Masses and envelope binding energies of primary stars at the onset of a common envelope

Marc van der Sluys*, Michael Politano† and Ronald E. Taam**

*University of Alberta, Canada; CITA National Fellow; now at R.U. Nijmegen, The Netherlands
†Marquette University, Milwaukee, Wi, U.S.A.
**Northwestern University; Academia Sinica Institute of Astronomy & Astrophysics/TIARA, Taipei, Taiwan

Abstract. We present basic properties of primary stars that initiate a common envelope (CE) in a binary, while on the giant branch. We use the population-synthesis code described in Politano et al. [1] and follow the evolution of a population of binary stars up to the point where the primary fills its Roche lobe and initiates a CE. We then collect the properties of each system, in particular the donor mass and the binding energy of the donor's envelope, which are important for the treatment of a CE. We find that for most CEs, the donor mass is sufficiently low to define the core-envelope boundary reasonably well. We compute the *envelope-structure parameter* $\lambda_{\rm env}$ from the binding energy and compare its distribution to typical assumptions that are made in population-synthesis codes. We conclude that $\lambda_{\rm env}$ varies appreciably and that the assumption of a constant value for this parameter results in typical errors of 20–50%. In addition, such an assumption may well result in the implicit assumption of unintended and/or unphysical values for the CE parameter $\alpha_{\rm CE}$. Finally, we discuss accurate existing analytic fits for the envelope binding energy, which make these oversimplified assumptions for $\lambda_{\rm env}$, and the use of $\lambda_{\rm env}$ in general, unnecessary.

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1. INTRODUCTION

Common envelopes (CEs) [2, 3, 4] constitute an important phase in the evolution of many binaries and are used to explain the formation of large numbers of observed compact binaries, such as X-ray binaries, cataclysmic variables and double white dwarfs. A CE is assumed to be initiated when a giant star with a deep convective envelope in orbit with a more compact, sufficiently low-mass companion fills its Roche lobe and the ensuing mass transfer is unstable [5]. The result is a fast expanding envelope which can quickly engulf the companion star. The core of the donor star and the companion star orbit each other inside this CE, and the friction and torques lead to the spiral-in of the orbit, resulting in a compact binary or a merger. The energy generated by the orbital decay is assumed to heat the CE and eventually expel it from the system on a time scale much shorter than the evolutionary timescales of stars ($\lesssim 10^3$ yr), leaving the secondary star unaffected [4].

Because of the large range of length scales involved, detailed modeling of the CE process is computationally expensive, and while three-dimensional models can follow about the first month of the process, the outcome of CEs cannot yet be predicted [*e.g.*, 6]. In practise, therefore, a cartoonish approach is often used to determine the orbital shrinkage during the spiral-in, where the energy needed to unbind the envelope is assumed to be dissipated from the orbit [7]. Hence, the decrease in orbital energy during the CE is related to the binding energy of the convective envelope of the donor star at the onset of the CE [3]:

$$E_{\rm bind} = -\alpha_{\rm CE}\left(\frac{GM_{1,\rm c}M_2}{2a_{\rm f}} - \frac{GM_1M_2}{2a_{\rm i}}\right). \tag{1}$$

In this equation, $E_{\rm bind}$ is the binding energy of the donor's convective envelope, M_1 is the mass of the donor at the onset of the CE, $M_{1,\rm c}$ is the helium-core mass of the donor and the mass of its remnant if the binary survives the CE, M_2 is the unchanged companion mass, $a_{\rm i,f}$ are the initial and final orbital separation, respectively, and $\alpha_{\rm CE}$ is the efficiency with which the orbital energy is used to expel the envelope.

We developed a population-synthesis code that is tailored to study the first CE event in a binary [1]. In addition, this code allows us to study the post-CE evolution of the merger remnant or compact binary. In these proceedings, we briefly describe our population-synthesis code in Sect. 2. In Section 3 we summarise our work on the present-day population of merger remnants that were formed through a CE, which we presented in Mykonos. The bulk of this

TABLE 1. Percentages of CEs that are initiated on the RGB or AGB, and either survive as a binary or merge, using 165,007 CEs.

	RGB	**AGB**	**Total**
Survivors	16.1%	31.8%	48.0%
Mergers	45.4%	6.6%	52.0%
Total	61.5%	38.5%	100.0%

paper, Section 4, consists of new results, where we present some basic properties of donor stars at the onset of a CE. In Section 5 we discuss these results and present our conclusions.

2. POPULATION-SYNTHESIS CODE

We developed a population-synthesis code that is specifically targeted at binaries in which the primary star fills its Roche lobe and causes a CE event [1]. For each binary formed at a random time during the last 10^{10} years, the code follows the evolution, including effects such as stellar wind and tides, and determines whether the stars merge during the CE. It can then follow the evolution of the merger remnants and create a population of merger products at the present epoch. As input for the code we use a grid of 32 low-mass/brown-dwarf models [8, 9] and 116 detailed stellar-evolution models, computed with the binary-evolution code `ev`[1], developed by Eggleton [10, 11, 12, and references therein] and updated as described in Pols et al. [13]. We define the helium-core mass $M_{\rm c}$ as the inner region of the star where the hydrogen abundance lies below 10% ($X < 0.1$). The detailed stellar-structure models allow us to compute the envelope binding energy accurately by integrating the stellar structure over the mass coordinate from the core-envelope boundary $M_{\rm c}$ to the stellar surface $M_{\rm s}$:

$$E_{\rm bind} = \int_{M_{\rm c}}^{M_{\rm s}} E_{\rm int}(m) - \frac{Gm}{r(m)}\,{\rm d}m, \tag{2}$$

where internal energy $E_{\rm int}$ contains the thermal and radiation energy of the gas, but not its recombination energy. More details of the code as used for these models are described in Politano et al. [1].

3. THE PRESENT-DAY POPULATION OF CE MERGERS

At the conference, we presented the results of our study of a population of 25,000 present-day merger remnants, descendant from an initial population of 10^7 ZAMS binaries with component masses up to $10\,M_\odot$ each. These results have recently been published in Politano et al. [1]. We will therefore only reiterate the most important conclusions here, and refer the reader to that paper for details. The main conclusions of our merger study are:

- Between 0.24% and 0.33% of the initial ZAMS binaries are visible today as non-degenerate merger products.
- Present-day merger remnants constitute of 37% RGB stars, 57% HB stars, and 6% AGB stars.
- RGB stars are under-represented w.r.t. HB+AGB stars in a merger population, compared to normal single stars.
- The median rotational velocity of the merger population is $16.2\,{\rm km\,s^{-1}}$, compared to $2.3\,{\rm km\,s^{-1}}$ for single stars.

4. PROPERTIES OF DONOR STARS AT THE ONSET OF A CE

In this main section of these proceedings, we will discuss the properties of the primary stars in our model binaries, at the moment they fill their Roche lobes and initiate a CE phase. The CE may result in either the survival of the binary system, or the merger of the two binary components. We consider a population of 10^6 ZAMS binaries with maximum

[1] The current version of `ev` is obtainable on request from `eggleton1@llnl.gov`, along with data files and a user manual.

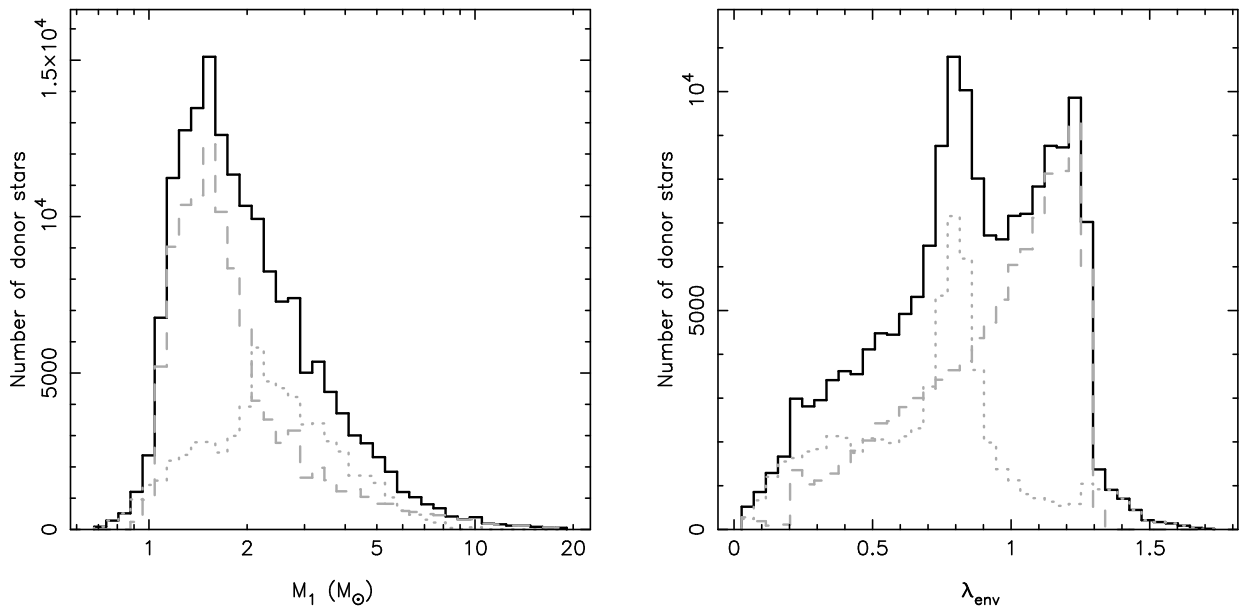

FIGURE 1. Distributions of selected properties of the primary stars in our binary population at the onset of a CE. *Left panel (a):* primary mass M_1. *Right panel (b):* envelope-structure parameter $\lambda_{\rm env}$. Dashed lines represent donors on the RGB, dotted lines donors on the AGB, and solid lines their sum.

component masses of $20\,M_\odot$ each and a uniform distribution of mass ratios, resulting in 165,007 CEs that occur when the donor star is on the RGB or AGB. Table 1 lists the fractions of these CEs that lead to survival of the binary or to merger for both RGB and AGB primaries. While Politano et al. [1] found that for CEs that lead to merger, the vast majority (87%) of CEs occur when the primary is on the RGB rather than AGB, we see that this majority drops to $\sim 62\%$ when both mergers and survivors are taken into account. Also, as one would intuitively expect because of the smaller orbital separations and higher binding energies, most (74%) RGB CEs lead to merger, whereas most (83%) CEs initiated when the primary is on the AGB allow the binary to survive.

4.1. Donor masses

While the combination of Eqs. 1 and 2 seems to fully determine the simplified CE treatment based on energy conservation for a given efficiency factor $\alpha_{\rm CE}$, the situation is not quite that simple. One of the major obstructions is the definition of the core-envelope boundary, and hence the helium-core mass $M_{\rm c}$, which is needed in Eq. 2 to compute the binding energy of the envelope. Dewi and Tauris [14] and Tauris and Dewi [15] compare five different definitions of the core-envelope boundary, and discard the two extreme cases, leaving the three definitions in the middle three rows of Table 1 of the latter paper (one of which is identical to the definition we use: $X < 0.1$). They find that for low-mass stars ($M \lesssim 7-10\,M_\odot$), where the gradients in the stellar structure are steep, the three definitions give similar results. However, for more-massive stars, the three results become very different and the exact definition of the core mass becomes uncertain by more than an order of magnitude for a $20\,M_\odot$ star. In fact, the exact separation between remnant and ejecta, and hence the mass of the remnant, will depend on the response of both the stellar structure and the orbit to the mass loss, and hence will depend on the secondary star as well [16, 17].

Figure 1a shows a histogram of donor masses (influenced by wind mass loss) at the onset of their CE for all donor stars, as well as the separate contributions from RGB and AGB stars. The median mass of all donors is $1.8\,M_\odot$ while the RGB donors contribute more to the low-mass part (median mass $1.6\,M_\odot$) and the AGB stars more to the high-mass end (median mass $2.3\,M_\odot$). We note that for 98.1% of the CEs in our models, the donor mass is lower than $7\,M_\odot$, while for 99.4% of the CEs, $M_1 < 10\,M_\odot$. This corresponds to $\sim 97.3\%$ and $\sim 98.6\%$ of all CEs respectively when allowing primary masses up to $\sim 200\,M_\odot$, assuming the same fraction of binaries undergo a CE. This suggests that for most *instances* of a CE, the core mass should be relatively well defined. However, when studying populations of massive stars, *e.g.* HMXBs, this is of course no longer true.

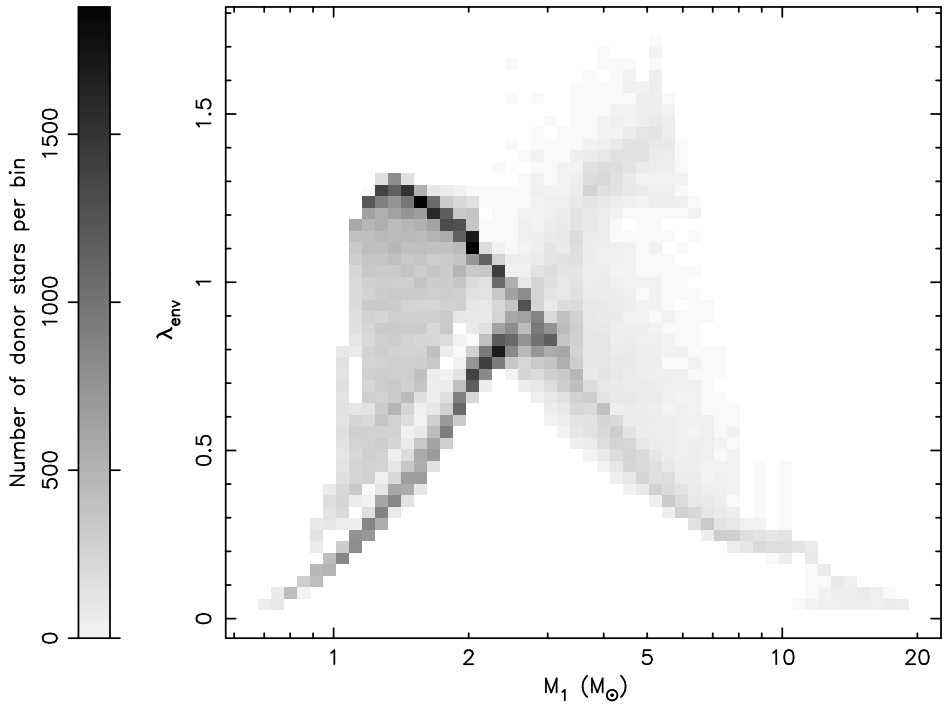

FIGURE 2. Two-dimensional histogram displaying $\lambda_{\rm env}$ versus M_1 for primary stars at the onset of a CE. Darker pixels indicate more donors in the bin, as indicated by the legend at the left. See Sect. 4.2 for more details. Note that the pattern looks like a λ ☺

4.2. The envelope-structure parameter $\lambda_{\rm env}$

In population-synthesis codes, where the evolution of millions of binaries must be computed, detailed stellar-structure models are computationally too expensive and hence have typically not been used. As a consequence, the binding energy of the donor's convective envelope cannot be computed exactly, and is often approximated using the so-called *envelope-structure parameter* $\lambda_{\rm env}$ [18], defined by

$$E_{\rm bind} = \frac{GM_1M_{1,\rm env}}{R_1\lambda_{\rm env}}, \tag{3}$$

where $M_{1,\rm env} \equiv M_1 - M_{\rm c,1}$ is the mass of the donor's envelope and R_1 the donor's radius. The value for this parameter is different for different stars and varies through the evolution of a star. This behaviour is presented in Dewi and Tauris [14] for high-mass stars, and in van der Sluys et al. [19] for lower-mass stars. Comparison of Eqs. 1 and 3 shows that the uncertainty in $\alpha_{\rm CE}$ and $\lambda_{\rm env}$ can be combined in their product $\alpha_{\rm CE}\lambda_{\rm env}$. Hence, as an alternative to assuming a constant value for $\lambda_{\rm env}$, in many population-synthesis studies a constant value for $\alpha_{\rm CE}\lambda_{\rm env}$ is used. For example, Nelemans et al. [20] and Hurley et al. [21] choose $\lambda_{\rm env} = 0.5$, while Belczynski et al. [22] assume $\alpha_{\rm CE}\lambda_{\rm env} = 0.5, 1.0$. In our models, the actual binding energy is known through Eq. 2, and therefore we can use Eq. 3 to compute the correct value for $\lambda_{\rm env}$ for each donor star that initiates a CE.

We present the distribution of $\lambda_{\rm env}$ for our data set of 165,007 CEs in Fig. 1b. The distribution is double peaked, where the peak below $\lambda_{\rm env} = 1$ is caused by AGB donors and the peak above that value by RGB stars. The range of values for $\lambda_{\rm env}$ is $0.027 - 1.73$ and the median value of the total distribution is 0.86, whereas the medians for the RGB and AGB sub-populations are 1.00 and 0.75 respectively. Using this distribution, we can derive that the median relative error made when assuming $\lambda_{\rm env} = 0.5$ is 47%, while that value is 22% for the assumption $\lambda_{\rm env} = 1.0$. In both cases the extreme errors exceed an order of magnitude.

Figure 2 shows a two-dimensional histogram of the $M_1 - \lambda_{\rm env}$ space. The arm in the χ-shape that runs from the upper left to the lower right, as well as the "slab" that extends down from the upper-left part, is formed by donor stars on the RGB. The other arm, running from the lower left to the upper right, and the thinly populated area between the two arms at $M_1 \gtrsim 3.5 M_\odot$, are formed by stars that initiate a CE on the AGB. Hence we find that for RGB stars, lower-mass donors have higher values for $\lambda_{\rm env}$, while for AGB donors, this trend is reversed.

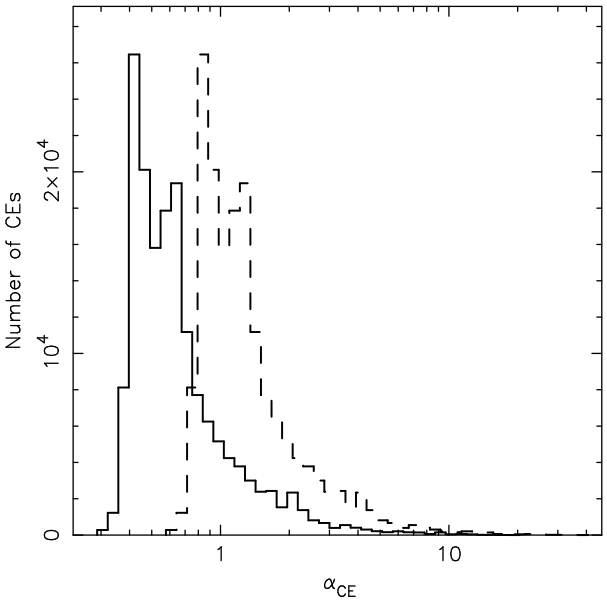

FIGURE 3. Distributions of the implicitly assumed common-envelope efficiency parameter $\alpha_{\rm CE}$ when using the premise $\alpha_{\rm CE}\lambda_{\rm env} = 0.5$ (solid line) or $\alpha_{\rm CE}\lambda_{\rm env} = 1.0$ (dashed line).

4.2.1. Implications for $\alpha_{\rm CE}$

Interestingly, we find that for 64.2% of our CE donors the envelope-structure parameter is smaller than unity and for 16.3% it is even smaller than 0.5. This means that the assumption $\alpha_{\rm CE}\lambda_{\rm env} = 1.0$ implies $\alpha_{\rm CE} > 1$ for most CEs, while the premise $\alpha_{\rm CE}\lambda_{\rm env} = 0.5$ (including *e.g.* $\alpha_{\rm CE} = 1.0; \lambda_{\rm env} = 0.5$) still gives values for the CE efficiency factor larger than unity for about one in six CEs. Figure 3 displays the distributions of the implicitly assumed values for $\alpha_{\rm CE}$ when using the premises $\alpha_{\rm CE}\lambda_{\rm env} = 0.5$ or $\alpha_{\rm CE}\lambda_{\rm env} = 1.0$. We find that $0.29 \lesssim \alpha_{\rm CE} \lesssim 18.3$ for the first assumption (the limiting values of this range are twice as high for the second assumption), indicating that a few extreme cases are quite unphysical indeed. From Fig. 2 and its description in Sect. 4.2, we find that extremely high values of $\alpha_{\rm CE}$ (when $\lambda_{\rm env}$ is low) occur on the AGB for low-mass stars and on the RGB for high-mass stars.

5. DISCUSSION AND CONCLUSIONS

In the previous section, we argued that although the exact core-envelope boundary is difficult to ascertain for massive stars, this will affect only $\sim 3\%$ or fewer of CEs, due to the initial-mass function. On the other hand, for the study of high-mass compact binaries, this effect may be quite important and even dominate the uncertainty in wind mass loss of massive stars for the determination of the envelope binding energy (see the discussion in Loveridge et al. [23]).

In addition, we presented realistic values for the envelope-structure parameter $\lambda_{\rm env}$, defined in Eq. 3, for stars that initiate a CE. We find that the actual value of this parameter is far from constant, but varies from star to star, as well as during the lifetime of a star. Assuming a constant value for either $\lambda_{\rm env}$ or $\alpha_{\rm CE}\lambda_{\rm env}$ introduces errors of typically $\sim 20-50\%$, but the error may be higher than an order of magnitude in some extreme cases. Furthermore, such an assumption for $\lambda_{\rm env}$ implicitly assumes values for $\alpha_{\rm CE}$ which can be very different from the intended value, and may be quite unphysical, reaching values of 10 or more.

It is therefore clear that a more realistic way of estimating the binding energy of the convective envelope of a giant star can significantly reduce much of the uncertainty with which the post-CE orbit can be determined, for a given value of $\alpha_{\rm CE}$. Fortunately, and not quite coincidentally (at least in one case), two studies have recently been published that offer a solution. Xu and Li [24] offer fits for stars of 14 different discrete masses between $1M_\odot$ and $20M_\odot$ for $Z = 0.001$ and $Z = 0.02$. The fits provide $\lambda_{\rm env}$ using the mass and radius (in some cases the core mass replaces the radius) of the star, which can then be used with Eq. 3 to compute the binding energy. Their expressions are simple and easy to implement, but unfortunately they do not provide the accuracy of their fits, so that it is difficult to assess to

what extent the errors discussed above are improved.

Independently, Loveridge et al. [23] provide fits for six different values of the metallicity between $Z = 10^{-4}$ and $Z = 0.03$, which directly give the envelope binding energy for giants with any mass between $0.8\,M_{\odot}$ and $100\,M_{\odot}$, as a function of Z, M and R. They show that the accuracies of their fits are better than 15% for 90% of their data points for all metallicities and evolutionary stages, and better than 10% for 90% of their data points for all cases, except the AGB for the three lowest metallicities in their data. This is a significant improvement from the accuracies found in Sect. 4.2, which can be expressed using the notation above as better than 22% and 47% for the assumptions $\alpha_{\rm CE}\lambda_{\rm env} = 0.5$ and $\alpha_{\rm CE}\lambda_{\rm env} = 1.0$ respectively, for only 50% of the data points. The fits are more complex in this study, but the fitting coefficients, and example routines that use them to compute the envelope binding energy for a number of cases, are available electronically [25]. Thus, we conclude that there is no longer a need to make oversimplified assumptions for $\lambda_{\rm env}$, or in fact no longer a need for the parameter $\lambda_{\rm env}$ itself, since more accurate alternatives exist to approximate the binding energy of the convective envelope for any giant star that can initiate a CE.

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