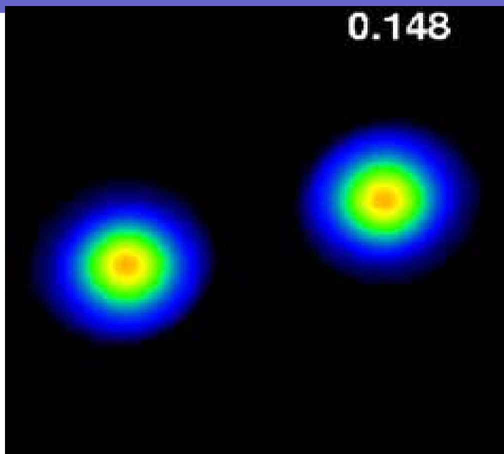
Sources of Gravitational Waves

2nd talk

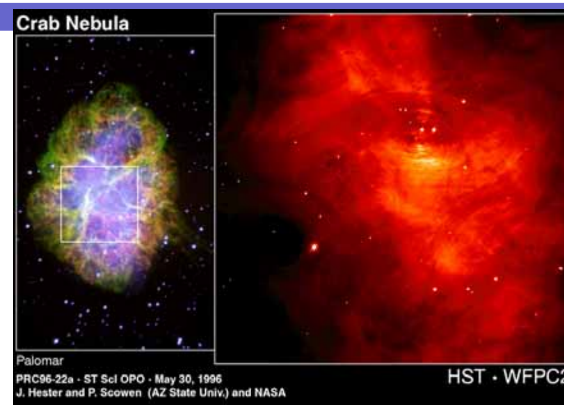
Astrophysical Sources of GWs

- Binary systems (NS/NS, NS/BH, BH/BH)
- Supernova
  - Bounce
  - Fall back
  - Oscillations & Instabilities
- Old and Isolated NS
- Cosmological origin

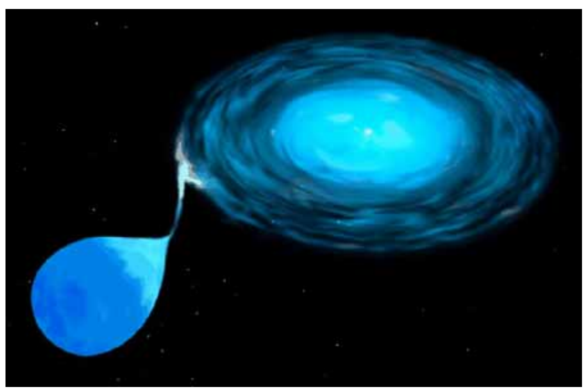
GW sources in ground-based detectors



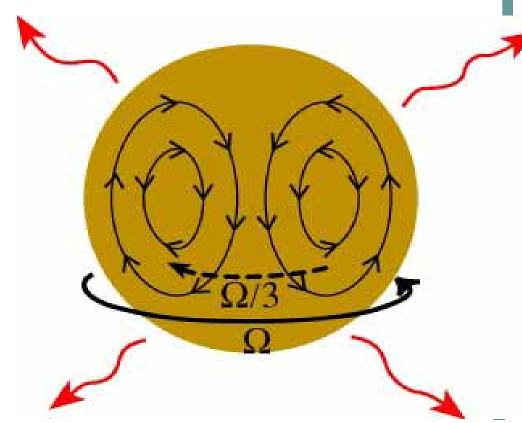
BH and NS Binaries



Supernovae, BH / NS formation

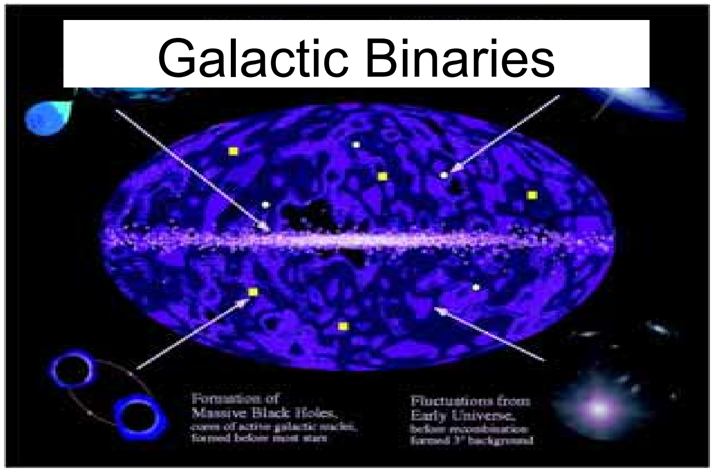


Spinning neutron stars in X-ray binaries

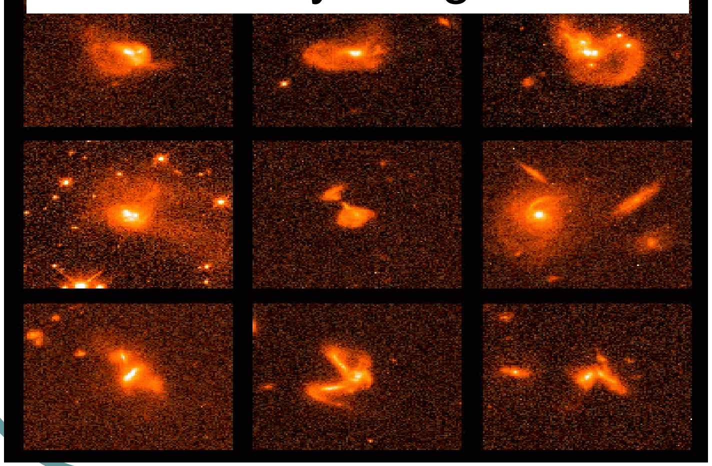


Young Neutron Stars

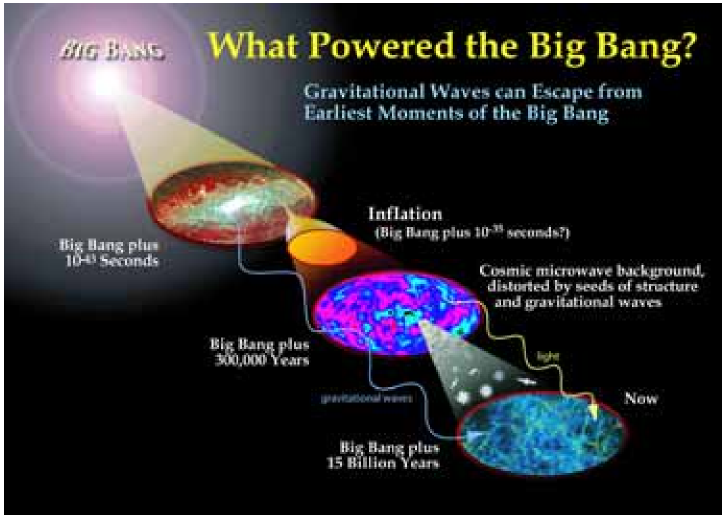
Sources in LISA

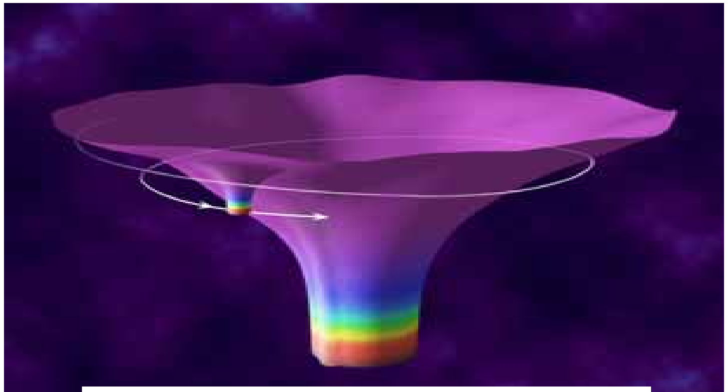


Galactic Binaries



Galaxy mergers

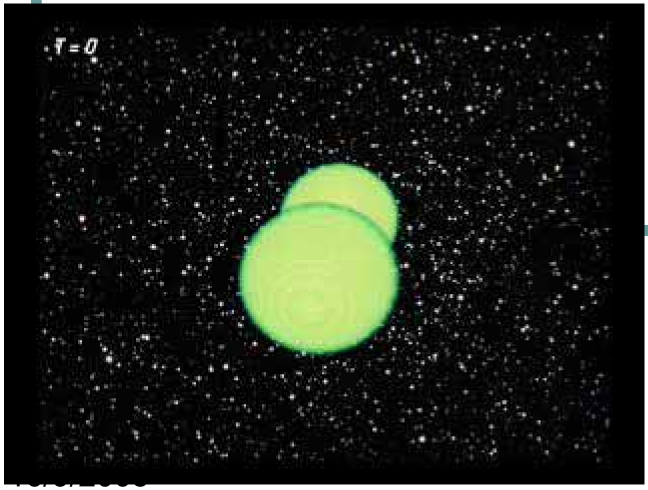


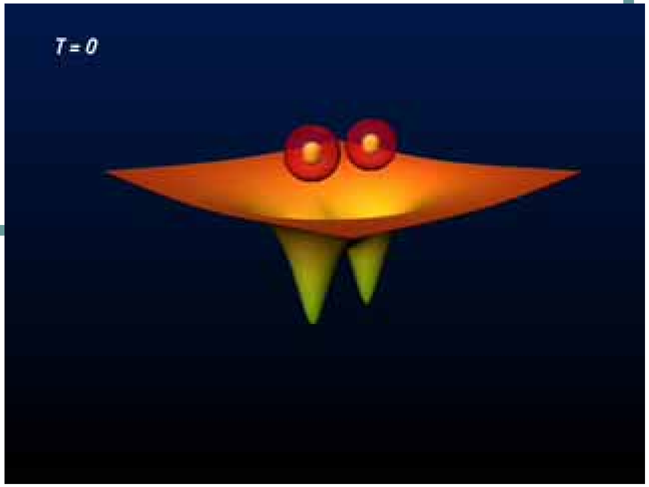


Capture orbits

Binary systems (NS/NS, NS/BH, BH/BH)

The best candidates and most reliable sources for broad band detectors





Coalescence of Compact Binaries

events/year	LIGO-I	LIGO-II
NS/NS	~0.05	~60-500
BH/NS	~0.02	~80
BH/BH	~0.8	~2000
Total	0.8	$\gtrsim$2000

- During the frequency change from 100-200Hz GWs carry away $5\times10^{-3} M_{\odot}c^2$.
- In LIGOs band
  - **NS/NS** (~16000 cycles)
  - **NS/BH**(~3500 cycles)
  - **BH/BH**(~600 cycles)
- The GW amplitude is:

$$h \approx 7.5\times10^{-23}\left(\frac{M}{2.8M_{\odot}}\right)^{2/3}\left(\frac{\mu}{0.7M_{\odot}}\right)\left(\frac{f}{100Hz}\right)^{2/3}\left(\frac{100Mpc}{r}\right)$$

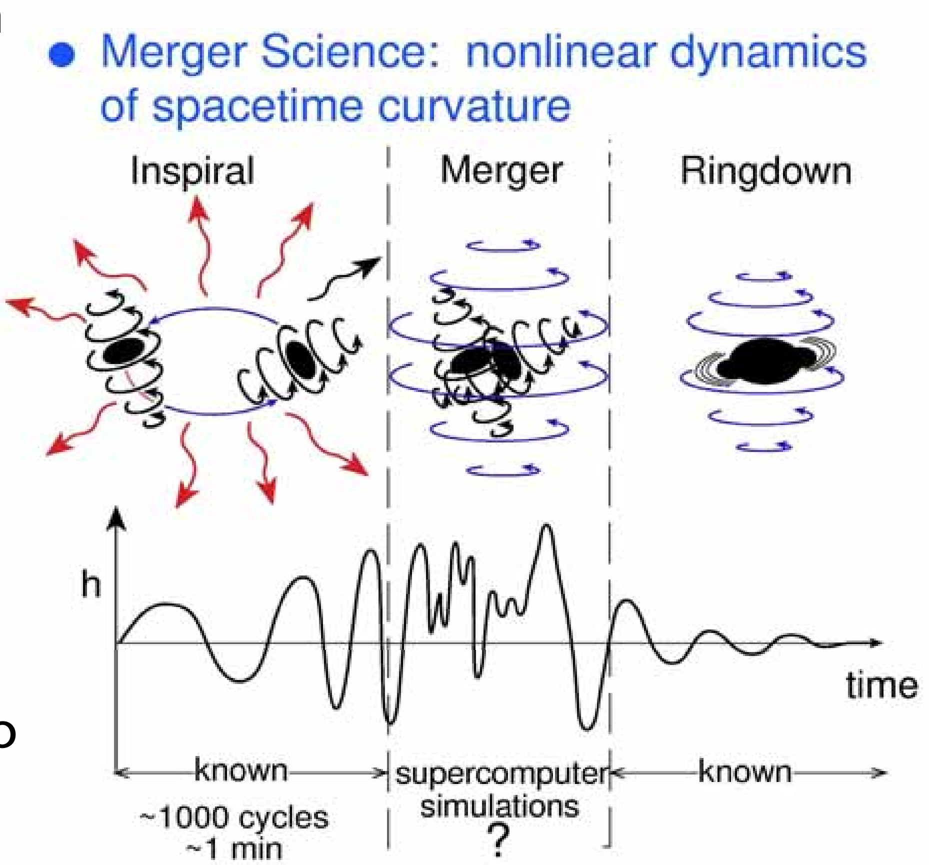
- Larger total mass improves detection probability.

•**Phase effects are important**, if the signal and the template get out of phase their cross correlation will be reduced.
•**High accuracy templates** are needed for accurate detection.

Gravitational Waves from Binaries

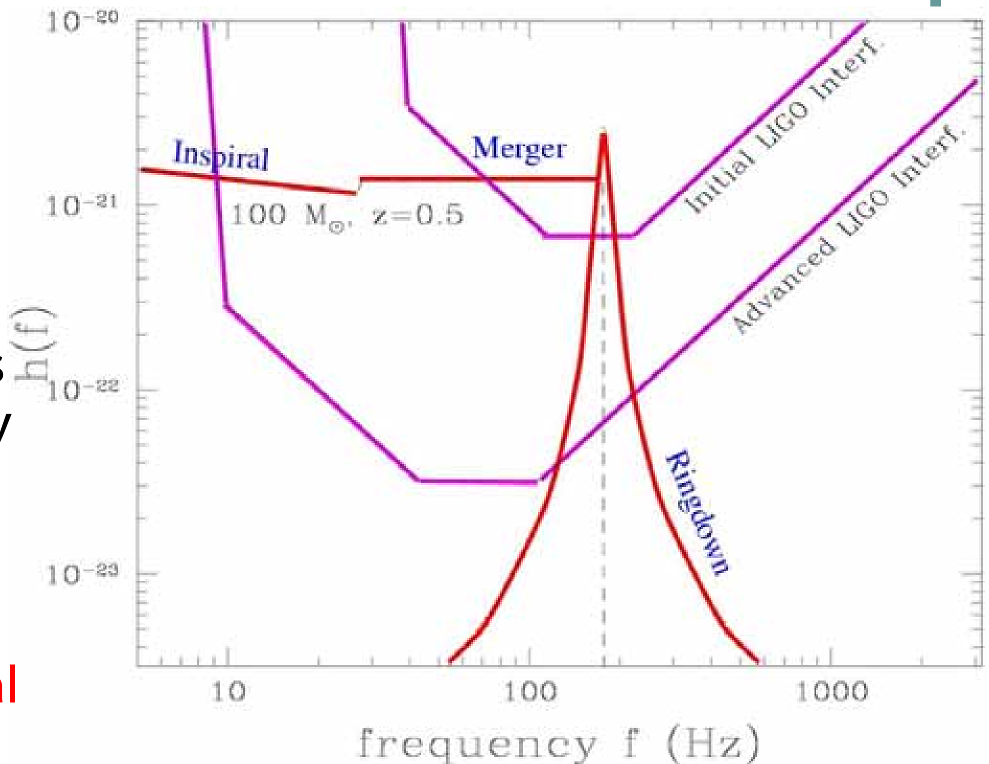
Generically, there are 3 regimes in which black holes radiate:

- **Orbital in-spiral**: **PN-approximations** or **point-particle orbits.**
- **Plunge/merger** after the last stable orbit: **numerical simulations** or **point-particle orbits**.
- **Ring-down** of the disturbed black hole as it settles down to a Kerr hole: **perturbation theory** of black holes.



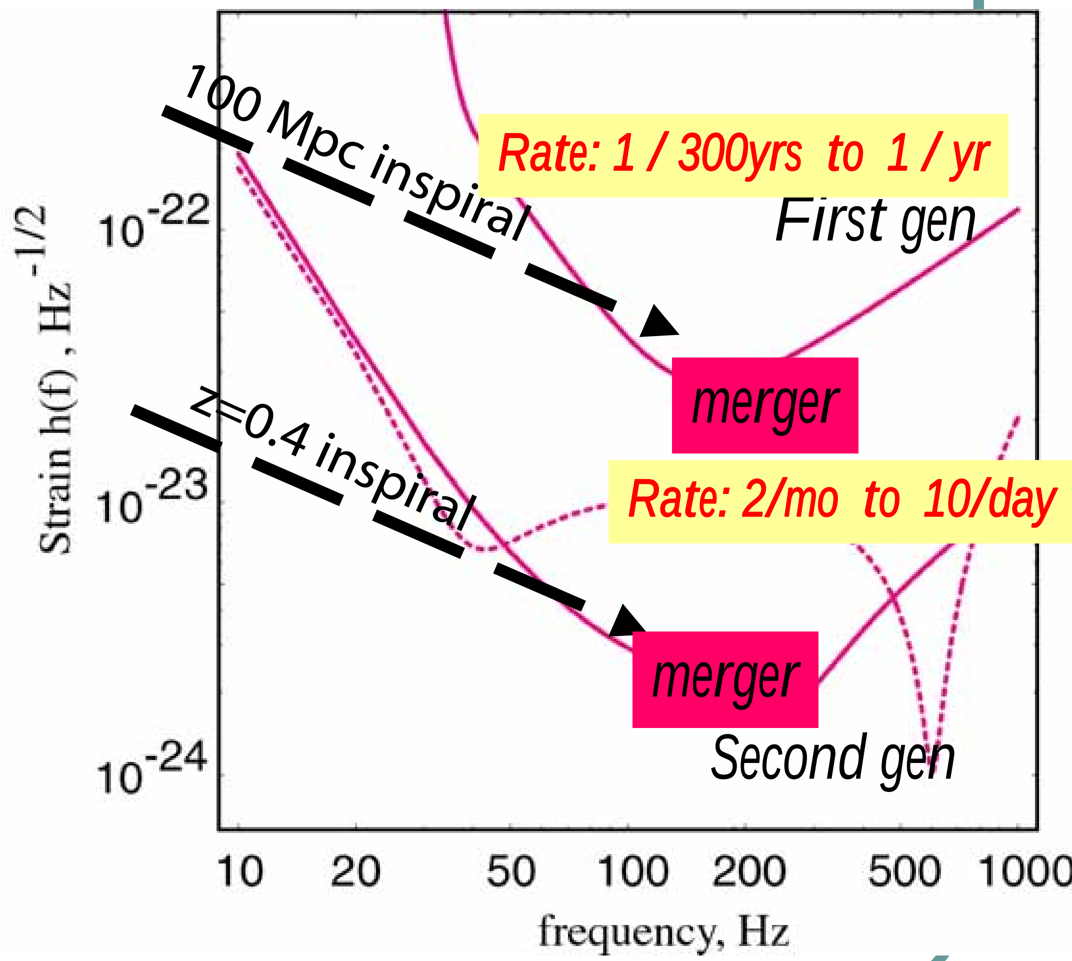
BH/BH coalescence

- The inspiral, merger, and ringdown waves from **$50M_\odot$ BH binaries** as observed by initial and advanced LIGO.
- The energy spectra are coming from crude estimates (10% of the total mass energy is radiated in merger waves and 3% in ringdown waves).
- We observe that the inspiral phase is not visible with initial LIGO, for this case Numerical Relativity is important.



Possible First Source: Binary Black Hole Coalescence

- $10 M_\odot + 10 M_\odot$ BH/BH binary
- Event rates based on population synthesis,
- mostly globular cluster binaries.
- Totally quiet!!

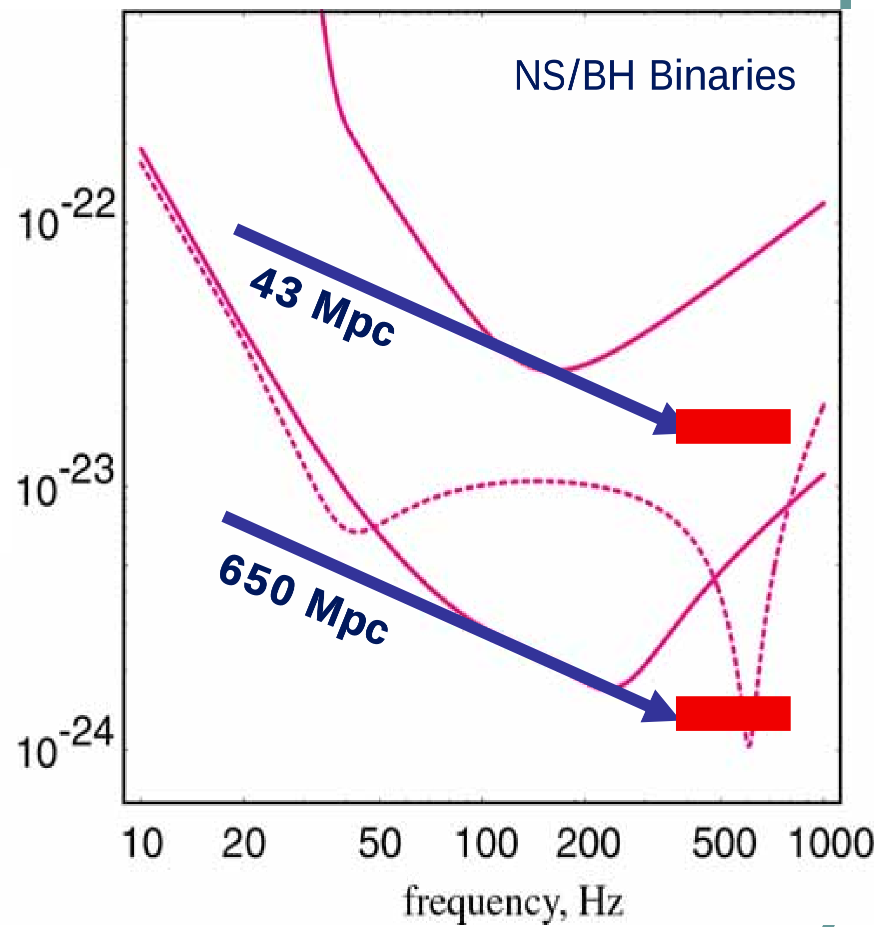


(Kip Thorne)

NS-BH inspiral and NS Tidal Disruption

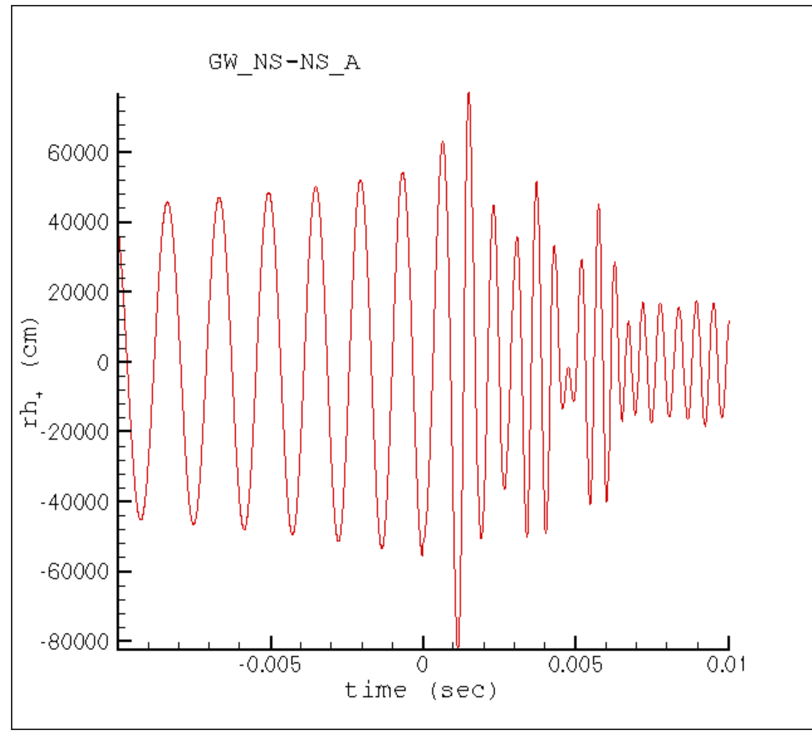
NS-BH Event rates

- Based on *Population Synthesis*
- Initial interferometers
  - Range: 43 Mpc
  - 1/1000 yrs to 1per yr
- Advanced interferometers
  - Range: 650 Mpc
  - 2 per yr to several per day



Merging phase: NS/NS & BH/NS

- **Tidal disruption of a NS by a BH (Vallisneri)**
  - GWs could carry information about the EOS of NS eg. estimation of NS radius (15% error).
  - The disruption waves lie in the band 300-1000Hz
  - A few events per year at 140Mpc (LIGO-II)
- **Merging of NS-NS (Rasio et al)**
  - Imprint of the NS radii just before merging (***f*** $\lesssim$ **1kHz**)
  - During the merging we could get important information about the EOS (***f*** $\gtrsim$ **1kHz**)



Core-collapse Supernova

The most spectacular astronomical event with exciting physics

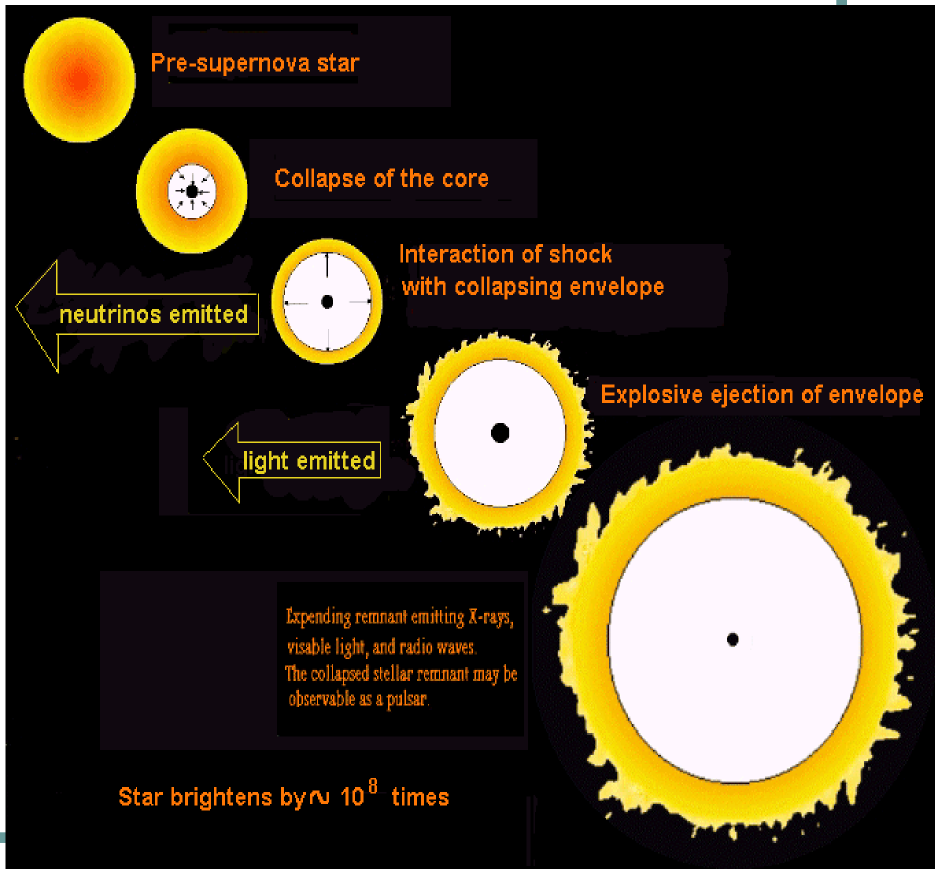
Supernovae/gravitational collapse

Supernova core collapse was the primary source of GW detectors.
GW amplitude uncertain by factors of 1,000's?

Rate 1/30yr in typical galaxy

Detection would provide unique insight into SN physics:
- optical signal hours after collapse
- neutrinos after several seconds
- GWs emitted <u>during</u> collapse

Simulations suggest low level of radiation ($<10^{-6}\ M_{\odot}c^2$?), but
- rotational instabilities possible
- observational evidence for asymmetry from speeding final neutron stars (release of $10^{-6}\ M_{\odot}c^2$ could explain 1000 km/s?)
- convective "boiling" observable to LMC



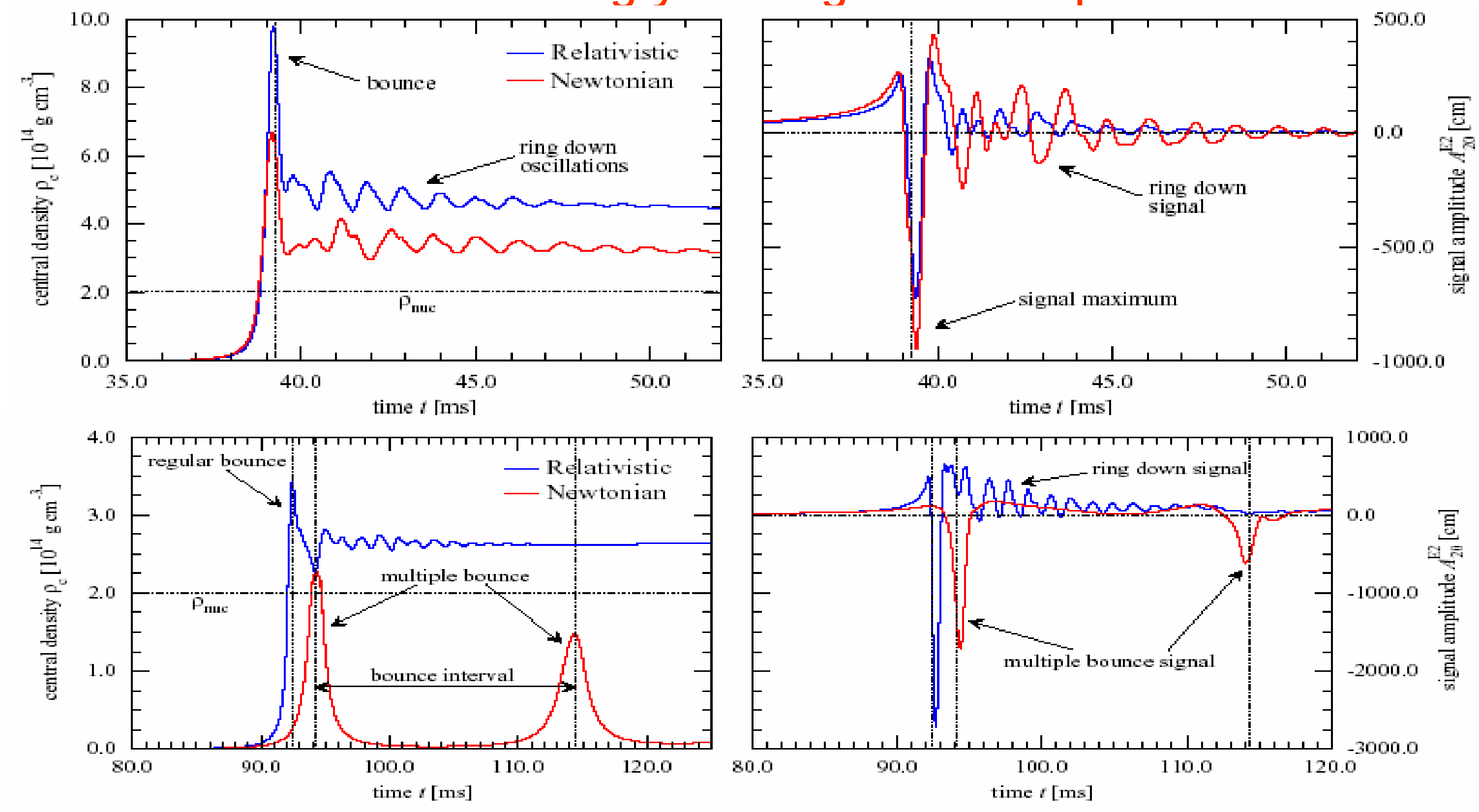
Core-Collapse Supernovae I

- **Stars more massive than ~8$M_\odot$ end in core collapse (~90% are stars with masses ~8-20$M_\odot$).**
- **Most of the material is ejected**
- **If $M>20M_\odot$ more than 10% falls back and pushes the PNS above the maximum NS mass leading to the formation of BHs (type II collapsars).**
- **If $M>40M_\odot$ no supernova is launched and the star collapses to form a BH (type I collapsars)**
- **Formation rate**:
  - 1-2 per century in the Galaxy (**Cappellaro & Turatto**)
  - 5-40% of them produce BHs through the fall back material
  - Limited knowledge of the rotation rate! Initial periods probably <20ms.
  - **Chernoff & Cordes** fit the initial spin with a Gaussian distribution peaked at 7ms. This means that 10% of pulsars are born spinning with millisecond periods.

Core-Collapse Supernovae II

Dimmelmeir, Font & Muller 2002

- Rotation increases strongly during the collapse.



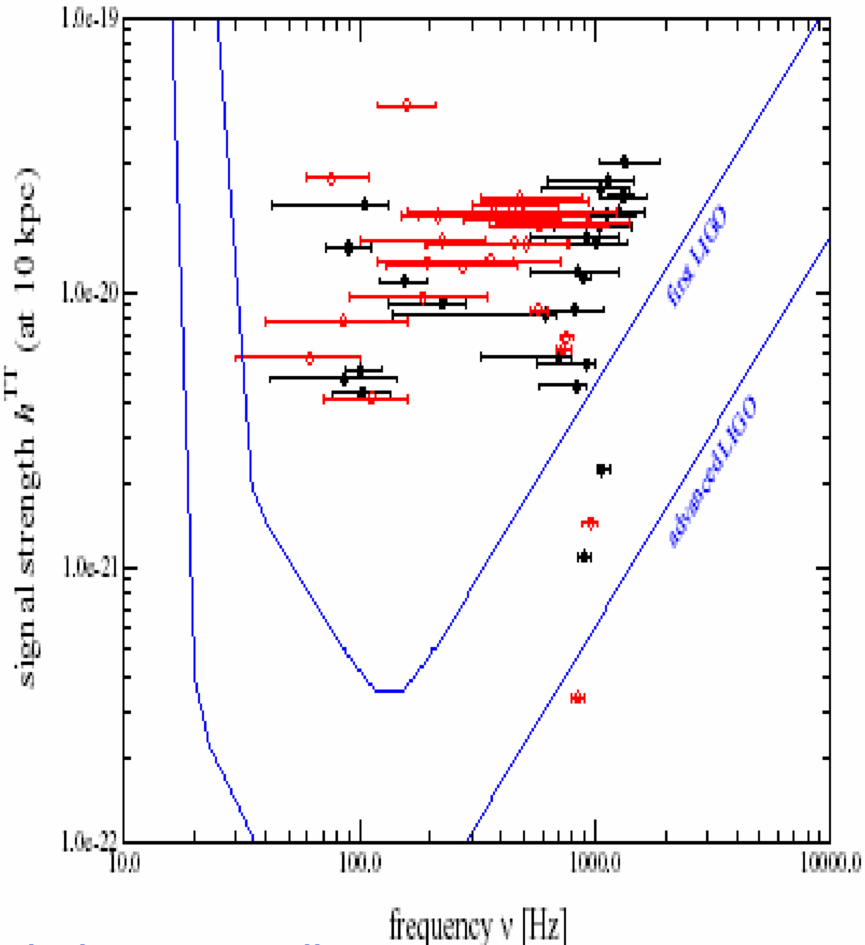
•Multiple bounces are possible for a few models.

Core-Collapse Supernovae III

- GW amplitude

$$h^{TT} \simeq 10^{-23} \frac{10Mpc}{d}$$

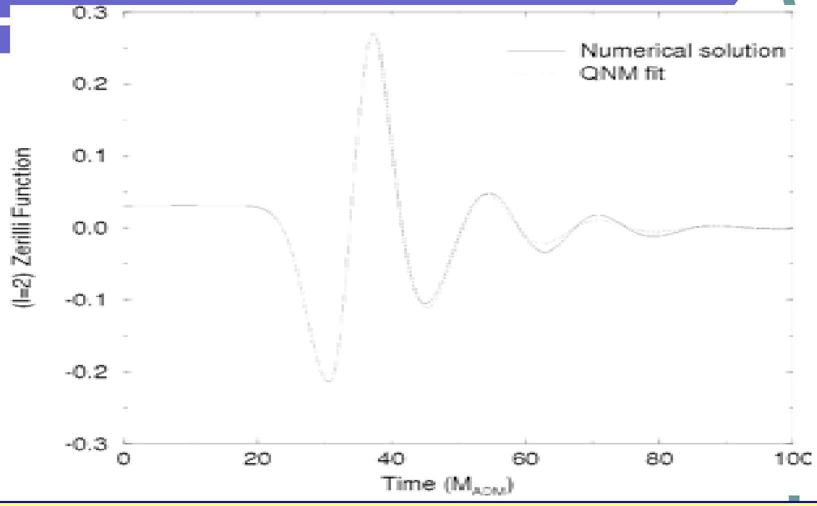
- Signals from Galactic supernova **detectable.**
- Frequencies **~1 kHz**
- The numerical estimates **are not conclusive.** A number of effects *(GR, secular evolution, non-axisymmetric instabilities)* **have been neglected!** (Axisymmetric collapse, Mathews-Wilson approximation…)
- Kicks suggest that a fraction of newly born NSs (and BHs) may be strongly asymmetric.
- Polarization of the light spectra in SN indication of asymmetry.



Dimmelmeir, Font & Muller 2002

Black-Hole Ringing I

- **The newly formed BH is ringing till settles down to the stationary Kerr state (QNMs).**
- **The ringing due to the excitation by the fallback material might last for secs**
- **Typical frequencies: ~1-3kHz**

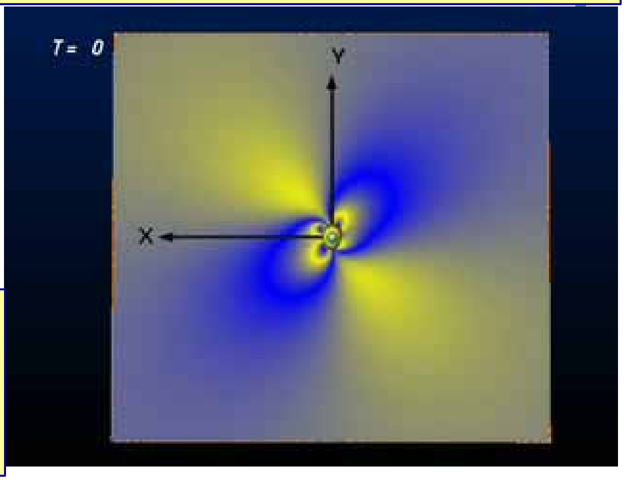


$$f_{m=2} \simeq 3.2\text{kHz}\ M_{10}^{-1}[1-0.63(1-a/M)^{3/10}]$$

$$Q = \pi f \tau \approx 2(1-a)^{-9/20}$$

- **The amplitude of the ringdown waves and their energy depends on the distortion of the BH.**
- **Energy emitted in GWs by the falling material:** $\Delta E > 0.01\mu c^2(\mu/M)$

$$h_c \approx 2\times10^{-21}\left(\frac{\varepsilon}{0.01}\right)\left(\frac{d}{10Mpc}\right)^{-1}\left(\frac{\mu}{M_\odot}\right)$$

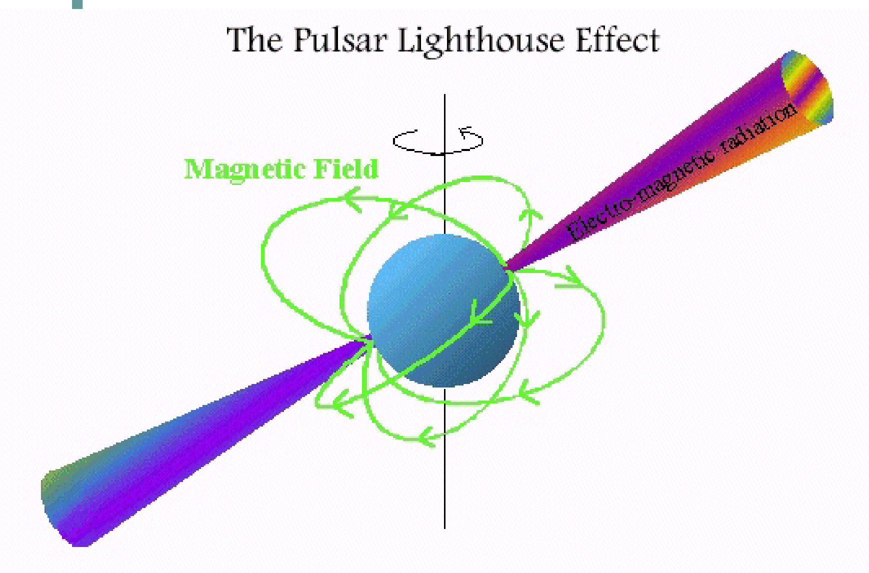


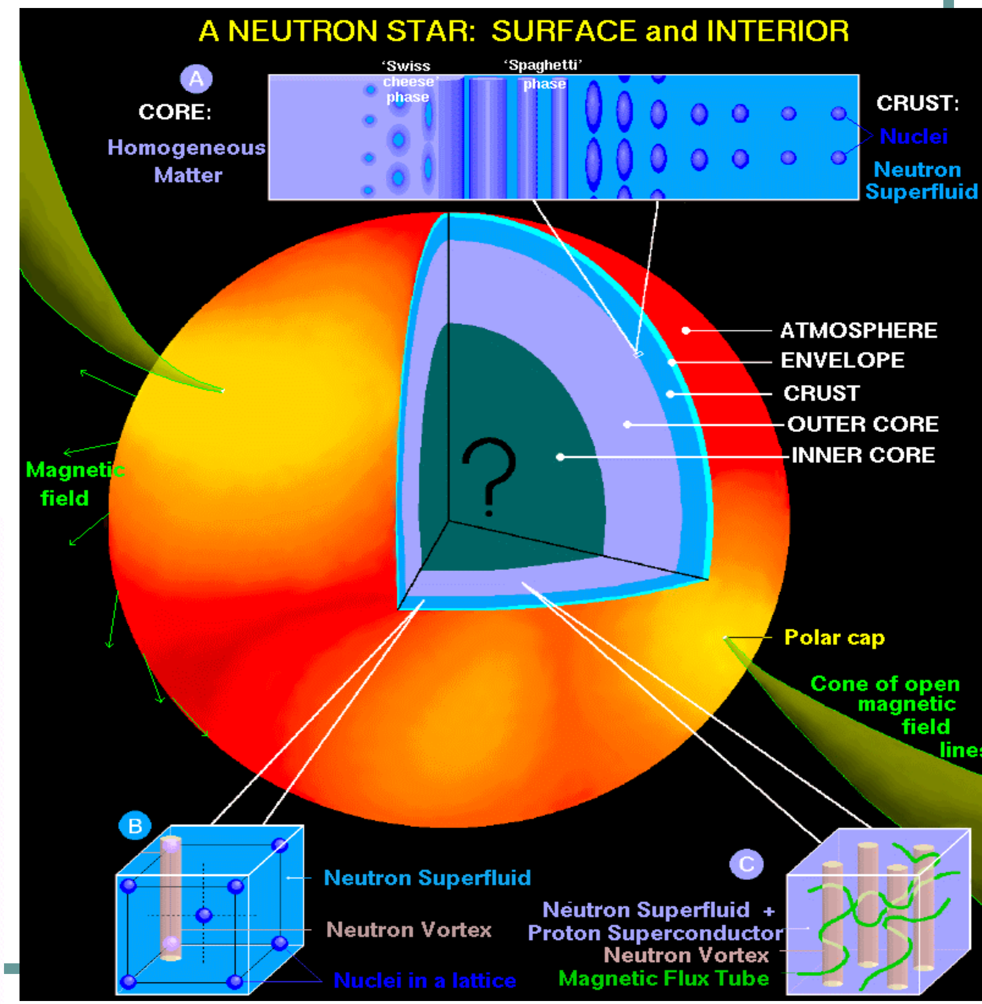
Oscillations & Instabilities

The end product of gravitational collapse

Neutron Stars

- Suggested: 1932
- Discovered: 1967
- Known: 1070+
- Mass: ~ 1.3-1.8 $M_{\odot}$
- Radius: ~ 8-14 Km
- Density: $\sim 10^{15} gr/cm^3$

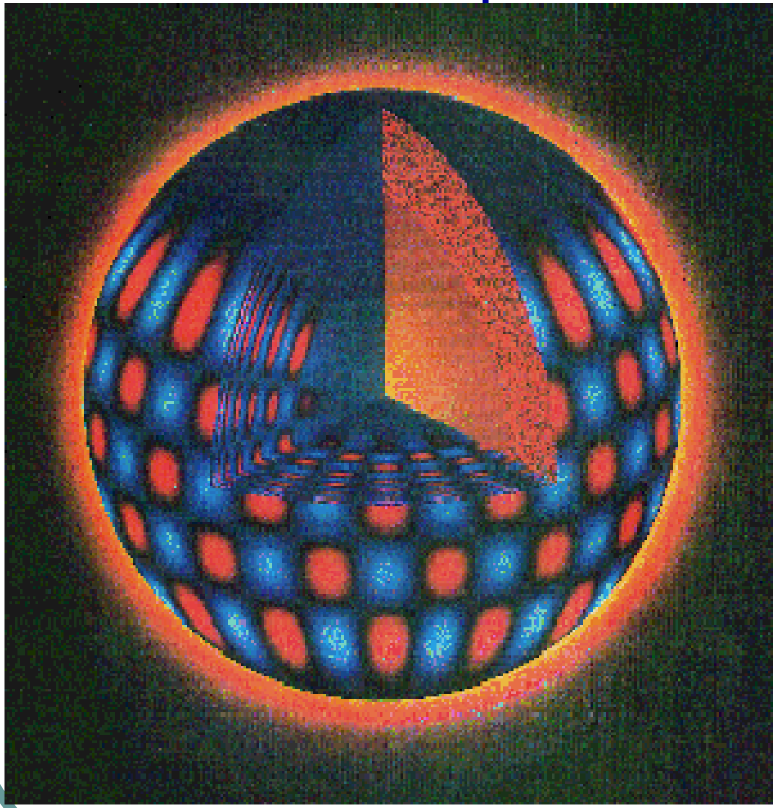




Stellar pulsation primer

For spherical stars we can (in the Cowling approximation) write the Euler equations as

$$\frac{\partial^2 \xi^i}{\partial t^2} = -\nabla^i \left(\frac{\delta p}{\rho} \right) + \frac{p\Gamma_1}{\rho} A^i (\nabla_j \xi^j)$$



Two main restoring forces, the pressure and the buoyancy associated with internal composition /temperature gradients, lead to:

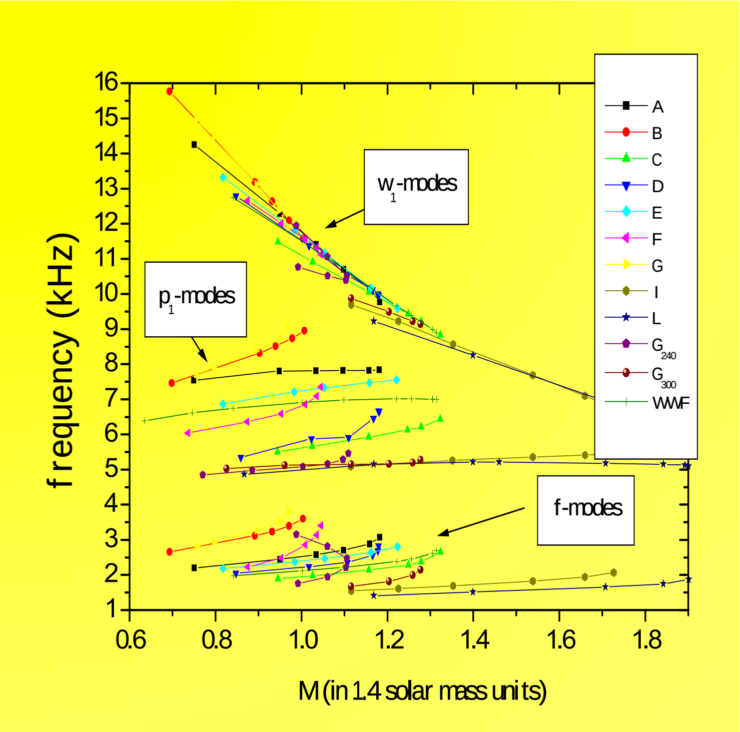
$$(\delta p \propto Y_{lm}(\theta,\varphi))$$

$$\omega^2 \approx \frac{l(l+1)c_s^2}{r^2}$$ p-modes

$$\omega^2 \approx -gA = \frac{A_i \nabla^i p}{\rho}$$ g-modes

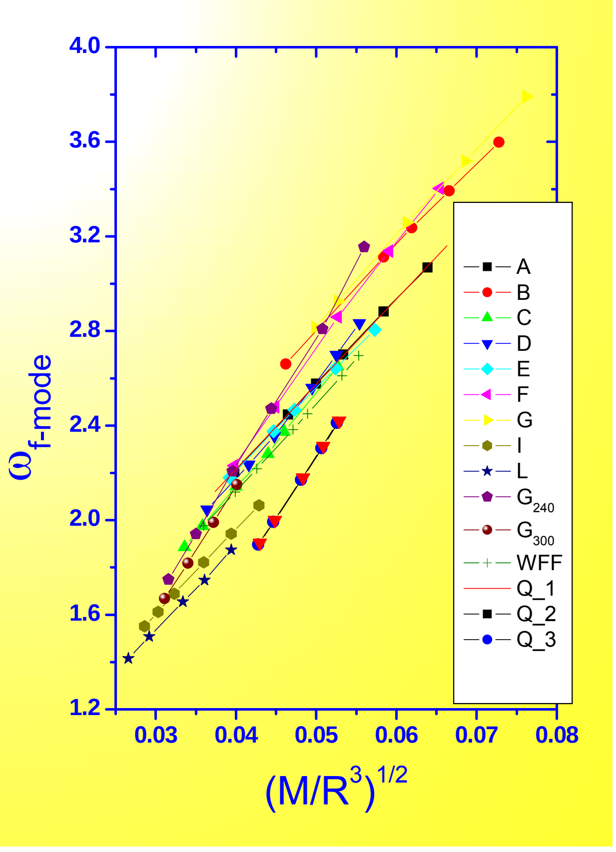
NS ringing : Stellar Modes

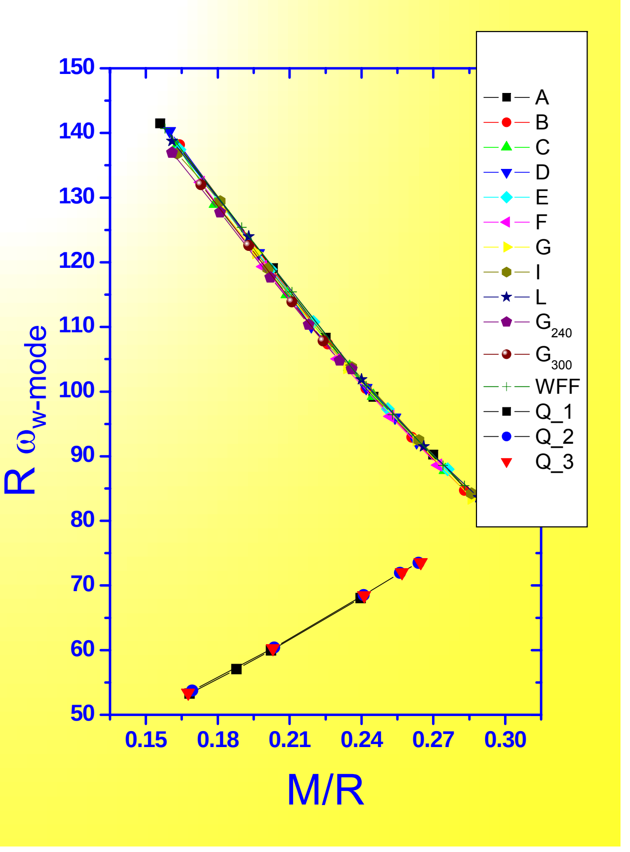
- **P-modes: main restoring force is the pressure**
- G-modes: main restoring force is the buoyancy force
- **F-mode: has an inter-mediate character of p- and g-mode**
- **W-modes: pure space-time modes (only in GR) (KK & Schutz)**
- **Inertial modes (r-modes) :main restoring force is the Coriolis force**
- Superfluid modes: Deviation from chemical equilibrium provides the main restoring agent



Each type of mode is sensitive to the physical conditions where the amplitude of the mode is greatest.

Grav. Wave Asteroseismology

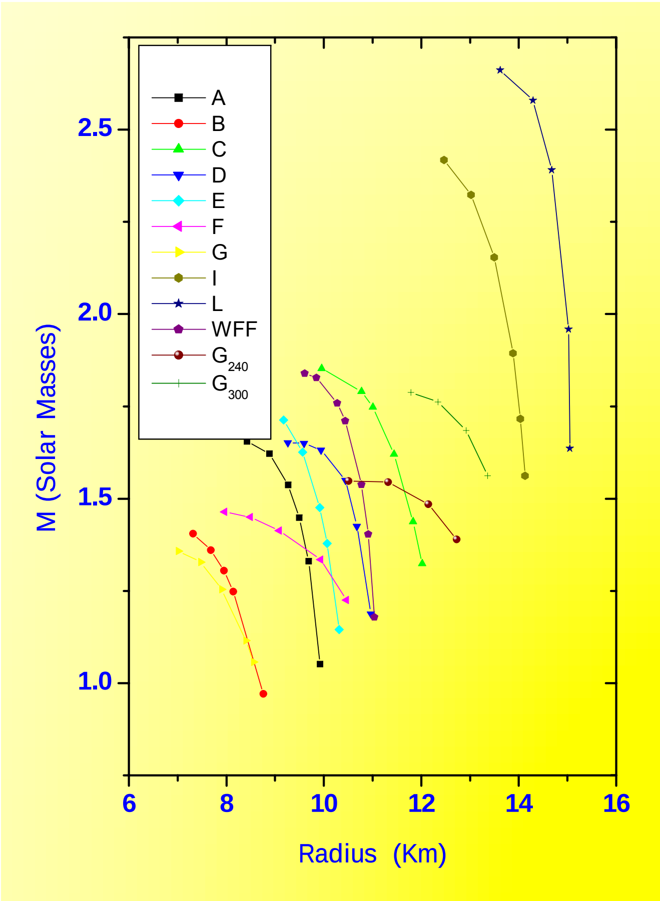


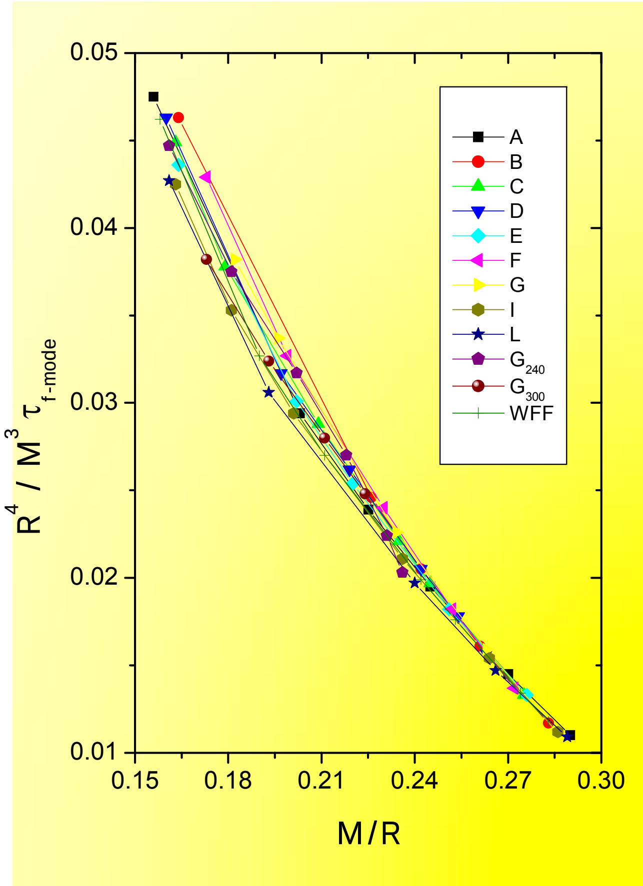


$$\omega_f(kHz) \approx 0.78 + 1.637\left(\frac{M_{1.4}}{R_{10}{}^3}\right)^{1/2}$$

$$\omega_w(kHz) \approx \frac{1}{R_{10}}\left[20.92 - 9.14\frac{M_{1.4}}{R_{10}}\right]$$

Grav. Wave Asteroseismology





***Unique estimation of* Mass *and* Radius *and* EoS**

$$\frac{1}{\tau_f}(\text{sec}) \approx \frac{M_{1.4}^3}{R_{10}^4}\left[22.85 - 14.65\frac{M_{1.4}}{R_{10}}\right]$$

Stability of Rotating Stars

Non-Axisymmetric Perturbations

A general criterion is: $\beta = \frac{T}{W}$

T : rot. kinetic energy
W : grav. binding energy

Dynamical Instabilities

- Driven by hydrodynamical forces (bar-mode instability)
- Develop at a time scale of about one rotation period

$$\beta \geq 0.27$$

Secular Instabilities

- **Driven by dissipative forces (*viscosity, gravitational radiation*)**
- **Develop at a time scale of several rotation periods.**
- **Viscosity driven instability causes a Maclaurin spheroid to evolve into a non-axisymmetric Jacobi ellipsoid.**
- **Gravitational radiation driven instability causes a Maclaurin spheroid to evolve into a stationary but non-axisymmetric Dedekind ellipsoid.**

$$\beta \geq 0.14$$

The bar-mode instability I

For rapidly (differentially!) rotating stars with:

$$\beta = \frac{T}{|W|} > \beta_{dyn} \approx 0.27$$

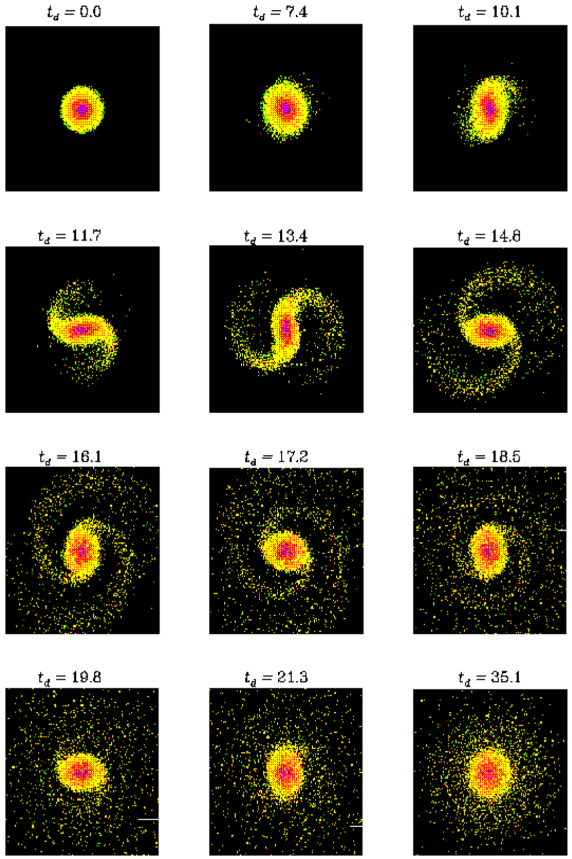
the "bar-mode" grows on a dynamical timescale.

$$h \approx 9\times10^{-23}\left(\frac{\varepsilon}{0.2}\right)\left(\frac{f}{3\text{ kHz}}\right)^2\left(\frac{15\text{ Mpc}}{\text{d}}\right)M_{1.4}R_{10}^2$$

If the bar persists for many (~10-100) rotation periods, the signal will be easily detectable from at least Virgo cluster.

–A considerable number of events per year in Virgo: ≤10^{-2} /yr/Galaxy

–**Frequencies ~1.5-3.5kHz**



Remember mini-Grail: f_0~3.2kHz

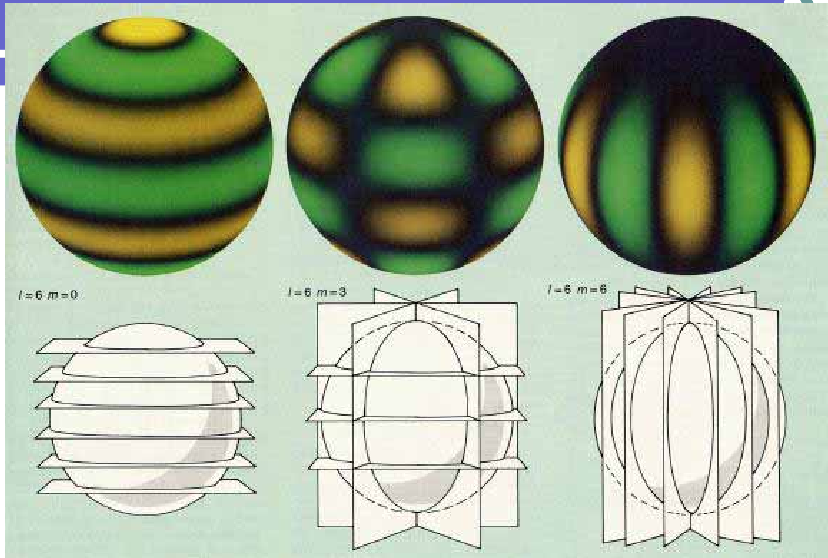
The pattern speed

- The pattern speed σ of a mode is:

$$\frac{d\varphi}{dt} = -\frac{\omega}{m} = \sigma$$

$$\omega_{\text{inert}} = \omega_{\text{rot}} + m\Omega$$
$$\sigma_{\text{inert}} = \sigma_{\text{rot}} + \Omega$$

- If a star rotates very fast, a backward moving mode, might change to move forward, *according to an inertial observer.*

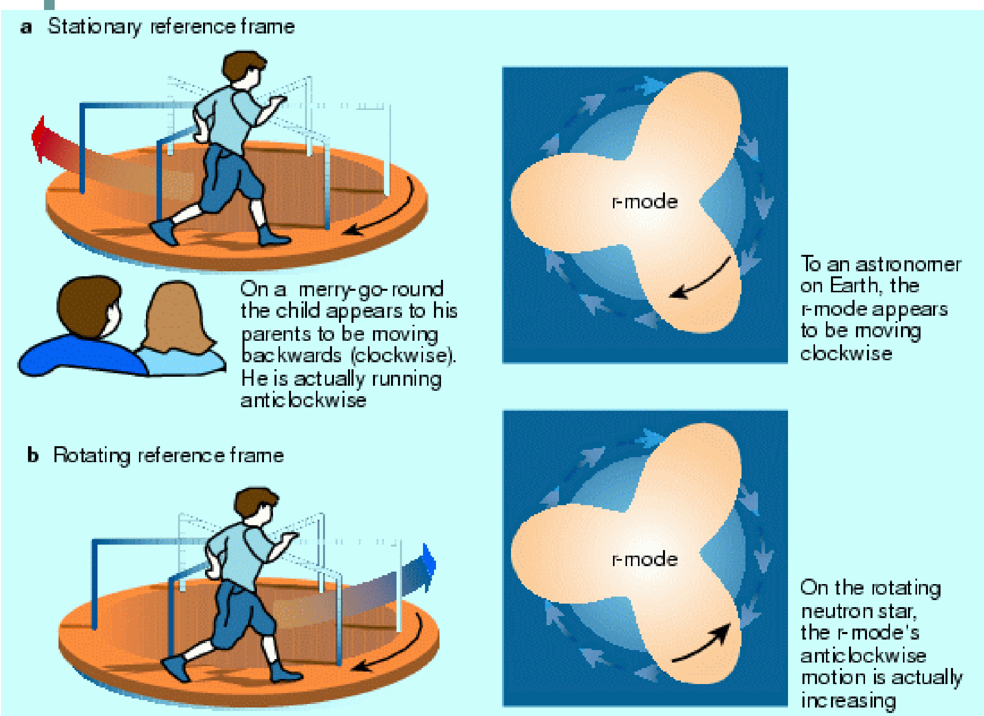


The CFS instability

Chandrasekhar 1969: Gravitational waves lead to a secular instability

Friedman & Schutz 1978: The instability is generic, modes with sufficiently large *m* are unstable.

A **neutral mode** of oscillation signals the onset of CFS instability.



- Radiation drives a mode unstable if the mode pattern moves backwards according to an observer on the star ($J_{rot}<0$), but forwards according to someone far away ($J_{rot}>0$).
- They radiate positive angular momentum, thus in the rotating frame the angular momentum of the mode increases leading to an increase in mode's amplitude.

$$\frac{\omega_{\text{in}}}{m} = -\frac{\omega_{\text{rot}}}{m} + \Omega$$

F-mode-(I)

- F-mode is the fundamental pressure mode of the star
- It corresponds to polar perturbations
- Frequency for uniform density stars

$$\omega^2 = \frac{2l(l-1)}{2l+1}\frac{GM}{R^3}$$

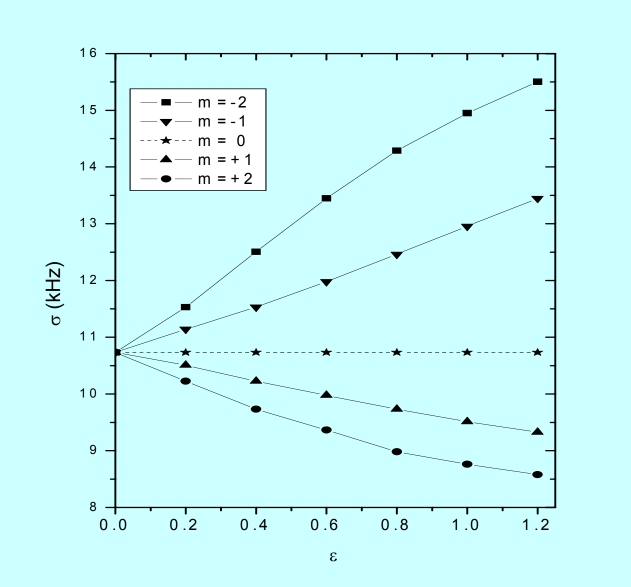
growth time(if unstable)

$$t_{GW} \approx f(l)R\left(\frac{R}{M}\right)^{l+1} \sim 0.07\left(\frac{1.4M_\odot}{M}\right)^3\left(\frac{R}{10km}\right)^4 \text{ sec}$$

- For ℓ=2 is ~2-4kHz

- Rotation breaks the symmetry: the various -ℓ≤m≤ℓ decouple
- There is coupling between the polar and axial modes
- The frequency shifts:

$$\omega_{\text{inert}}(\Omega) = \omega(\Omega=0) + \kappa m\Omega$$



The r-mode-(I)

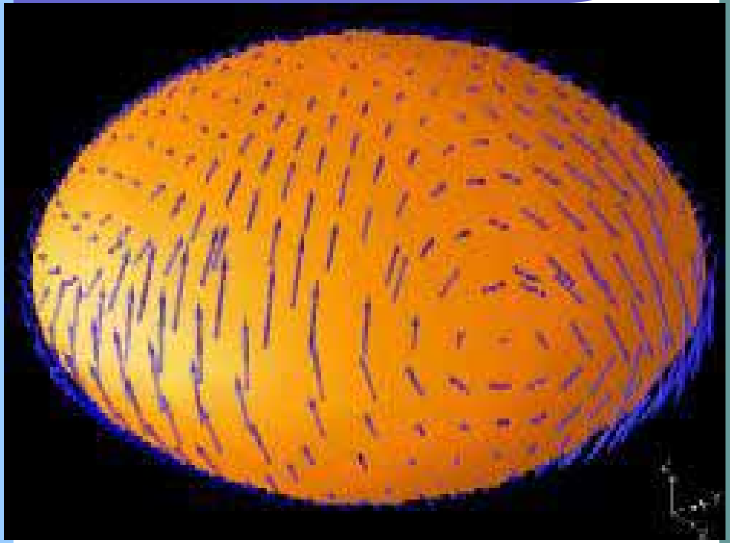
- A non-rotating star has only trivial axial modes
- Rotation provides a restoring force (Coriolis) and leads in the appearance of the inertial modes.
- The *l=m=2* inertial mode is called r-mode
- In a frame rotating with the star, the r-modes have frequency

$$\omega_{\text{rot}} = \frac{2m}{l(l+1)}\Omega$$

- Meanwhile in the inertial frame

$$\frac{\omega_{\text{inertial}}}{m} = -\frac{\omega_{\text{rot}}}{m} + \Omega = \Omega\left(1 - \frac{2}{l(l+1)}\right)$$

- r-modes are appear retrograde in the rotating system while in the inertial frame the prograde at all rotation rates!



R-modes have:

$$\delta u^{\varphi} \sim \Omega$$

$$\delta u^{r}, \delta u^{\theta}, \delta\rho \sim \Omega^{2}$$

Growth vs Damping

- Viscosity tends to suppress a GW instability.
- An instability is only relevant if it grows sufficiently fast that is not completely damped by viscosity
- **Bulk viscosity**: arises because the pressure and density variations associated with the mode oscillation drive the fluid away from beta equilibrium. It corresponds to an estimate of the extent to which energy is dissipated (via neutrino emission) from the fluid motion as the weak interaction tries to re-establish equilibrium.
- **Shear viscosity**: in matter hotter than superfluid transition temperature T~10^9 K, due to neutron-neutron scattering, and for superfluids, due to electron-electron scattering

$$\frac{1}{2E}\frac{dE}{dt} = -\frac{1}{\tau_{GW}} + \frac{1}{\tau_{BV}} + \frac{1}{\tau_{SV}}$$

$$E = \frac{1}{2}\int \rho \left|\dot{\xi}\right|^2 dV$$

Timescales

- Dissipation due to bulk viscosity

$$\left(\frac{dE}{dt}\right)_{\mathrm{BV}} = \int \zeta \left|\delta\sigma\right|^2, \quad \delta\sigma = -i(\omega + m\Omega)\frac{\Delta p}{\Gamma p}, \quad \zeta \sim \left(\frac{T}{10^9 K}\right)^6$$

- Dissipation due to shear viscosity

$$\left(\frac{dE}{dt}\right)_{\mathrm{SV}} = -2\int \eta\,\delta\sigma^{ab}\delta\sigma^*_{ab}\,dV$$

$$\delta\sigma = -i\frac{(\omega + m\Omega)}{2}\left(\nabla_a\xi_b + \nabla_b\xi_a - 2g_{ab}\nabla_c\xi^c\right), \quad \eta \sim \left(\frac{T}{10^9 K}\right)^{-2}$$

- Dissipation/growth due to gravitational radiation

$$\left(\frac{dE}{dt}\right)_{\mathrm{GW}} = -(\omega + m\Omega)\sum_{l=2}^{\infty} N_l\,\omega^{2l+1}\left(\left|\delta D_{lm}\right|^2 + \left|\delta J_{lm}\right|^2\right)$$

$$\delta D_{lm} = \int \delta\rho\, r^l Y^*_{lm}\,dV, \quad \delta J_{lm} = 2\left(\frac{l}{l+1}\right)^{1/2}\int r^l\left(\rho\delta\upsilon + \upsilon\delta\rho\right)\bar{Y}^{*B}_{lm}\,dV$$

R-mode: Instability window

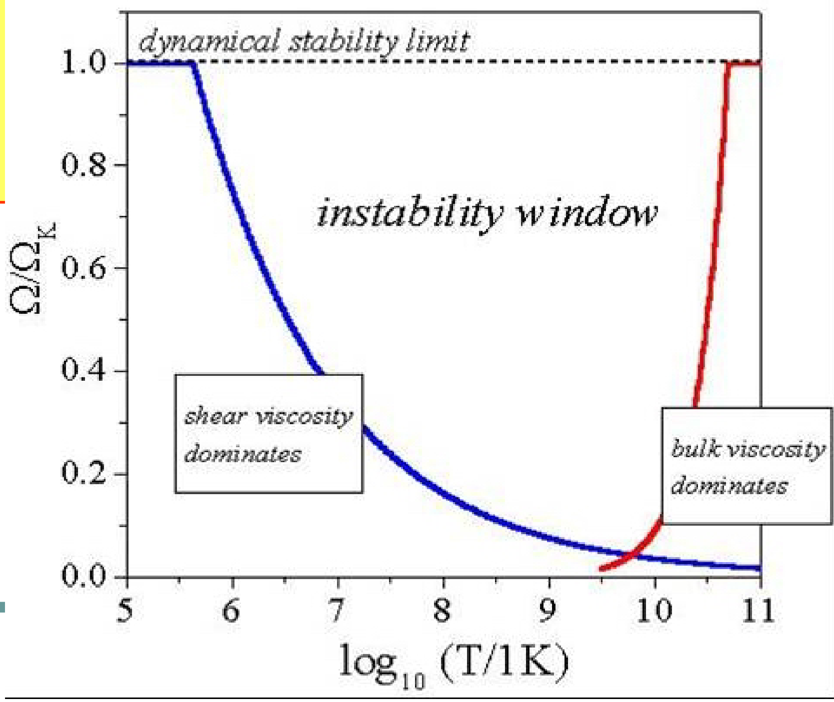
- For the r-mode (ℓ=2) we get:

$$\tau_{BV} \approx 2.4\times10^{10}\left(\frac{1.4M_\odot}{M}\right)\left(\frac{R}{10\mathrm{km}}\right)^5\left(\frac{10^9K}{T}\right)^6\left(\frac{P}{1\mathrm{ms}}\right)^2 \mathrm{sec}$$

$$\tau_{SV} \approx 1.2\times10^{8}\left(\frac{1.4M_\odot}{M}\right)^{5/4}\left(\frac{R}{10\mathrm{km}}\right)^{23/4}\left(\frac{T}{10^9K}\right)^2 \mathrm{sec}$$

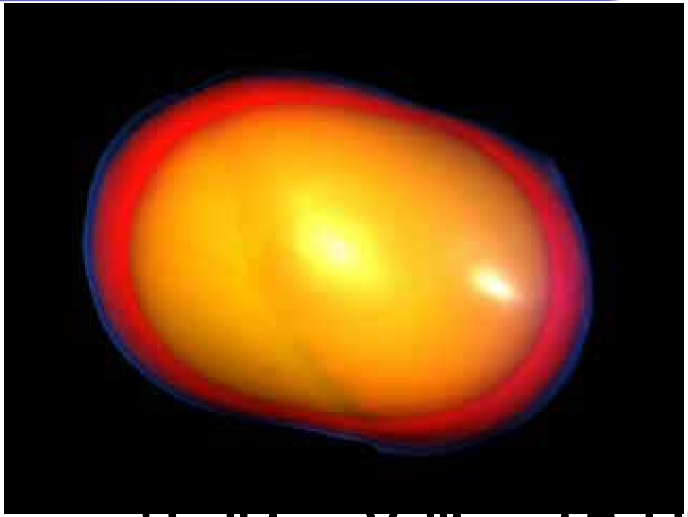
$$\tau_{GW} \approx -22\left(\frac{1.4M_\odot}{M}\right)\left(\frac{R}{10\mathrm{km}}\right)^{-4}\left(\frac{P}{1\mathrm{ms}}\right)^6 \mathrm{sec}$$

- Instability window
- Many astrophysical applications both on newly born and old NS



R-modes (astrophysics)

- **GW amplitude depends on α (the saturation amplitude).**
- **Mode coupling might not allow the growth of instability to high amplitudes (Schenk etal)**
- **The existense of *crust*, hyperons in the core, magnetic fields, affect the efficiency of the instability.**
- **For newly born neutron stars might be quite weak ; unless we have the creation of a strange star**
- **Old accreting neutron (or strange) stars, probably the best source! (400-600Hz)**



Lindblom-Vallisneri-Tohline

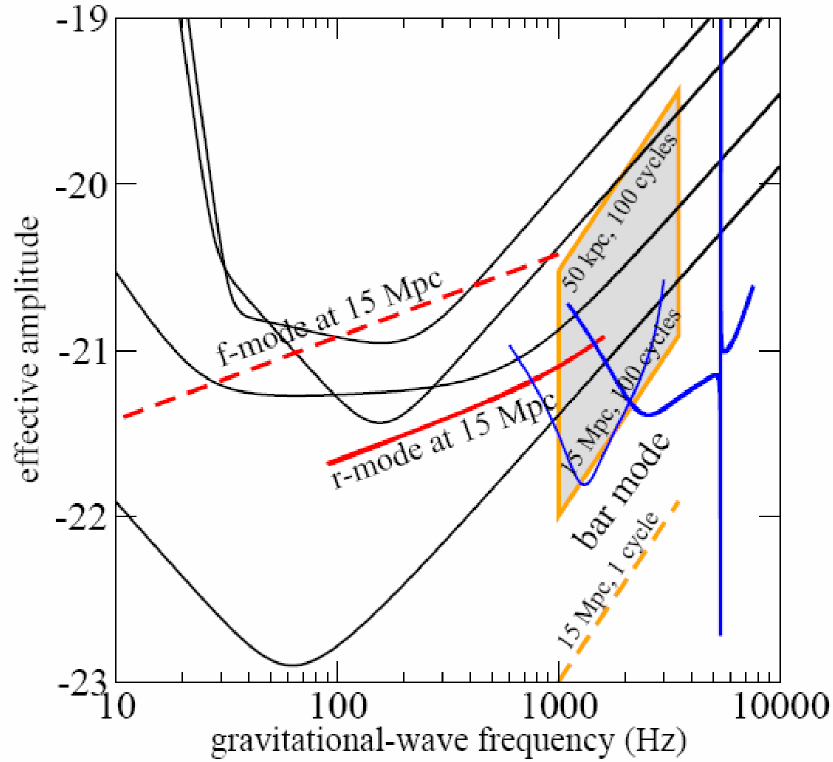
$$h(t) \approx 10^{-22} \alpha \left(\frac{\Omega}{1 \text{ kHz}} \right) \left(\frac{1\text{Mpc}}{d} \right)$$

$$\alpha \simeq 10^{-2} - 10^{-3}$$

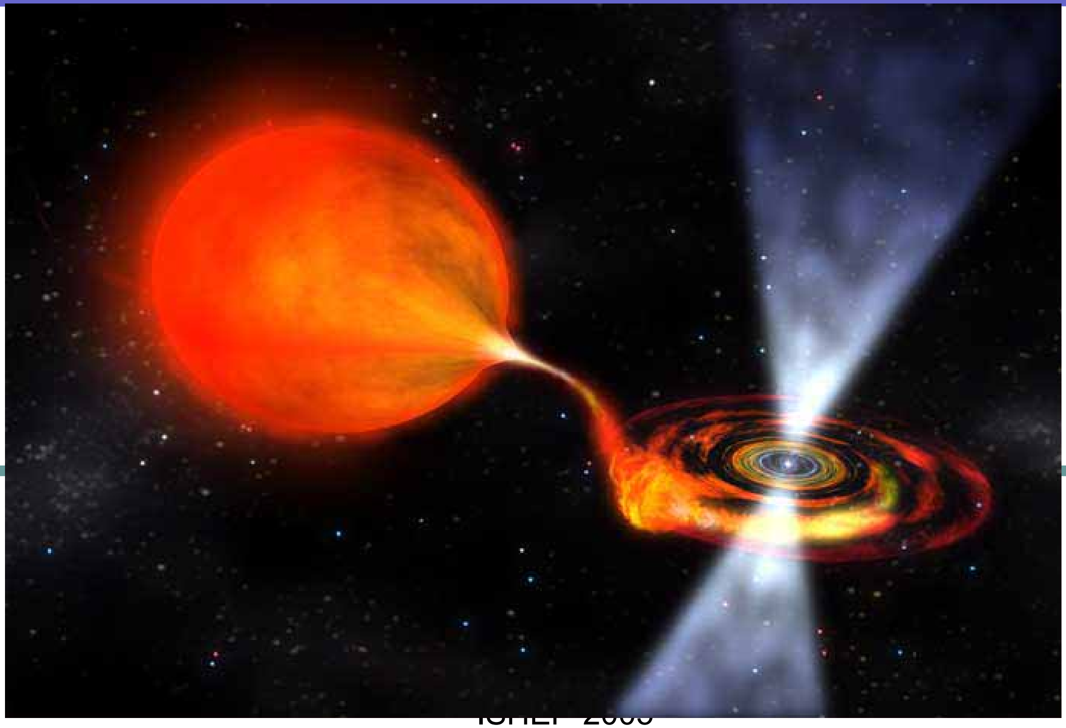
F-mode (astrophysics)

- F-mode is naturally excited in any process.
- In GR the *m=2* mode becomes unstable for $\Omega > 0.85\Omega_{Kepler}$ or $\beta > 0.06\text{-}0.08$
- The instability window significantly smaller than the r-mode
- Detectable from as far as 15Mpc (LIGO-I), 100Mpc (LIGO-II) (*depending on the saturation amplitude*).
- Differential rotation affects the onset of the instability
- Recent non-linear calculations by Shibata & Karino (2004) suggest that:
  - Up to 10% of energy and angular momentum will be dissipated by GWs.
  - Amplitude (ar ~500Hz):

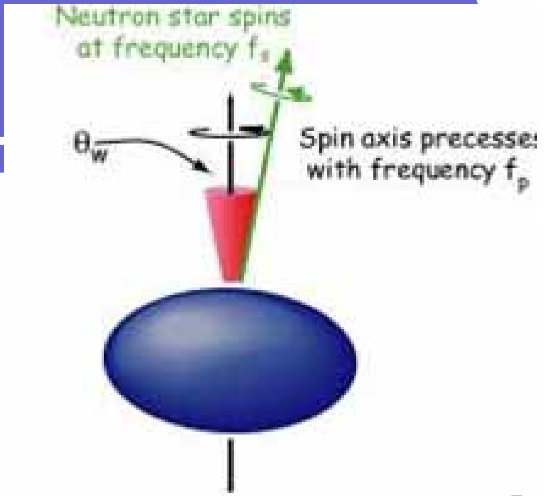
$$h_{\text{eff}} \sim 5\times10^{-22}\left(\frac{R_e}{20km}\right)^{1/4}\left(\frac{M}{1.4M_\odot}\right)^{3/4}\left(\frac{100\text{Mpc}}{r}\right)$$



Isolated & Old NS



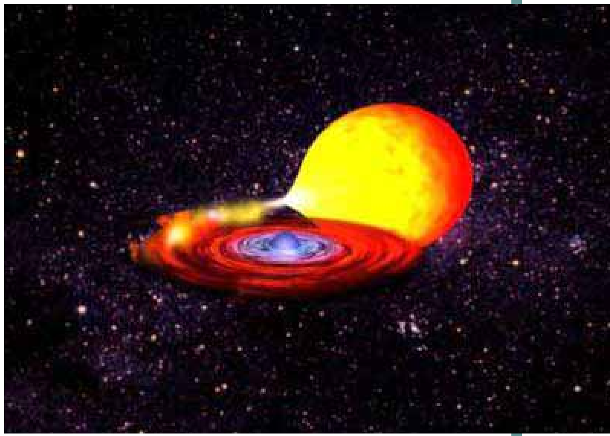
Isolated NS



- Wobbling or Deformed NS (many interesting features but highly uncertain the degree of deformation)

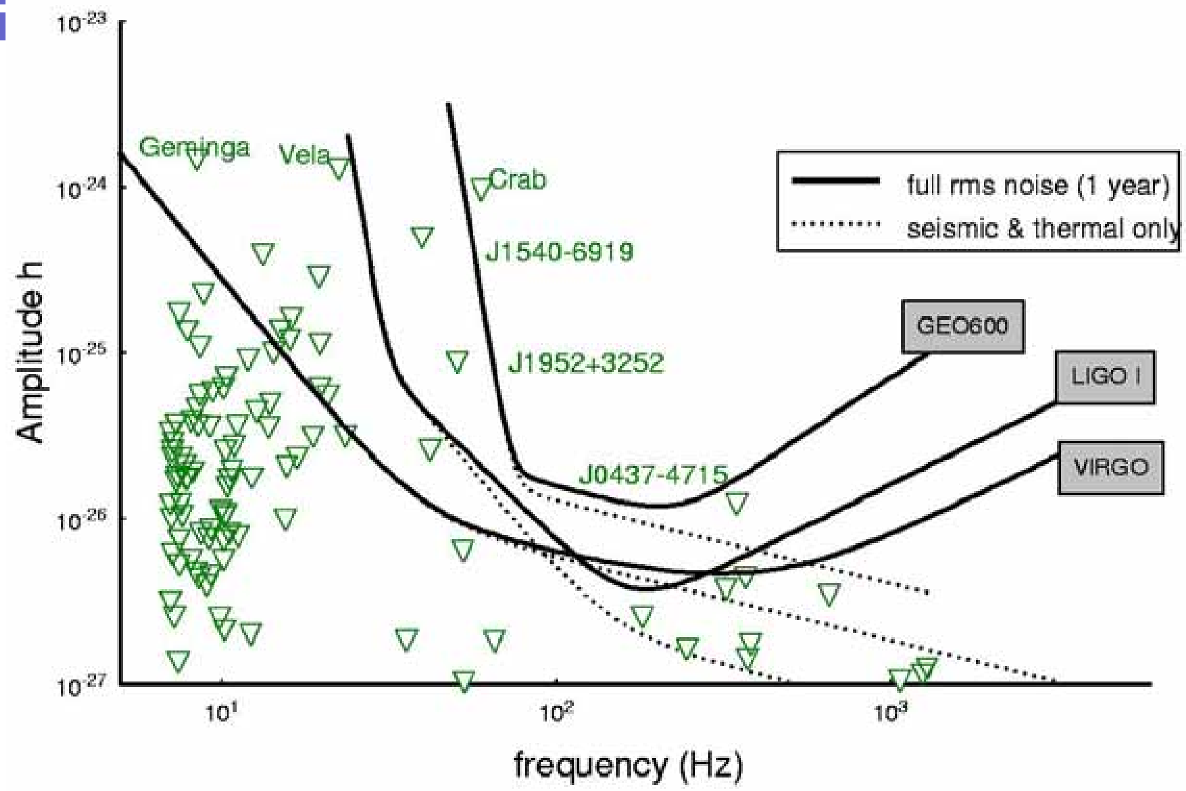
$$\varepsilon \geq 2\times10^{-8}\left(\frac{1kHz}{f}\right)^2\left(\frac{r}{10kpc}\right)$$

- LMXBs : if accretion spin-up torque on NS is counterbalanced by GW emission then Sco X-1 and a few more might be detectable around 500-700 Hz.



LMXBs might be as robust source of GWs as the binary systems!

Slowdown from pulsar



- Upper limits on amplitudes from known pulsars, set by assuming spindown due to the emission of gw energy. The points represent all pulsars with gravitational wave frequencies above 7 Hz and amplitudes above 10^{-27}.
- Expected sensitivities of three first-generation interferometers in a one-year observation, and the thermal noise limits on narrow-banding (dotted lines).

The Wagoner mechanism (1984)
Papaloizou &Pringle (1978)

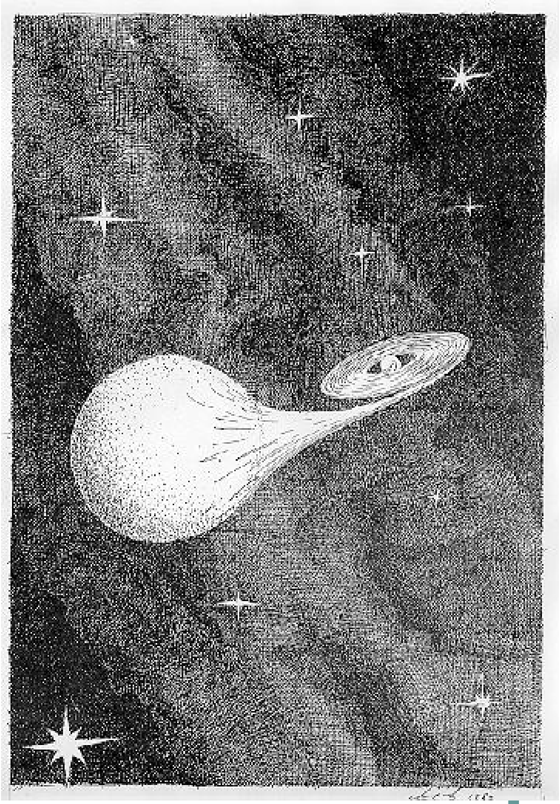
Key idea: Emission of GW balances accretion torque. Strength of waves can be inferred from X-ray flux.

Requires deformation:

$$\varepsilon = 4.5\times10^{-8}\left(\frac{\dot{M}}{10^{-9}M_{\odot}/yr}\right)^{1/2}\left(\frac{300\ \mathrm{Hz}}{\nu_s}\right)^{5/2}$$

Observational evidence (?):

clustering of spin-frequencies in LMXB (250-590 Hz)



Possible GW mechanisms:

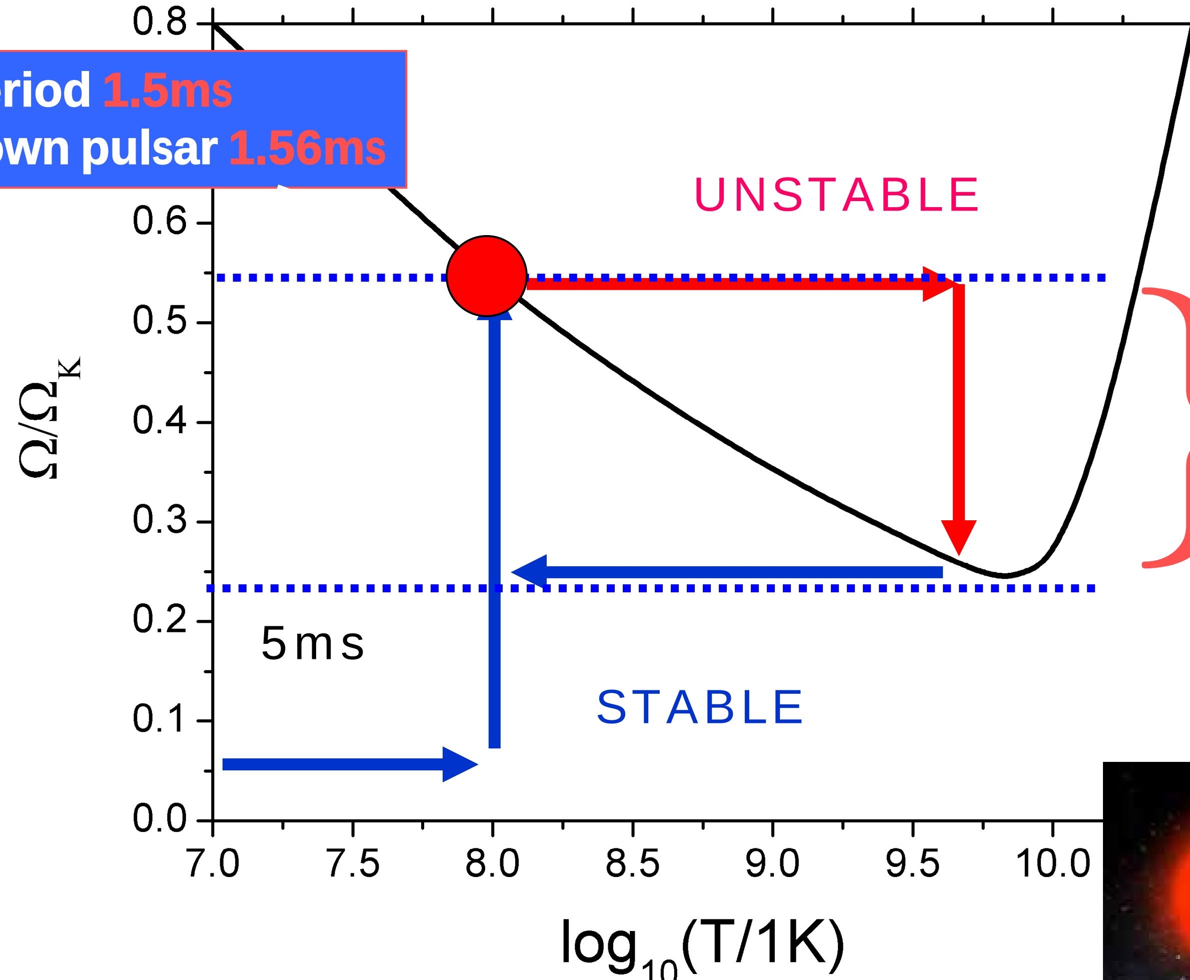
– accretion induced asymmetry

– unstable r-modes: strong bulk viscosity may shift instability window to lower temperatures; accreting stars can reach quasi-equilibrium state

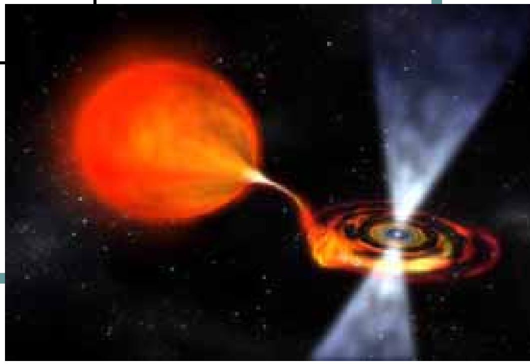
Variable accretion rate: coherent integration of signal only meaningful for 20 days or so.

LMXBs & r-modes

Limiting Period 1.5ms
Fastest known pulsar 1.56ms



Period clustering of ms pulsars



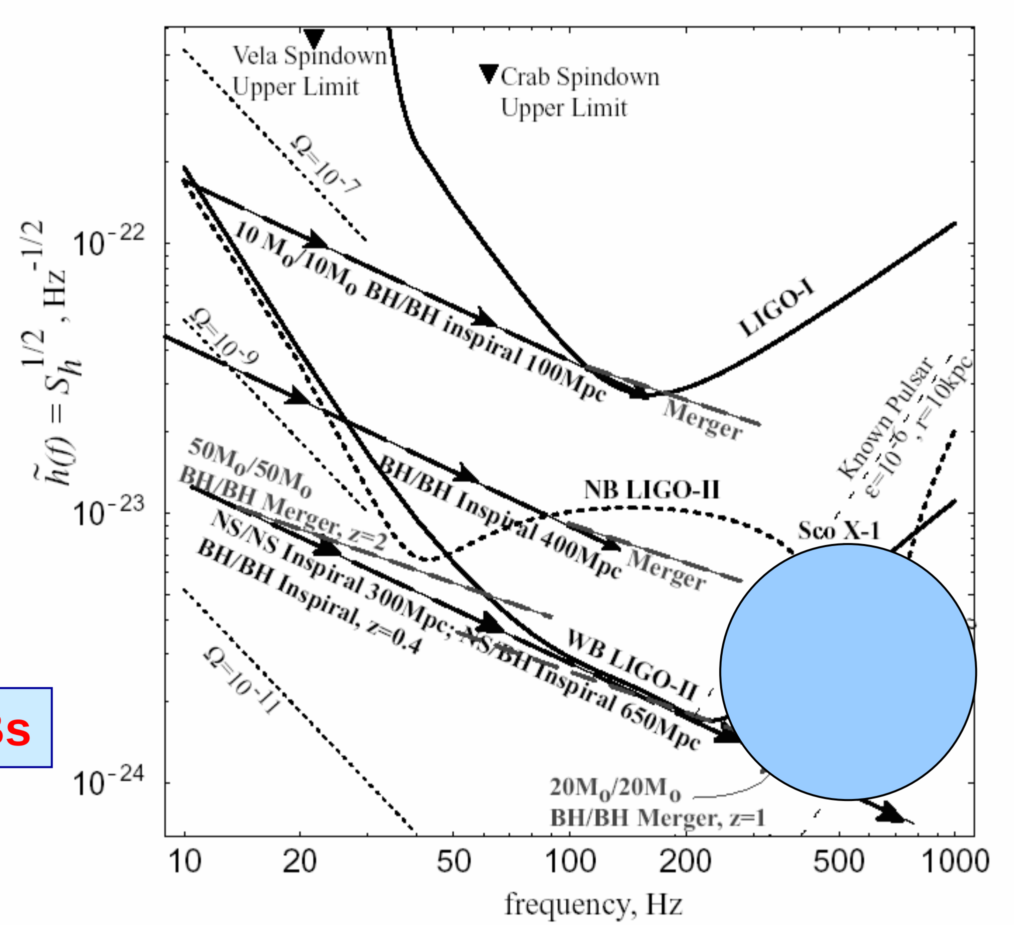
Andersson, KK, Stergioulas `99
Andersson, Jones, KK, Stergioulas `00

LIGO narrow banding

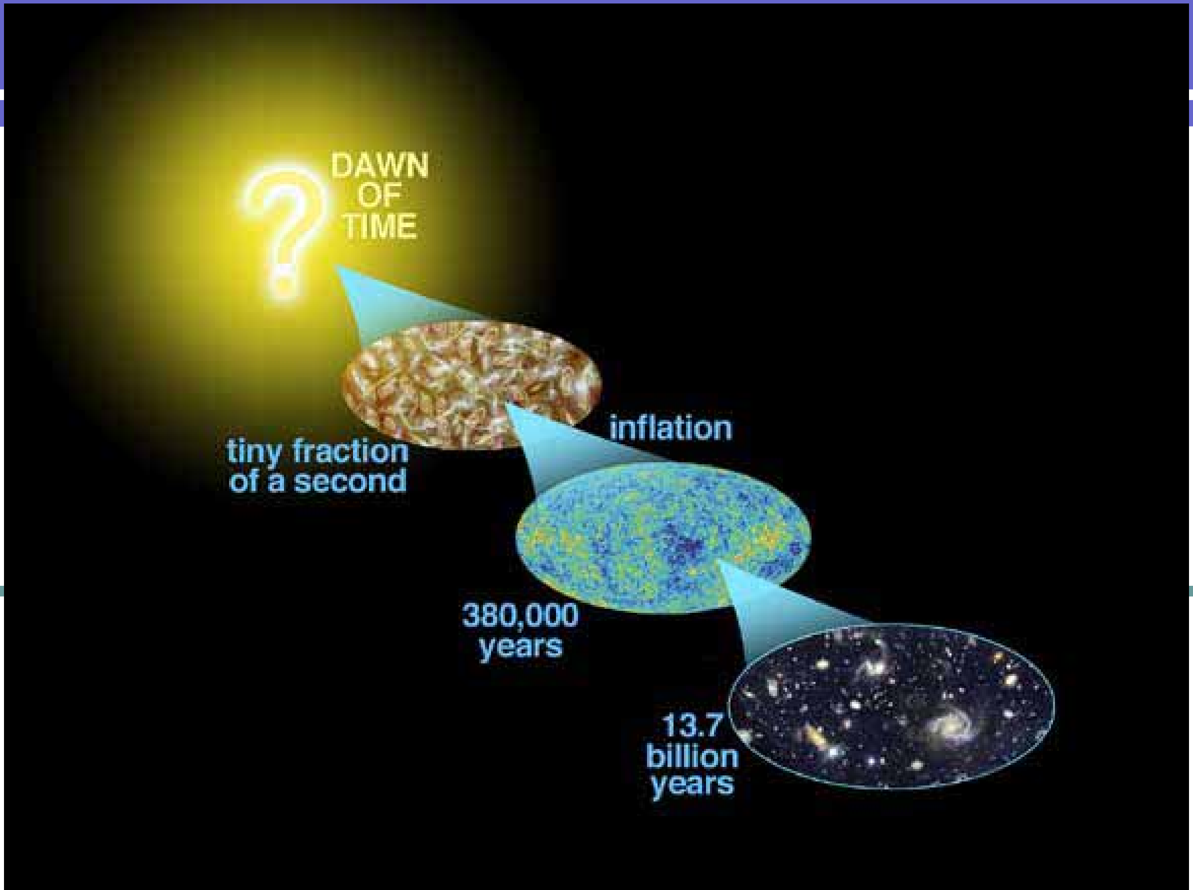
- **LIGO-I phase**
  - The only detectable source is BBHs (10M⊙)

- **LIGO-II phase (2006)**
  - Many sources…

Narrow banding for LMXBs



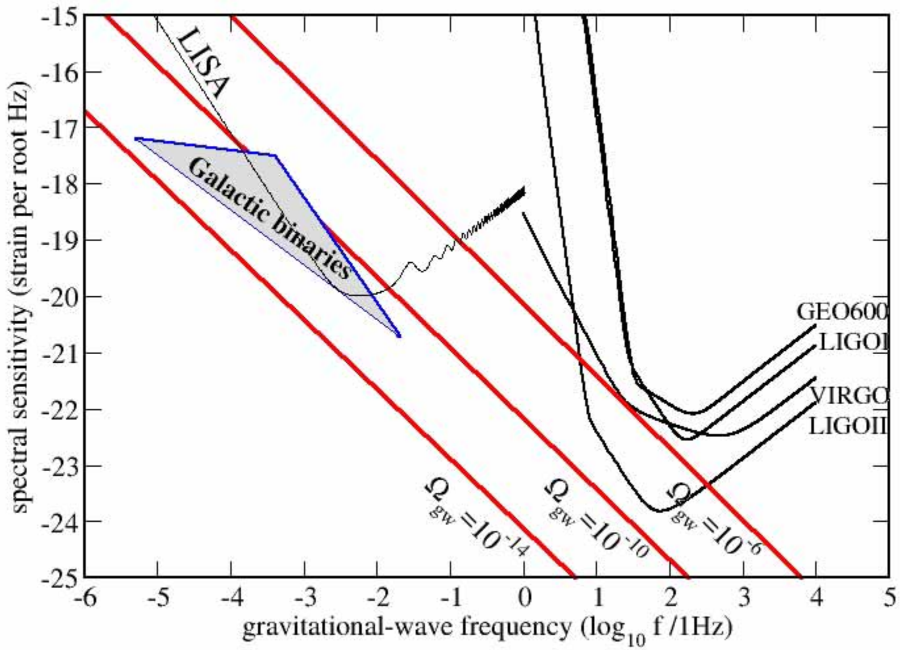
Stochastic Background



GW from the Big Bang

Stochastic background reflecting fundamental physics in the early universe;

- Phase transitions
- Inflation
- Topological defects
- String-inspired cosmology
- Higher dimensions



After the Big Bang, photons decoupled after 10^5 years, neutrino after 1s, GWs before 10^{-24} s!

Strength expressed as fraction of closure energy density;

$$10^{-14} \approx \Omega_{gw} < 10^{-5}$$

simple inflation nucleosynthesis

Detection: Requires cross-correlation of detectors.
Best window, free of "local" GW sources, is around 0.1-1 Hz.
Need LISA follow-on mission?

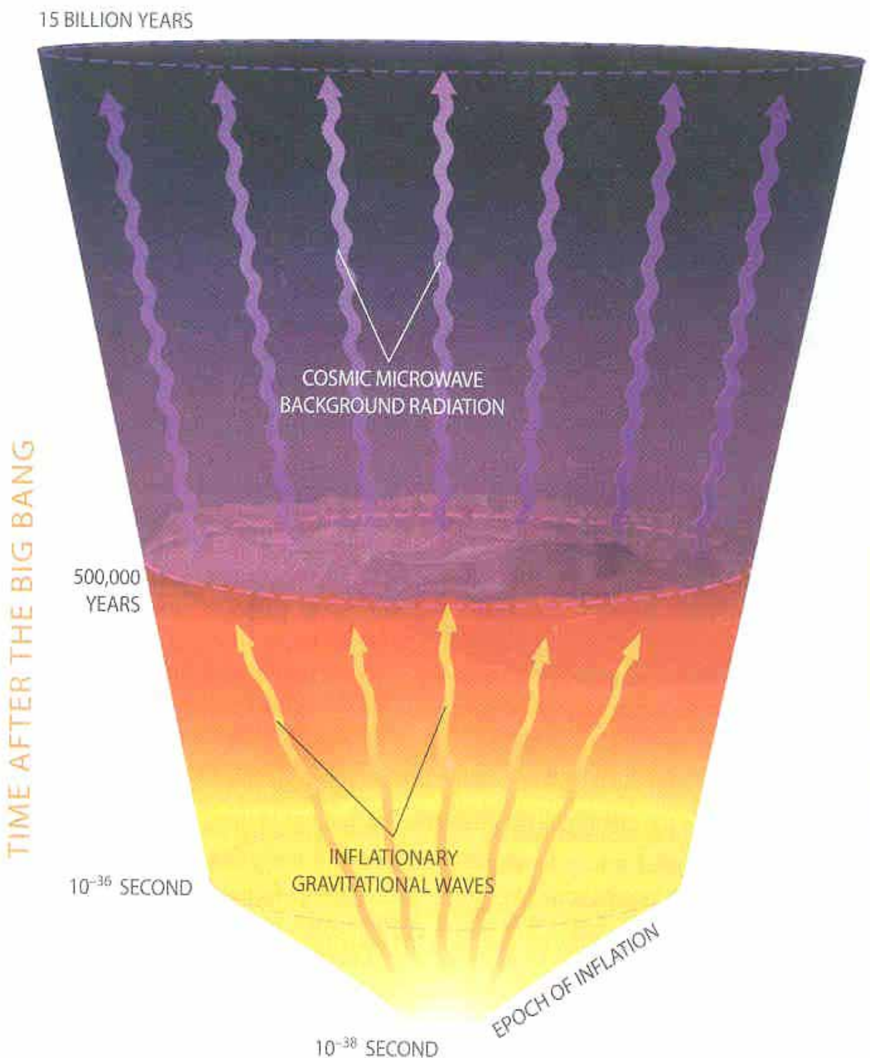
$$\Omega_{gw} = \frac{f}{\rho_c}\frac{d\rho_{gw}}{df}$$

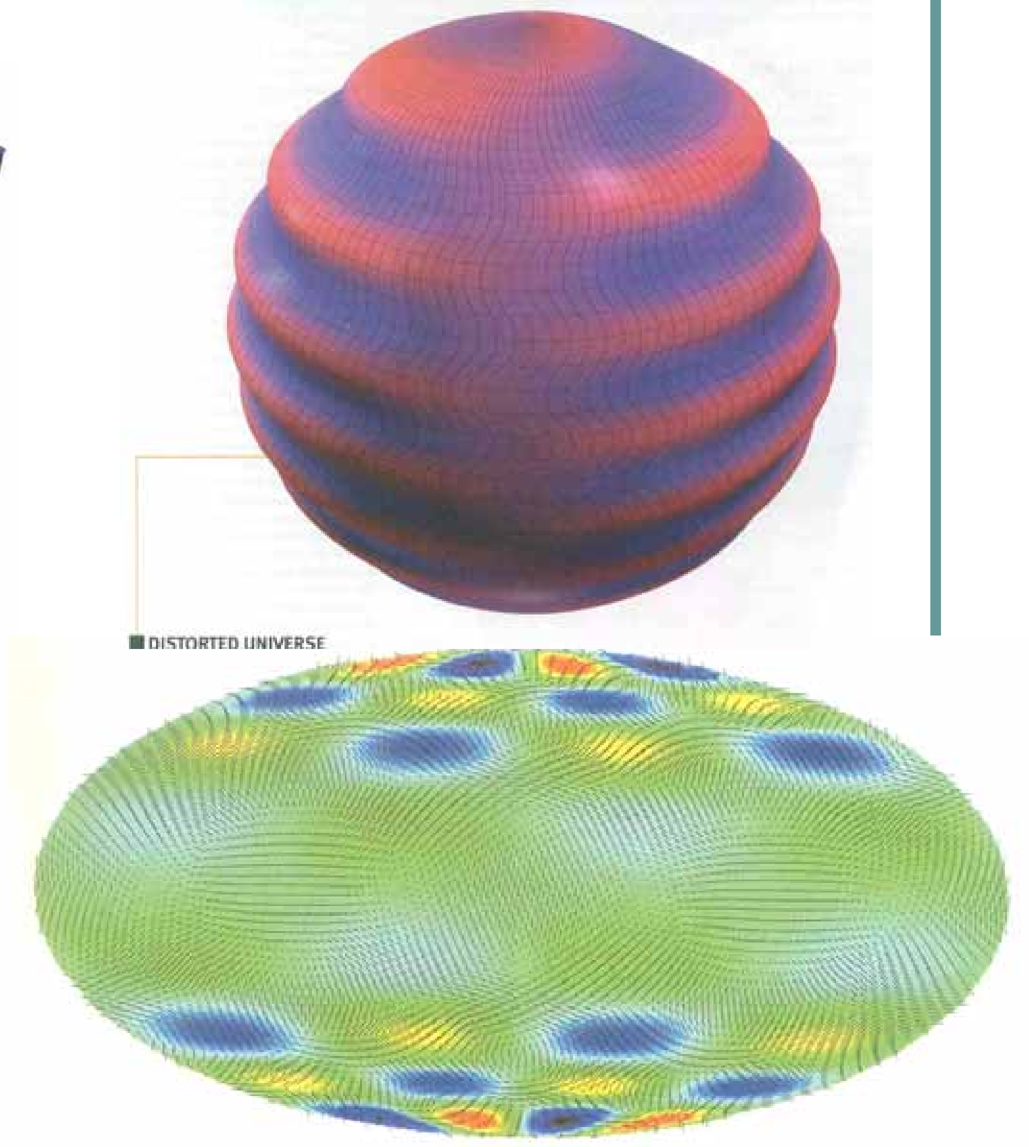
$$\rho_c = \frac{3H_0^2}{8\pi G}$$

$$h_c \simeq 10^{-18}\left(\frac{1\text{Hz}}{f}\right)\sqrt{h_0^2\Omega_{gw}(f)}$$

One of the most fundamental observations possible!

GW from Inflation



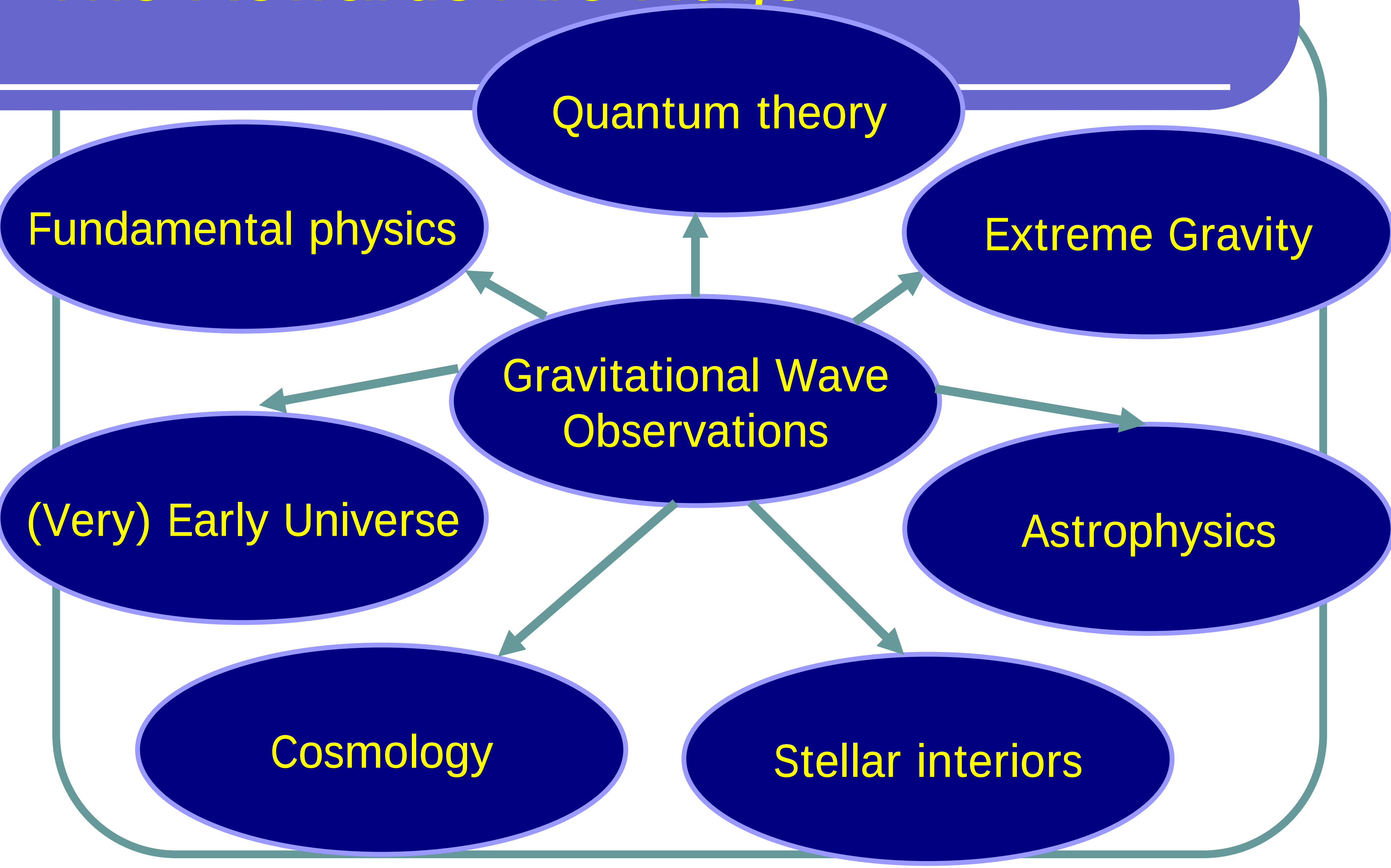


The Dark Side of the Universe

-- Kip Thorne

- Our present understanding of the Universe is based almost entirely on electromagnetic radiation.
- Black holes can emit only gravitational radiation.
- More than 90% of the Universe is **dark**, but it still interacts by gravity.
- There are 5-10 times as many dark baryons as luminous ones.
- If part of the dark matter forms compact clumps, then gravitational wave detectors will be the only way to see it directly.

The Rewards Are Huge



The END