# Human gravity-gradient noise in interferometric gravitational-wave detectors

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Among all forms of routine human activity, the one which produces the strongest gravity-gradient noise in interferometric gravitational-wave detectors (e.g. LIGO) is the beginning and end of weight transfer from one foot to the other during walking. The beginning and end of weight transfer entail sharp changes (time scale $\tau \sim 20$ msec) in the horizontal jerk (first time derivative of acceleration) of a person's center of mass. These jerk pairs, occurring about twice per second, will produce gravity-gradient noise in LIGO in the frequency band 2.5 Hz$\lesssim f \lesssim 1/(2\tau) \simeq 25$ Hz with the form $\sqrt{S_h(f)} \sim 0.6\times 10^{-23}\ \mathrm{Hz}^{-1/2}(f/10\ \mathrm{Hz})^{-6}[\Sigma_i(r_i/10\ \mathrm{m})^{-6}]^{1/2}$. Here the sum is over all the walking people, $r_i$ is the distance of the $i$'th person from the nearest interferometer test mass, and we estimate this formula to be accurate to within a factor 3. To ensure that this noise is negligible in advanced LIGO interferometers, people should be prevented from coming nearer to the test masses than $r \simeq 10$ m. A $r \simeq 10$ m exclusion zone will also reduce to an acceptable level gravity gradient noise from the slamming of a door and the striking of a fist against a wall. The dominant gravity-gradient noise from automobiles and other vehicles is probably that from decelerating to rest. To keep this below the sensitivity of advanced LIGO interferometers will require keeping vehicles at least 30 m from all test masses.
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## I. INTRODUCTION AND SUMMARY

Time-changing Newtonian gravitational forces, acting on the test masses of an interferometric gravitational-wave detector [e.g. in the Laser Interferometric Gravitational Wave Observatory (LIGO)], produce noise. This noise is conventionally called ''gravity-gradient noise'' because the interferometer measures the differences in the gravitational forces acting on the test masses. In a previous paper, Thorne and Hughes [1] gave a general overview of gravity-gradient noise and analyzed in detail the gravity-gradient noise in LIGO due to density fluctuations in the earth caused by ambient seismic waves. In this paper we focus on gravity-gradient noise due to human activity.

It has long been recognized that gravitational forces from moving humans can produce significant noise in high-precision gravitational experiments. Roll, Krotkov and Dicke [2] took great care to eliminate such forces in their classic Eötvös experiment, and Dicke [2,3] has raised the possibility that such noise was a serious factor in Baron Roland von Eötvös's original versions of that experiment [4]. In the early years of the LIGO Project, Robert Spero and others [8] made rough estimates of the magnitude of human (and other animal) gravity-gradient noise in LIGO and the distances to which humans (and other animals) should be relegated to control it. While these estimates were sufficiently accurate for their purposes, no analyses until ours seem to have identified the form of routine human activity that will dominate the noise (human walking), nor the spectrum of this dominant noise, $\sqrt{S_h} \propto f^{-6}$ for 2.5 Hz$\lesssim f \lesssim$25 Hz.

A first version of our analysis was carried out in summer 1995 [5], when the buildings that house LIGO's test masses were being designed. Our goal was to make sure that the building design would keep humans sufficiently far from the test masses, during routine LIGO observations, for human gravitational noise to be unimportant. The press of other research delayed until now our finalizing and publishing this analysis.

In our 1995 document [5], we focused on the gravitational effects of a walking person's horizontal center-of-mass motion, which was measured using force plates mounted in a floor. (The horizontal force exerted on the plates by the person's feet is equal and opposite to his or her center-of-mass acceleration.) We based our 1995 analysis on two force-plate experiments taken from the Biokinesiology (motion-in-biological-systems) literature; cf. Sec. II A below. For the present paper, we have augmented our center-of-mass-motion data base with new force-plate measurements on three different persons; and using data from the Biokinesiology literature, we have verified our original guess that the motions of a walking person's arms and legs produce gravitational noise small compared to that from center-of-mass motion (Sec. II C). This extended analysis has not changed significantly any of our 1995 conclusions.

Our analysis (Sec. II) produces the following estimate for gravity-gradient noise in LIGO due to walking people:

$$\sqrt{S_h(f)} \simeq \frac{0.6\times 10^{-23}}{\sqrt{\mathrm{Hz}}}\left(\frac{10\ \mathrm{Hz}}{f}\right)^6 \left[\sum_i \left(\frac{10\ \mathrm{m}}{r_i}\right)^6\right]^{1/2} \quad \text{at} \quad 2.5\ \mathrm{Hz} \lesssim f \lesssim 25\ \mathrm{Hz}. \tag{1}$$

Here $S_h(f)$ is the one-sided spectral density of the interferometer's output gravitational-wave signal $h = \Delta L/L$ (with $L = 4$ km the interferometer arm length and $\Delta L$ the difference

 

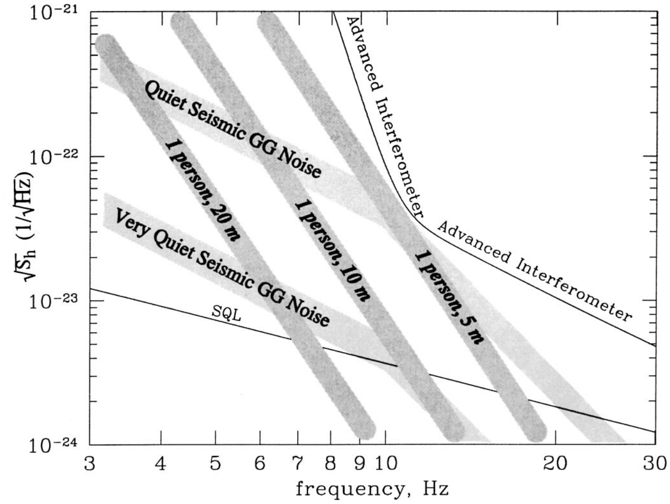


FIG. 1. The predicted spectrum of noise in LIGO's 4-km-long interferometers due to human gravity gradients (dark strips) and seismic gravity gradients (light strips), compared with the benchmark noise curve for advanced LIGO interferometers and with the standard quantum limit (SQL) in interferometers with 1 ton test masses. The thickness of the strips indicates the estimated uncertainties in our analysis.

in arm lengths, which fluctuates due to the gravitational forces from the walking people); also $r_i$ is the distance from person $i$ to the nearest interferometer test mass.

We believe this estimate to be accurate to within a factor 3. A factor $\sim 2$ uncertainty arises from the angular location and direction of motion of each person [the factor $\alpha$ in Eq. (12) below], and another factor that on occasion may be as large as $\sim 2$ arises from the gravitational forces of compressional and shear waves in the floor and ground, produced by the person's walking (Sec. II B). Somewhat smaller than this are uncertainties due to variations in the gait (walking) pattern from one person to another, and for each person, from one step to another (Sec. II A 2). Adding our two factor $\sim 2$ uncertainties in quadrature, we get our net factor $\sim 3$ uncertainty.

The $1/r_i^3$ dependence of the noise (1) results from the fact that it is produced by changes in the distance $r_i$ to the person's center of mass, and thus by the person's changing dipole gravitational field. The $1/f^6$ dependence results from two facts: (i) The gravitational force produces a test-mass acceleration, which means a second time derivative of $\Delta L$ and thence a second time derivative of $h$, and thence a $1/f^2$ in the amplitude spectrum $\sqrt{S_h}$. (ii) Force-plate measurements reveal that the fourth time derivative of the center-of-mass position is the lowest-order derivative with a delta-function-like behavior on time scales short enough to produce noise at frequences $f \sim 10$ Hz; those four time derivatives produce an additional $1/f^4$ in the spectrum.

Figure 1 shows the noise (1) for a single person at various distances from the nearest test mass. For comparison we also show (i) the benchmark noise curve for a broad-band ''advanced'' interferometer (which might operate in LIGO in the $\sim$2010 time frame) [6], (ii) the standard quantum limit (SQL) for a LIGO interferometer with 1 ton test masses, and (iii) the estimated seismic gravity-gradient noise from ambient earth motions [1] at ''quiet times'' (assuming the ''standard LIGO seismic spectrum'') and at ''very quiet times'' (assuming a seismic spectrum 10 times lower than the standard one—a level that might occur during wind-free nights).

Figure 1 shows that a single person walking at a distance of 5 m from a test mass could significantly increase the noise in an advanced interferometer, and several people would be correspondingly more serious.

The LIGO corner building (the only one with extensive human activity) has been designed to keep people at least 10 m from all test masses during normal operations. This provides an adequate safety factor for advanced interferometers; if the noise is three times as large as our estimate, then 10 people at 10 m distance would increase an advanced interferometer's noise by only a few tens of percent near the most sensitive frequency, 10 Hz.

It is conceivable that in $\sim$2010 or later interferometers will be operated in LIGO with good performance at frequencies below 10 Hz—e.g., as low as $\sim 3$ Hz. (Adalberto Giazzotto and colleagues of the VIRGO Project have developed seismic isolation systems that can go down to frequencies as low as 3 Hz [7].) Figure 1 indicates that, if such interferometers are ever operated in LIGO, it will be necessary to expand the people-free zone around each corner test mass.

The LIGO end and mid-station buildings (which have little human activity) are designed to keep all humans at least 5 m from the end test masses during normal operations. This provides an adequate safety margin for the first and enhanced LIGO interferometers, which will operate in the early and mid 2000's; but when advanced interferometers begin to operate, it will be necessary to expand the people-free zone, most especially at the end of each end building.

Robert Spero's early estimates of human gravity gradient noise focused on the slamming of a door or the striking of a fist against a wall [8]. Since this is more readily suppressed (by warnings and viscous door stops) than human walking, we have regarded it as a less serious and pervasive noise source. However, whenever a door slams or a fist strikes, the magnitude of the resulting gravitational ''signal'' in an advanced LIGO interferometer will be comparable to that from people walking.

In Sec. III, by a variant of Spero's analysis we derive the following expression for the Fourier transform of the signal $h(t)$ produced by a mass $M$ striking a building wall and coming suddenly to rest:

$$|\tilde{h}| = \frac{GM|\alpha \Delta v|}{L r^3 (2\pi f)^4}. \tag{2}$$

Here $\Delta v$ is the object's sudden change of speed, $r$ is its distance from the nearest interferometer test mass, $L = 4$ km is the interferometer arm length, and $\alpha$ is a coefficient in the range $-2 \lesssim \alpha \lesssim +2$ that depends on the object's angular location. Following Spero, we show that, with optimal signal processing, this ''signal'' would produce the following amplitude signal-to-noise ratio in an advanced LIGO interferometer:

$$\frac{S}{N} \simeq 1 \left( \frac{M\Delta v}{5 \ \text{kg m/s}} \right) \left( \frac{10 \ \text{m}}{r} \right)^3. \tag{3}$$

Here our fiducial value, 5 kg m/s, for $M\Delta v$ corresponds to a 5 kg door slamming shut at a speed of 1 m/s, or a 2 kg fist and forearm striking a wall at a speed of 2.5 m/s.

Thus, slamming doors and striking fists, like walking people, must be kept at distances $r \gtrsim 10$ m from the test masses of advanced LIGO interferometers.

A third type of human activity that can produce strong gravitational noise is the motion of automobiles and other vehicles. In Sec. III we argue that the dominant vehicle gravity-gradient noise, in the critical frequency region $f \sim 10$ Hz, is due to a vehicle's sudden change of acceleration when it comes to rest, e.g. when parking. If $\Delta a$ is the vehicle's change of acceleration, $M$ is its mass, and $r$ is its distance from the nearest interferometer test mass, then in an advanced LIGO interferometer the vehicle will produce a "signal" $h(t)$ with Fourier transform in our frequency band

$$|\tilde{h}| = \frac{GM|\alpha\Delta a|}{Lr^3(2\pi f)^5}. \tag{4}$$

Here $\alpha$ is the same angle-dependent coefficient with range $-2 \lesssim \alpha \lesssim +2$ as appears in the door-fist analysis. With optimal signal processing, this "signal" would produce the following amplitude signal-to-noise ratio in an advanced LIGO interferometer:

$$\frac{S}{N} \simeq 1 \left( \frac{M}{2 \ \text{ton}} \right) \left( \frac{|\Delta a|}{0.6g} \right) \left( \frac{30 \ \text{m}}{r} \right)^3. \tag{5}$$

Here $g = 9.8 \ \text{ms}^{-2}$ is the acceleration of gravity and we have set $|\alpha|$ to a representative value, $\sqrt{2}$.

The LIGO service roads come no closer than 40 meters to a LIGO corner test mass, but they approach to within 15 m of the corner- and mid-station test masses.

The product $M|\Delta a| = 2 \ \text{ton} \times 0.6g$ used in Eq. (5) is in the upper range of what one might expect for stopping vehicles. Two tonnes is a modest vehicle mass; $0.6g$ is the deceleration at which a vehicle begins to skid on dry asphalt. Thus, when advanced interferometers are operating in LIGO it will be necessary to increase the radius of the vehicle-free zone at the corner and midstations to $\gtrsim 30$ m.

The body of this paper is organized as follows: At the beginning of Sec. II we briefly explain why human walking is the dominant source of human gravity gradient noise in interferometers, and why, at the frequencies of interest, the noise comes predominantly from sudden changes in motion. Then in Sec. II A we compute the gravity-gradient noise in an interferometer due to a person's center-of-mass motion, and sum over a population of people to get Eq. (1) (discussed above). Momentum conservation implies that any sudden change in a person's center-of-mass motion will produce a corresponding sudden displacement of the floor and the ground beneath the floor; in Sec. II B and an Appendix we show that gravity gradient noise from this floor/ground motion will *not* cancel that from the person, but *might*, on occasion, cancel as much as half of it (thereby introducing a factor $\sim 2$ uncertainty in the net noise). In Sec. II C we compute the gravity-gradient noise due to the motions of a person's limbs (arms and legs; including, most importantly, the sudden changes of leg motion when a heel strikes the floor). We show that the limb motions produce noise that is smaller by a factor $\sim 0.1(10 \ \text{m}/r)(f/10 \ \text{Hz})$ than the noise from center-of-mass motion; here $r$ is the distance between the person and the nearest test mass. In Sec. III we analyze the gravity-gradient noise due to the sudden stopping of a moving mass—a slamming door, a fist striking a wall, or a parking vehicle—arriving at Eqs. (3) and (5), discussed above. In Sec. IV we make some concluding remarks.

## II. HUMAN WALKING

Consider a person (or vehicle) moving in the vicinity of a LIGO test mass. Denote by $\Phi$ the person's Newtonian gravitational potential in the test-mass vicinity, and by $\vec{x}'(t)$ and $r'(t) = |\vec{x}'|$ the vector and distance from the person's center of mass ("c.m.") to the test mass at time $t$. Expand the Newtonian potential in multipole moments around the person's (moving) c.m.:

$$\Phi = -\frac{GM}{r'} - \frac{3}{2}\frac{G\mathcal{I}_{jk}}{r'^5}x'_j x'_k + \cdots. \tag{6}$$

Here $\mathcal{I}_{jk}(t)$, the person's quadrupole moment relative to his c.m., is the means by which his moving limbs produce gravity-gradient noise, repeated indices ($j$ and $k$) are to be summed, and we use Cartesian coordinates so it doesn't matter whether tensor indices are up or down.

In Sec. II A we examine the first (monopolar) term in Eq. (6). This is the dominant gravitational effect of the c.m. motion. In Sec. II C we examine the second (quadrupolar) term—including its time dependence due both to limb motions and to center-of-mass motions—and we show that its influence on an interferometer's noise is small compared to that of the monopolar term.

Before presenting these analyses, it may be useful to comment on the importance of sudden changes (jerkiness) in the human (or vehicular) motion.

Smooth (non-jerky) motion produces gravitational forces that are concentrated at frequencies $f \sim 1$ Hz, well below those, $f \gtrsim 10$ Hz, of concern for the interferometers. For example, the period of the normal human gait cycle (two steps, one left and one right) is about 1 sec (frequency 1 Hz); and an automobile moving at speed 30 km/hr at a distance 15 m from a test mass travels through an angle $90^o$ as seen by the test mass in about 2 sec (frequency $\sim 0.5$ Hz). If the motion is sufficiently smooth, the Fourier transform of such motions will fall off with frequency exponentially, becoming totally negligible at $f \sim 10$ Hz.

By contrast, if the $n$'th time derivative of the motion changes significantly on a time scale $\tau \lesssim 50$ msec, then the $n+1$'th time derivative will have a sharp, delta-function-like peak with time width $\tau$, and that will produce a Fourier transform of the motion that falls off as $1/f^{n+1}$ up to frequencies $f \sim 0.5/\tau \gtrsim 10$ Hz. Such a power-law fall-off can produce much larger gravity-gradient noise near 10 Hz than the exponential fall-off caused by smooth motion.

### A. Motion of center of mass

In this section we examine the gravitational noise produced by the monopolar (center-of-mass) term in Eq. (6).

#### *1. General formulas for noise*

We focus on a time duration of one gait cycle $\simeq 1$ sec, centered on time $t=0$. We introduce Cartesian coordinates $x_j$ attached to the floor, with origin at the location of the person's c.m. at time $t=0$; and we denote by $\vec{\xi}(t)$ the motion of the c.m. relative to this origin (so $\vec{\xi}=0$ at $t=0$). Then the vector and distance $\vec{x}'$ and $r'$ from the person's c.m. to the test mass are

$$\vec{x}'=\vec{x}-\vec{\xi},$$

$$r'=|\vec{x}-\vec{\xi}|=r-\hat{n}\cdot\vec{\xi}+\frac{\vec{\xi}^2-(\hat{n}\cdot\vec{\xi})^2}{2r}+\cdots. \tag{7}$$

Here $\vec{x}$ and $r$ are the vector and distance from the origin of coordinates to the test mass, and $\hat{n}=\vec{x}/r$ is the unit vector pointing from the origin to the test mass. Correspondingly, we can write the center-of-mass piece of the gravitational potential (6) as

$$\Phi_{\rm c.m.}\equiv-\frac{GM}{r'}=-\frac{GM}{r}-\frac{GM\xi_j n_j}{r^2}-\frac{3}{2}\frac{GM\left(\xi_j\xi_k-\frac{1}{3}\left|\vec{\xi}\right|^2\delta_{jk}\right)n_j n_k}{r^3}+\cdots. \tag{8}$$

The first term (monopolar about our fixed center of coordinates) is constant in time and thus can produce no gravity-gradient noise, so we shall ignore it. The second term (dipolar about our fixed center of coordinates) produces the gravity-gradient noise via sudden changes of the c.m. location $\vec{\xi}$. The third term (quadrupolar about our fixed center of coordinates) is smaller than the second, dipolar term by $\sim|\vec{\xi}|/r\lesssim 0.7$ m/10 m$\sim 0.1$, and its gravity-gradient noise is correspondingly smaller in our frequency band, so we shall ignore it. The gravitational acceleration of the test mass, produced by the person's c.m. motion, is then minus the gradient of the second, dipolar term:

$$g_j=\frac{GM}{r^3}(\xi_j-3n_j n_k\xi_k). \tag{9}$$

The interferometer has four test masses labeled $A=1,2,3,4$, each of which experiences an acceleration of the form (9) due to the c.m. motion. The resulting output signal of the interferometer, $h(t)=\Delta L(t)/L$, has a second time derivative given by the sum of these accelerations projected onto unit vectors $\hat{m}_A$ that point along the interferometer arms as shown in Fig. 2:

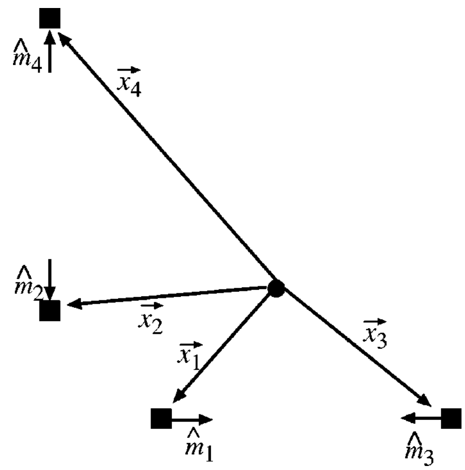


FIG. 2. Geometry of the LIGO test masses (solid squares) and the location (large dot) of the center of mass of a person at time $t=0$.

$$\frac{d^2h}{dt^2}=\frac{1}{L}\sum_A \vec{g}_A\cdot\hat{m}_A$$

$$=\sum_A\frac{GM}{Lr_A^3}[\vec{\xi}\cdot\hat{m}_A-3(\hat{n}_A\cdot\hat{m}_A)(\hat{n}_A\cdot\vec{\xi})]. \tag{10}$$

Since the interferometer's laser beams are horizontal, the vectors $\hat{m}_A$ are all horizontal; and since the person's c.m. is at the same height as the test masses to within $<1$ m and the person is at a distance $\sim 10$ m from the nearest test mass, the vertical component of $\hat{n}_A$ is $\lesssim 0.1$ of the horizontal component. This means [cf. Eq. (10)] that the vertical component of $\vec{\xi}$ produces significantly less gravitational noise than the horizontal component: its contribution to $\sqrt{S_h}$ is less by a factor $\lesssim 0.1|\tilde{\xi}_{\rm v}|/|\tilde{\xi}_{\rm h}|\sim 0.3$. (Here the tilde denotes a Fourier transform, v and h denote vertical and horizontal components, and we have used $|\tilde{\xi}_{\rm v}|/|\tilde{\xi}_{\rm h}|\sim 3$ at $f\sim 10$ Hz as inferred from force-plate measurements; see Table I below.) On this basis, we shall ignore the vertical component of the c.m. motion and approximate $\vec{\xi}$ and $\hat{n}_A$ as purely horizontal.

It will be convenient below to rewrite Eq. (10) in the form

$$\frac{d^2h}{dt^2}=\alpha\frac{GM\xi}{Lr^3}, \tag{11}$$

where $r$ is the distance from the center of coordinates to the nearest test mass, $\xi(t)$ (scalar, not vector) is the distance the c.m. has traveled since $t=0$ ($\xi=-|\vec{\xi}|$ for negative times and $+|\vec{\xi}|$ for positive), and $\alpha$ is a dimensionless coefficient given by

$$\alpha=\sum_A\left(\frac{r}{r_A}\right)^3[\hat{\xi}\cdot\hat{m}_A-3(\hat{n}_A\cdot\hat{m}_A)(\hat{n}_A\cdot\hat{\xi})]. \tag{12}$$

Here $\hat{\xi}=\vec{\xi}/\xi$ is the unit vector along the c.m.'s direction of motion, which we regard as constant during one gait cycle.

For people near an end mass (mass $A=3$ or 4 in Fig. 2), only that mass gives a significant contribution to the coefficient $\alpha$, and it is easy to verify that $\alpha$ ranges from $-2$ to $+2$, depending on the angular location of the person. The extremal values $\pm 2$ are reached when the person's c.m. is along the interferometer arm and the person is moving toward or away from the test mass ($\vec{\xi}$, $\hat{m}_A$, and $\hat{n}_A$ all parallel or antiparallel).

For people in LIGO's corner station, both corner masses contribute strongly to $\alpha$. This is because the distance between the two corner masses, $l \simeq 5$ m, is comparable to the distance between the person and the nearest test mass, $r \sim 10$ m. A straightforward numerical exploration shows, in this case, that $\alpha$ ranges from about $-2.2$ to about $+2.2$ depending on the person's location.

A representative value for $|\alpha|$, which we use in our final noise estimates [e.g., Eq. (1) above], is $|\alpha|_{\rm representative}=\sqrt{2}$.

Equation (11) implies that the noise spectrum $\sqrt{S_h(f)}$ will scale with frequency as $|\tilde{\xi}(f)|/f^2$, where $\tilde{\xi}$ is the Fourier transform of the distance traveled $\xi(t)$; and the discussion just before the beginning of this section implies that $\tilde{\xi}(f)$ will be governed primarily by the lowest order derivative of $\xi(t)$ that has sudden changes on a time scale $\tau \lesssim 50$ msec. To identify that derivative and the details of the sudden changes, we rely on experimental data from the field of Biokinesiology (the study of motion in biological systems).

Since $\xi(t)$ is distance traveled by the c.m., the only way that it or its derivatives can change jerkily is by the application of a sharply changing, horizontal external force. The only such force, as a person walks, is the horizontal force of the floor on the person's feet. The negative of that force [the horizontal force $F(t)$ of the feet on the floor] is measured by biokinesiologists, using force plates (pp. 414–418 of Ref. [9]; Sec. 4.2 of Ref. [10]). By momentum conservation, this measured force is $F=-Md^2\xi/dt^2$, and correspondingly the gravitational noise is related to the measured horizontal force by

$$\frac{d^4h}{dt^4}=\alpha\frac{GF}{Lr^3}. \tag{13}$$

#### 2. Force-plate measurements

Figure 3 shows the measured horizontal force $F(t)$ exerted on the floor by a woman weighing 73 kg during a full gait cycle. These data were obtained as follows:

Our colleague, Earnest L. Bontrager, placed two force plates in the floor of his laboratory at Rancho Los Amigos Medical Center, Downey, California. The force plates were so located that in normal walking a person will encounter them during one gait cycle, with the right foot landing on the first plate and then the left foot on the second. Each plate was equipped with piezo-electric transducers that measured the foot's forward horizontal (''progressive'') force, its vertical force, and its sideward horizontal (''medial'') force. Figure 3 is based on the progressive force measurements. The transducer outputs were sampled and recorded at 2500 samples per sec and were then averaged over 0.01 sec intervals to produce, for each measured half-gait cycle, a single data set. Figure 3 is based on Bontrager's data set 3506a5 (data set a5 for person 3506) [11].

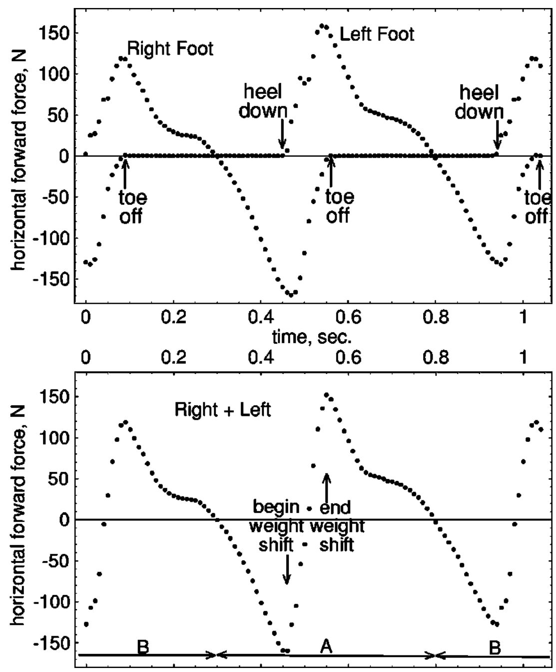


FIG. 3. The forward, horizontal force exerted on the floor by a woman weighing 73 kg, as measured using dual (two-feet) force plates by Bontrager [11] (data set 3506a5). Upper panel: The force as a function of time exerted by each foot. Lower panel: The sum of the forces from the two feet; the full gait cycle is divided into two half cycles, A and B.

For Fig. 3 we modified the data set by adding, at the beginning, the last 9 points from the left-foot measurement; and at the end, the first 11 points from the right-foot measurement (under the plausible assumption that the unmeasured end of the previous left-foot gait cycle is the same as the measured cycle, and the unmeasured beginning of the next right-foot cycle is the same as the measured cycle). We have divided the full gait cycle into two half cycles A and B, as shown in Fig. 3.

Equation (13) implies that the noise spectrum $S_h(f)$ is proportional to $|\tilde{F}(f)|/f^4$, where $\tilde{F}$ is the Fourier transform of $F(t)$, the sum of the forces from the two feet (bottom panel of Fig. 3). As was discussed at the beginning of Sec. II, the only features of $F(t)$ that can contribute significantly at frequencies $f \sim 10$ Hz are those that change on time scales $\tau \lesssim 50$ msec. The net force $F(t)$ varies only modestly on such short time scales, but its first time derivative $dF/dt \equiv \dot{F}$ varies strongly: Shortly after placing her heel on the floor (''heel down''), the person begins to transfer weight from her trailing foot to her leading foot; this beginning of weight transfer entails a change $\Delta\dot{F} \simeq 4000$ N/s in the slope of the force curve (change of jerk) on a very short timescale $\tau \sim 20$ msec. A time $\delta t \simeq 90$ msec later, just before the trailing toe lifts off the ground (''toe off''), the weight transfer ends with a second sharp change of jerk $-\Delta\dot{F}$.

These two jerk changes during each half gait cycle give rise to the following (approximate) form of the half-cycle's Fourier transform $\tilde{F}_{1/2}$ in the frequency band of interest, 2.5 Hz$\lesssim f \lesssim 25$ Hz$\simeq 0.5/\tau$:

$$|\tilde{F}_{1/2}| = \frac{1}{(2\pi f)^2} 2\sin(\pi f \delta t)\Delta\dot{F}. \tag{14}$$

A numerical Fourier transform (FFT) of $F(t)$ (Fig. 3) reveals that other features with timescales $\gtrsim \delta t = 90$ msec produce modulations of $\tilde{F}(f)$ analogous to but no larger than the $\sin(\pi f \delta t)$. We shall ignore these modulations and correspondingly shall approximate $\sin(\pi f \delta t)$ by its rms value, $1/\sqrt{2}$, and therefore shall rewrite the above formula as

$$|\tilde{F}_{1/2}| = \frac{\sqrt{2}\Delta\dot{F}}{(2\pi f)^2}. \tag{15}$$

Equation (15) is a fairly accurate representation of the half-gait-cycle Fourier transform, not just for the data set in Fig. 3, but for all force-plate data that we have examined—data in the Biokinesiology literature [9,10], and unpublished data on two other subjects in Bontrager's laboratory [11].

In all of Bontrager's data sets except 3506a5 (Fig. 3), only one force plate was used rather than two, so the data sets show only the horizontal force produced on the floor by one foot during a half gait cycle, along with the times of heel down and toe off for the second foot as measured by a switch attached to the foot. To compute the force of the second foot, we assumed (in accord with Bontrager's advice) that its force history was the same as that measured for the first, but displaced in time as shown on the foot-switch recordings. We thereby modified each data set to include the force of the second foot, and computed the total force of the two feet (analog of segment A and segment B of Fig. 3).

We have fit Eq. (15) to the Fourier transforms of each of Bontrager's data sets for the total force of both feet during a half gait cycle. For each single-force-plate data set, and for the two half-gait-cycles (A and B) of dual-plate set 3506a5, we did a least-squares solution for $\Delta\dot{F} = |\tilde{F}_{1/2}|(2\pi f)^2/\sqrt{2}$, and its rms fluctuations in the frequency range 2.5 to 20 Hz. The results are shown in Table I.

TABLE I. Sudden changes of jerk, and ratio of vertical to horizontal force spectra for a half gait cycle, as inferred from Bontrager's data sets [11].

| Subject | Plates | $\Delta dF/dt$ | $\lvert\tilde{F}_{\rm v}\rvert/\lvert\tilde{F}_{\rm h}\rvert$ |
|---|---|---|---|
| Male 82 kg | | | |
| 3700c5 | single | $6400 \pm 1900$ | 1.7 |
| 3700c7 | single | $6000 \pm 3400$ | 2.6 |
| Male 78 kg | | | |
| 3772c5 | single | $4600 \pm 1300$ | 3 |
| 3772c7 | single | $7100 \pm 2600$ | 3 |
| Female 73 kg | | | |
| 3506a5 — A | dual | $4400 \pm 2200$ | 5 |
| 3506a5 — B | dual | $3700 \pm 1900$ | 4 |

Note that in each measured spectrum there are fluctuations of typical magnitude 30–60 percent around the $1/f^2$ law of Eq. (15); and the fluctuations in the inferred $\Delta\dot{F}$ from one gait cycle to another and from person to person are of order 30 percent. If a person were to run rather than walk, the resulting $\Delta\dot{F}$ might be larger, but presumably not by more than a factor $\sim 2$; and running in the vicinity of a LIGO test mass should be much less common than walking. These variations in $\Delta\dot{F}$ are modest contributors to our overall factor $\sim 3$ uncertainty in the gravity gradient noise. Based on Table I, we shall use the value

$$\Delta\dot{F} = 5500 \ \ \text{N/s} \tag{16}$$

in our noise evaluations.

For Bontrager's data sets we have also evaluated the ratio $|\tilde{F}_{\rm v}|/|\tilde{F}_{\rm h}|$ of the Fourier transforms of vertical force and horizontal force in the vicinity of 10 Hz; see the last column of Table I. The vertical spectra vary by a large factor from one person to another: male 3700 walks rather smoothly; female 3506 strikes the floor sharply with her heel at heel down. Correspondingly, female 3506 produces a large change of force $\Delta F_{\rm v}$ at heel down on a short enough timescale to dominate the vertical spectrum, $\tilde{F}_{\rm v} \simeq (2\pi f)^{-1}\Delta F_{\rm v}$; whereas male 3700 (and also male 3772) has a vertical spectrum dominated by a change of jerk and therefore falling off more rapidly with frequency, $|\tilde{F}_{\rm v}| \simeq (2\pi f)^{-2}\Delta\dot{F}_{\rm v}$. As a result, female 3506 has a significantly larger ratio of vertical to horizontal $|\tilde{F}|$ than the males. However, even for her large heel-induced $|\tilde{F}_{\rm v}|$, the resulting vertical contribution to the gravity gradient noise is smaller than the horizontal contribution—smaller by the factor $\sim 0.1|\tilde{F}_{\rm v}|/|\tilde{F}_{\rm h}| \sim 0.5$ discussed in Sec. II A 1.

#### *3. Noise spectrum*

For people walking in the vicinity of LIGO test masses, the sharp changes of jerk are not likely to occur in a periodic fashion to within a period accuracy of 0.05 sec, and correspondingly, in the vicinity of 10 Hz the jerks are not likely to superpose coherently. Therefore, we can approximate the sharp changes of jerk as constituting a random shot noise, for which the spectral density of the gravitational-wave noise from a single person will be

$$\sqrt{S_h} = \left(2\frac{2}{P_{\rm gait}}\right)^{1/2} |\tilde{h}_{1/2}| = \frac{2\sqrt{2}\alpha G \Delta\dot{F}}{L\sqrt{P_{\rm gait}} r^3 (2\pi f)^6}. \tag{17}$$

Here $P_{\rm gait}$ is the gait-cycle period (about 1 sec). In the first expression $2/P_{\rm gait}$ is the rate of (dual-jerk) ''shots'' (half gait cycles) for each of which $\tilde{h}_{1/2}$ is the Fourier transform of $h(t)$, and the second expression follows from Eqs. (13) and (15).

For a number of walking people, each at a different distance $r_i$ from the test mass and with a different angular location and direction of motion and corresponding factor $\alpha_i$, the noises will add in quadrature, producing

$$\sqrt{S_h}=\frac{2\sqrt{2}G\Delta\dot{F}}{L\sqrt{P_{\rm gait}}(2\pi f)^6}\left(\sum_i \frac{\alpha_i^2}{r_i^6}\right)^{1/2}. \tag{18}$$

Inserting the numerical values (discussed above) $|\alpha_i| = |\alpha|_{\rm representative} = \sqrt{2}$, $\Delta\dot{F}=5500$ N/s, $L=4$ km, $P_{\rm gait}=1$ s, we obtain the noise spectrum (1) discussed in the Introduction and the abstract.

### B. Motion of floor and ground

Sharp changes in the horizontal force $F(t)$ will produce deformations of the floor and ground, which become seismic waves as they propagate out through the earth. These deformations will produce gravity gradient noise correlated with that from the person's c.m. In this section we shall estimate this noise.

We begin with the standard expression [1] for the gravitational potential at test-mass location $\vec{x}_A$, produced by the displacement $\vec{\zeta}$ of the floor and ground:

$$\Phi=-\int_{\mathcal{V}}\frac{-G\ \vec{\nabla}\cdot(\rho\vec{\zeta})}{|\vec{x}-\vec{x}_A|}d^3x-\int_{\partial\mathcal{V}}\frac{G\ \ \rho\vec{\zeta}\cdot d\vec{A}}{|\vec{x}-\vec{x}_A|}. \tag{19}$$

Here $-\vec{\nabla}\cdot(\rho\vec{\zeta})$ is the Eulerian change in density induced by the displacement $\vec{\zeta}$, the first integral is over the interior $\mathcal{V}$ of the floor and ground, and the second integral is over the surface layer of mass produced by $\vec{\zeta}$ on the surface $\partial\mathcal{V}$ of the floor and the adjoining ground outside the LIGO buildings.

Integrating the first term by parts and cancelling the resulting surface term against the second term of Eq. (19), we obtain

$$\Phi=-\int_{\mathcal{V}}G\rho\zeta_j\left(\frac{1}{|\vec{x}-\vec{x}_A|}\right)_{,j}d^3x. \tag{20}$$

The gravitational acceleration on test-mass A is minus the gradient of this with respect to $\vec{x}_A$. Since the only dependence of $\Phi$ on $\vec{x}_A$ is through the combination $\vec{x}-\vec{x}_A$, the gradient can be replaced by a derivative under the integral with respect to $-\vec{x}$, thereby giving

$$g_i=-\int_{\mathcal{V}}G\rho\zeta_j\left(\frac{1}{|\vec{x}-\vec{x}_A|}\right)_{,ji}d^3x. \tag{21}$$

This is the dipolar floor-ground analog of Eq. (9) for the gravitational acceleration produced by the person's c.m. By the procedure that led from Eq. (9) to Eq. (10), we obtain the gravitational-wave noise $d^2h/dt^2$ produced by the floor/ground motion, to which we add the person's c.m. noise (10). Differentiating the result once in time, we obtain

$$\frac{d^3h}{dt^3}=-\frac{G}{L}\sum_A m_{Ai}\left[M\dot{\xi}_j\left(\frac{1}{|\vec{x}-\vec{x}_A|}\right)_{,ji}+\int_{\mathcal{V}}\rho\dot{\zeta}_j\left(\frac{1}{|\vec{x}-\vec{x}_A|}\right)_{,ji}d^3x\right]. \tag{22}$$

Here (as should be obvious) in the first term $\vec{x}$ is the person's c.m. location and in the second it is a location in the floor or ground.

This equation expresses the gravitational noise in terms of sharp changes in the person's c.m. momentum and the momentum density of the ground. By momentum conservation, any change of the c.m. momentum $M\dot{\xi}_j$ must be accompanied by an equal and opposite change of the total floor-ground momentum $\int_{\mathcal{V}}\rho\dot{\zeta}_j d^3x$. If the suddenly deposited momentum remains close to the person [within a distance $\ll r$ = (person's distance to nearest test mass)] during a time $\tau \simeq 1/(2f) \simeq 50$ msec, then the gravity-gradient noise from the floor-ground will nearly cancel that from the person's c.m. If the deposited momentum spreads out over a distance $\gg r$ in the time $\tau$, then it will produce a neglible gravitational force on the test mass, and negligible gravity gradient noise.

The deposited momentum moves outward through the floor and ground with seismic-wave speeds; it resides in a spreading, widening shell whose sharp outer edge moves at the seismic P-wave speed $c_P$ and fuzzy inner edge at a little less than the seismic S-wave speed $c_S$ (see the Appendix).

The floor on which the person walks is a slab of reinforced concrete 20 cm thick. In each corner and end station this slab begins about 6 m from the test mass and extends outward to about 18 m from the test mass, and transversely about 12 m in each direction. In the corner station the slab begins about 10 m from the nearest test mass and extends on outward an additional 10 to several 10's of meters, in a complicated shape. The concrete has $c_P \simeq 3700$ m/s and $c_S \simeq 1700$ m/s; so in $\tau \simeq 50$ msec, the outer edge of the spreading shell could move a horizontal distance of 180 m and the inner edge, 85 m if the slab were that large. To the extent, then, that the deposited momentum gets trapped in the slab for $\tau \sim 50$ msec, it spreads through the whole slab; and since most of the slab is somewhat farther from the test mass than the nearest person (about 10 m) and in somewhat different directions (off to the sides), the slab's sudden momentum change will produce a considerably weaker gravitational force on the test mass than is produced by the person's sudden momentum change.

The momentum that passes through the thin floor and into the ground below spreads through the ground at much lower speeds than that confined to the floor: $c_S \simeq 270$ m/s; $c_P \simeq 520$ m/s in Hanford's dry soils and 1700 m/s in Livingston's water-saturated soils; cf. Tables II and IV of Ref. [1]. Correspondingly, in $\tau = 50$ msec time, the inner edge of the spreading momentum shell travels a distance $\simeq 13$ m; and the outer edge, $\simeq 25$ m at Hanford and $\simeq 80$ m at Livingston. These distances, being comparable to, and much larger than the person's $\sim 10$ m separation from the test mass, will cause the momentum suddenly deposited in the ground to produce a somewhat smaller gravitational noise than that of the person; there is no possibility for a strong cancellation.

On the other hand, in some cases the Green's function for the momentum spread (cf. Appendix, and Figs. 2–4 of Ref. [13]) is strongly localized near the inner edge of the spreading shell. In such cases, with the momentum having spread only a distance $\sim 15$ m compared to the person's $\sim 10$ m distance from the test mass, the ground's gravitational force on the test mass might be as much as half that of the person, thereby reducing the net noise by a factor 2 relative to that of the person alone. This is a significant contributor to our estimated factor 3 uncertainty in the net noise.

### C. Motion of limbs

Turn, next, to the noise produced by the person's quadrupolar gravitational field

$$\Phi_{\rm Quad} = -\frac{3}{2}\frac{G\mathcal{I}_{jk}}{r'^5}x'_j x'_k \tag{23}$$

[Eq. (6)]. Here $\mathcal{I}_{jk}$ is the quadrupole moment about the person's moving c.m., and $x'_j$ and $r' = |\vec{x}'|$ are the vector and distance from the c.m. to a test mass. Replacing $x'_j$ by $x_j - \xi_j$, where $x_j$ reaches from the (fixed) c.m. location at the center of a gait cycle to the test mass, and $\xi_j(t)$ is the displacement of the c.m. relative to that fixed point, we obtain

$$\Phi_{\rm Quad} = -\frac{3}{2}\frac{G\mathcal{I}_{jk}n_j n_k}{r^3} - \frac{3}{2}\frac{G\mathcal{I}_{jk}}{r^4}(3n_j n_k \xi^{||} - 2n_j \xi^{\perp}_k), \tag{24}$$

where $\xi^{||}$ is the component of $\vec{\xi}$ along $\hat{n} = \vec{x}/r$, the direction to the test mass, and $\xi^{\perp}_k$ is its projection orthogonal to $\hat{n}$.

This quadrupolar gravitational field produces interferometer noise via jerky changes of $\mathcal{I}_{jk}$ (noise "intrinsic" to the person's quadrupole moment) and via jerky changes of $\vec{\xi}$ ("extrinsic" noise).

The extrinsic noise is readily seen to be very small compared to that from the person's c.m. gravitational field $\Phi_{\rm c.m.} = -GM\xi^{||}/r^2$:

$$\frac{\sqrt{S_h^{\rm ext}}}{\sqrt{S_h^{\rm c.m.}}} \sim \frac{\Phi_{\rm ext}}{\Phi_{\rm c.m.}} \lesssim \frac{9}{2}\frac{\mathcal{I}_{jk}}{Mr^2} \sim 0.01\left(\frac{10\ \text{m}}{r}\right)^2. \tag{25}$$

Here we have used an obvious estimate of the person's quadrupole moment.

The intrinsic noise, arising from jerky changes of $\mathcal{I}_{jk}$, has contributions from both terms in Eq. (24). That from the second term is smaller by $\sim 2\xi/r \lesssim 0.15(10\ \text{m/r})$ than that from the first term, so we shall ignore it. Taking the gradient of the first term to obtain the gravitational acceleration on the test mass, and proceeding as in the derivation of Eq. (10), we obtain the following expression for the quadrupolar noise in the interferometer as a sum over contributions from the test masses $A = 1,2,3,4$:

$$\frac{d^2h}{dt^2} = \sum_A \frac{3G}{Lr_A^4}\left(\mathcal{I}_{jk}m_{Aj}n_{Ak} - \frac{5}{2}\mathcal{I}_{ij}n_{Ai}n_{Aj}m_{Ak}n_{Ak}\right). \tag{26}$$

We choose the $x$ axis along the progressive direction (direction $\hat{\xi}$ of motion), the $y$ axis along the medial (transverse horizontal) direction, and the $z$ axis vertically upward. Then, because $\hat{m}_A$ and $\hat{n}_A$ are horizontal vectors, and the body's jerky motion is in the $x$-$z$ plane, the noise (26) arises solely from two components of the quadrupole moment,

$$\mathcal{I}_{xx} = \frac{1}{3}(2I_{xx} - I_{zz}),\ \ \mathcal{I}_{yy} = -\frac{1}{3}(I_{xx} + I_{zz}). \tag{27}$$

Here $I_{jk}$ is the second moment of the body's mass distribution (the integral of $\rho x'_j x'_k$ over the body).

As the person walks, the dominant contributors to jerky changes of the quadrupole moment are the motions of his legs. (His arms are less massive and jerk less.) We divide each leg into two parts, the thigh (reaching from hip to knee) and the shank (reaching from knee to ankle); and we approximate each of these as a point mass located at its center of mass, thereby making an acceptably small error. Measurements discussed below show that the quadrupolar noise at frequencies $f \sim 10$ Hz arises primarily from sudden ($\tau \sim 0.5/f \sim 50$ msec) changes of the thigh and shank accelerations $\Delta a_k$ each time the person's heel strikes the floor (heel down). The corresponding sudden change of $\ddot{I}_{jk}$ (second time derivative of $I_{jk}$) is

$$\Delta\ddot{I}_{jk} = 2m^s x'^s_j \Delta a^s_k + 2m^t x'^t_j \Delta a^t_k, \tag{28}$$

where the superscripts $s$ and $t$ denote shank and thigh, $m$ is the mass of shank or thigh, and $x'_j$ is the vector from the person's c.m. to the center of mass of shank or thigh at heel down.

Biokinesiologists measure the motions of thigh and shank in two ways: by videotaping markers placed on them (position measurements), and via accelerometers placed on them (acceleration measurements); for a pedagogical discussion see [12]. Because of noise introduced when taking time derivatives of the position data, the position measurements cannot give reliable measures of acceleration on the short timescales $\tau \lesssim 50$ ms of concern to us [12]. Therefore, for the thigh and shank accelerations we have relied on accelerometer measurements as reported by Wu in Fig. 16-11 of Ref. [12]. It is obvious from that figure that the dominant contributions to the Fourier transform $\tilde{a}^b_k$ of $a^b_k(t)$ (for $k = x,y,z,\ b = t,s$) arise from sharp changes at heel down. We have Fourier transformed $a^b_k(t)$ and found that, to within a factor $\sim 2$ over the range 2.5 Hz$\lesssim f \lesssim$25 Hz, $|\tilde{a}^b_k| \propto 1/f$. This implies that, to adequate accuracy for our purposes (factor 2) we can regard the quadrupolar noise as due to sudden changes of acceleration $|\Delta a^b_k| = 2\pi f|\tilde{a}^b_k|$ at heel down.

Table II shows the values of $|\Delta a^b_k|$ at heel down for Wu's typical 60 kg individual, as inferred from our Fourier transforms of her Fig. 16-11; it also shows the values $x'^b_k$ of the center of mass location of leg and shank at heel down for a

TABLE II. Thigh and shank properties at heel down. Masses $m$ (in kg) and positions $x',z'$ relative to the body's center of mass (in m) were taken from Appendix A of [10]. Changes $|\Delta a_j|$ of acceleration (in m/s$^2$), on the time scales $\tau \lesssim 50$ ms, were computed from Fourier transforms of Fig. 11-16 of [12].

| | $m$ | $x'$ | $z'$ | $\|\Delta a_x\|$ | $\|\Delta a_z\|$ |
|---|---|---|---|---|---|
| Thigh | 5.7 | 0.08 | −0.3 | 15 | 6 |
| Shank | 3.5 | 0.2 | −0.6 | 11 | 6 |

typical individual, as given in Appendix A of Ref. [10].[1] We expect these numbers to vary, from one adult individual to another, by no more than a factor $\sim 2$.

By inserting the numbers from Table II into Eq. (28) and thence into Eq. (27), and adding terms in absolute value so as to get upper bounds, we obtain for the sudden (time scale $\lesssim 50$ ms) heel-down changes of the second time derivative of the person's quadrupole moment:

$$|\Delta \ddot{\mathcal{I}}_{xx}| \lesssim 35 \ \text{kg m}^2\text{s}^{-2}, \quad |\Delta \ddot{\mathcal{I}}_{yy}| \lesssim 25 \ \text{kg m}^2\text{s}^{-2}. \quad (29)$$

By then Fourier transforming Eq. (26), we obtain the following upper limit on the quadrupolar noise $|\tilde{h}_{\Delta a}|$ induced by the heel-down changes of thigh/shank acceleration:

$$|\tilde{h}_{\Delta a}| \lesssim \frac{9G|\Delta \ddot{\mathcal{I}}_{xx}|}{2(2\pi f)^5 L r^4}, \quad (30)$$

where $r$ is the distance to the nearest test mass.

During half a gait cycle (one heel down), the Fourier transform of the person's c.m.-induced noise is [Eqs. (13) and (15)]

$$|\tilde{h}_{\text{c.m.}}| = \frac{2G|\Delta \dot{F}|}{L r^3 (2\pi f)^6}, \quad (31)$$

where we have set the angle-dependent factor $|\alpha|$ to its representative value, $\sqrt{2}$. Near 10 Hz, the ratio of the quadrupolar noise (30) to this c.m. noise is

$$\frac{|\tilde{h}_{\Delta a}|}{|\tilde{h}_{\text{c.m.}}|} \lesssim \frac{9}{4}\,\frac{2\pi f}{r}\,\frac{|\Delta \ddot{\mathcal{I}}_{xx}|}{|\Delta \dot{F}|} \sim \frac{1}{10}\left(\frac{10 \ \text{m}}{r}\right)\left(\frac{f}{10 \ \text{Hz}}\right). \quad (32)$$

This is significantly less than one independent of the factor $\sim 2$ errors and variabilities of both noises. Thus, as was asserted in the Introduction, the dominant noise is caused by jerkiness in the person's c.m. motion.

### III. DOORS, FISTS, AND VEHICLES

We turn, now, to gravity gradient noise produced by the impulsive stopping of a massive, horizontally moving object—most especially a slamming door, a fist striking a wall, or a stopping automobile. [The impulsive stopping of vertical motion produces a much weaker signal than horizontal motion; cf. the discussion following Eq. (10).] Our analysis is a variant of that originally given by Spero [8] and reaches the same conclusions.

Since these impulsive events are not likely to occur repetitively and continually (by contrast with the gait cycles of human walking), they are more appropriately analyzed as a single impulsive gravitational-wave signal than as a stochastic noise. The standard formula for the amplitude signal to noise ratio $S/N$ produced by such an impulsive signal $h(t)$ in a LIGO interferometer is (e.g., Eq. (29) of [14] with a factor 2 correction)

$$\frac{S^2}{N^2} = \int_0^\infty \frac{4|\tilde{h}|^2}{S_h(f)} df, \quad (33)$$

where $S_h$ is the one-sided spectral density of the interferometer's total noise and $\tilde{h}$ is the Fourier transform of the signal $h(t)$.

The signal is given by

$$\frac{d^2 h}{dt^2} = \alpha \frac{GM\xi}{Lr^3} \quad (34)$$

[Eq. (11)], where $\alpha$ is the same angle-dependent factor as we met for a person's c.m. motion [Eq. (12)], $M$ is the object's mass, $\xi$ is its displacement while stopping, and $r$ is its distance mass from the nearest interferometer test mass.

For a slamming door or a fist striking a wall, it is the velocity $v = \dot{\xi}$ that changes suddenly, by some amount $\Delta v$. Correspondingly, the Fourier transform of the signal $h(t)$ is

$$|\tilde{h}| = \frac{GM|\alpha \Delta v|}{Lr^3 (2\pi f)^4}. \quad (35)$$

For the benchmark ''advanced'' LIGO interferometer, we can approximate the noise curve (Fig. 1) by $S_h = S_o[(f_o/f)^4 + (f_o/f)^{20}]$, where $S_o = 10^{-45}$/Hz and $f_o = 10$ Hz. Inserting this $S_h$ and expression (35) into Eq. (33) and integrating, we obtain

$$\frac{S}{N} \simeq \frac{1.2GM|\alpha \Delta v|}{2\pi L r^3 (2\pi f_o)^3 \sqrt{S_o f_o}} \simeq 1\left(\frac{M\Delta v}{5 \ \text{kg m/s}}\right)\left(\frac{10 \ \text{m}}{r}\right)^3. \quad (36)$$

Here we have used our representative value $\sqrt{2}$ for $|\alpha|$. This is the noise level discussed in the Introduction.

Turn to automobiles and other vehicles. Under normal (non-collisional) motion, the velocity of a vehicle cannot change significantly on a timescale of 50 ms. The acceleration, however, *can* so change, and will change most strongly when the vehicle comes to a stop, e.g. when parking.

The sudden change $\Delta a = \Delta \ddot{\xi}$ of acceleration, when the vehicle comes to rest, will produce the following Fourier transform of $h(t)$:

[1] Beware: biokinesiologists (influenced by the biomechanics literature) use different axis conventions from physicists: $y$ and $z$ are interchanged so their $y$ is vertical and $z$ is medial.

$$|\tilde{h}| = \frac{GM|\alpha\Delta a|}{Lr^3(2\pi f)^5}. \tag{37}$$

Inserting this $|\tilde{h}|$ and the advanced-interferometer $S_h(f)$ into Eq. (33) and integrating, we obtain

$$\frac{S}{N} \simeq \frac{GM|\alpha\Delta a|}{2\pi Lr^3(2\pi f_o)^4\sqrt{S_o f_o}} \simeq 1\left(\frac{M}{2\ \text{ton}}\right)\left(\frac{|\Delta a|}{0.6\ \text{g}}\right)\left(\frac{30\ \text{m}}{r}\right)^3. \tag{38}$$

Here we have used our representative value $\sqrt{2}$ for $|\alpha|$, and $g$ is the acceleration of gravity. This is the noise level discussed in the Introduction.

The gravitational signal from a slamming door, striking fist, or stopping vehicle will be mitigated to some modest extent by an opposite signal produced by the momentum deposited in the ''reaction mass'' (the wall, floor, and/or ground). However, as for human walking, the deposited momentum spreads over such a large spatial region in a time $1/2f \sim 50$ ms, that the mitigation will not be significant; cf. Sec. II B and the Appendix.

## IV. CONCLUSIONS

In this paper we have identified what we believe to be the dominant gravity gradient noise due to normal human activities; we have estimated its spectrum and its strength to within accuracies of a factor $\sim 3$; and we have discussed the implications of this noise for the size of the human exclusion zones around LIGO's test masses in the era, ca. 2010, of ''advanced'' interferometers. Until that era, human gravity gradient noise is not likely to be a serious issue for LIGO.

Our formulas and estimates can provide a basis for the design of the facilities of other earth-based gravitational-wave detectors.

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## APPENDIX: GREEN'S FUNCTION FOR SPREADING MOMENTUM IN FLOOR AND GROUND

When a time varying force $F_j(t)$ is applied to the surface of the earth at a location $\vec{x}'$, the time varying displacement that it produces in the ground at a location $\vec{x}$ is given by

$$\zeta_i(\vec{x},t) = \frac{\partial}{\partial t}\int_{-\infty}^{+\infty} g_{ij}^H(\vec{x},t-t';\vec{x}')F_j(t')dt'. \tag{A1}$$

Here $g_{ij}^H$ is the elastodynamic Green's function for a unit-step-function (Heaviside-function) force, $\vec{F}(t') = H(t')\vec{e}_j$. Seismologists focus on $g_{ij}^H$ rather than on the physicists' usual delta-function-sourced Green's function $g_{ij} \equiv \partial g_{ij}^H/\partial t$ because $g_{ij}^H$'s Heaviside steps at the various seismic propagation fronts are more easily visualized and compared with each other than $g_{ij}$'s delta-function spikes.

The Green's function $g_{ij}^H$ for a homogeneous medium (''homogeneous half space'') has been computed analytically by Johnson [13], up to a complicated integral; and Johnson has evaluated it numerically for several representative geometries; see his Figs. 2–4. Chao [15] has derived expressions for $g_{ij}^H$ when both force point and field point, $\vec{x}'$ and $\vec{x}$, are at the surface of a homogeneous half space; and Ma and Huang [16] have computed $g_{ij}^H$ for layered media.

Regardless of the nature of the medium, momentum conservation requires that

$$\begin{aligned} F_j(t) &= \frac{d}{dt}\int_{\mathcal{V}} \rho \frac{\partial \zeta_j}{\partial t} d^3x \\ &= \int_{-\infty}^{+\infty} \frac{d^3}{dt^3}\left[\int_{\mathcal{V}} \rho g_{jk}^H(\vec{x},t-t',\vec{x}')d^3x\right] F_k(t')dt', \end{aligned} \tag{A2}$$

where $\mathcal{V}$ is the entire volume of the medium. Since this must be true for every applied force, it must be that

$$\frac{d^3}{dt^3}\int_{\mathcal{V}} \rho g_{jk}^H(\vec{x},t-t';\vec{x}')d^3x = \delta(t-t')\,\delta_{jk}. \tag{A3}$$

Causality requires that $g_{ij}^H$ vanish everywhere for $t<t'$ and be nonzero for $\vec{x}$ arbitrarily near $\vec{x}'$ when $t-t'$ is arbitrarily small but positive; these facts, combined with Eq. (A3) imply

$$\int_{\mathcal{V}} \rho g_{ij}^H(\vec{x},t-t';\vec{x}')d^3x = \frac{(t-t')^2}{2}H(t-t')\,\delta_{ij}. \tag{A4}$$

Now consider the noise $h(t)$ produced in a gravitational-wave interferometer by a walking person, whose feet at location $\vec{x}'$ produce a horizontal force $F_j(t)$ on the floor and thence on the ground beneath the floor. By (i) taking three time derivatives of Eq. (22), with the floor and ground displacement $\vec{\zeta}$ expressed as an integral over the elastodynamic Green's function [Eq. (A1)], (ii) using force balance $d(M\dot{\xi}_j)/dt = -F_j$ for the floor and person, and (iii) setting $d^2F_i(t)/dt^2 = \Delta\dot{F}_i\delta(t)$, where $t=0$ is a time of sharp change of jerk at the beginning or end of the walking person's weight transfer (cf. Sec. II A 2), we obtain the following:

$$\frac{d^6h}{dt^6}=\frac{G}{L}\sum_A m_{Ai}\Delta\dot{F}_k\left\{\delta(t)\,\delta_{jk}\left(\frac{1}{|\vec{x}'-\vec{x}_A|}\right)_{,i'j'}-\frac{d^3}{dt^3}\left[\int_{\mathcal{V}}\rho g^H_{jk}(\vec{x},t;\vec{x}')\left(\frac{1}{|\vec{x}-\vec{x}_A|}\right)_{,ij}d^3x\right]\right\}. \tag{A5}$$

This equation exhibits the momentum-flow features discussed in the text following Eq. (22): The third time derivative of the Green's function $g^H_{jk}(\vec{x},t;\vec{x}')$ is significantly nonzero only in the expanding shell discussed in the text—a shell whose sharp outer edge travels at speed $c_P$ and fuzzy inner edge a bit slower than $c_S$; cf. Figs. 2–4 of Johnson [13]. Correspondingly, the contribution of the floor and earth to $h$ is confined to that shell. When that shell is small compared to the separation $|\vec{x}'-\vec{x}_A|$ between the person and the nearest test mass $A$, the double gradients in Eq. (A5) are nearly equal, and momentum conservation as embodied in Eq. (A3) guarantees that the two terms in Eq. (A5) (the person noise and the floor and ground noise) will nearly cancel. When the shell is comparable in size to the separation $|\vec{x}'-\vec{x}_A|$, the two terms will cancel partially but not strongly. When the shell is large compared to $|\vec{x}'-\vec{x}_A|$, the second term (floor and ground noise) will be negligible compared to the first (person noise).

Equation (A5), together with the explicit expressions for the Green's functions in Refs. [13,15,16], could be used to compute quantitatively the partial cancellation of person noise and floor and ground noise. We have not done so, since uncertainties elsewhere in our modeling are comparable to or larger than the errors in the above rough estimates.


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