Advanced Virgo : cryostat designs
Some thermal aspects

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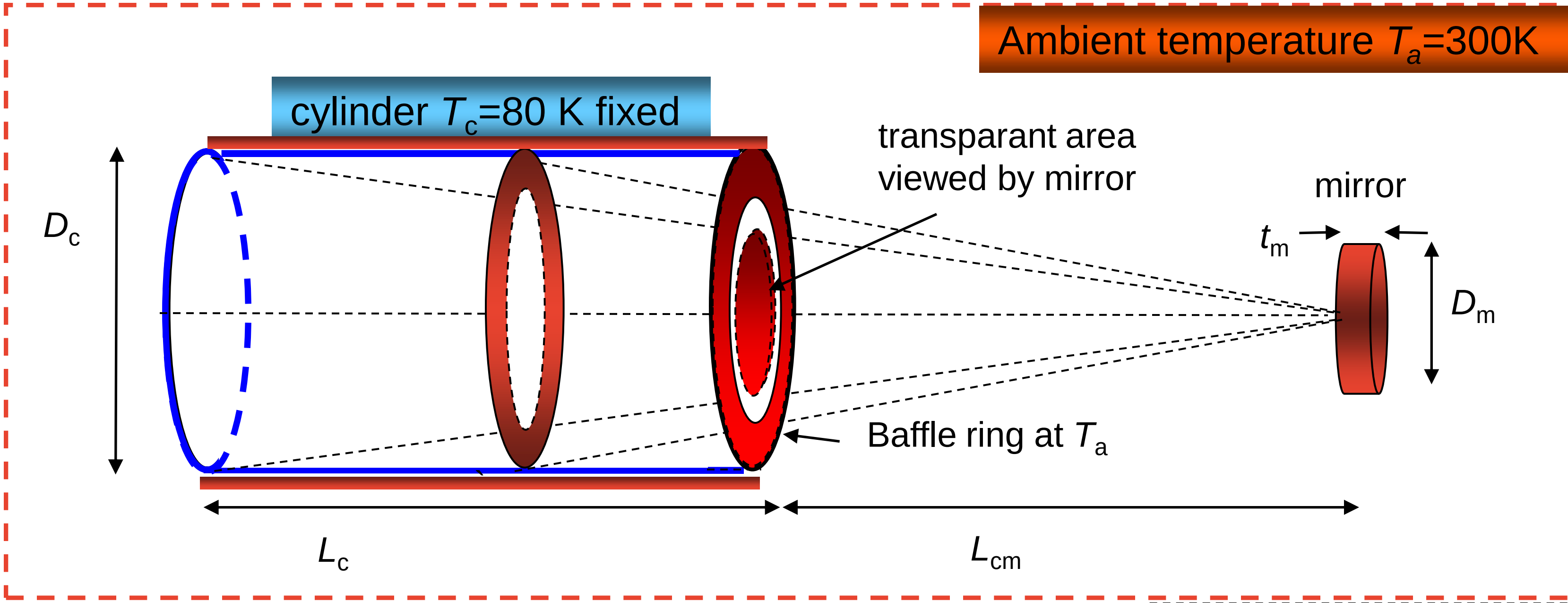
Gravitational Waves group Amsterdam

Nikhef-VU-UvA

Cascina, Februari 3, 2009

Simplified geometry



Cylinder:

D_c = 0.65 or 1.0 m

L_{vc} = 2.5 m

$A_c = \pi D_c L_c$

Exterior: isolated

Distance:

L_{cm}= 2.5 m

Mirror:

D_m = 0.4 m

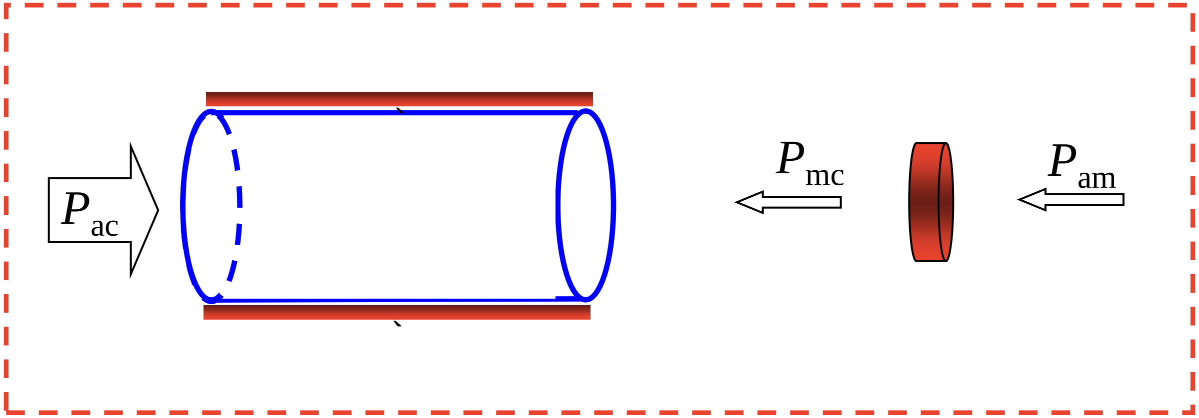
t_m = 0.1 m

$A_m = \pi(D_m t_m + D_m^2/2)$

$R_m = D_m/2$

To be estimated

- Cryostat power consumption P_{ac} due to radiation
- Radiative heat flow $P_{\mathrm{mc}} = P_{\mathrm{am}}$ from and to mirror (in equilibrium)
- Mirror temperature distribution, without or with baffle(s)
- Effect T-distribution on mirror optics (optical path)
- (Design of baffle heating)



Thermal material properties involved

Mirror:	Thermal conductivity	: k	= 1.36 W/Km
	Thermo-optic coefficient	: dn/dT	= 0.98 10-5 K^{-1}
	Thermal Expansivity	: α	= 0.54 μm//Km
	Emissivity:	: ε_m	= 0.89
Cryostate	Emissivity:	: ε_c	= 0.1 (initial)
			= 0.2 (nominal, t< 2 years)
			= 0.9 (worst “icing” case)
Baffle	“	: ε_b	= 0.12
Ambiance	“	: ε_{amb}	= 1.0 (“black”)

Reflection of radiation : diffuse, i.e. non-specular

Modeling method / tools 1

Mirror temperature distribution : FEM (COMSOL)

Domain : $\Delta T=0$

Boundary : $\nabla.\mathbf{n}=J$ (outward conducted power flux)

Radiation heat exchange

general : FEM

Isothermal mirror equilibrium : analytical approximation

cooling power: $$P_{ca} \cong \varepsilon_c F_{ca} A_c E_{ac} \quad , \quad E_{ac} = \sigma\left(T_a^4 - T_c^4\right)$$

ambiance to mirror: $$P_{ma} \cong \varepsilon_m F_{ma} A_m E_{am}$$

mirror to cryostat: $$P_{mc} \cong \frac{\varepsilon_c \varepsilon_m F_{mc} A_m E_{mc}}{1 - F_{mc}(1-\varepsilon_c)(1-\varepsilon_m)F_{cm} - F_{cc}(1-\varepsilon_c)}$$

solving T_m (equilibrium): $P_{mc} = P_{am}$

Modeling method / tools 2

View factor calculation

general : FEM (COMSOL)

2 simple bodies ($A_m << A_c$) : analytical approximation:

general: $A_1F_{12}=A_2F_{21}$

$$1-F_{ca} \cong F_{cc} = 1+\frac{L_c}{D_c}-\sqrt{1+\frac{L_c^2}{D_c^2}}$$

$$1-F_{ma} = F_{mc} \cong \frac{\pi D_m^4/4}{A_m}\left(F^{cc}(D_m, D_c, L_{cm}) - F^{cc}(D_m, D_c, L_{cm}+L_c)\right)$$

with F^{cc}: view factor between two circular faces 1 and 2 at distance d:

$$F^{cc}(D_1, D_2, d) = \frac{1}{2}\left(X-\sqrt{X^2-4\left(\frac{D_2}{D_1}\right)^2}\right), \quad X = 1+\left(\frac{2d}{D_1}\right)^2+\left(\frac{D_2}{D_1}\right)^2$$

Modeling method / tools 3

Mirror geometrical properties change (mirror initially flat)

mirror thickness change due to thermal expansion:

$$\Delta t_m(r) \cong \alpha \int_0^{t_m} \Delta T(z,r)dz$$

mirror surface radii of curvature: FEM mechanical analysis

Thermal lensing

change in optical path:

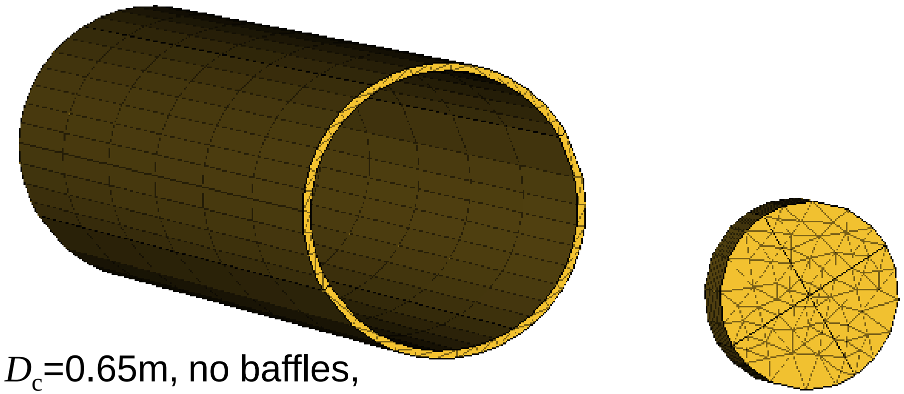
$$\Delta s(r) = (n-1)\Delta t_m(r) + \frac{dn}{dT}\int_0^{t_m} \Delta T(z,r)dz$$

$$\cong \frac{dn}{dT} t_m \left(\Delta T(t_m, r) - \Delta T(0,r)\right)$$

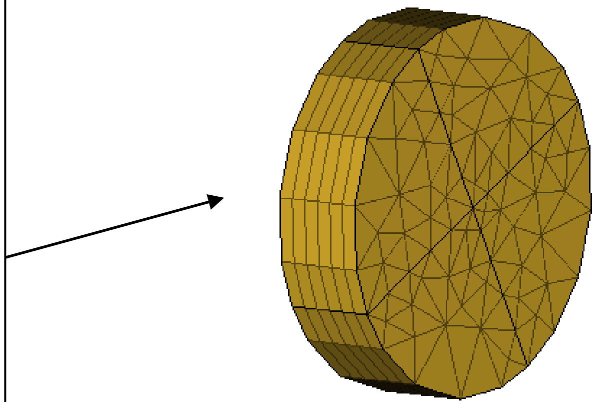
radius of curvature of equivalent isothermal lens (one side flat):

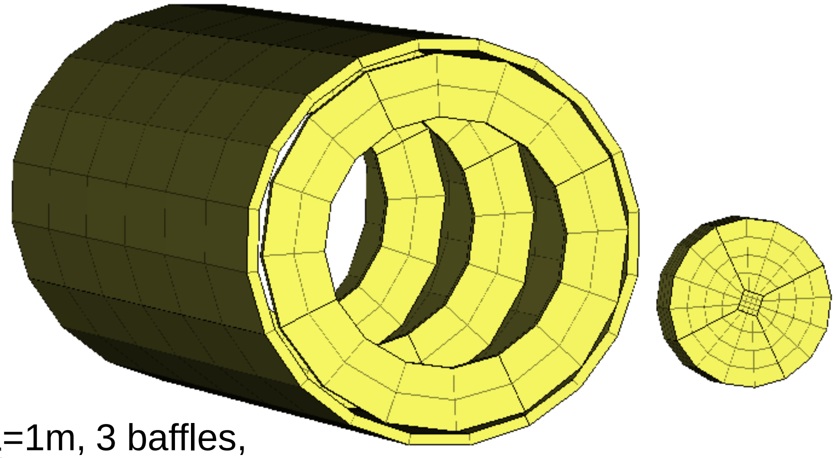
$$R_{thermo-optic} = \frac{(n-1)R_m^2}{2\left(\Delta s(R_m) - \Delta s(0)\right)}$$

FEM models (thermal & thermo-mechanical)

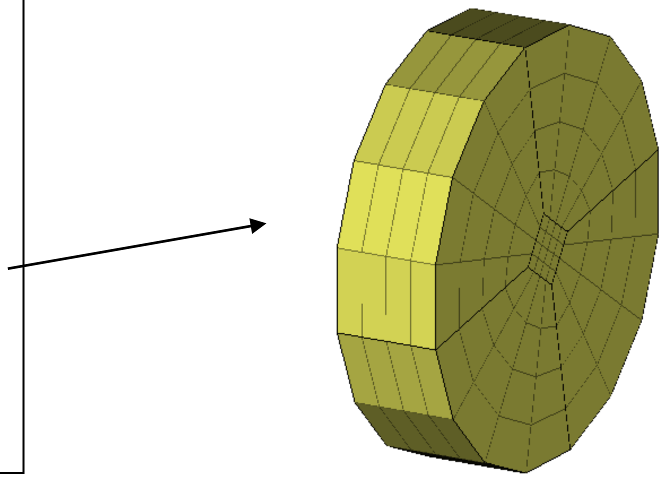


D_c=0.65m, no baffles,
linear prism elements





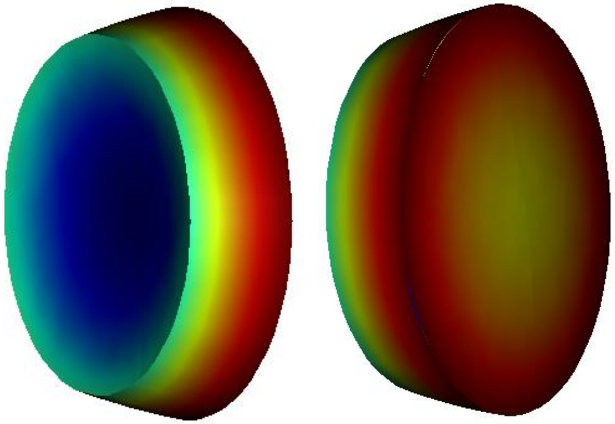
D_c=1m, 3 baffles,
quadratic hex elements



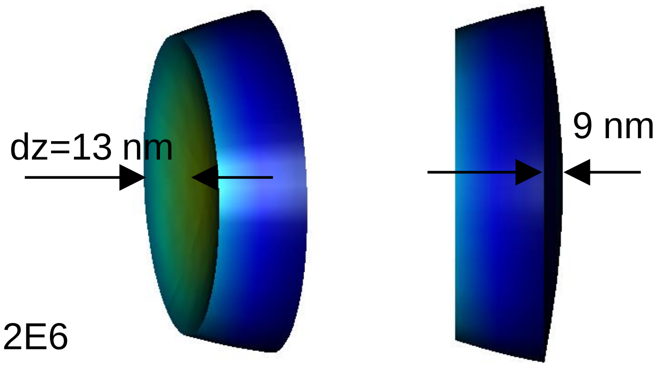
Results small cryostate (ε_c=0.2), no baffles

Cryostat	power	P_{ac}	180	W
	Self-irradiance	F_{cc}	0.87	
Mirror	view factor	F_{mc}	0.0123	
	power	P_{mc}	0.22	W
	Av. cooling	ΔT_m	0.23	
	front-back	ΔT_{fb}	0.10	K
	mid-edge	ΔT_{me}	0.06	
	thickness	$\Delta t_m(R_m) - \Delta t_m(0)$	4	
displacement differences	front	$\Delta z_f(R_m) - \Delta z_f(0)$	13	nm
	back	$\Delta z_b(R_m) - \Delta z_b(0)$	9	
Radius of curvature	front surface	R_{front}	1500	
	back surface	R_{back}	2200	km
	refractive	$R_{thermo\text{-}optic}$	120	

Front Back



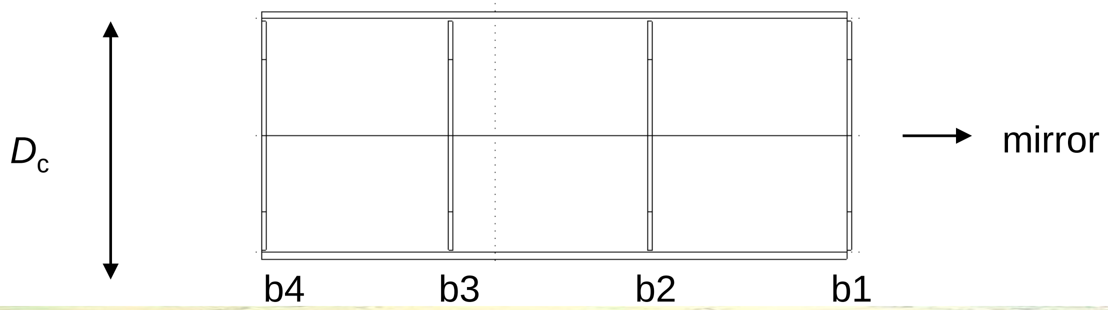
Temperature profile (299.70 – 299.86 K)



Enlargement factor : 2E6

Summary results for ε_c=0.2

$D_c(m)$	baffles	P_c(W)	P_m(W)	ΔT_m(K)	$R_{thermo\text{-}optic}$(km)
0.65	no	180	0.42	0.21	120
1.0	no	370	0.8	0.43	60
	b1	320	0.24	0.12	220
	b1+b2	340	0.23	0.11	250
	b1…..b3	350	0.31	0.16	170
	b1…..b4	305	0.40	0.19	100
	b1+b4	260	0.44	0.21	120



Cryostat power for several emissivities ε_c

	E_c:	*0.1*	0.2	0.9
$D_c(m)$	baffles	P_c(W)		
1.0	no	245	370	675
	b1	221	320	530
	b1+b4	190	260	370
0.65	no	130	180	295

Final remarks & conclusion

• All radii of curvature are larger than 100 km and exceed by far those predicted for beam power absorption (see van Putten et al.)
Conclusion: no mirror optics problems expected for any proposed design

• Lowest cryostat power is obtained using (only) 2 baffles on either side

• Including the recoil mass into the models will result into a smaller radial temperature gradient, and consequently, into even larger radii of curvature