response simulated by a finite element program (COSMOS /M). This single stage provides an attenuation of around 30 dB; the agreement between the model and the experimental measurements is extremely satisfactory.

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Off-Line Subtraction of Seismic Newtonian Noise

G. Cella

Abstract. A fundamental limit for the sensitivity of an Interferometric Gravitational Wave detector in the low frequency region is imposed by the effect of environmental mass density fluctuations. These fluctuations generate stochastic gravitational fields which couple directly to the apparatus test masses, bypassing seismic isolation systems. In this paper the results of a preliminary investigation on the possibility of reducing this kind of noise are reported. We focus on the feasibility of off-line noise subtraction. In this approach the mass density fluctuations are monitored by an appropriate set of measurement devices. The resulting signals are combined linearly with the output of the interferometer to obtain partial cancellation of the noise.

1 Introduction

Density mass fluctuations are generated continuously in the environment by a variety of mechanisms. Examples include atmospheric pressure fluctuations, infrastructure movements, human activities and seismic fluctuations of the ground.

These mass fluctuations are the sources of a gravitational field which, though very weak, couples directly to the test masses of an interferometric gravitational wave detector. The effect of this coupling on the sensitivity curve of a gravitational wave detector was estimated in a series of works [1–5]. The main result is that this source of noise could be relevant, given the planned sensitivity of the current generation interferometers, in the frequency band between 1 and 10 Hz. Below this range it is overwhelmed rapidly by seismic noise, above by thermal noise. For a concrete example see Fig. 1, where the more important source of noise for the VIRGO interferometer are compared.

All these estimates are normalized to the power spectrum amplitude of seismic motion, which, in some cases, is not known very well and probably overestimated [6]. An attempt to explore the feasibility of the reduction of this "Newtonian noise" is in our opinion worth doing, especially in the perspective of second generation cryogenic detectors. For these Newtonian noise could become the fundamental limitation in the low frequency range.

In the following we will focus on seismic generated Newtonian noise. Preliminary, unpublished results show that atmospheric generated Newtonian noise could also be relevant, in fact, the most relevant. We expect that it will be possible to carry out a similar analysis in this case as well.

Different strategies can be elaborated to reduce the effect of seismic originated Newtonian noise. The more obvious is the construction of a mold of appropriate depth around test masses, in order to isolate a sufficiently large ground volume from seismic motion. Accurate modelling is required to estimate the effectiveness of this

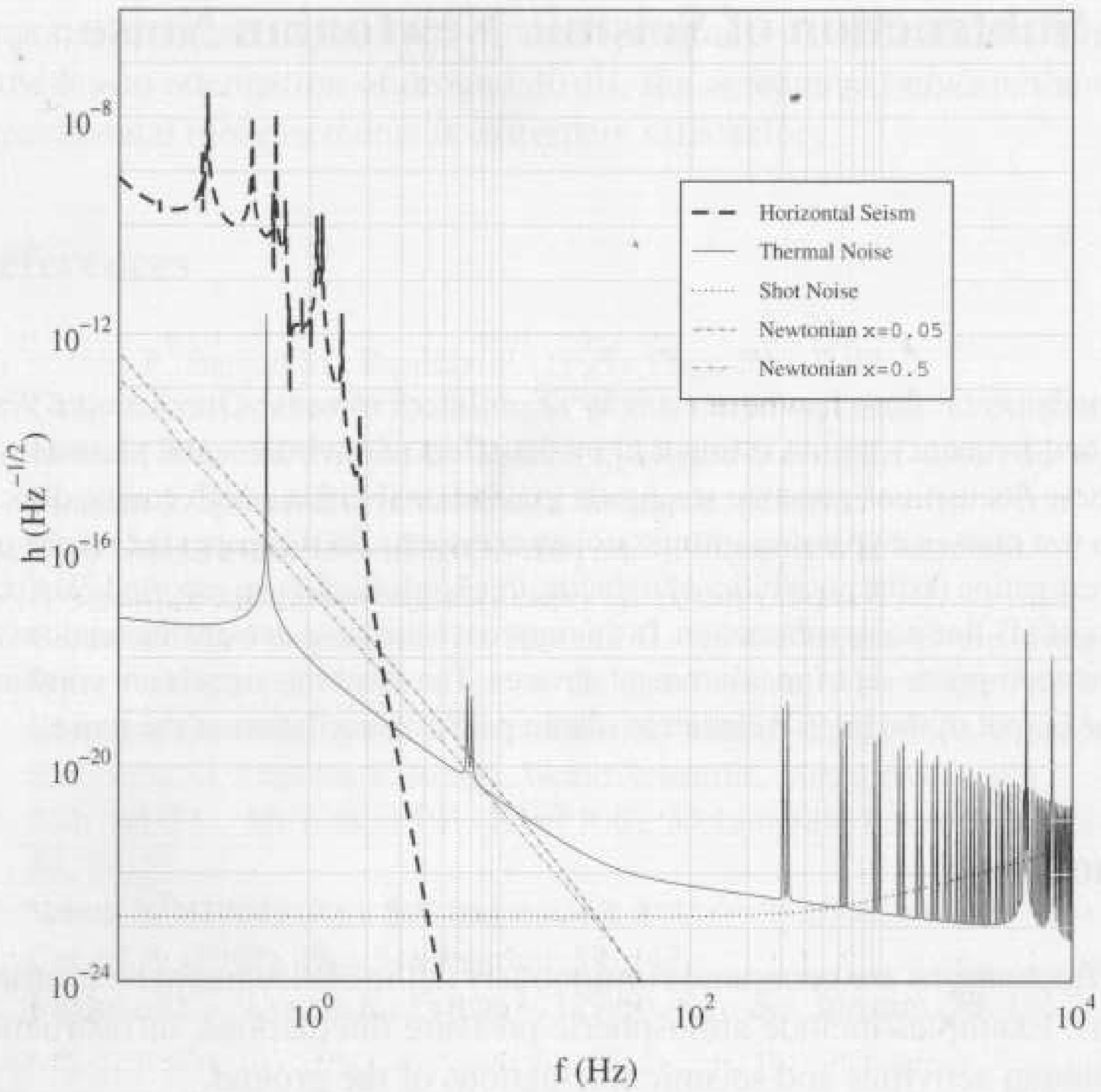

Fig. 1. Sensitivity curve for the VIRGO interferometer. The two different extimates for the Newtonian noise correspond to two different values of the ratio between transverse and longitudinal sound speed in the terrain considered

approach. This must take into account, for example, the effect of the diffraction of elastic seismic waves on the mold.

It is also possible to contemplate the construction of dynamic structures which could provide a screening effect. However this approach appears to be very tricky and, to obtain relevant effects, these dynamical structures would probably have to be very finely tuned.

Another important point is that these solutions require us to introduce permanent modifications in the apparatus, which should be avoided as much as possible in order not to add non-controlled, systematic effects.

In this preliminary stage of investigation it seems wiser to concentrate our attention on a third approach, which does not require apparatus modifications. This is based on the possibility of monitoring the environmental sources of Newtonian noise and correcting "off-line" the output of the experiment using the information obtained.

2 Off-line subtraction: generalities

Newtonian noise, seen as a random process in the time domain, is a linear function of the mass density fluctuations in the environment. Suppose that we know, at each

point in time, the displacement of each point in the ground from its equilibrium position $\mathbf{u}(\mathbf{x}, t)$ The force experienced by an isolated test mass in $\mathbf{x}_0$ can be written as

$$\mathbf{F}(t) = G \int \varrho(\mathbf{x}) \frac{\mathbf{x} - \mathbf{x}_0}{|\mathbf{x} - \mathbf{x}_0|^3} \nabla \cdot \mathbf{u}(\mathbf{x}, t) dV . \tag{2.1}$$

In the general case we can write, for the Newtonian part of the output signal of an interferometer, $N(t) = \widetilde{(N|B)}$, where $|B)$ is the state of the ground (explicitly, the displacement field $\mathbf{u}$) and $(N|$ some linear operator which depends on the geometry of the interferometer. Suppose now that we place a measure instrument, for example, an accelerometer, at a point $\mathbf{x}$ of the ground. The part of the instrument output correlated with seismic motion will also be a linear function of the ground fluctuations, which we can write in the form $\widetilde{(a(x)|B)}(t)$.

More generally, we write the output of the interferometer in the form

$$\widetilde{H}(t) = \widetilde{(N|B)}(t) + \widetilde{h}(t), \quad \text{with} \quad E[\widetilde{(N|B)}\widetilde{h}] = 0 \tag{2.2}$$

with the assumption that both $\widetilde{(N|B)}$ and $\widetilde{h}$ are stationary, zero-mean stochastic processes. The $E[\cdots]$ average must be appropriately defined. For example, it could be the usual $T \to \infty$ limit of the ensemble average over strips of length T. In Eq. (2.2) $\widetilde{h}$ represents the fraction of the output uncorrelated with seismic motion, as, for example, thermal noise, shot noise, gravitational wave signals. In the same way we write the output of an accelerometer as

$$\widetilde{A}_i(t) = \widetilde{(a_i|B)}(t) + \widetilde{\eta}_i, \quad \text{with} \quad E[\widetilde{(a_i|N)}\widetilde{\eta}_i] = 0 \tag{2.3}$$

and, in this case, $\widetilde{\eta}$ represents the intrinsic instrument noise. We assume that, if the output of n different accelerometers is known, the maximum reduction of Newtonian noise could be obtained by constructing a "subtracted signal" H_S that can be written as

$$\widetilde{H}_S(t) = \widetilde{H}(t) - \sum_i \int_{-\infty}^{t} \widetilde{w}_i(t - t')(N|a_i)\widetilde{A}_i(t')dt' \tag{2.4}$$

or, in the frequency domain,

$$\begin{aligned} H_S(\omega) &= H(\omega) - \sum_i w_i(\omega)(N|a_i)A_i(\omega) \\ &= H(\omega) - \sum_i w_i(\omega)(N|a_i)\left[(a_i|B)(\omega) + \eta_i(\omega)\right] . \end{aligned} \tag{2.5}$$

Here the $w_i(\omega)$ are a set of functions (one for each accelerometer) which must be calculated. The factor $(N|a_i)$ is not a stochastic process, and was introduced for convenience. It represents the coupling of the fluctuations measured by a given accelerometer to the Newtonian noise, and depends only on the interferometer geometry and on the accelerometer position relative to it. With its aid Eq. (2.5) suggest that what we are doing is subtracting from H the Newtonian noise signal due to the density fluctuation fraction controlled by our acceleration measurements.

It is also evident that, in the limit of an infinite number of accelerometers, it is always possible to write $(N|B) = \sum_i w_i(N|a_i)(a_i|B)$, so that, in this case, the subtraction of Newtonian noise should be complete in the absence of instrumental noise.

If the noise is stationary, and if the processes $(a_i|B)$ are Gaussian, the subtraction procedure (2.5) is optimal. The ω dependency of the weights w_i is due to the fact that the optimal signal to subtract encodes the information about seismic dynamics, and this means that it must have memory.

In order to determine the functions w_i we have to fix a well-defined quantity that we want to minimize. In the stationary case it is natural to use the frequency integral of the interferometer output power spectrum density, weighted by a function $g(\omega)$ which selects the frequency band of interest,

$$\Gamma[w_1, \cdots, w_n] = \int g(\omega)\overline{|H_S|^2}d\omega. \qquad (2.6)$$

As indicated, this is a (quadratic) functional of the unknown functions $w_i(\omega)$. In order to simplify our expression we now specialize to the case $g(\omega) = \delta(\omega - \omega_0)$, so that the expression to be minimized is

$$\Gamma = f_i f_j^\star(C_{ij} + R_{ij})w_i(\omega_0)w_j^\star - f_i^\star(N_i + Z_i)w_i^\star(\omega_0) - f_i(N_i + Z_i)^\star w_i(\omega_0). \qquad (2.7)$$

Summation over a repeated index, which labels the accelerometers, is understood. Each quantity which appears in this expression admits a simple interpretation. The factor $f_i = (N|a_i)$ has been described earlier. The array $C_{ij} = E[(a_i|B)(B|a_j)]$ is simply the statistical correlation between the outputs of i-th and j-th accelerometers, in the absence of instrumental noise. To obtain the correlation between the outputs of two real instruments we must add the $R_{ij} = E[\eta_i\eta_j^\star]$ array, which is the correlation of the intrinsic noises of accelerometers i and j.

The vector $N_i = E[(N|B)(B|a_i)]$ is the statistical correlation between the fraction of the interferometer output and of the i-th accelerometer output of seismic origin. Finally the vector $Z_i = E[\widehat{H}\eta_i^\star]$ is the correlation between the intrinsic noise of the i-th instrument and the fraction of the interferometer output uncorrelated with seismic origins.

Minimizing Γ we easily find the optimal weights

$$w_i^{\text{opt}}(\omega_0) = f_i^{-1}(C_{ij} + R_{ij})^{-1}(N_i + Z_i)^\star \qquad (2.8)$$

and the reduction of noise power spectrum in ω_0

$$|H|^2 - |H_S|^2 = (C_{ij} + R_{ij})^{-1}(N_i + Z_i)(N_j + Z_j)^\star. \qquad (2.9)$$

This last expression is a nonlinear function of the position and orientation of each accelerometer. Our objective is to optimize these parameters for a given number of instruments, to understand what is the best result we can achieve.

The quantities C_{ij} and N_i depend on the properties of seismic noise, which are connected in turn to the dynamics of the background motion. While it is relatively easy to measure C_{ij}, to obtain experimental information on N_i we need the sensitivity of the complete interferometer. However both C_{ij} and N_i can be calculated starting from a theoretical model of the ground seismic motion. Some of these calculations can be found in [3], and a detailed account will be published elsewhere [7, 8]. For what concerns the terms connected with instrumental noise we note that the non-diagonal part of the R_{ij} matrix and the vector Z_i should be negligible.

An intuitive understanding of the result of the optimization procedure can be obtained by looking at Eq. (2.9). In order to maximize this expression we can try to move all the accelerometers to the position which is maximally coupled to Newtonian noise. In this way the N_i terms grow. However the accelerometers cannot become too near to each other, because in this case the C_{ij} terms grow as well. The optimal configuration is obtained by balancing these two factors. Note that if, the intrinsic noise R_{ij} is large, the C_{ij} terms become less important, and all the accelerometers will try to cluster in similar positions and orientations, in such a way as to improve the statistics.

3 Some results

We studied Eq. (2.9) numerically, finding the optimal configurations for a fixed number of accelerometers. The starting point was knowing C_{ij} and N_i as functions of the positions of the accelerometers. Next we minimized numerically the optimal weight subtracted power spectrum by adjusting the accelerometer positions. In Fig. 3 we show the convergence of the mininization algorithm, plotting the relative power spectrum reduction against the iterative procedure step. The three different curves are the results for three different initial conditions. In particular the case with the best initial condition corresponds to a regularly spaced lattice of accelerometers. It is apparent that this configuration is also quite suboptimal. It is also evident that the final result is independent on the initial condition. In some cases (large number of accelerometers) this result can be achieved, avoiding local minima, with a simulated annealing procedure.

In Fig. 3 we plot the results obtained for two simple models. The first is the model used by Saulson to obtain an early estimate of Newtonian noise [1]. Its main assumption is that seismic motion can be modelled by a partition of the ground into cubic cells which oscillate coherently, but are completely uncorrelated to each other. The second is a slight modification of Saulson's model (which we will call the coherent model) which takes mass conservation into account. From our point of view this means that the motions of nearby cells are no longer uncorrelated [3].

A first observation is that the subtraction procedure is less effective at higher frequencies. This fact has a simple explanation, because the coherence length of the seismic motion (the cell dimension of Saulson's model) is proportional to the inverse of the frequency. In order to control a given volume at some specified level, we need a number of accelerometers which grows as the third power of the frequency.

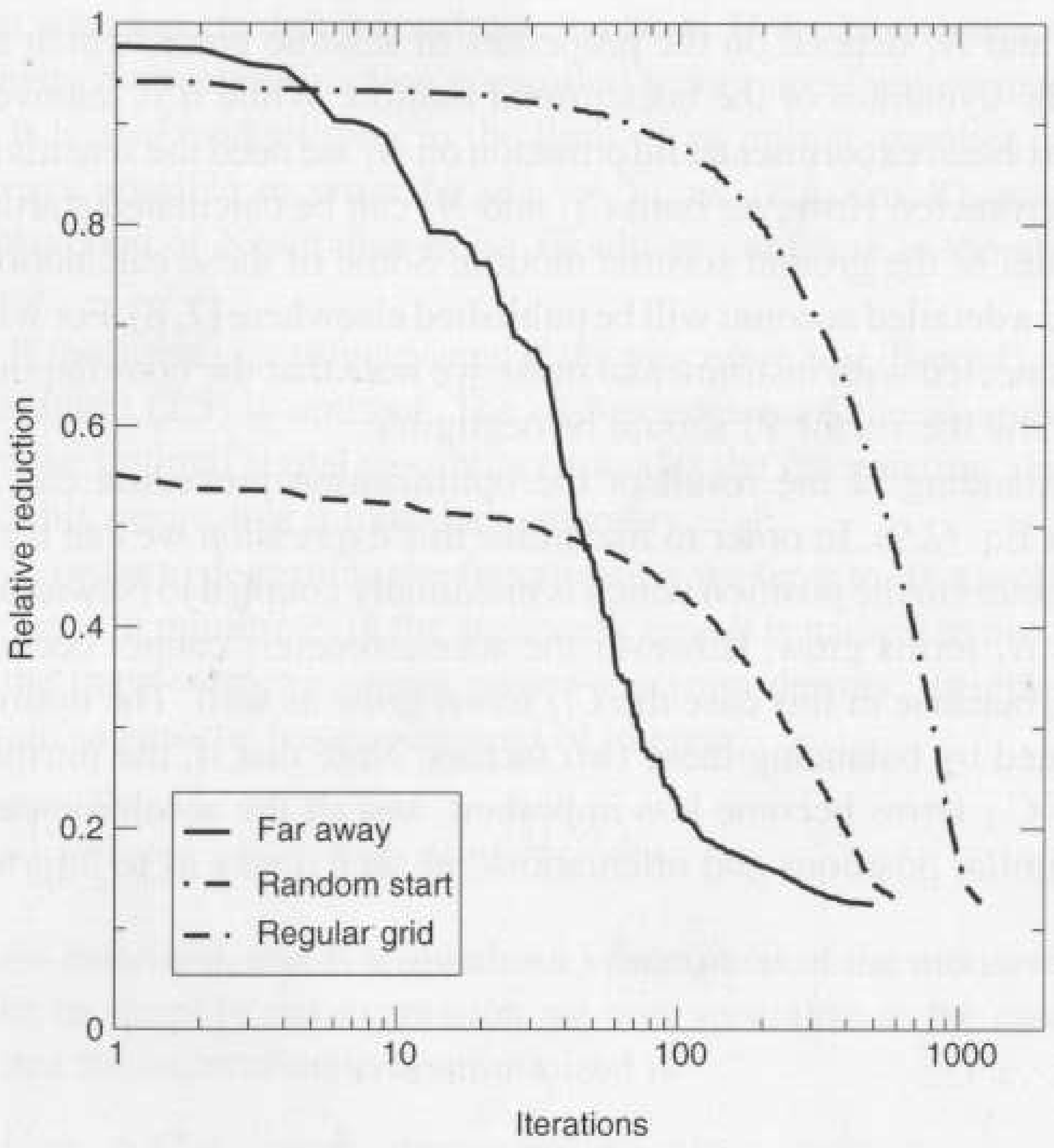

Fig. 2. Convergence of the algorithm for the minimization of the best weight subtracted power spectrum. The position of the accelerometers is adjusted, starting from some initial conditions, until the relative reduction of power spectrum converges. Organization of the accelerometers into a somewhat regular configuration can be observed. This configuration is model dependent and also frequency dependent

Another point is that we can use only accelerometers located on the surface without making worse the overall performance of our procedure, as long as the number of accelerometers is not too big (less than 40 for the Saulson's model). Note that the importance of underground measurements is greater for Saulson's model. This is a consequence of the fact that, in this case, surface measurements give us no information at all on the dynamics below the level of the first cell.

The performance of the subtraction procedure is not exceptional: for a reasonable number of accelerometers we can expect a relative reduction of an order of magnitude. We can see that a more coherent seismic dynamics improves the result, and that the two simple models we have used probably underestimate the coherence of the real dynamics. In addition they do not model the contributions to Newtonian noise due to the surface discontinuity, and it turns out that these are the most relevant ones.

We expect to obtain better results with more realistic models. We have evaluated C_{ij} and N_i for the elastic wave model used in [3] to predict Newtonian noise amplitude and extensive numerical simulations are currently in progress. However we do not expect that a relative reduction of Newtonian noise amplitude of greater than two orders of magnitude can be obtained.

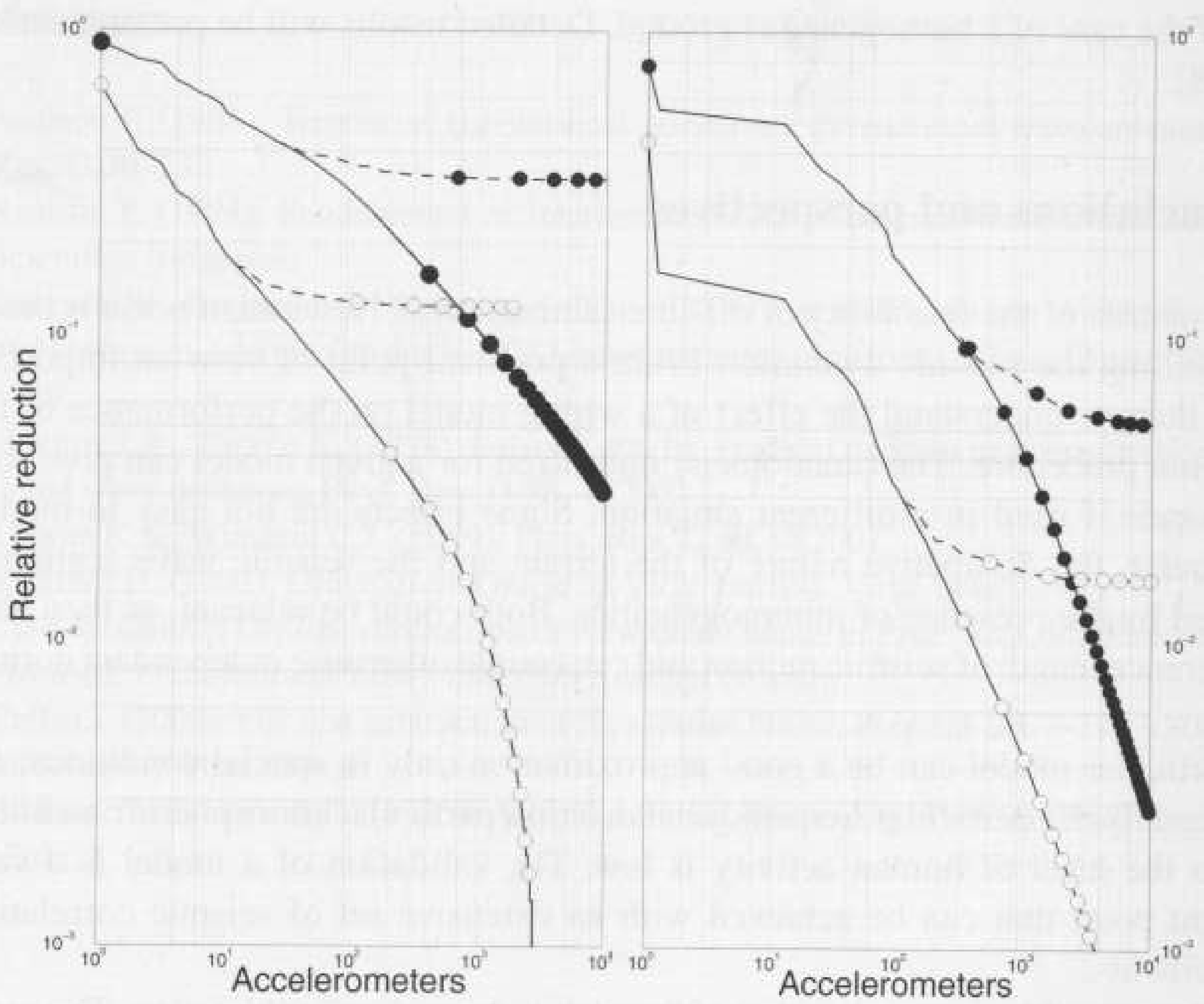

Fig. 3. Relative reduction of noise amplitude for Saulson's model (left) and the coherent model (right). The empty circles are for $f_0 = 1\,\text{Hz}$, the filled for $f_0 = 4\,\text{Hz}$. The plots with dotted lines were obtained by constraining the accelerometers to the surface

There is a general comment that can be made about the calculation of the functions C_{ij} and N_i. For any reasonable model of background seismic motion in the frequency range considered, it is important to consider the effect of dissipation on elastic modes. The main consequence is a modification of the correlation functions C_{ij} which becomes exponentially damped for large separation between points, with a finite coherence length ξ. Experimental measurements lead to the conclusion that ξ could also be of the order of magnitude of wavelength. This means that the local seismic noise level in the frequency band investigated is strongly influenced by local sources (wind, human activities, scattering processes) and an important assumption is that the frequency spectrum of these sources is wide-band.

In other words the typical expression for the amplitude of a given elastic mode, in the presence of dissipative effects, can be written as the convolution between a model dependent kernel and a source strength function. We can normalize all quantities to this unknown source strength if the convolution integral is dominated by the poles of the kernel. If this is not the case, then the structure of the excitation process becomes important (imagine, for example, a very narrow peak in the spectral distribution of the forces connected to the wind) and must be modeled.

By using this assumption the effect of dissipation can be included in the model for seismic dynamics and the seismic correlation functions can be evaluated in closed form for the case of a homogeneous ground. Detailed results will be presented elsewhere [8].

4 Conclusions and perspectives

The estimation of the feasibility of off-line subtraction of Newtonian noise is based on modellling the seismic dynamics. From a practical point of view an important issue is that we understand the effect of a wrong model on the performance of the subtraction procedure. The functions w_i optimized for a given model can give poor performance if used in a different situation. Some effects are not easy to model, in particular, the dissipative nature of the terrain and the seismic wave scattering generated by the presence of inhomogeneities. Both could be relevant, as they alter the coherence length of seismic motion and can couple otherwise independent normal modes.

A particular model can be a good approximation only in special conditions, for example, only in a particular frequency band, or in a particular atmospheric condition, or when the level of human activity is low. The validation of a model is a very important point that can be achieved with an extensive set of seismic correlation measurements.

Another possible approach is that of "model–independent" subtraction. The main point is the optimization of the functions w_i by the direct experimental estimation and minimization of the Newtonian noise power spectrum. This can be seen as the training problem for a M–adaline network with delays [9], and can be solved with standard adaptive techniques. Numerical simulations are in progress to test the effectiveness of this method. Note that experimentally it is easy to adapt the functions w_i, but it is not easy to adapt the positions and the orientations of the accelerometers. This means that a good theoretical model is in any case important in order to guess a good configuration for the instruments.

Our method for the determination of the optimal subtraction procedure is based on the assumption that the stochastic processes in which we are interested are stationary. If this is not true, then the main point is the precise determination of the quantity which we want to minimize, as the noise power spectrum is longer a useful concept. In some simple cases our formalism requires only minor modifications, for example, the redefinition of the $E[\cdots]$ average. Further investigations are needed.

As a final comment we stress that our method is independent on the particular instrument used to monitor density fluctuations. We have used accelerometers, but there are many alternative possibilities which can be investigated using the same formalism. A good instrument has a strong overlap N_i with the seismic modes which are maximally coupled to Newtonian noise, and an accurate choice can allow us to obtain major improvements.

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