$

- For which masses is a_min reached?

Conservative mass transfer

Note:

- If $M_\mathrm{d} > M_\mathrm{a} \;\rightarrow\; \dot{M}_\mathrm{d} < 0 \;\rightarrow\; \dot{a} < 0 \;\rightarrow$ orbit shrinks
- If $M_\mathrm{d} = M_\mathrm{a} \;\rightarrow\; \dot{a} = 0 \;\rightarrow\; a = a_\mathrm{min}$



Conservative mass transfer

Eggleton code + BinSim, R. Hynes, LSU

Stellar wind

Case 2: isotropic (fast) stellar wind from star 1 only:

Using Eq. 16 and $\dot{J} = \dot{J}_1 \neq 0$; $\dot{M}_\mathrm{T} = \dot{M}_1$; $\dot{M}_2 = 0$:

$$\frac{\dot{a}}{a} = 2\frac{\dot{J}}{J} - 2\frac{\dot{M}_\mathrm{T}}{M_1} + \frac{\dot{M}_\mathrm{T}}{M_\mathrm{T}} \tag{21}$$

Each gram of matter is lost with the specific AM of star 1 and using Eq. 15:

$$h_1 \equiv \frac{J_1}{M_1} = J_\mathrm{orb}\frac{M_2}{M_1\, M_\mathrm{T}}, \tag{22}$$

so that $\dot{J}_1 = h_1\, \dot{M}_1$, and

$$\left(\frac{\dot{J}}{J_\mathrm{orb}}\right)_\mathrm{wind} = \frac{M_2}{M_\mathrm{T}}\frac{\dot{M}_\mathrm{T}}{M_1} = \frac{1}{q_1}\frac{\dot{M}_\mathrm{T}}{M_\mathrm{T}}, \tag{23}$$

$$\frac{\dot{a}}{a} = 2\left(\frac{\dot{J}}{J_\mathrm{orb}}\right)_\mathrm{wind} - \frac{M_1 + 2M_2}{M_1}\frac{\dot{M}_\mathrm{T}}{M_\mathrm{T}} = -\frac{\dot{M}_\mathrm{T}}{M_\mathrm{T}}, \tag{24}$$

$$a \propto M_\mathrm{T}^{-1}. \tag{25}$$

Note that it does not matter *which* star loses the material.

Realistic mass transfer

1. Onset of MT $*1 \rightarrow *2$: $t = 0$, $R_* = R_{\rm Rl}$
2. $t = t + \Delta t$, $\dot{M}_1 < 0 \;\rightarrow\; M_1 \downarrow$ and
   1. orbit: $q \downarrow \rightarrow$ (assume $\dot{M}_2, \dot{M}_{\rm T}, \dot{J}$) $\rightarrow\; \Delta q, \Delta a \;\rightarrow\; \Delta R_{\rm Rl}$
   2. star: stellar structure $\rightarrow\; \Delta R_*$
3. Result:
   1. $R_{*,\rm new} = R_* + \Delta R_*$, $R_{\rm Rl\,,new} = R_{\rm Rl} + \Delta R_{\rm Rl} \;\rightarrow\; \dot{M}_1$

In general:

- $\dot{M} = f(R_* - R_{\rm Rl})$
- Important: $\dot{R}_*(\dot{M}_*)$ compared to $\dot{R}_{\rm Rl}(\dot{a}, \dot{M}_1, \dot{M}_2)$:
  - $\dot{R}_* > \dot{R}_{\rm Rl} \;\rightarrow\; \dot{M} \uparrow$
  - $\dot{R}_* < \dot{R}_{\rm Rl} \;\rightarrow\; \dot{M} \downarrow$
  - $\dot{R}_* > \dot{R}_{\rm Rl} \;\rightarrow\; \dot{M} \uparrow + \dot{R}_* > \dot{R}_{\rm Rl} \;\rightarrow$ unstable MT
  - $\dot{R}_* < \dot{R}_{\rm Rl} \;\rightarrow\; \dot{M} \downarrow + \dot{R}_* < \dot{R}_{\rm Rl} \;\rightarrow$ MT will stop
  - $\dot{M} \uparrow\downarrow \rightarrow\; \dot{R}_*, \dot{R}_{\rm Rl} \uparrow\downarrow \rightarrow\; \dot{M} \downarrow\uparrow \rightarrow$ equilibrium: MT $\sim$ constant

Non-conservative mass transfer:

A fraction β of the transferred matter is accreted, the rest is expelled with a fraction α of the specific AM of the *accretor*:

$$\left(\frac{\dot{J}}{J}\right)_{\rm MT} = -\alpha(1-\beta)\frac{M_{\rm d}}{M_{\rm a}}\frac{\dot{M}_{\rm d}}{M_{\rm T}}. \qquad (26)$$

Stability of mass transfer:

$$\zeta \equiv \left(\frac{d\log R}{d\log M}\right) \qquad (27)$$

Note that $d\log M < 0$ for the donor star! MT is stable (in fact, $\dot{M}$ does not grow) if

$$\zeta_{\rm d} \geq \zeta_{\rm Rl}(q, \beta). \qquad (28)$$

e.g. Hjellming & Webbink (1987); Soberman et al. (1997)

Eddington limit

Accretion onto a compact object (NS, BH) generates radiation. If the accretion luminosity $L_{\rm acc}$ becomes sufficiently high, it may prevent further accretion. Hence, there is a maximum accretion rate for an accretor, known as the *Eddington limit* ($\dot{M}_{\rm edd}$).
Assume that the accreted matter consists of "particles" with a proton mass $m_{\rm p}$ and the Thomson cross section of electrons $\sigma_{\rm T}$. If the luminosity force cancels out gravity on such a particle:

$$F_{\rm L} = F_{\rm g} \quad \rightarrow \quad \frac{L}{c}\frac{\sigma_{\rm T}}{4\pi r^2} = \frac{G\,M_*\,m_{\rm p}}{r^2}. \tag{29}$$

The *Eddington luminosity* ($L_{\rm edd}$) is defined as:

$$L_{\rm edd} \;=\; \frac{4\pi c\,G\,m_{\rm p}}{\sigma_{\rm T}}\,M_* \;\approx\; 3.3\times 10^4\left(\frac{M_*}{M_\odot}\right)L_\odot. \tag{30}$$

The *Eddington accretion limit* ($\dot{M}_{\rm edd}$) can be found from $L_{\rm edd} = L_{\rm acc} = \dfrac{G\,M_*\,\dot{M}}{R_*}$:

$$\dot{M}_{\rm edd} \;=\; \frac{4\pi c\,m_{\rm p}}{\sigma_{\rm T}}\,R_* \;\approx\; 1.5\times 10^{-8}\left(\frac{R_*}{10\,{\rm km}}\right)M_\odot\,{\rm yr}^{-1}. \tag{31}$$

Drives of mass transfer

The mass-transfer rate depends on the change of the radius of the donor, and on the change in orbit.

Classification of the drives of mass transfer:

Intrinsic drive: R_* changes due to a change of stellar structure caused by stellar evolution.

Extrinsic drive: the binary orbit changes (shrinks) due to loss of angular momentum: $\dot{J} \rightarrow \dot{a} \rightarrow \dot{R}_{\mathrm{Rl}}$.

Complication:

1. $\dot{M}_*$ is a function of $\dot{R}_*$, $\dot{R}_{\mathrm{Rl}}$
2. $\dot{R}_*$, $\dot{R}_{\mathrm{Rl}}$ are functions of $\dot{M}_*$!

Intrinsic drives of mass transfer

Mass transfer can take place on three timescales:

1. Nuclear-evolution timescale
2. Thermal timescale
3. Dynamical timescale

The applicable timescale depends on the structure and evolutionary phase of the donor star.

Reminder:

$$\zeta \equiv \left(\frac{d\log R}{d\log M}\right)$$

MT on the nuclear-evolution timescale:

- $\dot{R}_*$ due to nuclear evolution of the donor star
  - star is in thermal and hydrostatic equilibrium:

$$\zeta_{\rm d,th} > \zeta_{\rm Rl}(q, \beta = \beta_{\rm nuc}) \tag{32}$$

- $R_* \uparrow$, $\dot{M}$ such that $\dot{a}$, $\dot{R}_{\rm Rl}$ ensure that $R_{\rm Rl}$ follows R_*
- MS timescale:

$$\tau_{\rm nuc,MS} \sim \frac{0.1\,M_*}{L_*} \approx 10^{10}\,{\rm yr}\left(\frac{M_*}{M_\odot}\right)^{-2,-3} \tag{33}$$

- HG, GB timescale: $\tau_{\rm nuc,GB} \sim 0.01 - 0.1\tau_{\rm nuc,MS}$

$$\dot{M} \sim \frac{M_*}{\tau_{\rm nuc}} \tag{34}$$

Intrinsic drives of mass transfer

MT on the thermal timescale:

- Donor is in hydrostatic equilibrium, but not in thermal equilibrium:

$$\begin{aligned}\zeta_{\rm d,th} &< \zeta_{\rm Rl}(q, \beta = \beta_{\rm nuc}) \\ \zeta_{\rm d,ad} &> \zeta_{\rm Rl}(q, \beta = \beta_{\rm th})\end{aligned}$$

- Thermal equilibrium reached after $\tau_{\rm KH} \sim \frac{G\,M_*^2}{R_*\,L_*}$
- Hydrostatic equilibrium sets in at a much shorter timescale
- Hence, it is possible that (*e.g.* in the HG, on the GB):
  - $\dot{R}_* < \dot{R}_{\rm Rl}$ for $t < \tau_{\rm KH}$, but
  - $\dot{R}_* > \dot{R}_{\rm Rl}$ for $t \gtrsim \tau_{\rm KH}$
  - hence, $\dot{M} \downarrow$ first, but $\dot{M} \uparrow$ after $\tau_{\rm KH}$
    - $\rightarrow$ $\dot{M}$ driven by $\tau_{\rm KH}$:

$$\dot{M} \sim \frac{M_*}{\tau_{\rm KH}} \gg \dot{M}_{\rm nuc} \qquad (35)$$

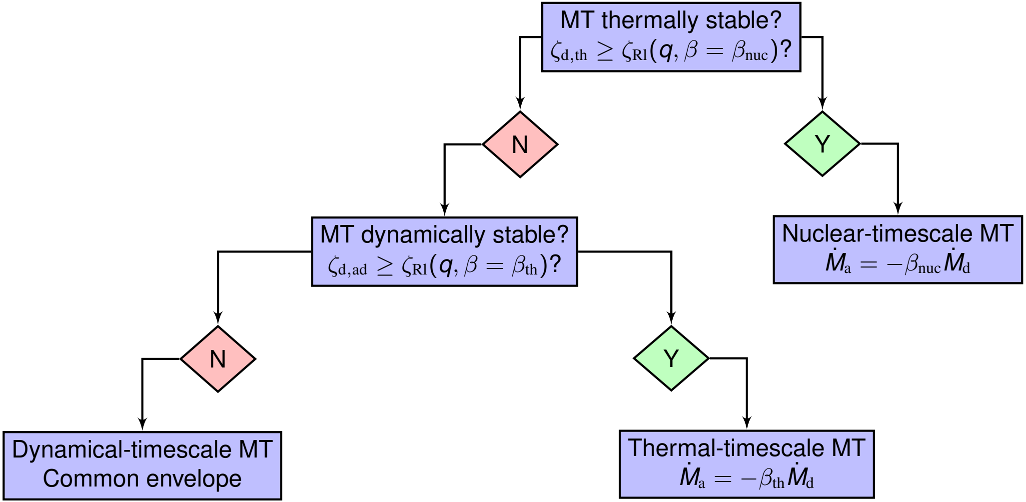
MT on the dynamical timescale:

- Donor is not in hydrostatic equilibrium:

$$\zeta_{\rm d,ad} < \zeta_{\rm Rl}(q, \beta = \beta_{\rm th})$$

- **very** short timescale (years – kyrs?)
- if $\dot{R}_* > \dot{R}_{\rm Rl}$, the MT is **dynamically unstable**
- this results in **runaway mass transfer** (common envelope?)

Intrinsic drives of mass transfer



Extrinsic drives of mass transfer

Extrinsic drives of MT:

1. Angular-momentum is removed from the binary orbit
2. The orbit shrinks
3. The donor star overfills its Roche lobe (more than before)

Extrinsic drivers of MT:

1. Emission of gravitational waves (GWs)
2. Magnetic braking

Gravitational waves:

$$\left(\frac{\dot{J}}{J}\right)_{\mathrm{GW}} = -\frac{32}{5}\frac{G^3}{c^5}\frac{M_1 M_2 M_{\mathrm{T}}}{a^4} \times f(e) \tag{36}$$

(Peters, 1964)

For a detached binary:

$$\tau_{\mathrm{GW}} \equiv \left|\frac{J}{\dot{J}}\right|_{\mathrm{GW}} \approx 380\,\mathrm{Gyr}\,\frac{(1+q)^2}{q}\left(\frac{M_{\mathrm{T}}}{M_\odot}\right)^{-5/3}\left(\frac{P}{\mathrm{day}}\right)^{8/3}, \tag{37}$$

$$t_{\mathrm{contact}} = \frac{\tau_{\mathrm{GW}}}{8}. \tag{38}$$

Examples: $q = 1$, $t_{\mathrm{contact}} = 10^{10}$ yr:

0.6+0.6 $M_\odot$	DWD	$P_{\mathrm{max}} \approx 0.37$ day (~ 9 h)
1.4+1.4 $M_\odot$	BNS	$P_{\mathrm{max}} \approx 0.63$ day (~ 15 h)
10+10 $M_\odot$	BBH	$P_{\mathrm{max}} \approx 2.1$ day

Why not MS+MS binary?

Extrinsic drives of mass transfer

GWs as MT driver: WD donor

1. Assume $R_* \propto M_*^{-1/3} \rightarrow \frac{\dot{R}}{R} = -\frac{1}{3}\frac{\dot{M}}{M}$ (not exactly true!)
2. Lower-mass WD fills RL (why?)
3. MT from low-mass to high-mass $\rightarrow q < 1, q\downarrow \rightarrow a\uparrow$
4. Smaller $q \rightarrow \dot{R}_* < \dot{R}_{\rm Rl} \rightarrow$ MT stops
5. But: $\dot{J}_{\rm GW} < 0 \rightarrow a\downarrow \rightarrow R_{\rm Rl}\downarrow \rightarrow$ RLOF $\rightarrow$ MT
6. Hence, MT is driven by GW, and

$$\dot{M} \propto \frac{M_{\rm d}}{\tau_{\rm GW}} = \frac{M_{\rm d}^2\, M_{\rm a}\, M_{\rm T}}{a^4} \quad (39)$$

Example 1: AM CVn stars (H-poor CVs):

- AM CVn stars evolve to large a / long P
- $M_{\rm d}\, M_{\rm a}\downarrow, a\uparrow \rightarrow \tau_{\rm GW}\uparrow\uparrow \rightarrow \dot{M}\downarrow$
- $\rightarrow$ evolution starts rapidly (high $|\dot{M}|, |\dot{a}|$), but slows down exponentially with time

Example 2: 0.3+0.6 $M_\odot$ MS+WD (CV):

- RLOF of $0.3 M_\odot$ MS
- $\dot{M} \rightarrow q\downarrow, a\uparrow, R_{\rm Rl}\uparrow + R_*\downarrow$ (since $R_* \propto M_*$)
- MT stops unless external $\dot{J}$ is present
- $\dot{M} \sim \frac{M_{\rm d}}{\tau_{\rm GW}};\quad M_{\rm d}, M_{\rm a}\downarrow, a\downarrow \rightarrow \tau_{\rm GW} \sim\downarrow, M_{\rm d} \sim\downarrow \rightarrow \dot{M} \sim$ constant

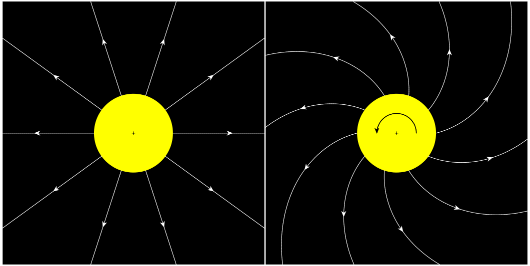
Extrinsic drives of mass transfer

Example 2 (cont.): observed CVs:

- short P ($\lesssim 2\,\text{hr}$): $\dot{M}_{\text{obs}} \sim 10^{-10}\, M_{\odot}/\text{yr}$; $\dot{M}_{\text{GW}} \sim 10^{-10}\, M_{\odot}/\text{yr}$ ✓
- but: long P ($\gtrsim 3\,\text{hr}$): $\dot{M}_{\text{obs}} \sim 10^{-8}\, M_{\odot}/\text{yr}$ ✗
- + LMXBs (NS+MS): $\dot{M}_{\text{obs}} \sim 10^{-8}\, M_{\odot}/\text{yr}$ ✗
- Hence: additional $\dot{J}$-mechanism needed!

Magnetic braking

- Rotating stars can have magnetic fields
- Evolved stars can have strong winds
- Stellar wind follows magnetic-field lines
- Star loses angular momentum efficiently
- Tidal coupling causes orbit to shrink in case of a binary



Extrinsic drives of mass transfer

Magnetic braking:

- Observed in single stars in clusters, field
- In binaries, tides cause corotation
  $\rightarrow \dot{J}_{\rm MB} = \dot{J}_{\rm orb}$

$$\dot{J}_{\rm MB} \approx -3.8 \times 10^{-30} M_* R_*^4 \omega^3 \text{ dyn cm.} \tag{40}$$

(Verbunt & Zwaan, 1981)

- Reasaonable values for $\dot{J}_{\rm MB}$ give $\dot{M} \sim 10^{-8} M_\odot$yr ✓
- However, is $\dot{J}_{\rm MB}$ observed in slowly rotating single stars applicable to short-period binaries?

Saturating magnetic braking:

$$\dot{J}_{\rm MB} = -K \left(\frac{R}{R_\odot}\right)^{\frac{1}{2}} \left(\frac{M}{M_\odot}\right)^{-\frac{1}{2}} \omega^3, \quad \omega \leq \omega_{\rm crit}$$
$$= -K \left(\frac{R}{R_\odot}\right)^{\frac{1}{2}} \left(\frac{M}{M_\odot}\right)^{-\frac{1}{2}} \omega\, \omega_{\rm crit}^2, \quad \omega > \omega_{\rm crit}$$

(Sills et al., 2000)

- Seems to give better agreement with observations for ultrashort periods

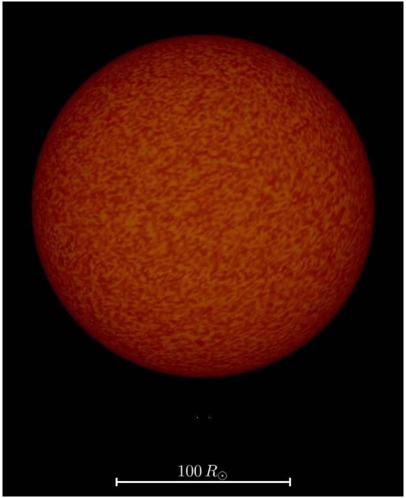
Observed double-lined double white dwarfs

System	P_{orb} (d)	a_{orb} ($R_\odot$)	M_1 ($M_\odot$)	M_2 ($M_\odot$)	q_2 (M_2/M_1)	$\Delta\tau$ (Myr)
WD 0135–052	1.556	5.63	0.52 ± 0.05	0.47 ± 0.05	0.90 ± 0.04	350
WD 0136+768	1.407	4.99	0.37	0.47	1.26 ± 0.03	450
WD 0957–666	0.061	0.58	0.32	0.37	1.13 ± 0.02	325
WD 1101+364	0.145	0.99	0.33	0.29	0.87 ± 0.03	215
PG 1115+116	30.09	46.9	0.7	0.7	0.84 ± 0.21	160
WD 1204+450	1.603	5.74	0.52	0.46	0.87 ± 0.03	80
WD 1349+144	2.209	6.59	0.44	0.44	1.26 ± 0.05	—
HE 1414–0848	0.518	2.93	0.55 ± 0.03	0.71 ± 0.03	1.28 ± 0.03	200
WD 1704+481a	0.145	1.14	0.56 ± 0.07	0.39 ± 0.05	0.70 ± 0.03	-20[a]
HE 2209–1444	0.277	1.88	0.58 ± 0.08	0.58 ± 0.03	1.00 ± 0.12	500

[a] Unclear which white dwarf is older

See references in: Maxted et al., 2002 and Nelemans & Tout, 2005.

Common envelopes (CEs)

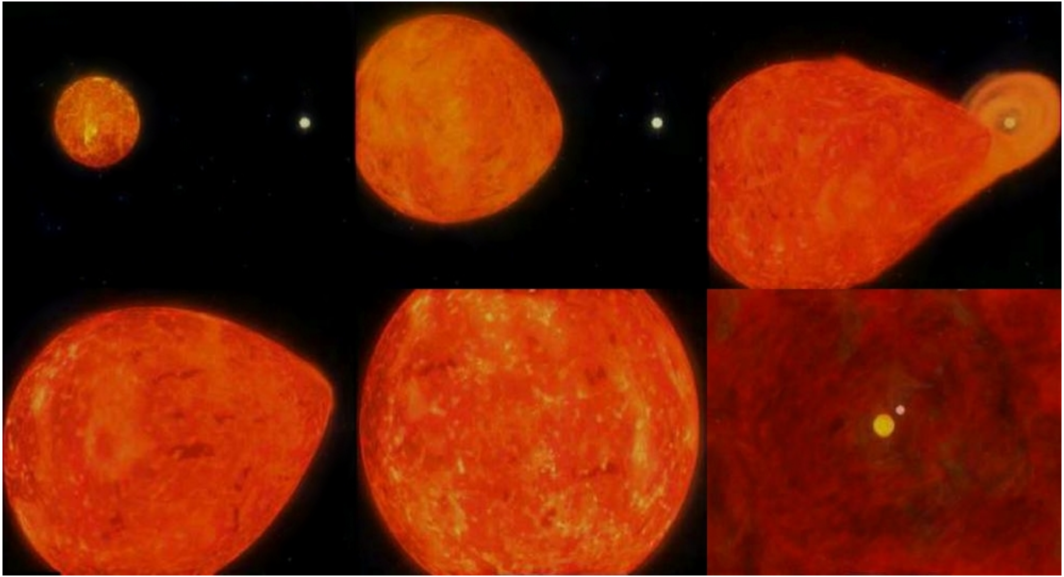


Reason CEs are needed:

- Average orbital separation: $\sim 7\,R_\odot$
- Typical progenitor: giant:
  - $M_c \gtrsim 0.3\,M_\odot$
  - $R_* \sim 100\,R_\odot$
  - $a \sim 250\,R_\odot$ (for $2 \times 1\,M_\odot$)
- Hence, large ΔJ_{orb}, Δa is required

- Idea: giant donor, dynamical MT, donor envelope expands rapidly, engulfs companion: Common Envelope (Paczyński, 1976)
- Inside the CE, the donor core and companion spiral in through friction, heating up and expelling the CE
- Fast: M_c does not grow during CE
- Fast: no accretion by companion during CE

Common envelope cartoon



Classical common envelope: energy balance

Change in orbital separation due to CE:

- Equate the binding energy of the giant's envelope to the change in orbital energy:

$$E_{\rm bind,env} = \alpha_{\rm CE} \left[\frac{G\, M_{1,\rm f}\, M_2}{2\, a_{\rm f}} - \frac{G\, M_{1,\rm i}\, M_2}{2\, a_{\rm i}} \right] \quad (41)$$

(Paczyński, 1976; Webbink, 1984)

- $\alpha_{\rm CE}$ is the efficiency factor ($0 < \alpha_{\rm CE} \leq 1$)
- The envelope binding energy is often parameterised: $E_{\rm bind,env} \approx \dfrac{G\, M_{\rm env}\, M_*}{\lambda\, R_*}$
- Result depends on exact definition of $M_{\rm c}$, $M_{\rm env}$
- Result depends on energy sources taken into account
  - *e.g.* recombination energy, nuclear energy star

Success of CEs:

- Often $a_{\rm f} \ll a_{\rm i}$, as needed
- Works well to explain *e.g.* WD-MS binaries

However, for DWDs:

- Expect CE1: large $a_{\rm i}$ $\rightarrow$ large R_* $\rightarrow$ large $M_{\rm c}$ $\rightarrow$ massive WD (M_1)
- Expect CE2: smaller $a_{\rm i}$ $\rightarrow$ smaller R_* $\rightarrow$ smaller $M_{\rm c}$ $\rightarrow$ low-mass WD (M_2)
- Hence, expect $M_1 > M_2$
- But: observe $M_1 \sim M_2$
- Apparently, $a_{\rm i} \sim a_{\rm f}$ in first MT phase — no CE! X
- CE works well for second MT phase ✓

Common envelope with AM balance

Envelope ejection with AM conservation:

- Idea: at onset of CE1, the AM is typically large, and the companion is massive
- Hence, the companion could spin up the envelope $\rightarrow$ no more friction
- Hence, no E balance, not necessarily a spiral-in
- Also, AM is harder to create or destroy than E

$$\frac{J_{\rm i} - J_{\rm f}}{J_{\rm i}} = \gamma \, \frac{M_{1,\rm i} - M_{1,\rm f}}{M_{\rm tot,i}} \tag{42}$$

Nelemans et al. (2000)

- $\gamma \sim 1.5$ is the average specific AM of the binary
- For $q \sim 1$, $\frac{\Delta M}{M}$ relatively small $\rightarrow$ ΔJ small $\rightarrow$ little orbital shrinkage
- For $q < 1$, $\frac{\Delta M}{M}$ large $\rightarrow$ ΔJ large $\rightarrow$ more orbital shrinkage
- This seems to explain the masses and mass ratios of the 10 observed double-lined DWDs (but not their age differences)

- However, why should $\gamma = 1.5$ (or any other value)? — little physical significance
- Much discussion about sensitivity of exact γ on outcome

Darwin instability

AM catastrophe:

If

1. one of the binary companions has a large spin AM,
2. the star is tidally locked to the orbit, *and*
3. the star expands (due to evolution),

then

1. the rotation of the expanding star slows down,
2. tidal locking speeds up the star's spin and drains AM from the orbit,
3. the orbit shrinks, tides become stronger,
4. a stable orbit is impossible, and the stars will merge.

This happens when

$$J_* \geq \frac{1}{3} J_{\mathrm{orb}} \tag{43}$$

(Darwin 1879)

Post-CE stars

After the CE:

- If $R_{\rm R1,acc} < R_{\rm a}$: merger
- If $|E_{\rm bind,env}| > E_{\rm orb}$: merger
- Else: binary survives CE: post-CE (compact) binary

Exercises

To get started, derive Equations 4, 6 (for a circular orbit) and 10.

Prove Equation 18 and derive Equations 16 and 21.

For more depth, derive Equations 11, 14, 19, 20, 25, 26, 31, 38 and 43.

You can use the corresponding subsections in Appendix A of *Binary evolution in a nutshell* for guidance.

Online material

http://www.astro.ru.nl/~sluys/

Teaching → Compact binaries 2021

1. These slides;
2. Document *Binary evolution in a nutshell* with more details, derivations and references.