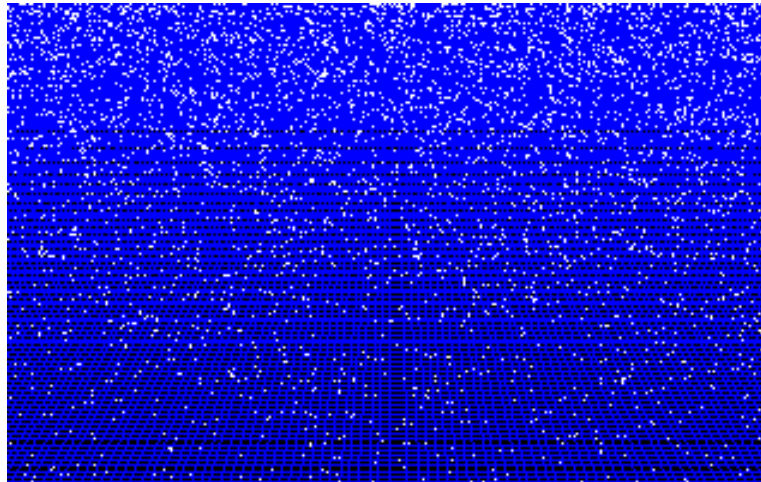


# Energie

## *FEW cursus*



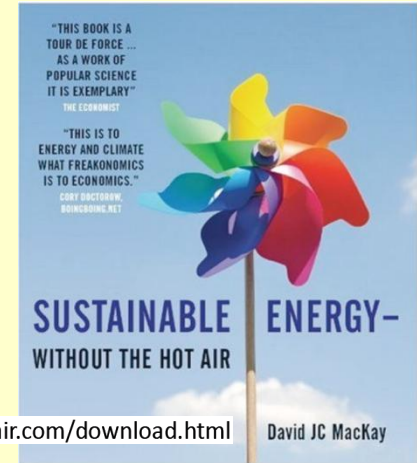
**Jo van den Brand en Jacco de Vries**

*[www.nikhef.nl/~jo/energie](http://www.nikhef.nl/~jo/energie)*

**9 mei 2014**

# Inhoud

- **Jo van den Brand**
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  - 0620 539 484 / 020 598 7900, Kamer T2.69
- **Jacco de Vries**
  - Email: [jdevries@nikhef.nl](mailto:jdevries@nikhef.nl)
- **Beoordeling**
  - Huiswerk (20%), scriptie (20%), tentamen (60%)
- **Boeken**
  - Energy Science, John Andrews & Nick Jelley
  - Sustainable Energy – without the hot air, David JC MacKay
  - Elmer E. Lewis, Fundamentals of Nuclear Reactor Physics
- **Inhoud van de cursus**
  - Week 1 Motivatie, exponentiële groei, CO2 toename, broeikaseffect, klimaat
  - Week 2 Energieverbruik: transport, verwarming, koeling, verlichting, landbouw, veeteelt, fabricage
  - Week 3 Kernenergie: kernfysica, splijting
  - Week 4 Kernenergie: reactorfysica
  - Week 5 Kernfusie
  - Week 6 Energie, thermodynamica
    - Entropie, enthalpie, Carnot, Otto, Rankine processen, informatie
    - Energiebronnen: fossiele brandstoffen (olie, gas, kolen), wind, zon (PV, thermisch, biomassa), waterkracht, geothermisch
  - Week 7 Fluctuaties: opslag (batterijen, water, waterstof), transport van energie, efficiëntie
    - Energie: scenario's voor Nederland, wereld, fysieke mogelijkheden, politiek, ethische vragen, economische aspecten



Gratis te downloaden

# Kernreactor

Stabiel bedrijf vereist *multiplicatiefactor*  $k = 1$ : per reactie moet gemiddeld 1 neutron weer een nieuwe kernsplijting induceren

Subkritisch (superkritisch):  $k < 1$  ( $k > 1$ )

Regelstaven van cadmium (of boron) absorberen neutronen en zorgen dat de reactor precies kritisch ( $k = 1$ ) blijft

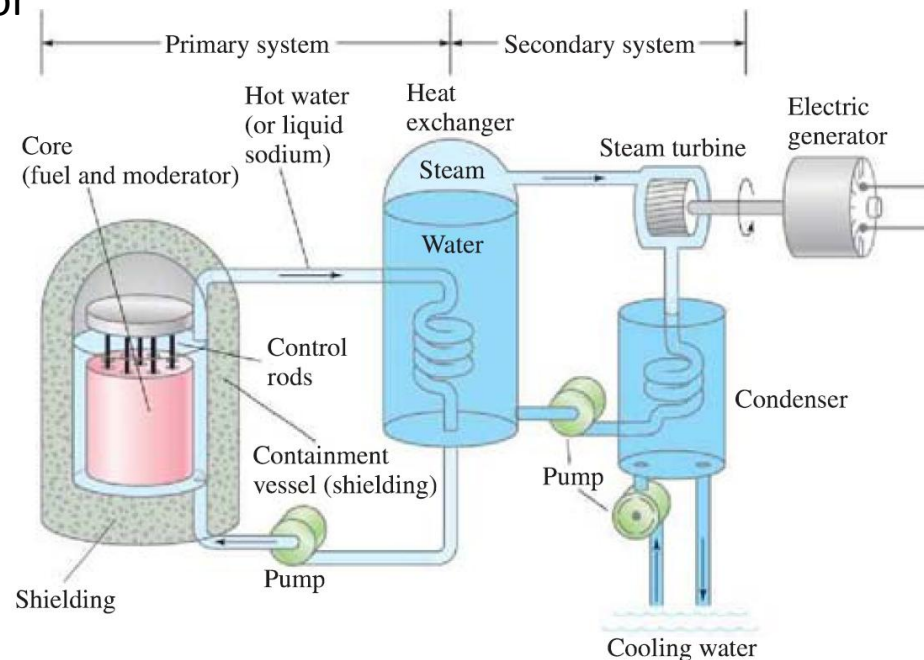
Regeling is enkel mogelijk dankzij een kleine fractie (1%) vertraagde neutronen afkomstig van kernverval met levensduur van enkele seconden

Reactor voor onderzoek: neutronenbron voor productie van isotopen

Reactor voor productie van energie

Verrijkt uranium van 2 – 4%

Water of vloeibaar zout onder hoge druk



# Vier-factoren formule

Vermenigvuldigingsfactor  $k_{\infty}$  kan inzichtelijk gemaakt worden

Er geldt 
$$k_{\infty} = \frac{\text{neutron productie door splijting in generatie } i}{\text{neutron absorptie in generatie } i-1}$$

Fast fission factor 
$$\varepsilon = \frac{\# \text{ snelle neutronen geproduceerd door alle splijtingen}}{\# \text{ snelle neutronen geproduceerd door thermische splijtingen}}$$

Resonance escape probability 
$$p = \frac{\# \text{ neutronen die thermische energie bereiken}}{\# \text{ snelle neutronen die met slow down beginnen}}$$

Thermal utilization factor 
$$f = \frac{\# \text{ thermische neutronen geabsorbeerd in fuel}}{\# \text{ thermische neutronen geabsorbeerd in alles}}$$

Reproduction factor 
$$\eta_T = \frac{\# \text{ snelle neutronen geproduceerd in thermische splijting}}{\# \text{ thermische neutronen geabsorbeerd in de fuel}}$$

Vier-factoren formule

$$k_{\infty} = \varepsilon p f \eta_T$$

|   |                                                          |                                                               |
|---|----------------------------------------------------------|---------------------------------------------------------------|
| F | $n \rightarrow (1) \rightarrow$                          | $\rightarrow \varepsilon n$                                   |
|   | $\uparrow \varepsilon p f \eta_T n$                      | $\downarrow$                                                  |
| I | $(4) \leftarrow \varepsilon (1-p)n$                      | $\leftarrow (2)$                                              |
|   | $\uparrow$                                               | $\downarrow$                                                  |
| T | $\leftarrow \leftarrow (3) \leftarrow \varepsilon p f n$ | $\leftarrow \varepsilon p n \rightarrow \varepsilon p (1-f)n$ |
|   | $\uparrow$                                               | $\downarrow$                                                  |
|   | Fuel                                                     | Moderator                                                     |

# Effectieve vermenigvuldigingsfactor

Effectieve vermenigvuldigingsfactor

$$k_{\text{eff}} = \frac{\text{neutron production from fission in one generation}}{\text{neutron absorption in the preceding generation} + \text{neutron leakage in the preceding generation}}$$

$$k_{\text{eff}} = k_{\infty} \mathcal{L}_f \mathcal{L}_t$$

Fast non-leakage probability

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

Thermal non-leakage probability

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

Totale “non-leakage” waarschijnlijkheid hangt af van temperatuur van koelmiddel via een negatieve temperatuure coefficient

Als temperatuur stijgt, dan zet het koelmiddel uit. Dichtheid van de moderator wordt kleiner; neutronen leggen grotere afstand af tijdens slow-down.

Zes-factoren formule

$$k_{\text{eff}} = \epsilon \mathcal{L}_f p \mathcal{L}_t f \eta$$

# Neutron life cycle in thermische reactor

Representative Four Factor and  $k_{\infty}$  Values for Thermal Reactors

|                | <i>PWR Pressurized-<br/>H<sub>2</sub>O Reactor</i> | <i>BWR Boiling-<br/>H<sub>2</sub>O Reactor</i> | <i>PHWR<br/>CANDU-D<sub>2</sub>O<br/>Reactor</i> | <i>HTGR<br/>C-Moderated<br/>Reactor</i> |
|----------------|----------------------------------------------------|------------------------------------------------|--------------------------------------------------|-----------------------------------------|
| $\epsilon$     | 1.27                                               | 1.28                                           | 1.08                                             | 1.20                                    |
| $p$            | 0.63                                               | 0.63                                           | 0.84                                             | 0.62                                    |
| $f$            | 0.94                                               | 0.94                                           | 0.97                                             | 0.98                                    |
| $\eta_T$       | 1.89                                               | 1.89                                           | 1.31                                             | 2.02                                    |
| $k_{\infty}^a$ | 1.41                                               | 1.40                                           | 1.12                                             | 1.47                                    |

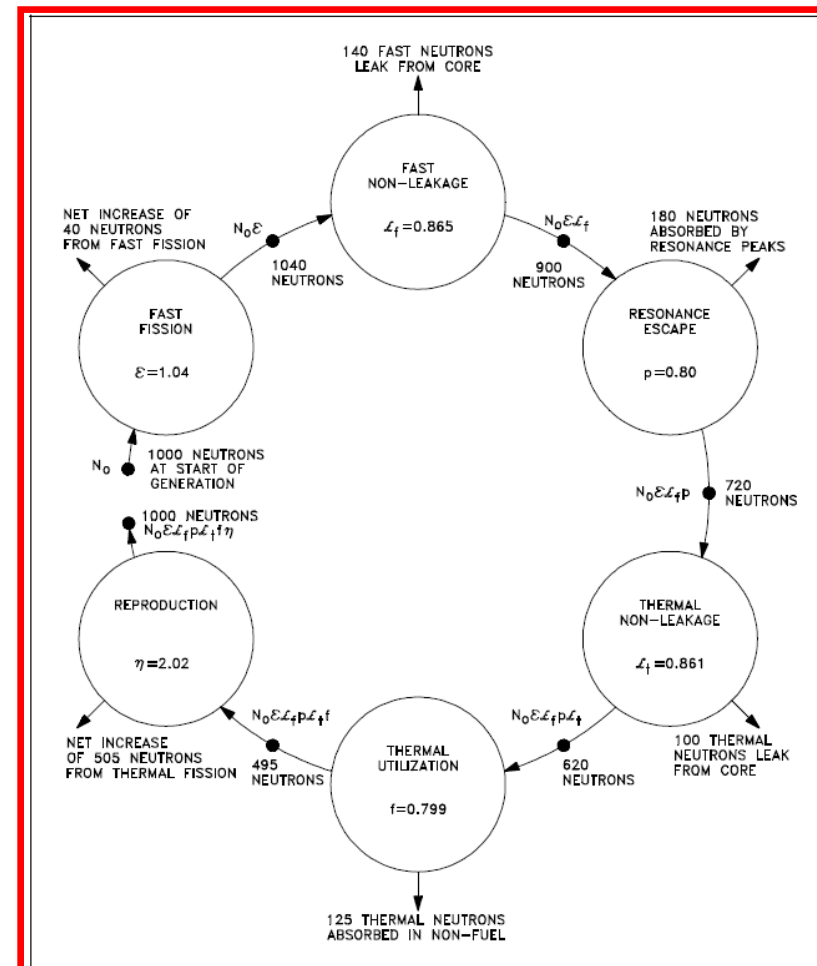
<sup>a</sup>Fresh fuel without neutron poisons.

Source: Data courtesy of W. S. Yang, Argonne National Laboratory.

Verrijking beïnvloedt  
thermal utilization  $f$   
reproduction factor  $\eta$   
resonance escape probability  $p$

Cyclus in een snelle kweekreactor is  
geheel anders

Energieverlies wordt geminimaliseerd  
en bijna alle splijtingen vinden plaats  
door snelle neutronen



# *Fast fission factor*

Fast fission factor  $\varepsilon = \frac{\text{\# snelle neutronen geproduceerd door alle splijtingen}}{\text{\# snelle neutronen geproduceerd door thermische splijtingen}}$

Er geldt

$$\varepsilon = \frac{\int_T \nu \Sigma_f^f(E) \phi_f(E) dE + \int_F \nu \Sigma_f^f(E) \phi_f(E) dE}{\int_T \nu \Sigma_f^f(E) \phi_f(E) dE} = 1 + \frac{\int_F \nu \Sigma_f^f(E) \phi_f(E) dE}{\underbrace{\int_T \nu \Sigma_f^f(E) \phi_f(E) dE}}$$

Varieert tussen 0.02 en 0.30

Afhankelijk van

Moderator materiaal

Verrijkingsgraad

# Resonance escape probability

We hadden 
$$p = \frac{\# \text{ neutronen die thermische energie bereiken}}{\# \text{ snelle neutronen die met slow down beginnen}}$$

Alle snelle neutronen die downward scatteren worden geabsorbeerd  
 In I-range door resonante capture door fuel  
 In T-range door fuel en moderator

Er geldt 
$$p = \frac{V_f \int_T \Sigma_a^f(E) \phi_f(E) dE + V_m \int_T \Sigma_a^m(E) \phi_m(E) dE}{V_f \left[ \int_T \Sigma_a^f(E) \phi_f(E) dE + \int_I \Sigma_a^f(E) \phi_f(E) dE \right] + V_m \int_T \Sigma_a^m(E) \phi_m(E) dE}$$

Logaritmische decrement  $\xi \approx \frac{6}{3A+1}$

Benadering 
$$p \cong e^{-\frac{2.4}{\xi} \sqrt{\frac{n_f}{n_s \sigma_s}}}$$

Dichtheid brandstof  $n_f$ , dichtheid moderator  $n_s$   
 Verstrooiingsdoorsnede  $\sigma_s$  in barns

Typisch geldt  $p \approx 0.7$

# Thermal utilization factor

Thermal utilization factor voor een *homogene* reactor

$$f = \frac{\text{rate of absorption of thermal neutrons by the fuel}}{\text{rate of absorption of thermal neutrons by all reactor materials}}$$

$$f = \frac{\sum_a^U \phi^U V^U}{\sum_a^U \phi^U V^U + \sum_a^m \phi^m V^m + \sum_a^p \phi^p V^p}$$

U, m en p voor uranium, moderator en poison

$$f = \frac{\sum_a^U}{\sum_a^U + \sum_a^m + \sum_a^p}$$

Homogene reactor (overal dezelfde flux en volume)

Example:

Calculate the thermal utilization factor for a homogeneous reactor. The macroscopic absorption cross section of the fuel is  $0.3020 \text{ cm}^{-1}$ , the macroscopic absorption cross section of the moderator is  $0.0104 \text{ cm}^{-1}$ , and the macroscopic absorption cross section of the poison is  $0.0118 \text{ cm}^{-1}$ .

Solution:

$$\begin{aligned} f &= \frac{\sum_a^U}{\sum_a^U + \sum_a^m + \sum_a^p} \\ &= \frac{0.3020 \text{ cm}^{-1}}{0.3020 \text{ cm}^{-1} + 0.0104 \text{ cm}^{-1} + 0.0118 \text{ cm}^{-1}} \\ &= 0.932 \end{aligned}$$

An increase in moderator temperature will have the following effects.

- Increase the thermal utilization factor
- Decrease resonance escape probability
- Decrease fast non-leakage probability
- Decrease thermal non-leakage probability

# Thermal utilization factor

Thermal utilization factor  $f = \frac{\# \text{ thermische neutronen geabsorbeerd in fuel}}{\# \text{ thermische neutronen geabsorbeerd in alles}}$

Alle thermische neutronen  
worden in fuel of moderator  
geabsorbeerd

$$f = \frac{V_f \int_T \Sigma_a^f(E) \varphi_f(E) dE}{V_f \int_T \Sigma_a^f(E) \varphi_f(E) dE + V_m \int_T \Sigma_a^m(E) \varphi_m(E) dE}$$

Definieer  $\bar{\varphi}_{fT} = \int_T \varphi_f(E) dE$ , en  $\bar{\varphi}_{mT} = \int_T \varphi_m(E) dE$  (ruimtelijk gemiddelde thermische fluxen)

Dan  $\bar{\Sigma}_{xT}^f(E) = \bar{\varphi}_{fT}^{-1} \int_T \Sigma_x^f(E) \varphi_f(E) dE$ , en  $\bar{\Sigma}_{xT}^m(E) = \bar{\varphi}_{mT}^{-1} \int_T \Sigma_x^m(E) \varphi_m(E) dE$

We vinden

$$f = \frac{1}{1 + \zeta \left( V_m \bar{\Sigma}_{aT}^m / V_f \bar{\Sigma}_{aT}^f \right)}$$

Met thermal disadvantage factor  $\zeta = \bar{\varphi}_{mT} / \bar{\varphi}_{fT}$

Hoe meer neutronen gecaptured worden in de moderator (vanwege de grotere flux daar), hoe minder er splijting kunnen veroorzaken in de fuel

# Reproduction factor

Reproduction factor  $\eta_T = \frac{\# \text{ snelle neutronen geproduceerd in thermische splijting}}{\# \text{ thermische neutronen geabsorbeerd in de fuel}}$

Er geldt 
$$\eta_T = \frac{\int_T \nu \Sigma_f^f(E) \phi_f(E) dE}{\int_T \Sigma_a^f(E) \phi_f(E) dE} \rightarrow \eta_T = \frac{\bar{\nu} \Sigma_{fT}^f}{\Sigma_{aT}^f}$$

$$\eta = \frac{\Sigma_f^U \phi^U \nu}{\Sigma^U \phi^U} \quad \text{fission reaction rate } (\Sigma_f^U \phi^U) \quad \text{average number of neutrons produced per fission } (\nu)$$

Example:

Calculate the reproduction factor for a reactor that uses 10% enriched uranium fuel. The microscopic absorption cross section for uranium-235 is 694 barns. The cross section for uranium-238 is 2.71 barns. The microscopic fission cross section for uranium-235 is 582 barns. The atom density of uranium-235 is  $4.83 \times 10^{21}$  atoms/cm<sup>3</sup>. The atom density of uranium-238 is  $4.35 \times 10^{22}$  atoms/cm<sup>3</sup>.  $\nu$  is 2.42.

$$\eta = \frac{N^{U-235} \sigma_f^{U-235} \nu^{U-235}}{N^{U-235} \sigma_a^{U-235} + N^{U-238} \sigma_a^{U-238}}$$

When core contains <sup>235</sup>U and <sup>238</sup>U

Solution:

Use Equation (3-2) to calculate the reproduction factor.

$$\begin{aligned} \eta &= \frac{N^{U-235} \sigma_f^{U-235} \nu^{U-235}}{N^{U-235} \sigma_a^{U-235} + N^{U-238} \sigma_a^{U-238}} \\ &= \frac{\left(4.83 \times 10^{21} \frac{\text{atoms}}{\text{cm}^3}\right) (582 \times 10^{-24} \text{ cm}^2) (2.42)}{\left(4.83 \times 10^{21} \frac{\text{atoms}}{\text{cm}^3}\right) (694 \times 10^{-24} \text{ cm}^2) + \left(4.35 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}\right) (2.71 \times 10^{-24} \text{ cm}^2)} \\ &= 1.96 \end{aligned}$$

**TABLE 1**  
Average Number of Neutrons Liberated in Fission

| Fissile Nucleus | Thermal Neutrons |        | Fast Neutrons |        |
|-----------------|------------------|--------|---------------|--------|
|                 | $\nu$            | $\eta$ | $\nu$         | $\eta$ |
| Uranium-233     | 2.49             | 2.29   | 2.58          | 2.40   |
| Uranium-235     | 2.42             | 2.07   | 2.51          | 2.35   |
| Plutonium-239   | 2.93             | 2.15   | 3.04          | 2.90   |

# *Reactor core*



# Reactor core

Samenstelling van de core wordt bepaald door

Behoud van criticality gedurende bedrijf

Transfer van thermische energie uit de core

## Configuraties

Gesmolten materiaal (vloeibare brandstof)

Pebble bed reactor

Meest voorkomend: cilindrische container met axiale koelkanalen

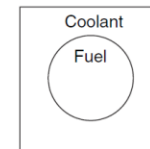
## Roosterstructuur van

Brandstof

Koelmiddel

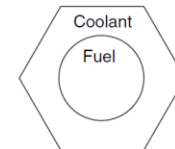
Moderator

H<sub>2</sub>O gekoeld



(a)

Fast reactor

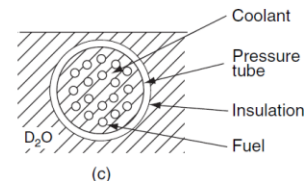


(b)

## Diameter brandstofstaven

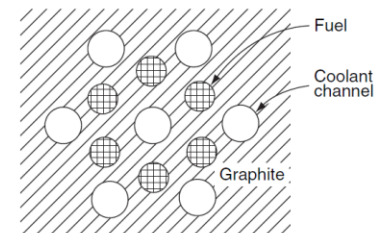
Warmte flux door oppervlak

Temperatuur in centerline (linear heat rate in de orde van ongeveer 10 kW/m)



(c)

CANDU D<sub>2</sub>O



(d)

HTGCR

## GW reactor

Duizenden brandstofstaven (fuel pins)

We moeten de verschillen in flux in rekening brengen

# Fuel assemblies

Plaats brandstofstaven bij elkaar in assemblies

Makkelijker dan verwisselen van duizenden individuele staven

Geometrie: vierkant of hexagonaal

Niet alle assemblies zijn gelijk: verrijking om power in core te homogeniseren

Gemiddelde vermogensdichtheid

Linear heat rate van brandstofstaven

Ratio volume van moderator / brandstof

Core volume is omgekeerd evenredig met de maximum vermogensdichtheid

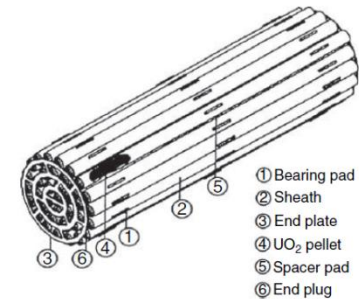
Structuur van core lattice

Maximaliseer de vermogensdichtheid bij gegeven koelcapaciteit

Er geldt  $P_{NL} \rightarrow 1$  bij groter core volume

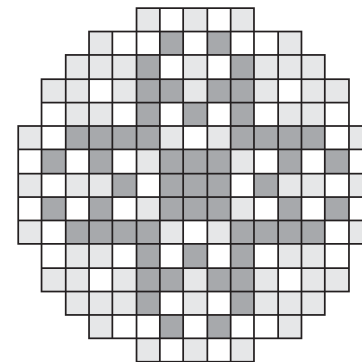
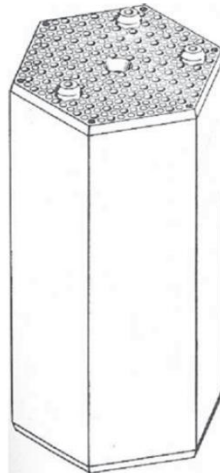


PWR

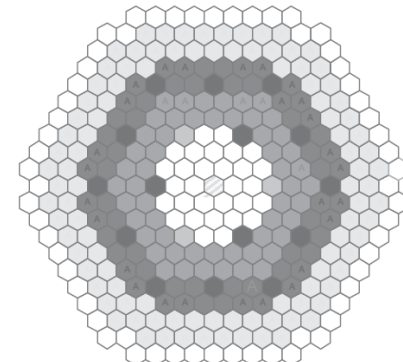


CANDU D<sub>2</sub>O

HTGCR



vierkant



hexagonaal

# Reactor core eigenschappen

## Water reactors

Representative Reactor Lattice Properties

|                                                         | <i>PWR</i><br><i>Pressurized-</i><br><i>H<sub>2</sub>O Reactor</i> | <i>BWR Boiling-</i><br><i>H<sub>2</sub>O Reactor</i> | <i>PHWR</i><br><i>CANDU-D<sub>2</sub>O</i><br><i>Reactor</i> | <i>HTGR</i><br><i>C-Moderated</i><br><i>Reactor</i> | <i>SFR</i><br><i>Na-Cooled</i><br><i>Fast Reactor</i> | <i>GCFR</i><br><i>He-Cooled</i><br><i>Fast Reactor</i> |
|---------------------------------------------------------|--------------------------------------------------------------------|------------------------------------------------------|--------------------------------------------------------------|-----------------------------------------------------|-------------------------------------------------------|--------------------------------------------------------|
| $\bar{q}'$ (kW/m) average linear heat rate              | 17.5                                                               | 20.7                                                 | 24.7                                                         | 3.7                                                 | 22.9                                                  | 17.0                                                   |
| $V_x/V_f$ volume ratio <sup>a</sup>                     | 1.95                                                               | 2.78                                                 | 17                                                           | 135                                                 | 1.25                                                  | 1.93                                                   |
| $\bar{P}'''$ (MW/m <sup>3</sup> ) average power density | 102                                                                | 56                                                   | 7.7                                                          | 6.6                                                 | 217                                                   | 115                                                    |
| $V$ (m <sup>3</sup> ) volume 3000 MW(t) reactor         | 29.4                                                               | 53.7                                                 | 390                                                          | 455                                                 | 13.8                                                  | 26.1                                                   |
| $\tilde{e}$ (weight %) enrichment                       | 4.2                                                                | 4.2                                                  | 0.7                                                          | 15                                                  | 19                                                    | 19                                                     |

<sup>a</sup>  $x$  = moderator for a thermal reactor and coolant for a fast reactor,  $f$  = fuel.

Source: Data courtesy of W. S. Yang, Argonne National Laboratory.

# LWR – light water reactors

## Water

Koelmiddel en moderator

Grootste slowing down power

Kleinste slowing down ratio

## Lattice

Compact en vierkant

Uranium-dioxide pellets

Enrichment: 2 – 5 %

Zirkonium cladding

Moderator – fuel volume: 2:1

Hoge power density

Klein core volume

## PWR

Druk 150 bar, temperatuur: 300 °C

Warmtewisselaar

## BWR

Druk 70 bar, temperatuur: 300 °C

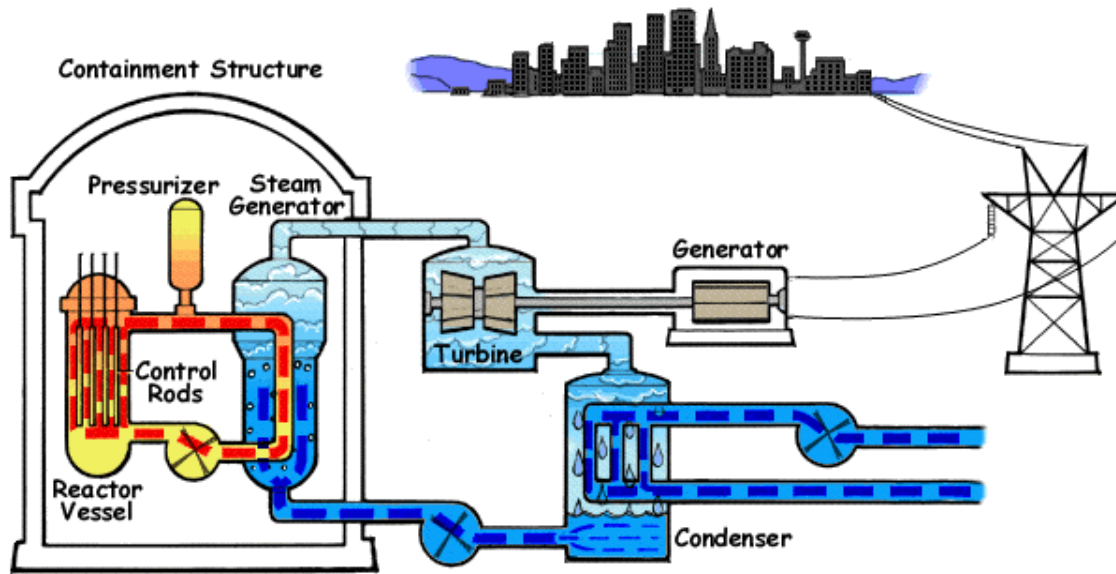
Water direct in reactor, stoom naar turbine (geen warmtewisselaar)

Representative Reactor Lattice Properties

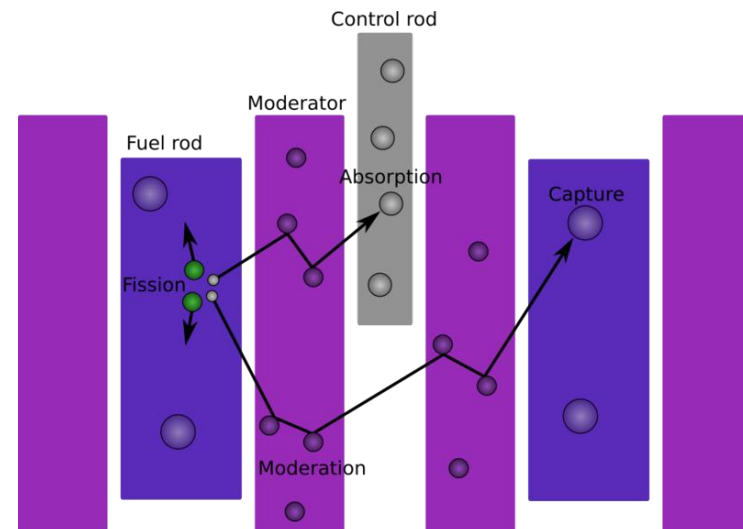
|                                                         | <i>PWR</i><br><i>Pressurized-</i><br><i>H<sub>2</sub>O</i> Reactor | <i>BWR Boiling-</i><br><i>H<sub>2</sub>O</i> Reactor |
|---------------------------------------------------------|--------------------------------------------------------------------|------------------------------------------------------|
| $\bar{q}'$ (kW/m) average linear heat rate              | 17.5                                                               | 20.7                                                 |
| $V_x/V_f$ volume ratio <sup>a</sup>                     | 1.95                                                               | 2.78                                                 |
| $\bar{P}'''$ (MW/m <sup>3</sup> ) average power density | 102                                                                | 56                                                   |
| $V$ (m <sup>3</sup> ) volume 3000 MW(t) reactor         | 29.4                                                               | 53.7                                                 |
| $\tilde{e}$ (weight %) enrichment                       | 4.2                                                                | 4.2                                                  |

<sup>a</sup>  $x$  = moderator for a thermal reactor and coolant for a fast reactor,  $f$  = fuel.  
Source: Data courtesy of W. S. Yang, Argonne National Laboratory.

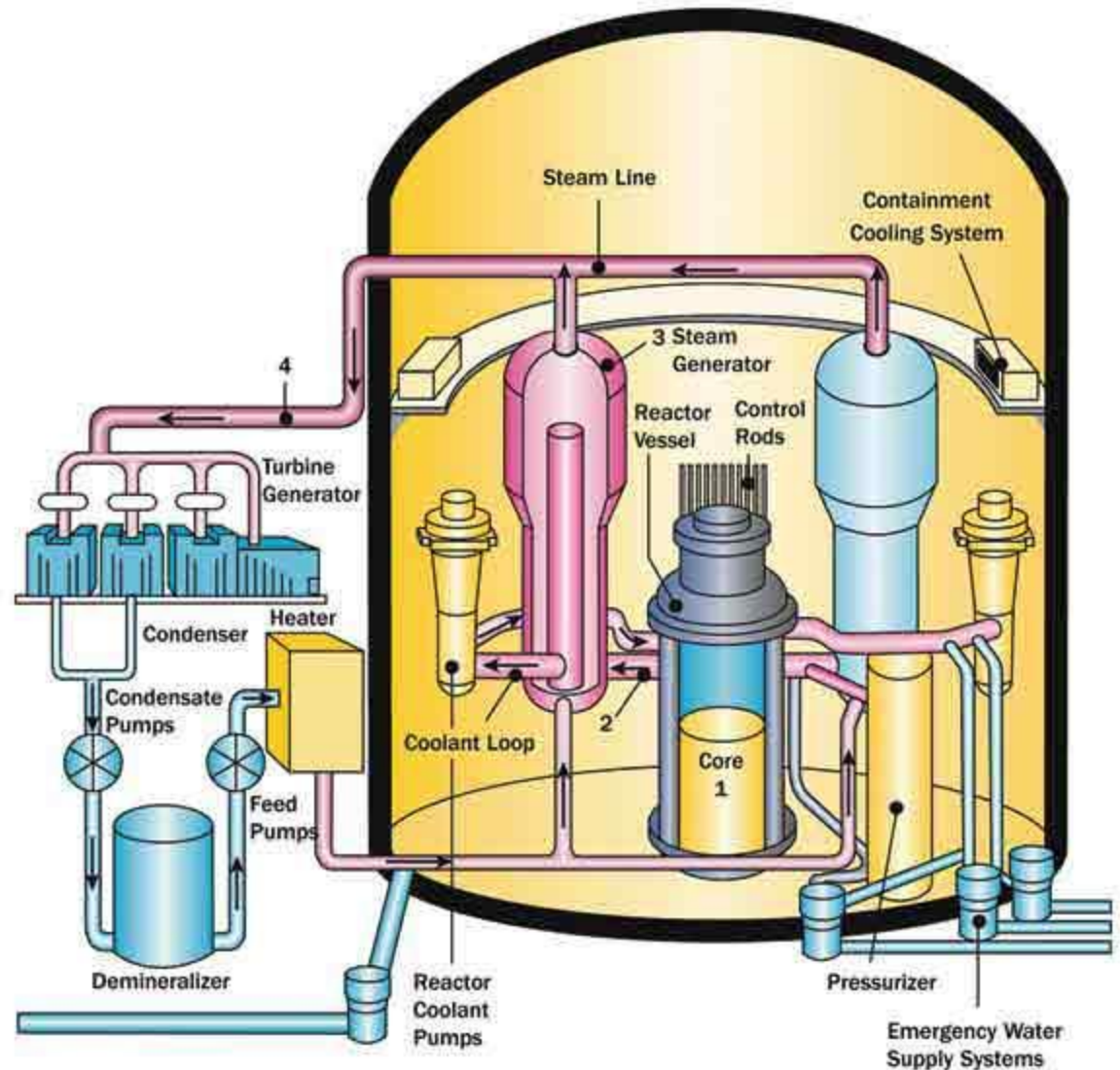
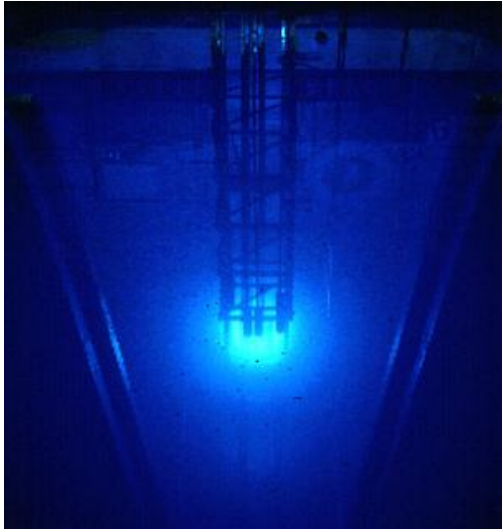
# PWR – Pressurized water reactor



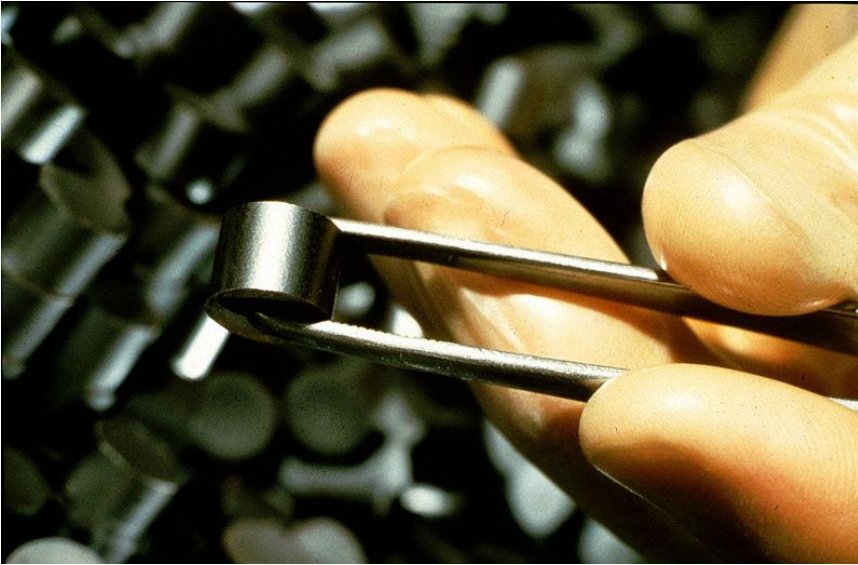
- PWR meest voorkomend reactortype (~1 GW) met thermische efficiëntie van ongeveer 30 %
- Houd water onder druk (~15 MPa) zodat het kan opwarmen (~315 °C), maar zonder te koken
- Water in de reactor en het water in de stoomgenerator (~5 MPa) mengen nooit. Op deze wijze blijft de meeste radioactiviteit in de core van de reactor
- Gebruik verrijkt uranium als brandstof
- Brandstof in staven zorgt voor toename in resonance escape probability  $p$  en fast fission factor  $\varepsilon$



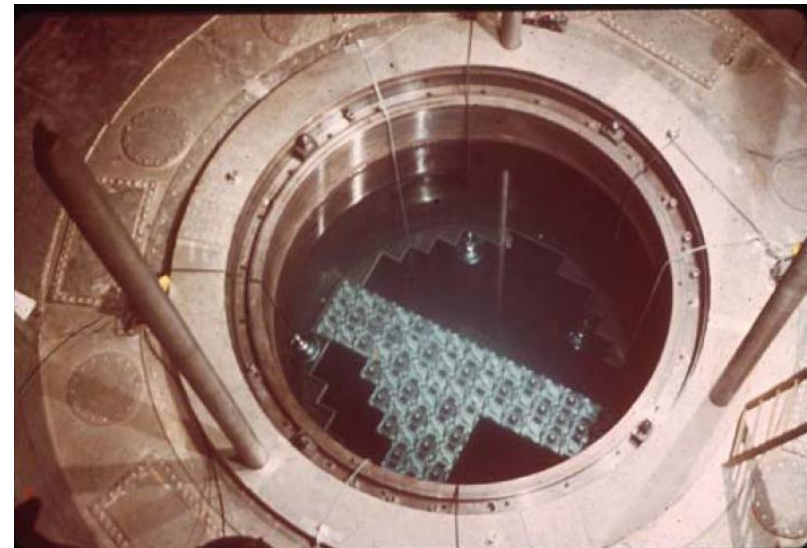
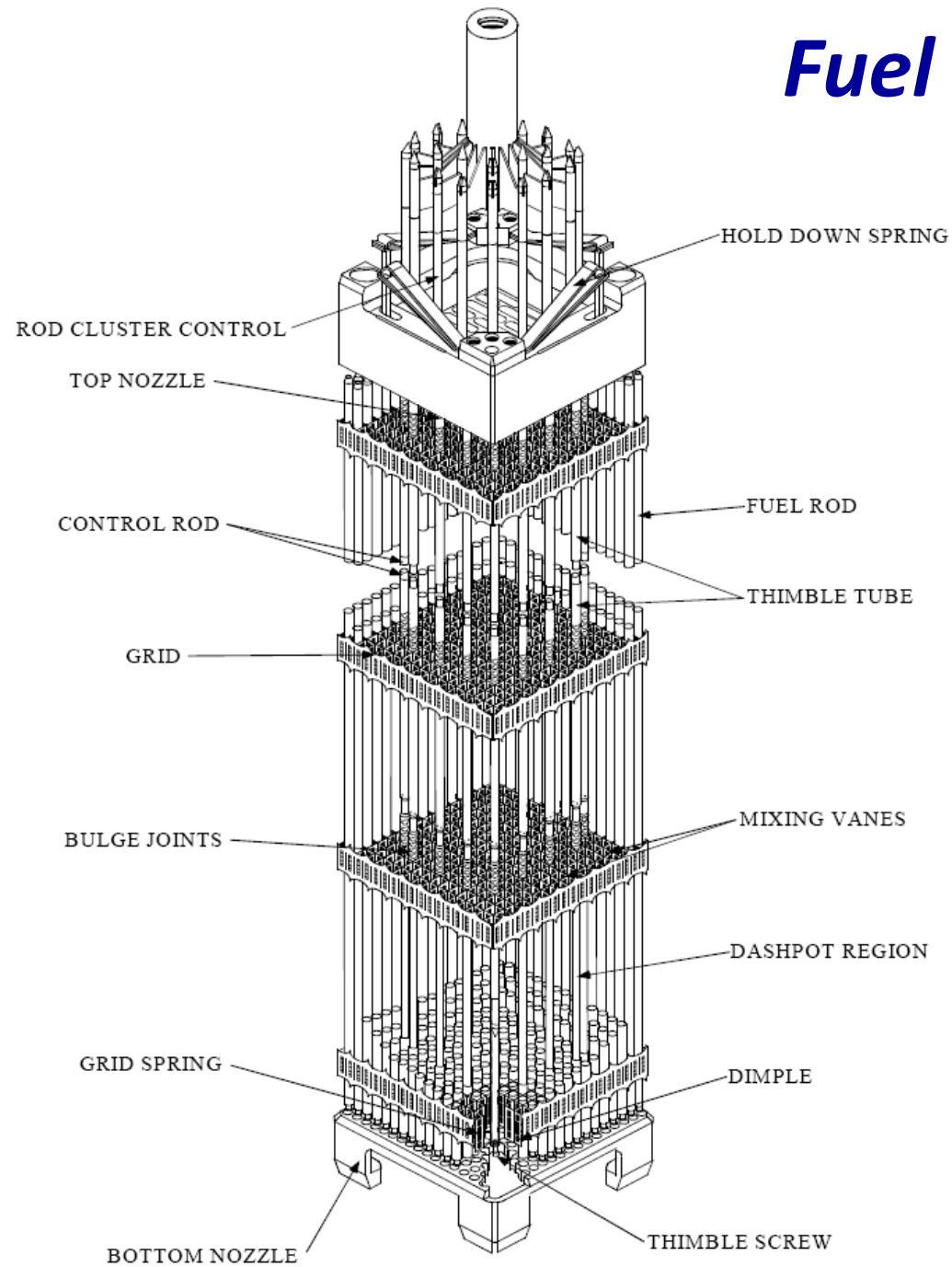
# *Pressurized water reactor*



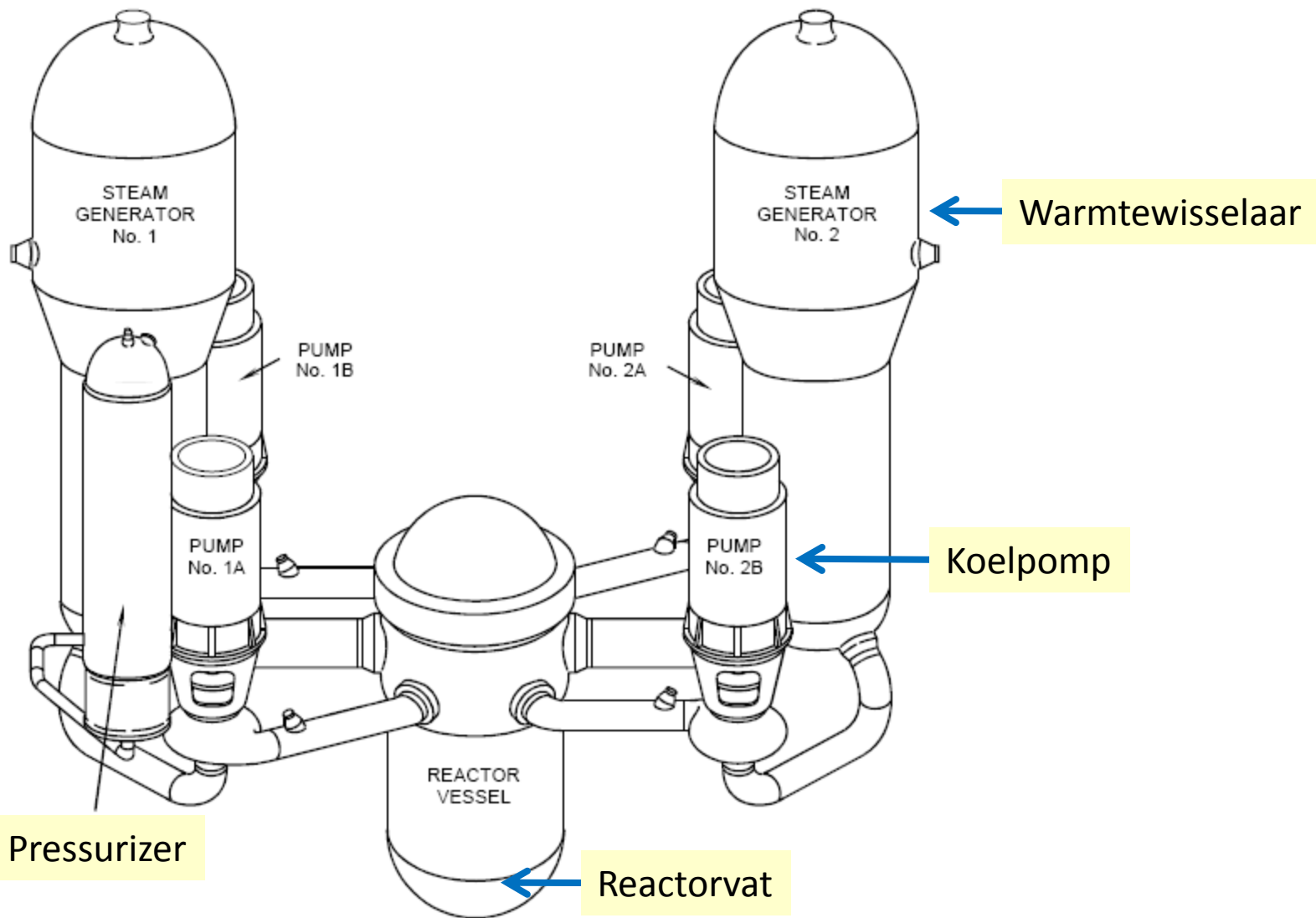
# *Fuel assembly*



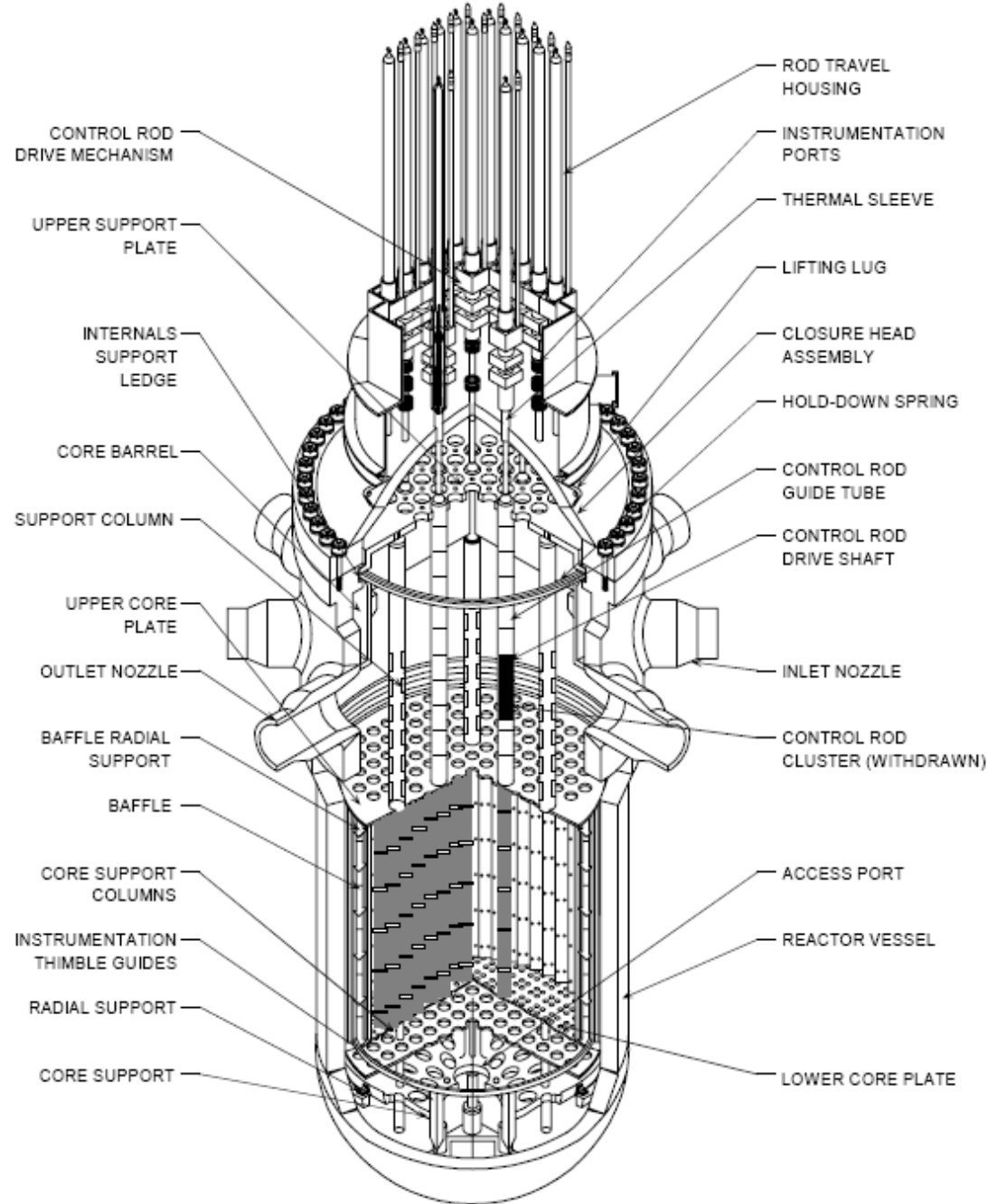
# *Fuel assembly*



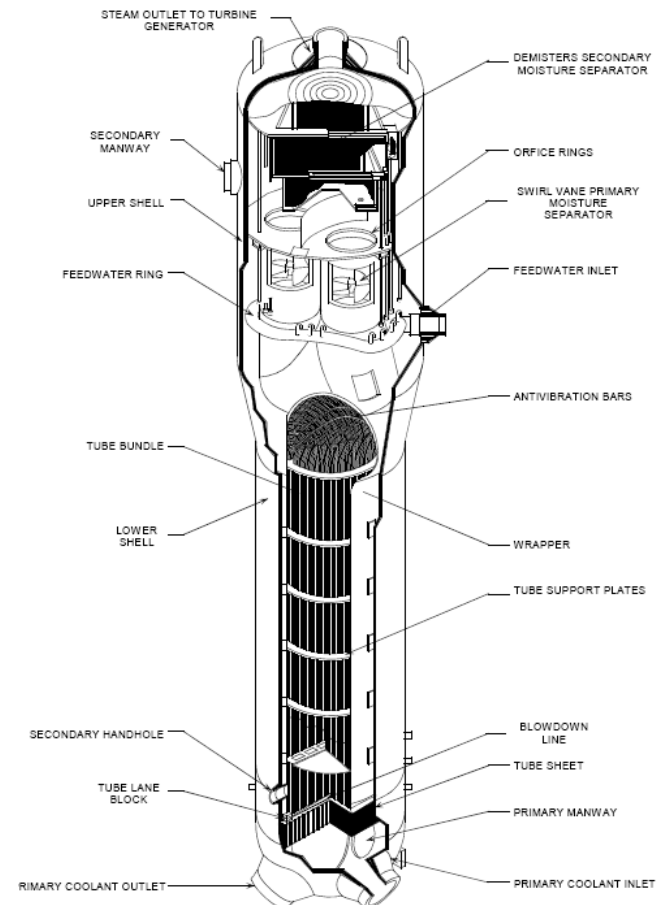
# *PWR opbouw*



# Reactorvat

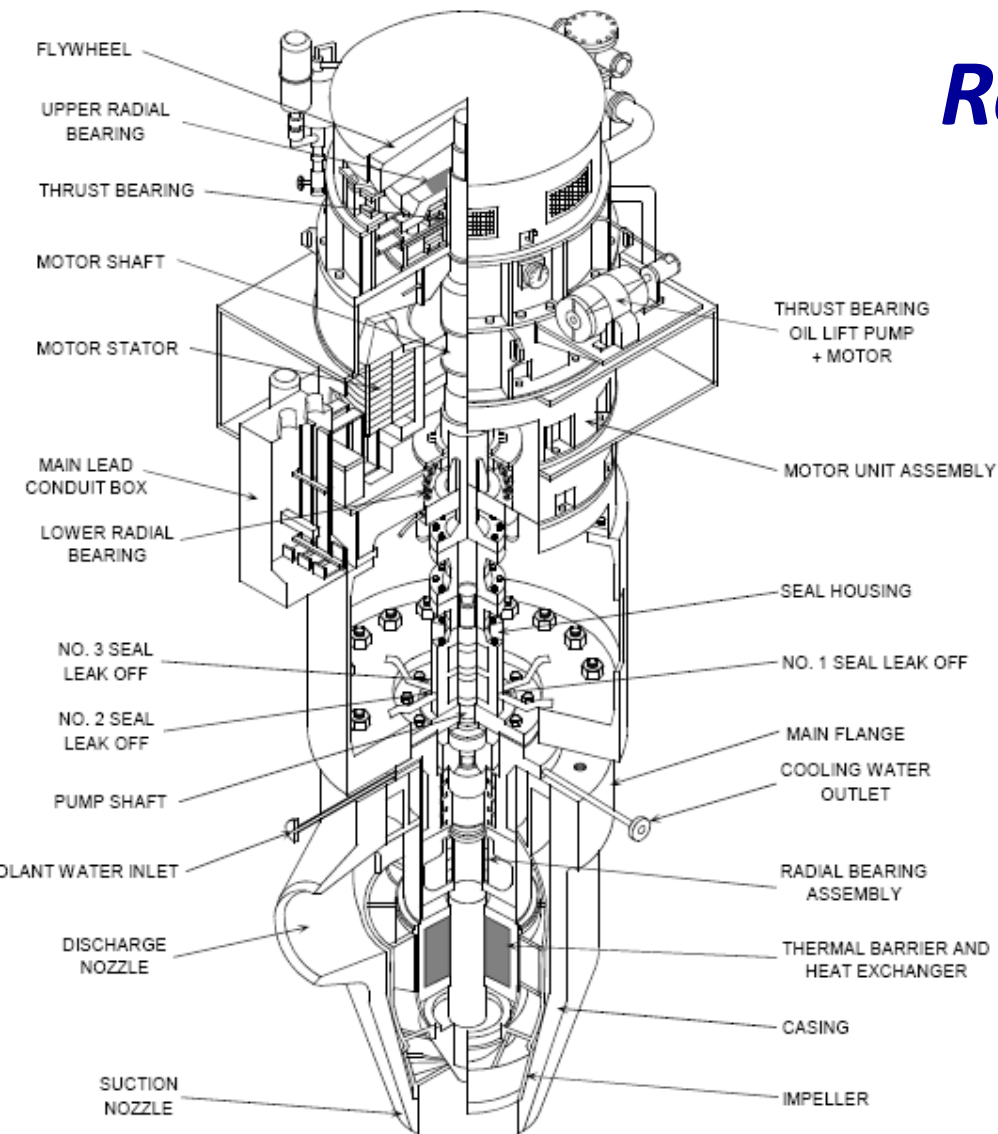


Doorsnede reactorvat

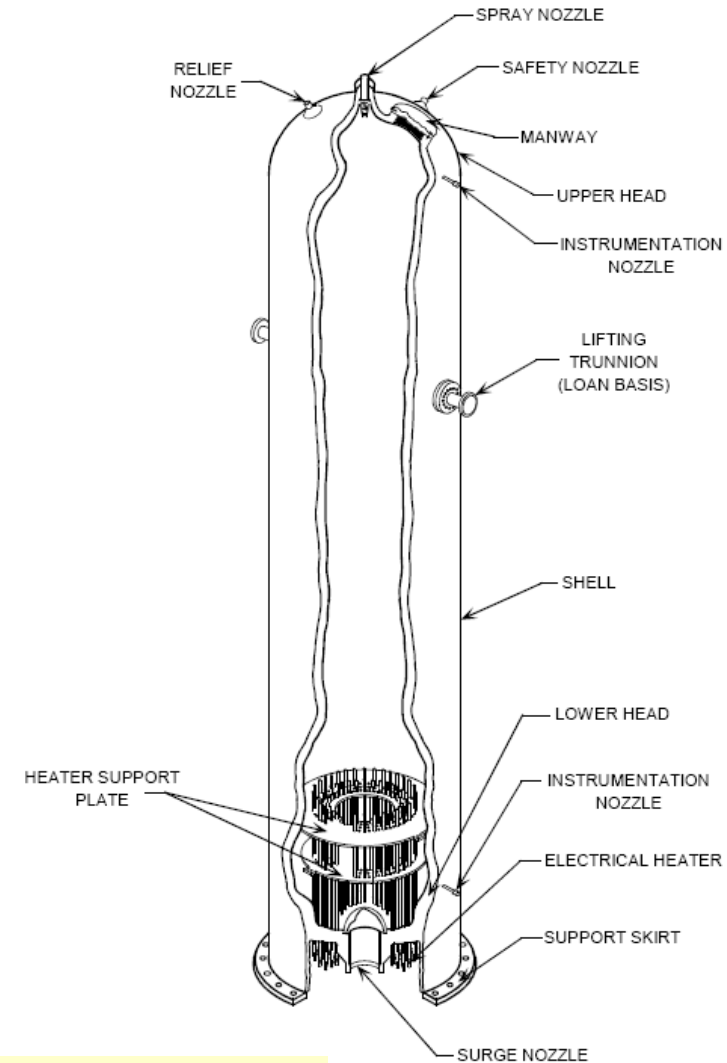


Doorsnede warmtewisselaar

# Reactor componenten

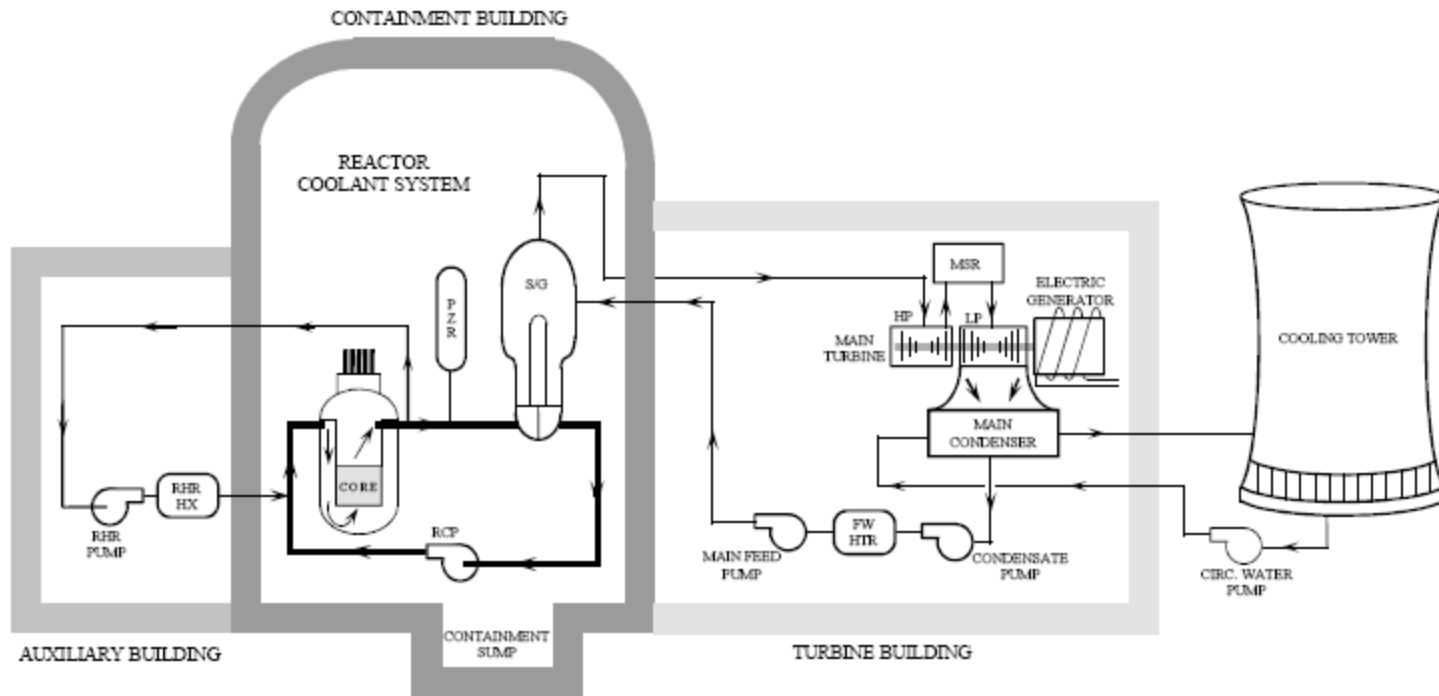


Doorsnede reactor koelpomp

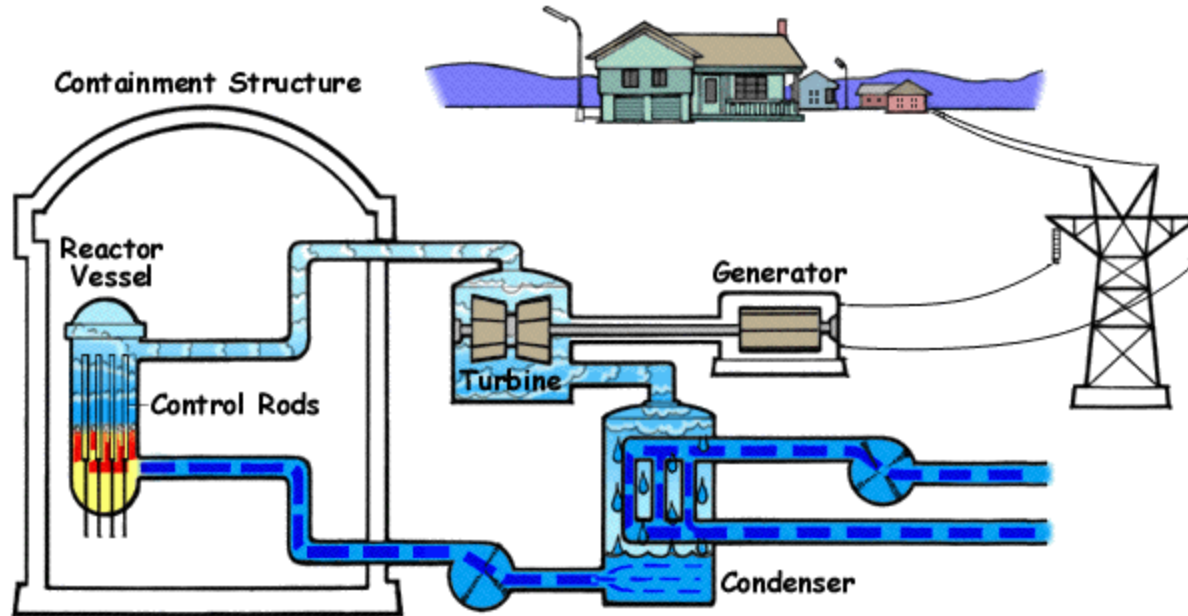


Doorsnede pressurizer

# ***PWR containment***



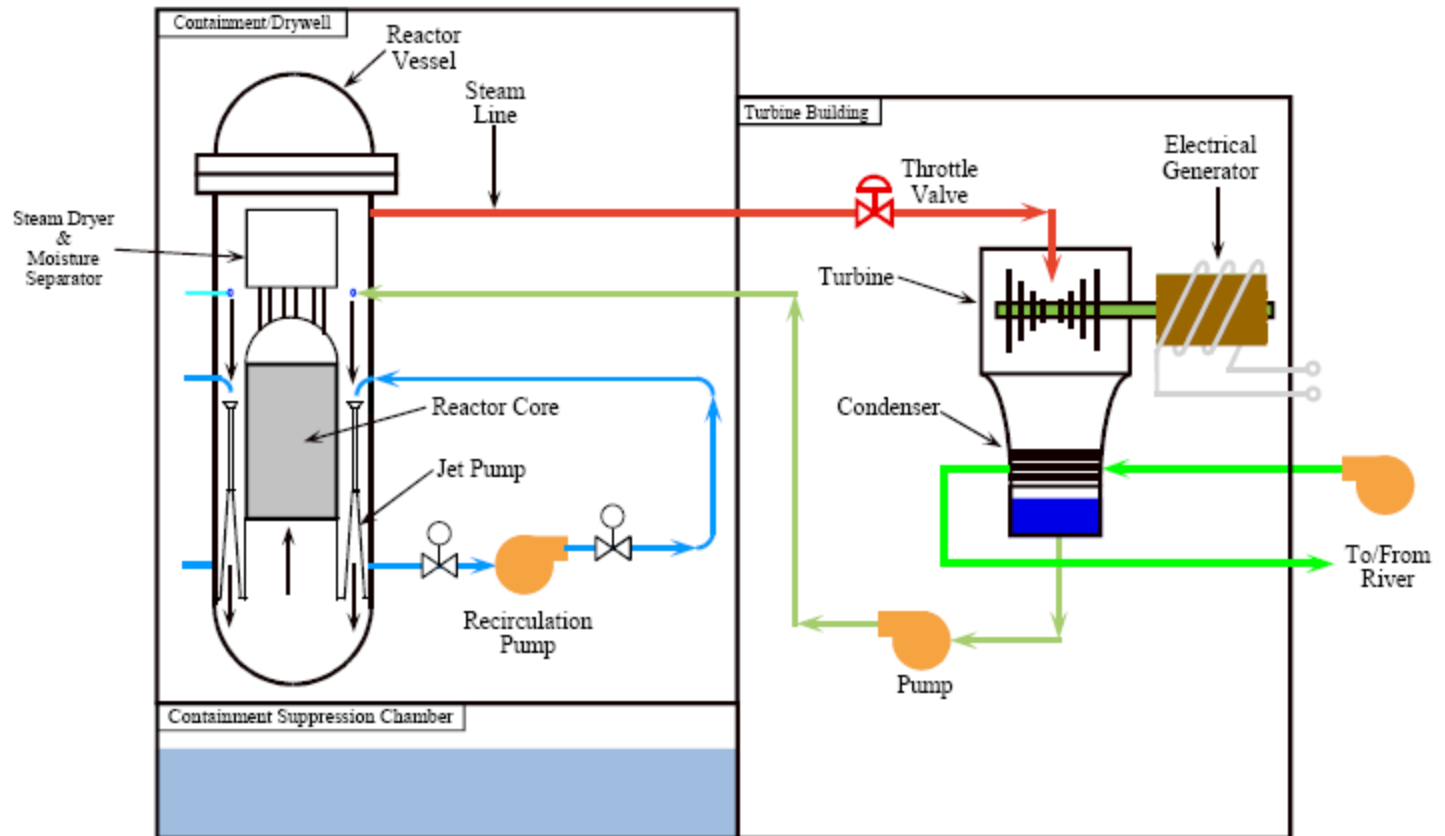
# ***BWR – Boiling water reactor***

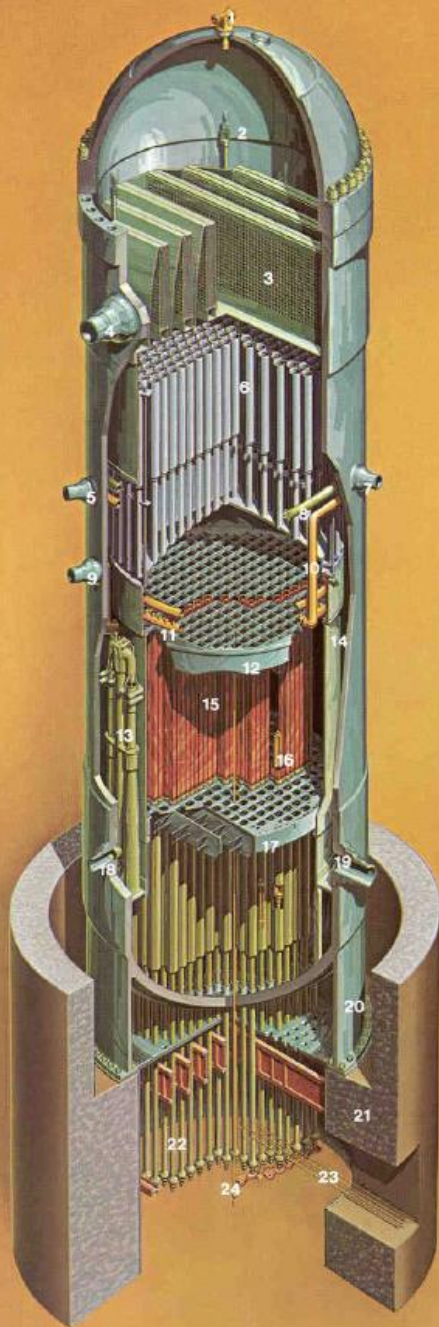


- In BWRs wordt water door splijting aan het koken gebracht en de stoom drijft een generator aan
- Eenvoudiger ontwerp en lagere bedrijfsdruk (7.5 MPa en 285 °C in core), dus commercieel aantrekkelijker
- Natuurlijke water circulatie wordt gebruikt
- Lagere stralingsbelasting op het reactorvat
- Veel groter drukvat dan voor PWR bij hetzelfde vermogen



# ***BWR containment***





## BWR/6 REACTOR ASSEMBLY

1. VENT AND HEAD SPRAY
2. STEAM DRYER LIFTING LUG
3. STEAM DRYER ASSEMBLY
4. STEAM OUTLET
5. CORE SPRAY INLET
6. STEAM SEPARATOR ASSEMBLY
7. FEEDWATER INLET
8. FEEDWATER SPARGER
9. LOW PRESSURE COOLANT INJECTION INLET
10. CORE SPRAY LINE
11. CORE SPRAY SPARGER
12. TOP GUIDE
13. JET PUMP ASSEMBLY
14. CORE SHROUD
15. FUEL ASSEMBLIES
16. CONTROL BLADE
17. CORE PLATE
18. JET PUMP / RECIRCULATION WATER INLET
19. RECIRCULATION WATER OUTLET
20. VESSEL SUPPORT SKIRT
21. SHIELD WALL
22. CONTROL ROD DRIVES
23. CONTROL ROD DRIVE HYDRAULIC LINES
24. IN-CORE FLUX MONITOR

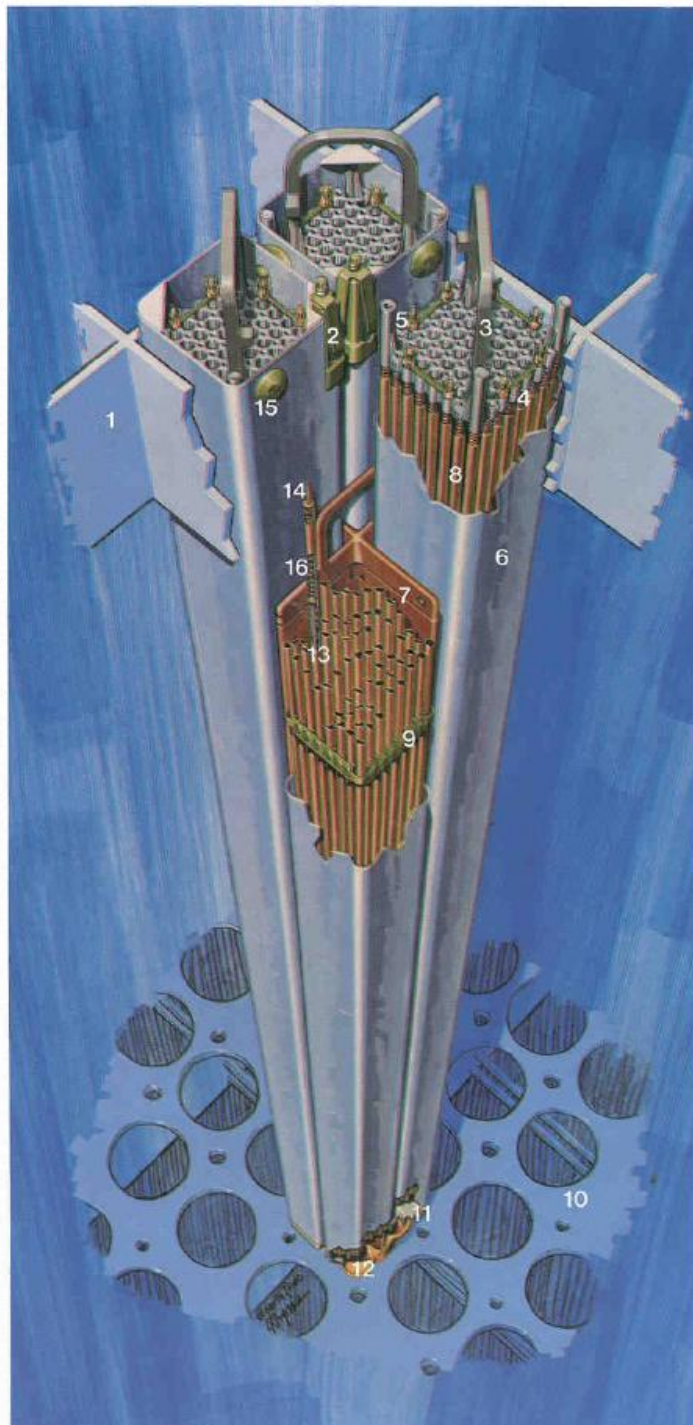
GENERAL  ELECTRIC

# BWR

# *BWR fuel*

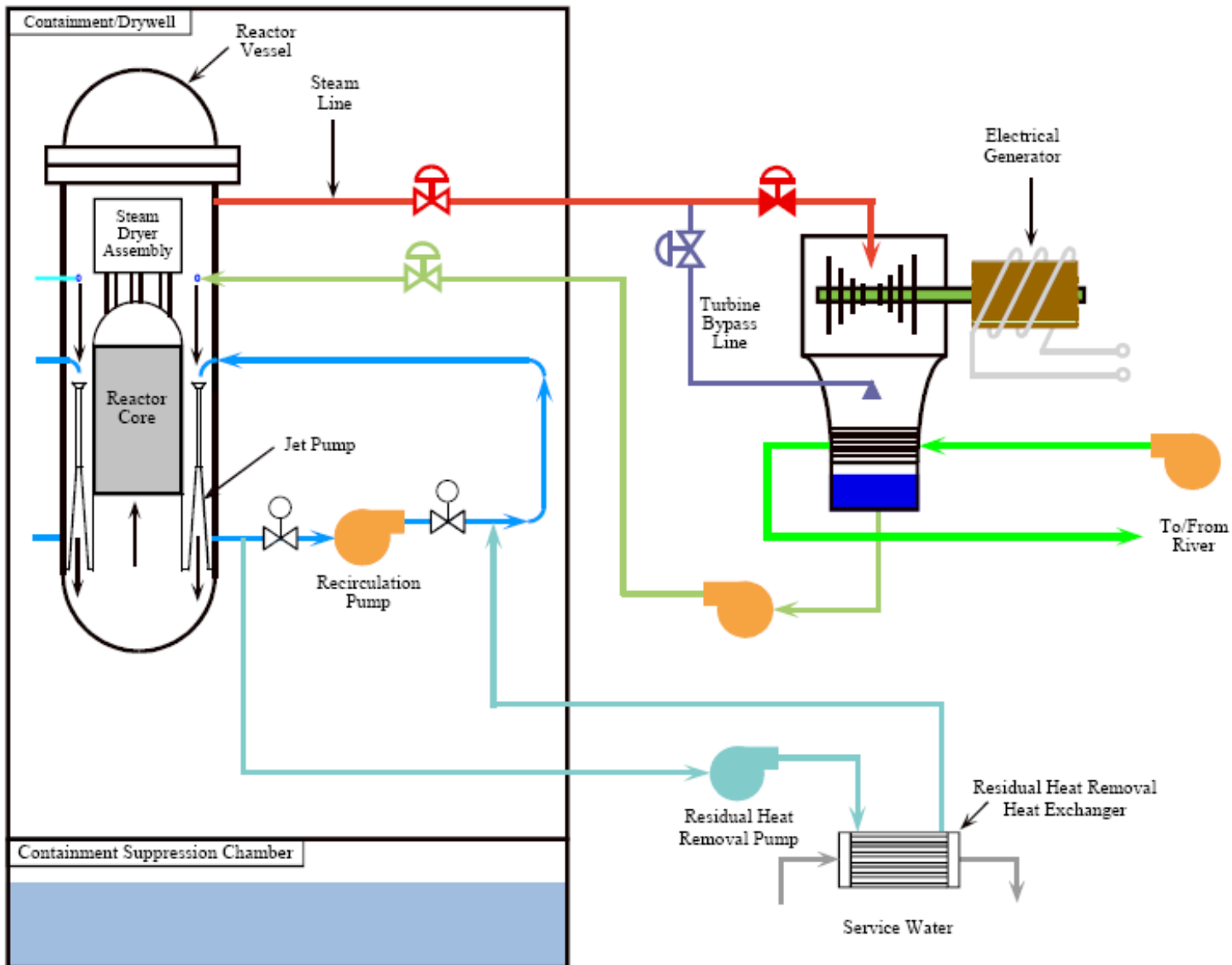
## **BWR/6 FUEL ASSEMBLIES & CONTROL ROD MODULE**

- 1.TOP FUEL GUIDE
- 2.CHANNEL  
FASTENER
- 3.UPPER TIE  
PLATE
- 4.EXPANSION  
SPRING
- 5.LOCKING TAB
- 6.CHANNEL
- 7.CONTROL ROD
- 8.FUEL ROD
- 9.SPACER
- 10.CORE PLATE  
ASSEMBLY
- 11.LOWER  
TIE PLATE
- 12.FUEL SUPPORT  
PIECE
- 13.FUEL PELLETS
- 14.END PLUG
- 15.CHANNEL  
SPACER
- 16.PLENUM  
SPRING

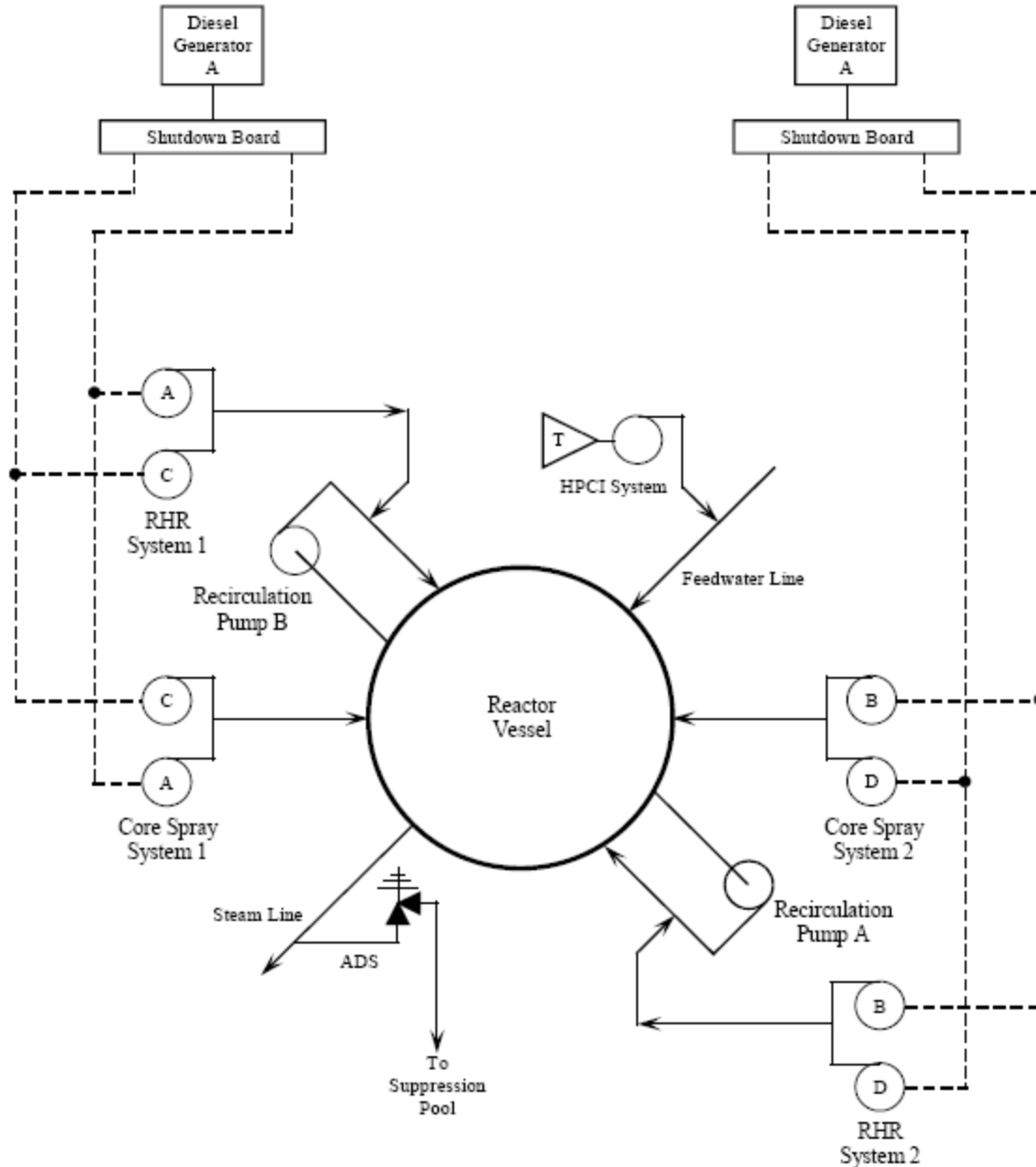


GENERAL  ELECTRIC

# ***BWR heat removal***



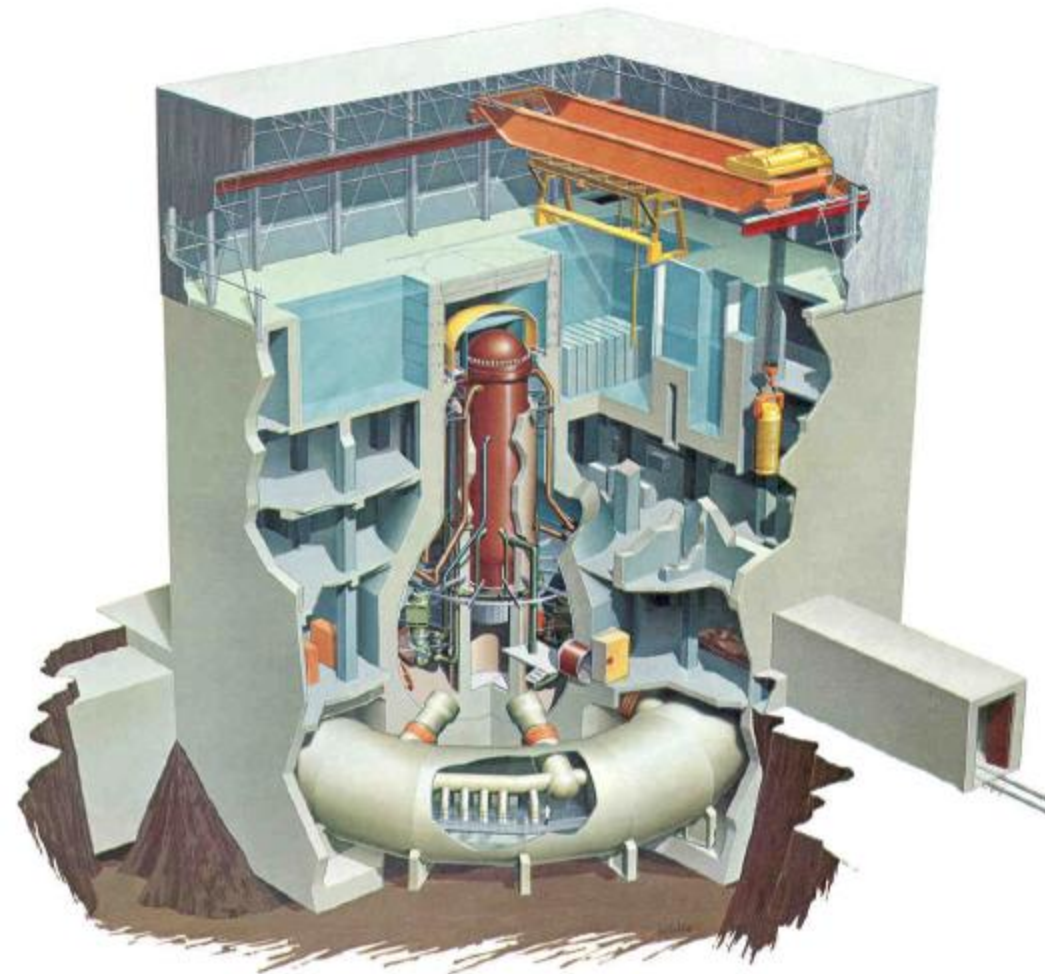
# ***BWR emergency core cooling***



# ***BWR buildings***

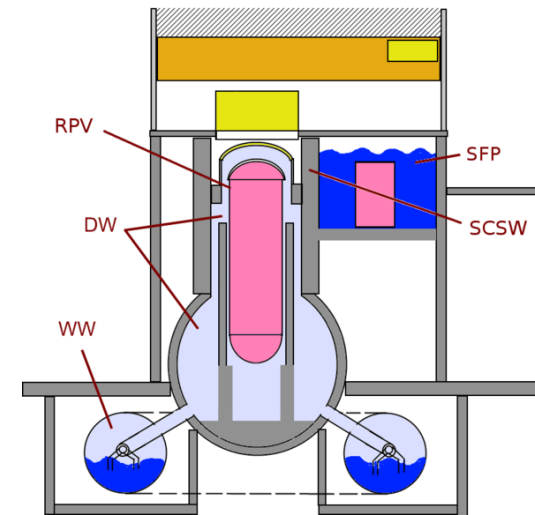
## Mark I containment

DW drywell  
WW wetwell torus  
RPV reactor pressure vessel  
SFP spent fuel pool  
SCSW secondary concrete shielding wall



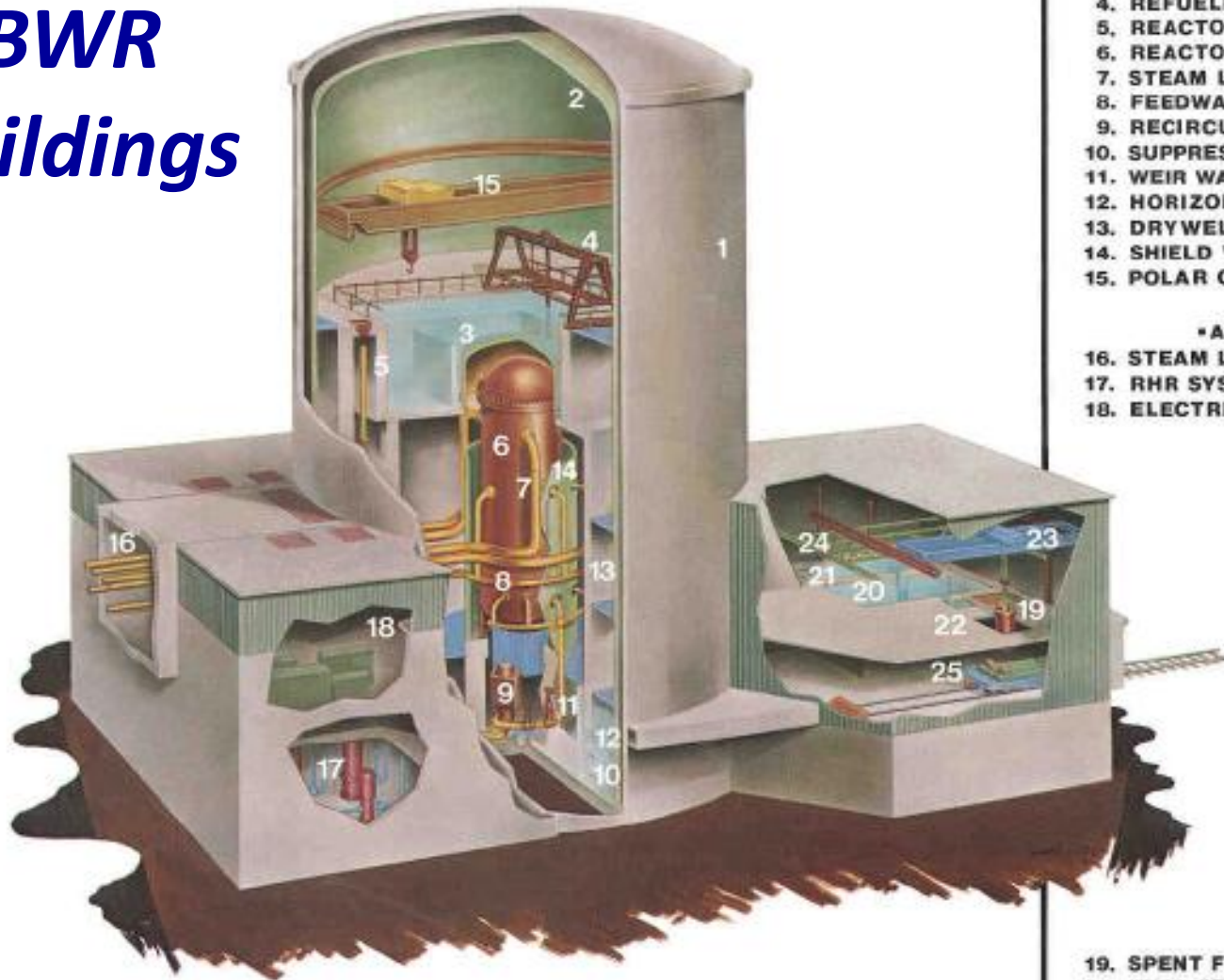
DRYWELL TORUS

GENERAL ELECTRIC



# MARK III CONTAINMENT

## *BWR buildings*



### • REACTOR BUILDING •

1. SHIELD BUILDING
2. FREESTANDING STEEL CONTAINMENT
3. UPPER POOL
4. REFUELING PLATFORM
5. REACTOR WATER CLEANUP
6. REACTOR VESSEL
7. STEAM LINE
8. FEEDWATER LINE
9. RECIRCULATION LOOP
10. SUPPRESSION POOL
11. WEIR WALL
12. HORIZONTAL VENT
13. DRYWELL
14. SHIELD WALL
15. POLAR CRANE

### • AUXILIARY BUILDING •

16. STEAM LINE TUNNEL
17. RHR SYSTEM
18. ELECTRICAL EQUIPMENT ROOM

### • FUEL BUILDING •

19. SPENT FUEL SHIPPING CASK
20. FUEL STORAGE POOL
21. FUEL TRANSFER POOL
22. CASK LOADING POOL
23. CASK HANDLING CRANE
24. FUEL TRANSFER BRIDGE
25. FUEL CASK SKID ON RAILROAD CAR

# Reactor core eigenschappen

Pressurized heavy water reactor

High temperature gas cooled reactor

Sodium cooled fast reactor

Gas cooled fast reactor

Representative Reactor Lattice Properties

|                                                         | <i>PWR</i><br><i>Pressurized-</i><br><i>H<sub>2</sub>O Reactor</i> | <i>BWR Boiling-</i><br><i>H<sub>2</sub>O Reactor</i> | <i>PHWR</i><br><i>CANDU-D<sub>2</sub>O</i><br><i>Reactor</i> | <i>HTGR</i><br><i>C-Moderated</i><br><i>Reactor</i> | <i>SFR</i><br><i>Na-Cooled</i><br><i>Fast Reactor</i> | <i>GCFR</i><br><i>He-Cooled</i><br><i>Fast Reactor</i> |
|---------------------------------------------------------|--------------------------------------------------------------------|------------------------------------------------------|--------------------------------------------------------------|-----------------------------------------------------|-------------------------------------------------------|--------------------------------------------------------|
| $\bar{q}'$ (kW/m) average linear heat rate              | 17.5                                                               | 20.7                                                 | 24.7                                                         | 3.7                                                 | 22.9                                                  | 17.0                                                   |
| $V_x/V_f$ volume ratio <sup>a</sup>                     | 1.95                                                               | 2.78                                                 | 17                                                           | 135                                                 | 1.25                                                  | 1.93                                                   |
| $\bar{P}'''$ (MW/m <sup>3</sup> ) average power density | 102                                                                | 56                                                   | 7.7                                                          | 6.6                                                 | 217                                                   | 115                                                    |
| $V$ (m <sup>3</sup> ) volume 3000 MW(t) reactor         | 29.4                                                               | 53.7                                                 | 390                                                          | 455                                                 | 13.8                                                  | 26.1                                                   |
| $\tilde{e}$ (weight %) enrichment                       | 4.2                                                                | 4.2                                                  | 0.7                                                          | 15                                                  | 19                                                    | 19                                                     |

<sup>a</sup>  $x$  = moderator for a thermal reactor and coolant for a fast reactor,  $f$  = fuel.

Source: Data courtesy of W. S. Yang, Argonne National Laboratory.

# PHWR – Pressurized heavy water reactor

CANDU reactor met D<sub>2</sub>O moderator en koelmiddel

Calandria (horizontale cylinder) met hoge-druk buizen

Buizen bevatten fuel bundels met UO<sub>2</sub> pellets

50 cm x 10 cm

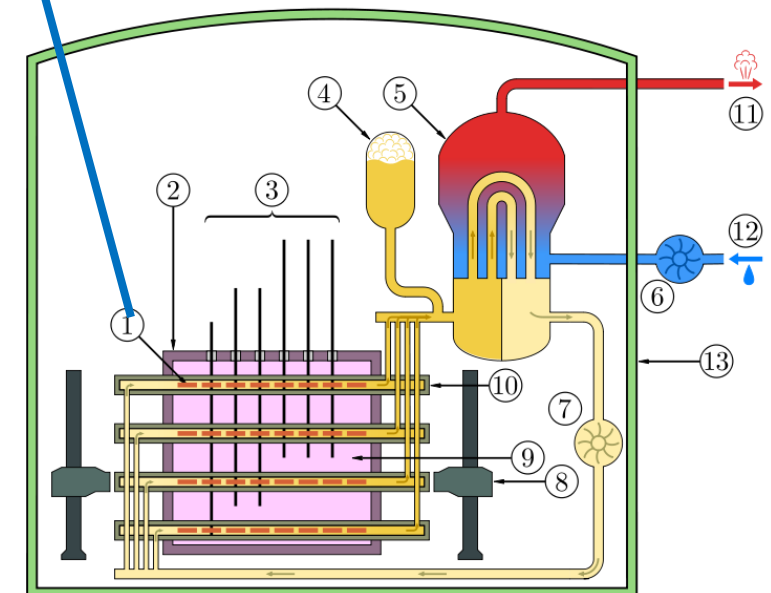
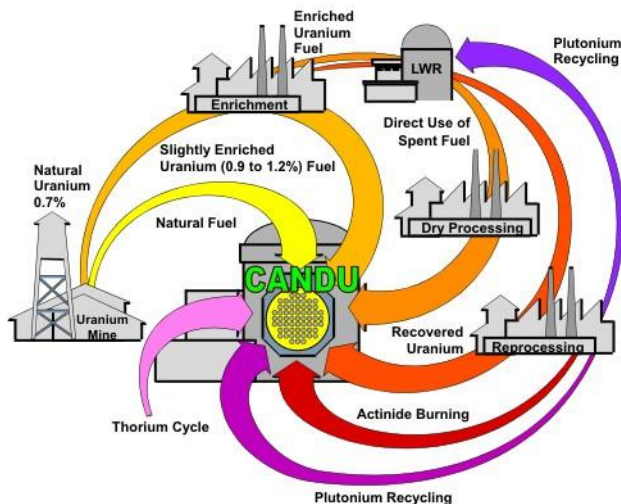


Qinshan - China

Grote moderator – fuel volume ratio

Natuurlijk uranium als brandstof mogelijk

Continue refueling (fuel burn up)



# ***HTGR– Graphite moderated reactor***

Grafiet: lage slowing down power, maar lage absorptie

Grote moderator – fuel volume ratio

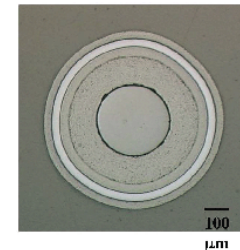
Reactortype met grootste volume

CO<sub>2</sub> koeling en natuurlijk uranium mogelijk

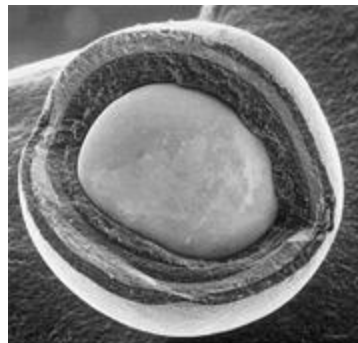
Helium koeling: HTGR

Uranium-carbide deeltjes in grafiet

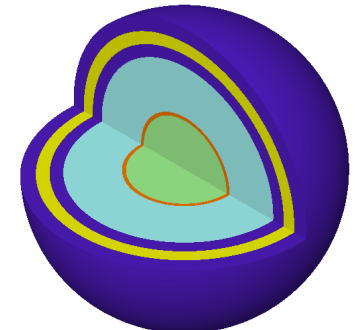
Pebble-bed reactor (Type IV)



Triso pebble  
Tri-layer isotropic



Quadriso pebble



# ***RBMK– H2O cooled graphite moderated***

RBMK is veel gebruikte Russische reactor

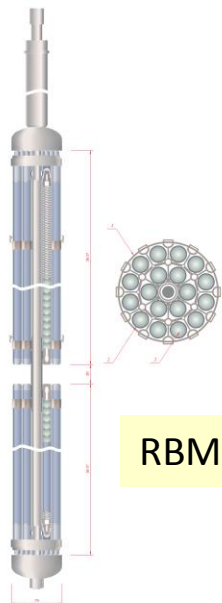
Nog 11 in gebruik in Rusland (type Chernobyl)

Grote moderator – fuel volume ratio

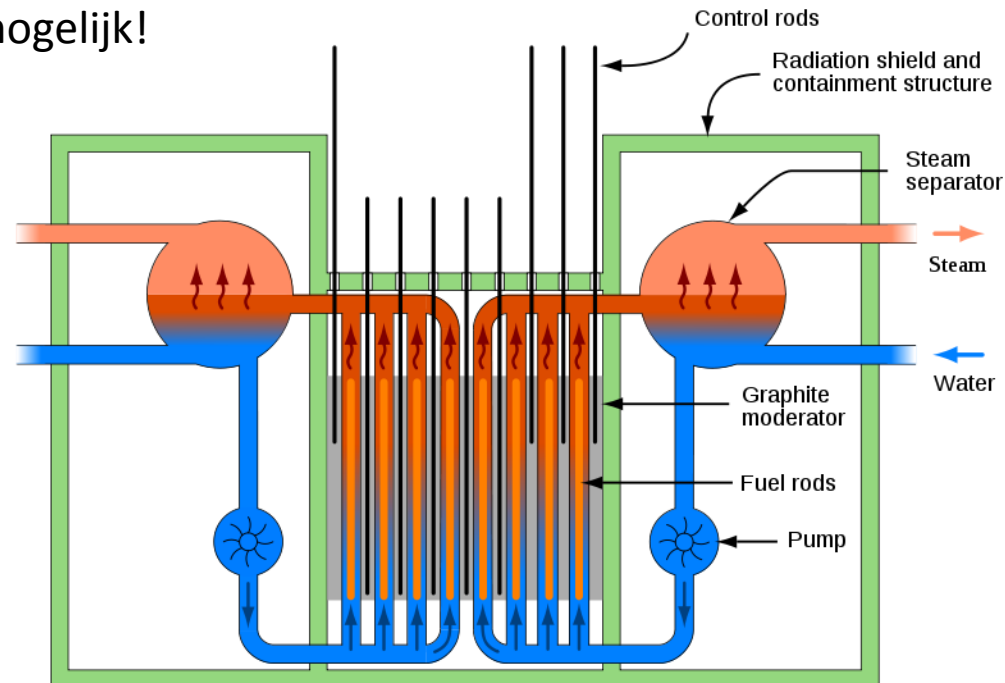
Volume reactors tot 1000 m<sup>3</sup>

Dit maakt het duur om meerdere containment gebouwen te construeren

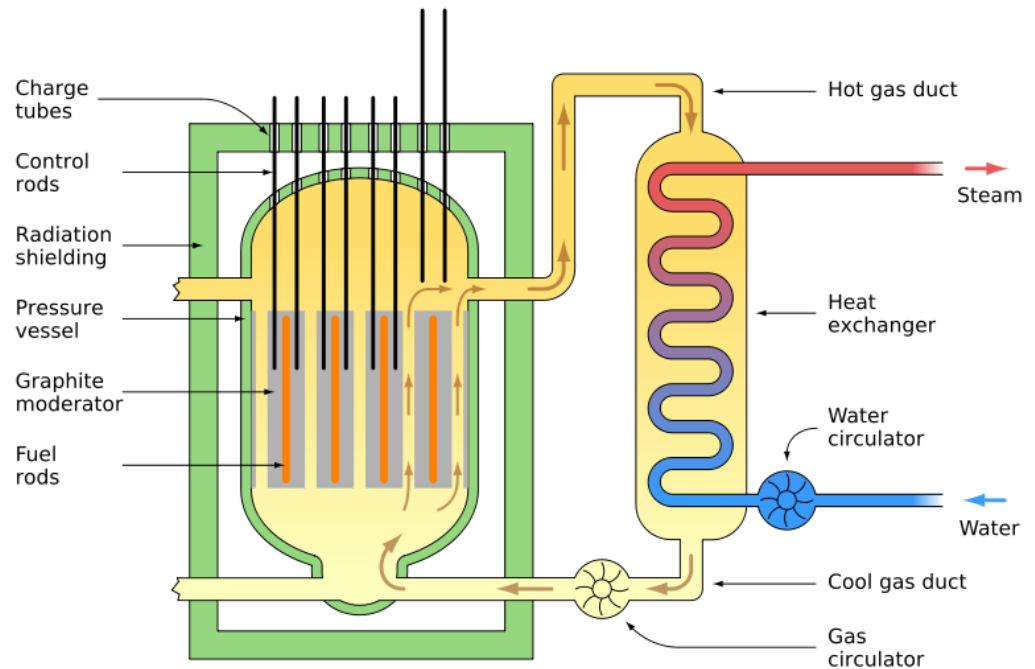
Normaal water en natuurlijk uranium mogelijk!



RBMK fuel rods



# Magnox and UNGG reactors



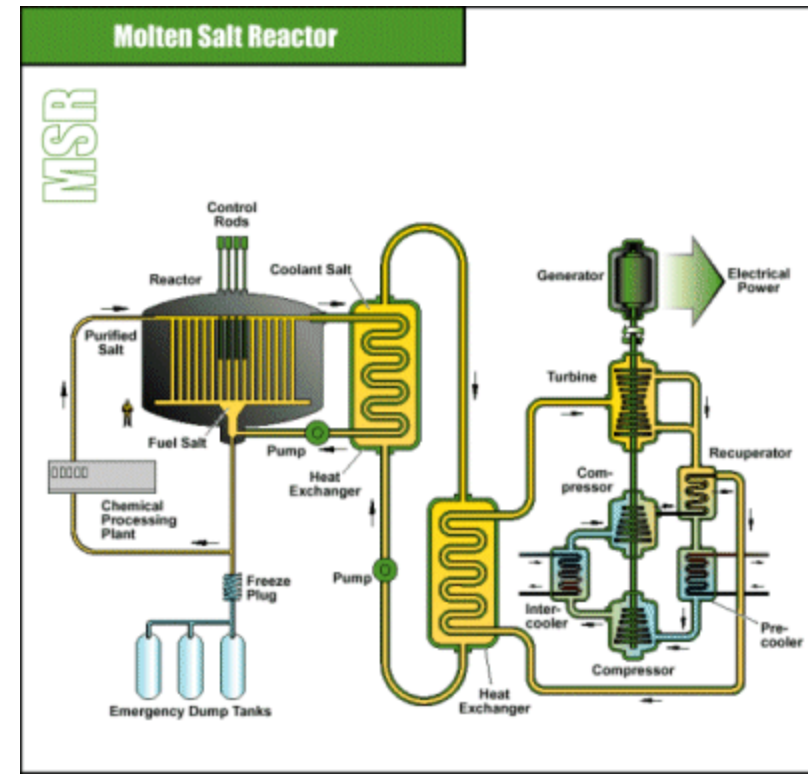
- Used in UK (26 units). Now obsolete type, but 2 in operation. Used for power and plutonium production. Magnox is now realized in N. Korea.
- Pressurized, CO<sub>2</sub> gas cooled, graphite moderated, natural uranium as fuel. Similar to France UNGG reactor: Uranium Naturel Graphite Gaz
- Coolant is a gas, so explosive pressure buildup from boiling (Chernobyl) is not possible.
- Magnesium non-oxidizing.



# MSR – Molten salt fast reactor

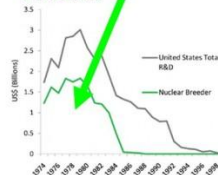
Superphenix

- Generation IV reactor: primary coolant is a molten salt.
- Nuclear fuel dissolved in the molten fluoride salt coolant ( $\text{LiF}$  and  $\text{BeF}_2$ ) as uranium tetrafluoride  $\text{UF}_4$ . Graphite core serves as the moderator.
- Low pressure: makes design simpler and safer, high temperature cooling: makes turbines more efficient.
- Compact: MSRE study to power aircraft.
- Inherently safe, but immature technology. Pressure explosion impossible, meltdown proof.
- Molten salt thorium breeders possible (thorium is abundant and cheap). Can operate decades without refueling.
- Co-locate with reprocessing facility.



But US advanced nuclear fission R&D has dropped – near zero for breeders.

**\$16 billion (\$2011)**  
Cumulative LMFBR investment.



2011 DOE Nuclear Energy budget items  
\$103 million  
NGNP high temperature gas reactor with TRISO fuel.  
\$22 million  
Advanced reactor concepts, principally fast reactors.  
\$40 million  
Advanced fuel cycles, but no liquid fuel, no closed fuel cycle.

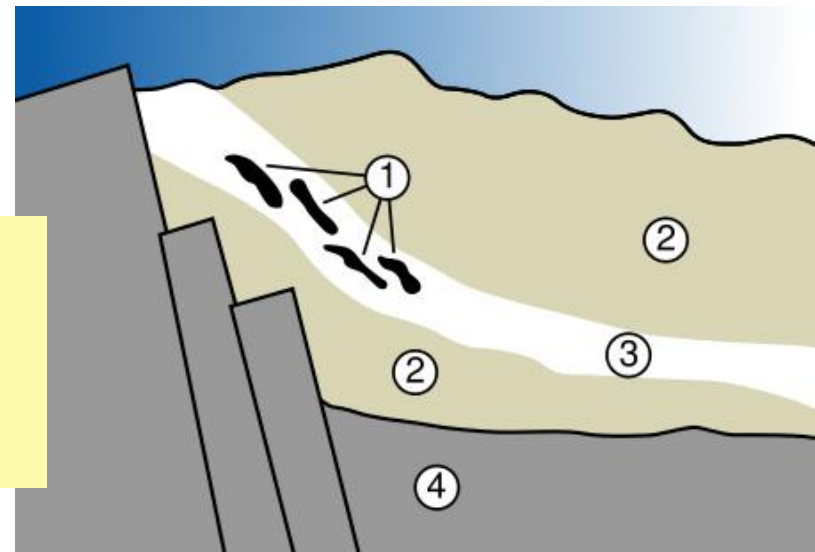
# ***Gabon natural fission reactors***

- Predicted by Paul Kuroda (Univ. of Arkansas) (1956).
- Fifteen natural reactors found (in 1972) at the Oklo mine in Gabon.
- Nuclear fission reactions took place 1.5 billion years ago, and ran for a few hundred thousand years (100 kW).
- Uranium-rich mineral deposit became inundated with groundwater that acted as a neutron moderator.
- Extensively studied by scientists interested in geologic radioactive waste disposal.



Geological situation in Gabon leading to natural nuclear fission reactors

1. Nuclear reactor zones
2. Sandstone
3. Uranium ore layer
4. Granite



# ***Reactor kinetics***

# Reactor kinetics

## Aannamen:

Neutron distributies en werkzame doorsneden gemiddeld over energie  
Verwaarloos neutron leakage uit eindige core

## Definities:

Totaal aantal neutronen op tijd  $t$  is  $n(t)$

Gemiddelde neutron snelheid is  $\bar{v}$

Energie-gemiddelde werkzame doorsnede voor reactie van type  $x$  is  $\Sigma_x$

## Infinite medium non-multiplying system

### Balansvergelijking

$$\frac{dn(t)}{dt} = S(t) - \Sigma_a \bar{v} n(t)$$

### Gemiddelde levensduur van neutronen

Neem aan  $n(0)$  neutronen op  $t = 0$

Neem aan dat er geen verdere neutronen geproduceerd worden, dus  $S(t) = 0$

# neutronen geabsorbeerd / s

# neutronen geproduceerd / s

$$\frac{dn(t)}{dt} = -\Sigma_a \bar{v} n(t) \rightarrow n(t) = n(0)e^{-t/l_\infty}, \text{ met } l_\infty = 1/\bar{v}\Sigma_a$$

En dus

$$\bar{t} \equiv \frac{\int_0^\infty t n(t) dt}{\int_0^\infty n(t) dt} = 1/\bar{v}\Sigma_a = l_\infty \quad \longrightarrow \quad n(t) = l_\infty S_0 [1 - e^{-t/l_\infty}], \text{ met } n(0) = 0$$

# Infinite medium multiplying systems

Aannamen:

Er is ook splijtbaar materiaal aanwezig

Verwaarloos neutron leakage uit eindige core

Infinite medium multiplying system

Balansvergelijking

Infinite medium multiplication  $k_{\infty} = \nu \Sigma_f / \Sigma_a$

Herschrijf tot 
$$\frac{dn(t)}{dt} = S(t) + \frac{(k_{\infty} - 1)}{l_{\infty}} n(t)$$

Aanname: enkel neutronen van splijting ( $S = 0$ )

Criticality voor  $k_{\infty} = 1$  (dan stabiele populatie)

We onderscheiden

Subcritical  $k_{\infty} < 1$

Critical  $k_{\infty} = 1$

Supercritical  $k_{\infty} > 1$

$$\frac{dn(t)}{dt} = S(t) + \nu \Sigma_f \bar{v} n(t) - \Sigma_a \bar{v} n(t)$$

# neutronen geabsorbeerd / s



# neutronen van splijting / s

# neutronen geproduceerd / s

$$\frac{dn(t)}{dt} = \frac{(k_{\infty} - 1)}{l_{\infty}} n(t)$$

# Finite multiplying systems

Aannamen:

- Er is ook splijtbaar materiaal aanwezig
- Er is neutron leakage uit eindige core

## Finite multiplying system

Balansvergelijking

Notatie: leakage evenredig met aantal absorberd

Neutronen

- Geboren in source  $S$  of in splijting
- Eindigen door absorptie of leakage

Waarschijnlijkheid op (non)leakage

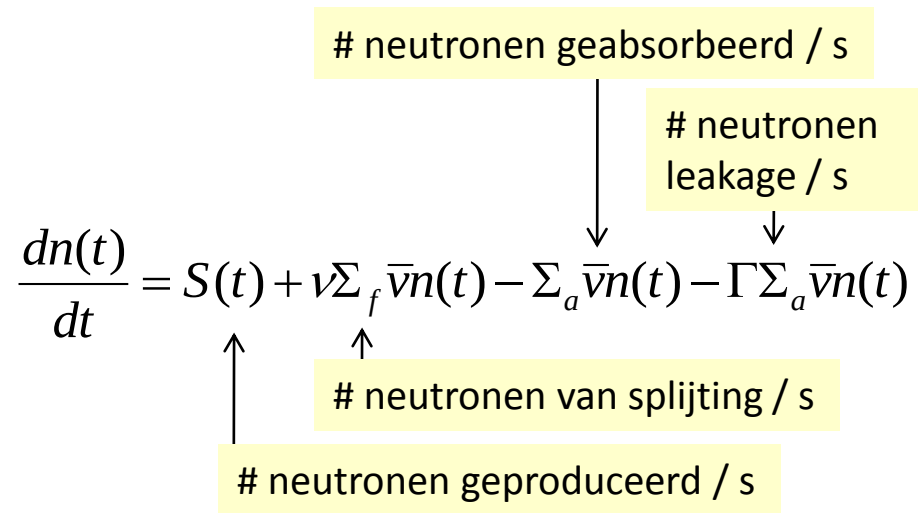
$$P_L = \frac{\Gamma \Sigma_a \bar{v}n}{\Sigma_a \bar{v}n + \Gamma \Sigma_a \bar{v}n} = \frac{\Gamma}{1 + \Gamma} \rightarrow P_{NL} = 1 - P_L = \frac{1}{1 + \Gamma}$$

We verwachten dat  $\Gamma$  afneemt met grootte van reactor

We schrijven 
$$P_{NL} \frac{dn(t)}{dt} = P_{NL} S(t) + P_{NL} (\nu \Sigma_f) \bar{v}n(t) - \Sigma_a \bar{v}n(t)$$

$$\Rightarrow P_{NL} \frac{dn(t)}{dt} = P_{NL} S(t) + \frac{(P_{NL} k_{\infty} - 1)}{l_{\infty}} n(t) \Rightarrow \frac{dn(t)}{dt} = S(t) + \frac{(k - 1)}{l} n(t)$$

Analoog aan infinite medium, met notatie  $k = P_{NL} k_{\infty}$ , en  $l = P_{NL} l_{\infty}$



# Gedrag multiplying systems

Criticality analyse:

Zet bronterm  $S(t) = 0$

Verwaarloos delayed neutrons

$$\frac{dn(t)}{dt} = \frac{(k-1)}{l} n(t)$$

Indien  $n(0) > 0$

$$n(t) = n(0)e^{\frac{(k-1)}{l}t}$$

Een systeem is critical als

Er een tijdonafhankelijke kettingreactie gaande is  
in afwezigheid van een bron  $S(t)$



$$k = 1$$

We onderscheiden weer

Subcritical  $k_{\infty} < 1$

Critical  $k_{\infty} = 1$

Supercritical  $k_{\infty} > 1$

Met bron  $n(0) = 0$ , en  $S(t) = S_0$

 
$$n(t) = \frac{lS_0}{(k-1)} \left\{ e^{\frac{(k-1)}{l}t} - 1 \right\}$$

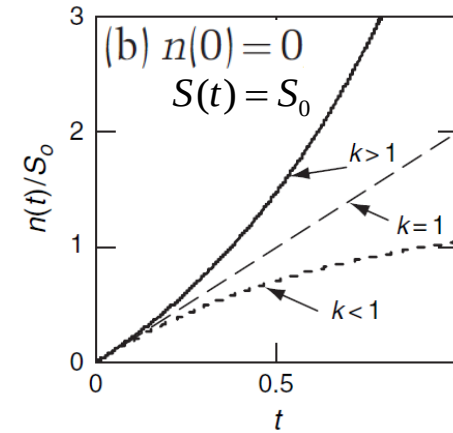
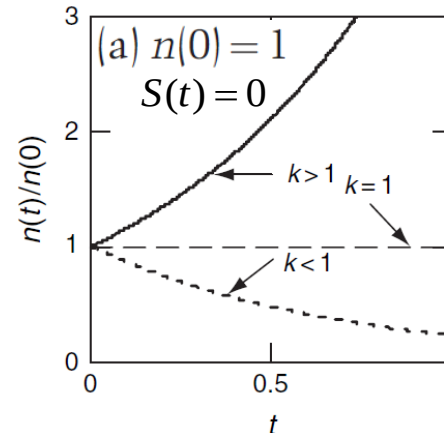
$$k < 1 \Rightarrow n(\infty) = \frac{lS_0}{(1-k)}$$

$$k \rightarrow 1 \Rightarrow n(t) = S_0 t$$

Neutronen populaties

(a) zonder bron

(b) met bron



Zeer snelle tijdvariaties:  $10^{-8}$  tot  $10^{-4}$  s

# Vertraagde neutronen

Meer dan 99% van alle splijtingsneutronen worden “*instantaan*” geproduceerd

Een kleine fractie  $\beta$  komt van het verval van splijtingsproducten

We onderscheiden

Er geldt  $\beta = \sum_{i=1}^6 \beta_i$

Gemiddelde halfwaardetijd  $t_{1/2} = \frac{1}{\beta} \sum_{i=1}^6 \beta_i t_{i1/2}$  }  $\frac{1}{\lambda} = \frac{1}{\beta} \sum_{i=1}^6 \beta_i \frac{1}{\lambda_i}$

Verder  $t_{i1/2} = 0.693 / \lambda_i$

Prompt neutron levensduur  $l$

Delayed neutron levensduur  $l_d = l + t_{1/2} / 0.693 = l + 1 / \lambda$

Gemiddelde neutron levensduur

$$\bar{l} = (1 - \beta)l + \beta l_d = l + \beta / \lambda$$

Bijdrage van delayed neutronen domineert de gemiddelde neutron levensduur, want  $\beta / \lambda \gg l$

We kunnen niet eenvoudig  $l$  door  $\bar{l}$  vervangen in uitdrukkingen

| Approximate<br>Half-life (sec) | Delayed Neutron Fraction |           |            |
|--------------------------------|--------------------------|-----------|------------|
|                                | $U^{233}$                | $U^{235}$ | $Pu^{239}$ |
| 56                             | 0.00023                  | 0.00021   | 0.00007    |
| 23                             | 0.00078                  | 0.00142   | 0.00063    |
| 6.2                            | 0.00064                  | 0.00128   | 0.00044    |
| 2.3                            | 0.00074                  | 0.00257   | 0.00069    |
| 0.61                           | 0.00014                  | 0.00075   | 0.00018    |
| 0.23                           | 0.00008                  | 0.00027   | 0.00009    |
| Total delayed fraction         | 0.00261                  | 0.00650   | 0.00210    |
| Total neutrons/fission         | 2.50                     | 2.43      | 2.90       |

# Vertraagde neutronen: dynamica

Kinetics equations

$$\frac{dn(t)}{dt} = S(t) + (1-\beta)\nu\Sigma_f \bar{v}n(t) + \sum_i \lambda_i C_i(t) - \Sigma_a \bar{v}n(t) - \Gamma\Sigma_a \bar{v}n(t)$$

Annotations for the kinetics equation:

- $S(t)$ : # neutronen geproduceerd / s
- $(1-\beta)\nu\Sigma_f \bar{v}n(t)$ : # neutronen van splijting / s
- $\sum_i \lambda_i C_i(t)$ : # delayed neutronen / s
- $\Sigma_a \bar{v}n(t)$ : # neutronen geabsorbeerd / s
- $\Gamma\Sigma_a \bar{v}n(t)$ : # neutronen leakage / s

Precursor concentraties

$$\frac{dC_i(t)}{dt} = \beta_i \nu \Sigma_f \bar{v} n(t) - \lambda_i C_i(t), \quad i = 1, 2, \dots, 6$$

Annotations for the precursor equation:

- $\beta_i \nu \Sigma_f \bar{v} n(t)$ : # precursors geproduceerd / s
- $\lambda_i C_i(t)$ : # precursors verval / s

Neutron kinetics equations herschrijven als

$$\frac{dn(t)}{dt} = S(t) + \frac{1}{l} [(1-\beta)k - 1] n(t) + \sum_i \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \beta_i \frac{k}{l} n(t) - \lambda_i C_i(t), \quad i = 1, 2, \dots, 6$$

Steady-state oplossing:

$$0 = S_0 + \frac{(k-1)}{l} n$$

Dus  $k = 1$  als  $S_0 = 0$

# Reactiviteit

Definitie van reactiviteit  $\rho \equiv \frac{k-1}{k}$

We onderscheiden weer

Subcritical  $\rho < 0$

Critical  $\rho = 0$

Supercritical  $\rho > 0$

Definitie: prompt generation time  $\Lambda \equiv l / k$

Neutron kinetics equations herschrijven als

Meestal  $\lambda_i \Lambda / \beta_i \ll 1$

Dan geldt  $C_i \gg n$

Aantal splijtingsproducten dat neutronen uitzendt is veel groter dan het aantal neutronen

Stapverandering in reactiviteit  $\pm 0.10\beta$

Neem aan  $\Lambda = 50 \times 10^{-6}$  s

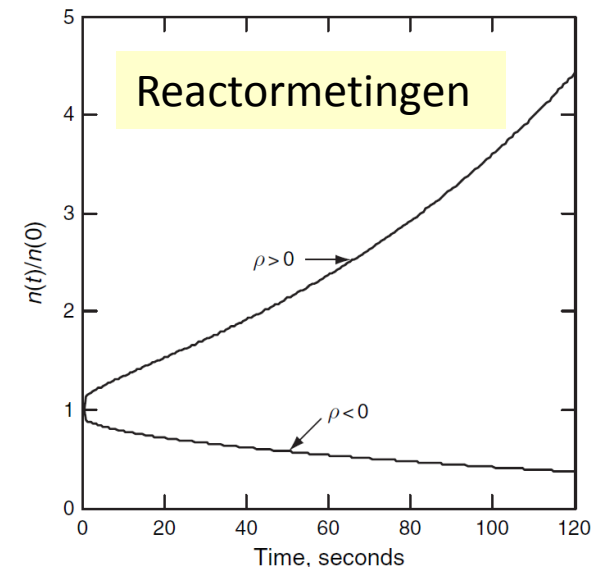
Levensduur van de splijtingsproducten die neutronen uitzenden bepalen de tijd response

Asymptotisch geldt  $n(t) \approx A_1 e^{t/T} \approx A_1 e^{(\rho\lambda)t/\beta}$

Reactor period  $T$

$$\frac{dn(t)}{dt} = S(t) + \frac{(\rho - \beta)}{\Lambda} n(t) + \sum_i \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t), \quad i = 1, 2, \dots, 6$$



# Reactor periode

Prompt critical conditie  $\rho = \beta$

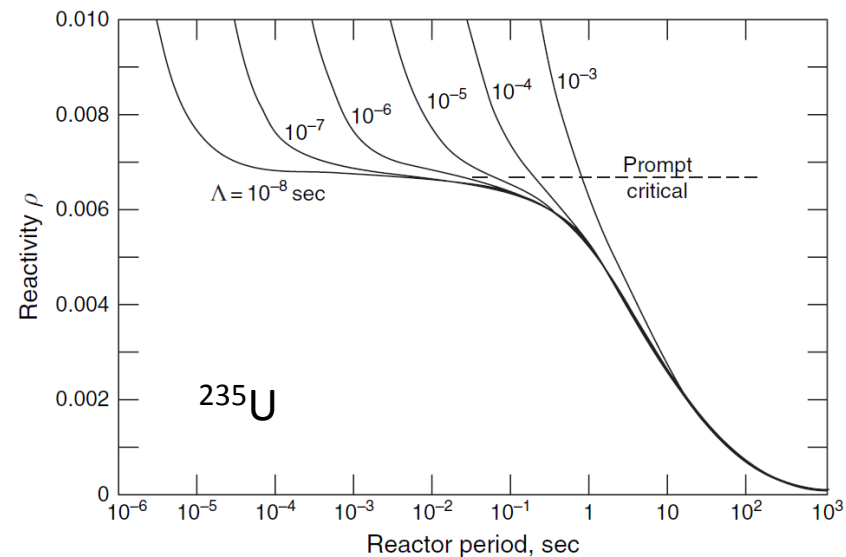
Voor  $\rho > \beta$  is kettingreactie mogelijk  
zonder delayed neutronen!

Prompt critical niet benaderen!

Reactor kan niet sneller uit dan in 56 s

Voor kleine reactivities  $T \approx \beta / (\rho \lambda)$

Vertraagde neutronen maken de dienst uit



# ***Lange termijn core gedrag***

# Lange termijn core gedrag

Lange termijn effecten:

Opbouw en verval van radioactieve splijtingsproducten

Depletie van brandstof

Opbouw van actiniden (veroorzaken neutron capture)

Vermenigvuldigingsfactor neemt af in de tijd  $k = \eta_T f \varepsilon p P_{NL}$

Fuel burnup en fission product buildup hebben effect op thermische werkzame doorsnede, en dus  $\eta_T$   $f$

Merk op  $f = \frac{1}{1 + \varsigma \left( V_m \bar{\Sigma}_{aT}^m / V_f \bar{\Sigma}_{aT}^f \right)}$   $\left. \begin{array}{l} \varsigma = \bar{\varphi}_{mT} / \bar{\varphi}_{fT} \\ \eta_T = \frac{\nu \bar{\Sigma}_{fT}^f}{\bar{\Sigma}_{aT}^f} \end{array} \right\} k = \frac{\nu \Sigma_f^f}{\Sigma_a^f + \varsigma (V_m / V_f) \Sigma_\gamma^m} \varepsilon p P_{NL}$

Fuel depletion  $\Sigma_f^f \rightarrow \Sigma_f^f(t)$

Splijtingsproducten (fp) die ontstaan  $\Sigma_a^f \rightarrow \Sigma_a^f(t) + \Sigma_a^{fp}(t)$

Reactor moet altijd kritisch blijven ( $k = 1$ ), dus voegen we poisons toe

Dus  $1 = \frac{\nu \Sigma_f^f(t)}{\Sigma_a^f(t) + \Sigma_a^{fp}(t) + \varsigma (V_m / V_f) \Sigma_\gamma^m + \Sigma_\gamma^{con}(t)} \varepsilon p P_{NL}$

# Splijtingsproducten: opbouw en verval

Vermenigvuldigingsfactor *zonder* poisons  $k(t) = \frac{\nu \Sigma_f^f(t)}{\Sigma_a^f(t) + \Sigma_\gamma^{fp}(t) + \varsigma(V_m/V_f) \Sigma_\gamma^m} \epsilon p P_{NL}$

Excess reactivity  $\rho_{ex}(t) = \frac{k(t) - 1}{k(t)} \approx k(t) - 1$

Fuel depletion en fission product buildup laten reactivity afnemen

Splijtingsproducten  $\frac{d}{dt} N(t) = \gamma_{fp} \bar{\Sigma}_f \phi - \lambda N(t) - \sigma_a N(t) \phi$



Fission rate: opbouw fp



fp verval



fp neutron absorptie

Splijtingsproducten als Xenon en Samarium hebben grote capture werkzame doorsnede

Herschrijf  $\frac{d}{dt} N(t) = \gamma_{fp} \bar{\Sigma}_f \phi - \lambda' N(t)$   $\lambda' = \lambda + \sigma_a \phi$

Oplossing  $N(t) = \frac{\gamma_{fp} \bar{\Sigma}_f \phi}{\lambda'} [1 - \exp(-\lambda' t)]$

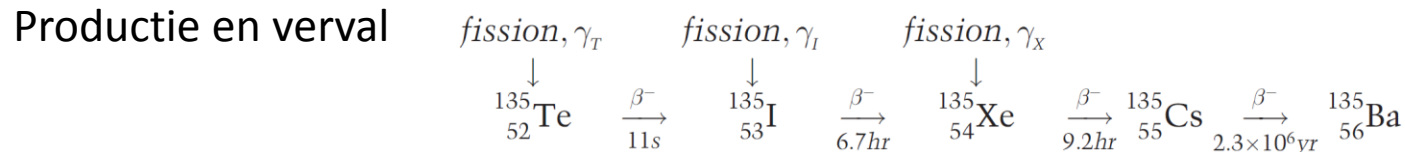
Voor korte tijden  $\lambda' t \ll 1$  geldt  $N(t) \approx \gamma_{fp} \bar{\Sigma}_f \phi t$

Voor lange tijden  $\lambda' t \gg 1$  geldt  $\gamma_{fp} \bar{\Sigma}_f \phi / \lambda'$

Halfwaardetijden: jodium-131 (8.0 dagen), cesium-137 (30.2 jaren)

# Xenon vergiftiging

Absorptie werkzame doorsnede  $2.65 \times 10^6 \text{ b}$



Thermal Fission Product Yields in Atoms per Fission

| Isotope             | Uranium-235 | Plutonium-239 | Uranium-233 |
|---------------------|-------------|---------------|-------------|
| ${}^{135}\text{I}$  | 0.0639      | 0.0604        | 0.0475      |
| ${}^{135}\text{Xe}$ | 0.00237     | 0.0105        | 0.0107      |
| ${}^{149}\text{Pm}$ | 0.01071     | 0.0121        | 0.00795     |

Source: M. E. Meek and B. F. Rider, "Compilation of Fission Product Yields," General Electric Company Report NEDO-12154, 1972.

Dan geldt

$$\frac{d}{dt}I(t) = \gamma_I \bar{\Sigma}_f \phi - \lambda_I I(t)$$

Neem tellurium-235 en jodium-135 samen

$$\frac{d}{dt}X(t) = \gamma_X \bar{\Sigma}_f \phi + \lambda_I I(t) - \lambda_X X(t) - \sigma_{aX} X(t) \phi$$

Verwaarloos verval van cesium, en geen absorptie door  ${}^{135}\text{I}$

Na reactor start-up bouwen de I en X concentraties op naar evenwicht

Evenwichtconcentraties  $I(\infty) = \gamma_I \bar{\Sigma}_f \phi / \lambda_I$   $X(\infty) = \frac{(\gamma_I + \gamma_X) \bar{\Sigma}_f \phi}{\lambda_X + \sigma_{aX} \phi}$

Voor hoge fluxen geldt  $\sigma_{aX} \phi \gg \lambda_X$

# Xenon en reactor shutdown

Tijdens shutdown hebben we concentraties  $I_o$  en  $X_o$

Stel  $\phi = 0$  in  $\frac{d}{dt}I(t) = \gamma_I \bar{\Sigma}_f \phi - \lambda_I I(t)$

Dan geldt  $I(t) = I_o \exp(-\lambda_I t)$

Invullen in  $\frac{d}{dt}X(t) = \gamma_X \bar{\Sigma}_f \phi + \lambda_I I(t) - \lambda_X X(t) - \sigma_{aX} X(t) \phi$

$$\rightarrow \frac{d}{dt}X(t) = \lambda_I I_o \exp(-\lambda_I t) - \lambda_X X(t)$$

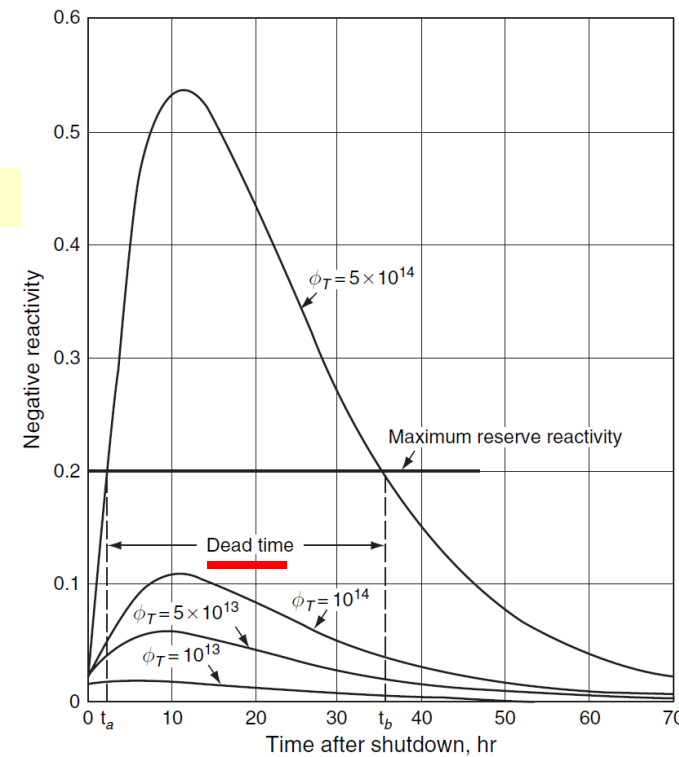
$$\rightarrow X(t) = \underbrace{X_o e^{-\lambda_X t}}_{\text{Xenon verval}} + \underbrace{\frac{\lambda_I}{\lambda_I - \lambda_X} I_o (e^{-\lambda_X t} - e^{-\lambda_I t})}_{\text{Xenon uit jodium verval}}$$

Na enkele dagen

$$X(t) = \bar{\Sigma}_f \phi \left[ \frac{(\gamma_I + \gamma_X)}{\lambda_X + \sigma_{aX} \phi} e^{-\lambda_X t} + \frac{\gamma_I}{\lambda_I - \lambda_X} (e^{-\lambda_X t} - e^{-\lambda_I t}) \right]$$

Negatieve reactiviteit bijdrage  $\sigma_{aX} X(t)$

$$k(t) = \frac{\nu \Sigma_f^f(t)}{\underbrace{\Sigma_a^f(t)}_{\Sigma_a^f \rightarrow \Sigma_a^f(t) + \Sigma_a^{fp}(t)} + \Sigma_\gamma^{fp}(t) + \varsigma(V_m/V_f) \Sigma_\gamma^m} \epsilon p P_{NL}$$



# Samarium vergiftiging

Werkzame doorsnede voor absorptie 41,000 b

Vervalreeks  $fission, \gamma_N$   

$$^{149}_{60}\text{Nd} \xrightarrow[1.7\text{hr}]{\beta^-} ^{149}_{61}\text{Pm} \xrightarrow[53\text{hr}]{\beta^-} ^{149}_{62}\text{Sm}$$

Er geldt  $\frac{d}{dt}P(t) = \gamma_P \bar{\Sigma}_f \phi - \lambda_P P(t)$



Promethium

$\frac{d}{dt}S(t) = \lambda_P P(t) - \sigma_{aS}(t)\phi$



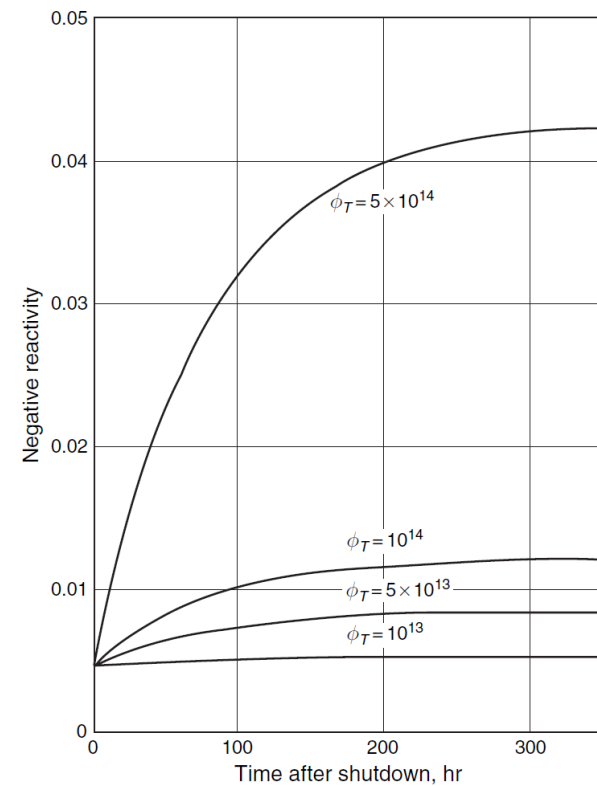
Samarium

Shutdown yield  $P(t) = P_o \exp(-\lambda_P t)$   
 $S(t) = S_o + P_o(1 - e^{-\lambda_P t})$

Combineren  $S(t) = \frac{\gamma_P \bar{\Sigma}_f}{\sigma_{aS}} + \frac{\gamma_P \bar{\Sigma}_f \phi}{\lambda_P} (1 - e^{-\lambda_P t})$

Na shutdown neemt de samarium concentratie toe met  $\gamma_P \bar{\Sigma}_f \phi / \lambda_P$

Forse extra reactivity nodig om te kunnen herstarten



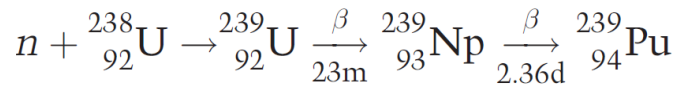
# Brandstofdepletie

Vermogensdichtheid  $P''' = \gamma \Sigma_f \phi$  opsplitsen  $P''' = \gamma \left[ \sigma_f^{25} N^{25}(t) + \sigma_f^{49} N^{49}(t) \right] \phi(t)$

Vergelijkingen  $\frac{d}{dt} N^{25}(t) = -N^{25}(t) \sigma_a^{25} \phi(t)$  Uranium-235

$\frac{d}{dt} N^{28}(t) = -N^{28}(t) \sigma_a^{28} \phi(t)$  Uranium-238

Plutonium  $\frac{d}{dt} N^{49}(t) = \sigma_\gamma^{28} \phi(t) N^{28}(t) - \sigma_a^{49} \phi(t) N^{49}(t)$



Integreer 25  $N^{25}(t) = N^{25}(0) e^{-\sigma_a^{25} \Phi(t)}$

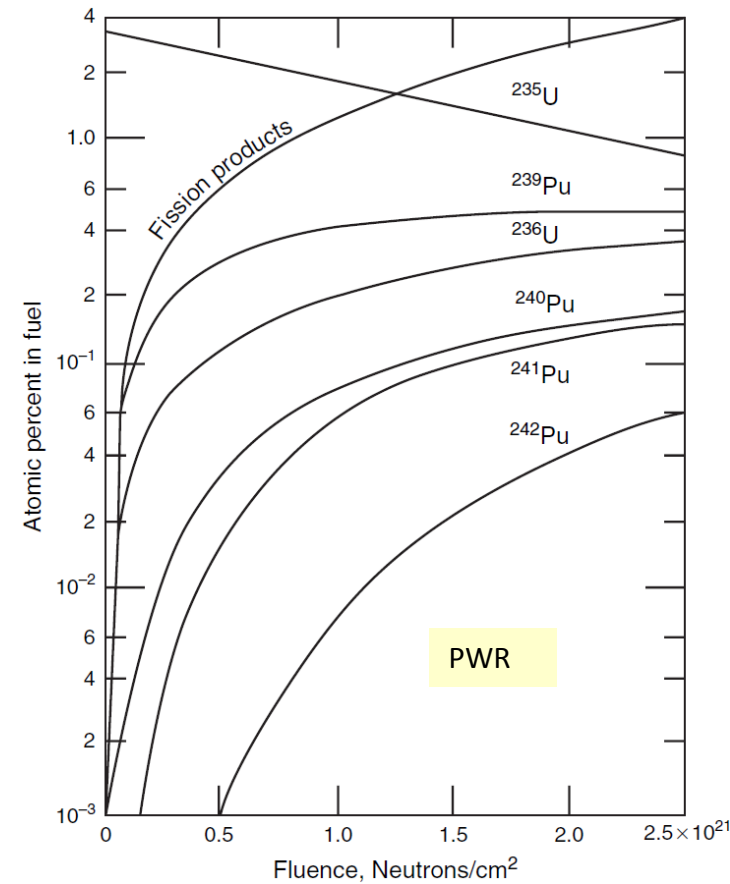
Fluence  $\Phi(t) = \int_0^t \phi(t') dt'$  Kleine absorptie

Evenzo 28  $N^{28}(t) = N^{28}(0) e^{-\sigma_a^{28} \Phi(t)} \approx N^{28}(0)$

We vinden  $N^{49}(t) = \frac{\sigma_\gamma^{28}}{\sigma_a^{49}} N^{28}(0) \left( 1 - e^{-\sigma_a^{49} \Phi(t)} \right)$

Breeding ratio  $CR(t) = \frac{\sigma_\gamma^{28} N^{28}(0)}{\sigma_a^{25} N^{25}(t) + \sigma_a^{49} N^{49}(t)}$

Verder  $k(t) = \frac{\nu \sigma_f^{25} N^{25}(t) + \nu \sigma_f^{49} N^{49}(t)}{\sigma_a^{25} N^{25}(t) + \sigma_a^{49} N^{49}(t) + \Sigma_\gamma^{28} + \Sigma_\gamma^{49}(t) + \varsigma(V_m/V_f) \Sigma_a^m} \epsilon p P_{NL}$



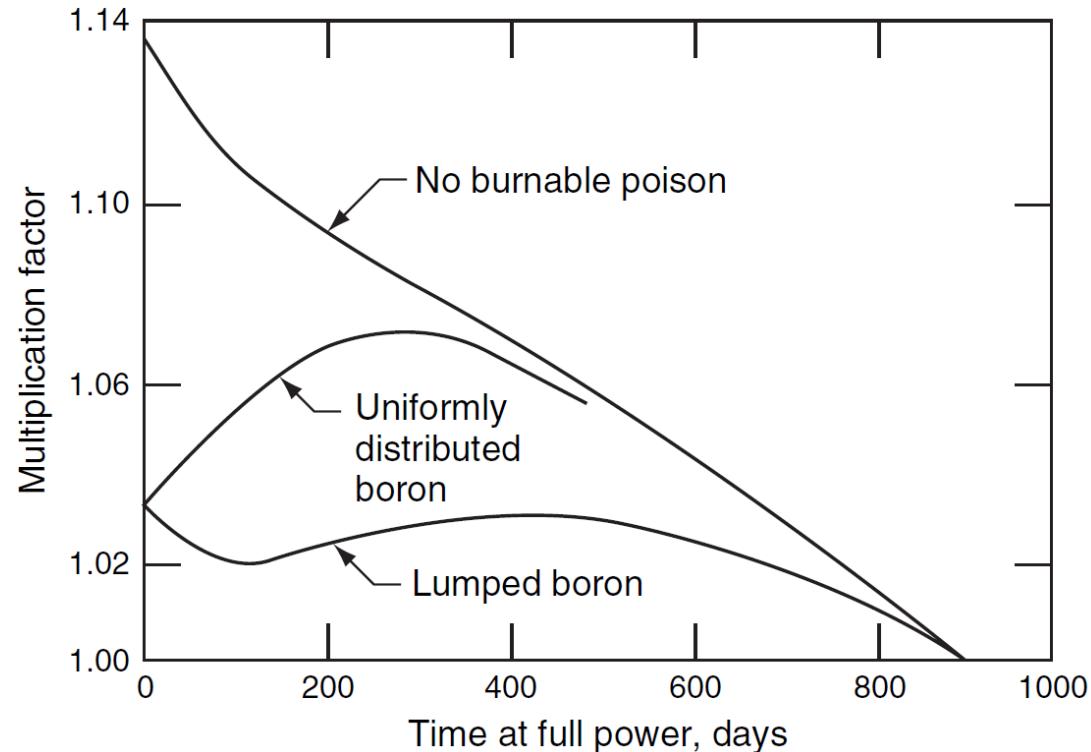
# ***Burnable poisons***

Los neutronabsorbers op in koelvloeistof

Beperk hiermee de excess reactivity

Deze materialen hebben een grote absorptie werkzame doorsnede, worden opgebrand, en zijn effectief in het begin van het reactor leven

Lumping leidt tot ruimtelijke self-shielding



# Splijtingsproducten en actiniden

Productie van splijtingsproducten is potentieel gezondheidsrisico  
Belangrijk zijn jodium, strontium en cesium

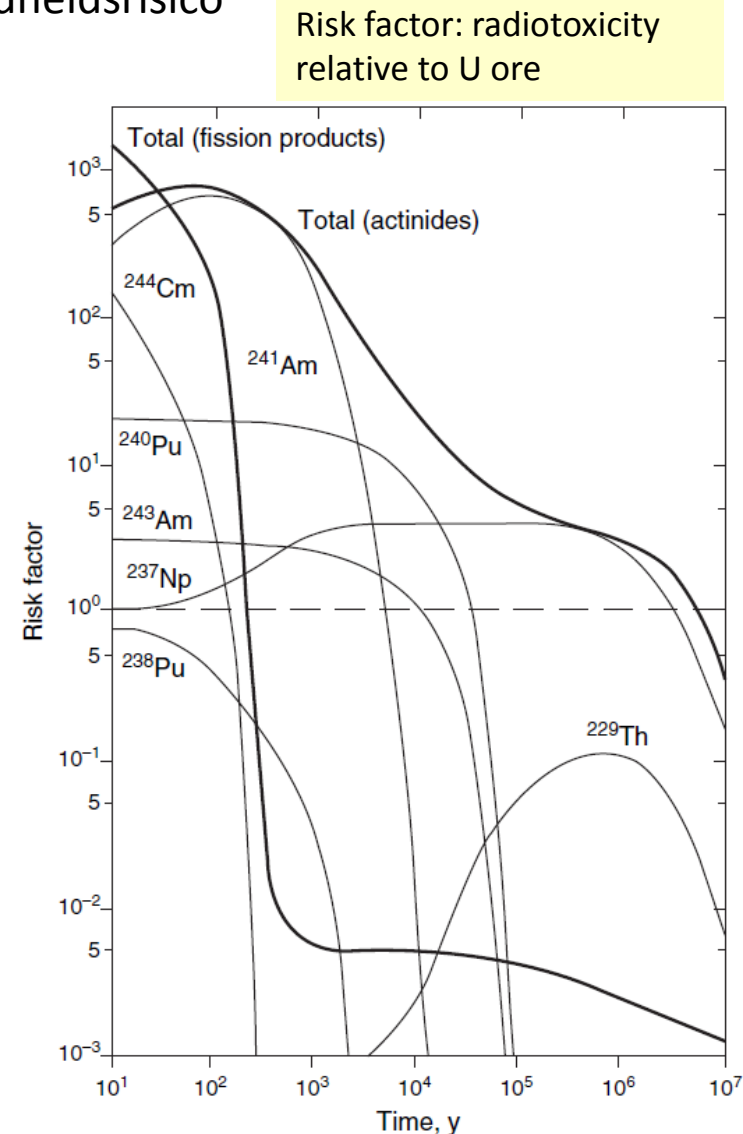
Na ongeveer een eeuw komt alle  
radioactiviteit van de actiniden en *niet*  
van de splijtingsproducten

Tim van der Hagen (TU Delft) over  
hoogradioactief afval. Bij 100% gebruik van  
kernenergie

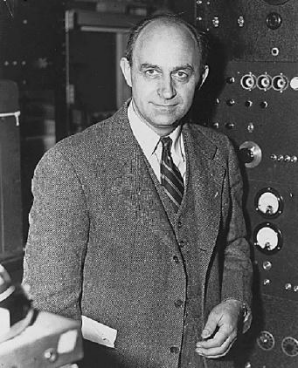
Afval per gezin 0.4 gram per jaar  
In een leven, 1 biljartbal per persoon

Borssele: 1.5 kubieke meter per jaar:  
140 kilo actiniden,  
450 kilo splijtingsproducten

Snelle reactoren (4e generatie) maken  
transmutatie mogelijk: reduceer levensduur  
van 220.000 jaar tot 500 – 5000 jaar

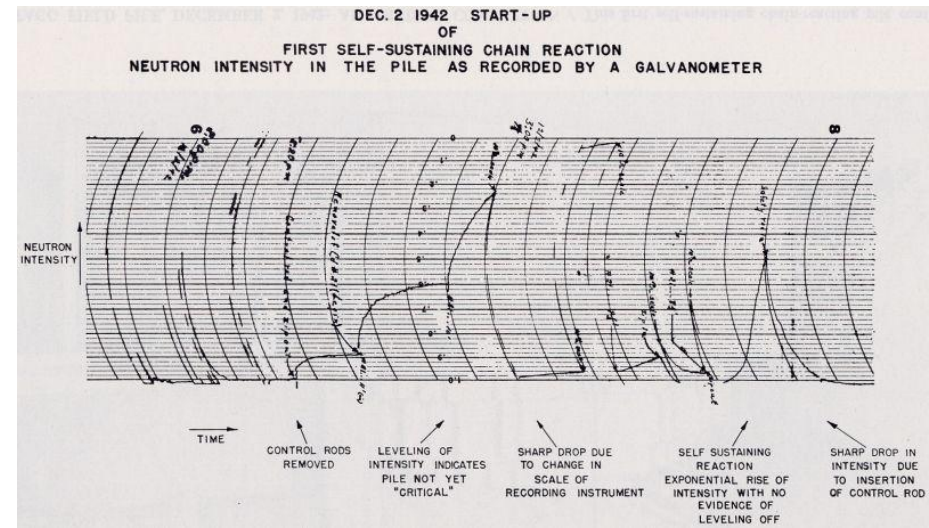
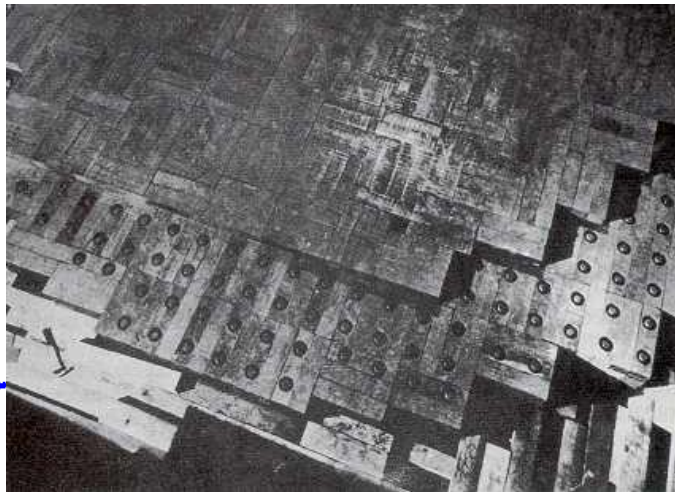


# ***Andere aspecten***



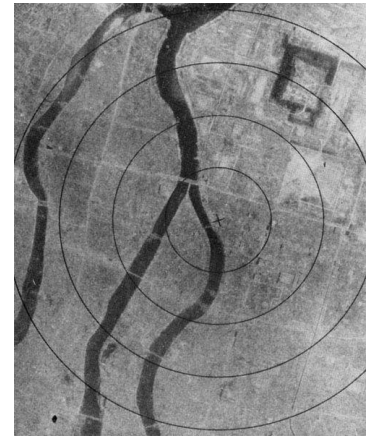
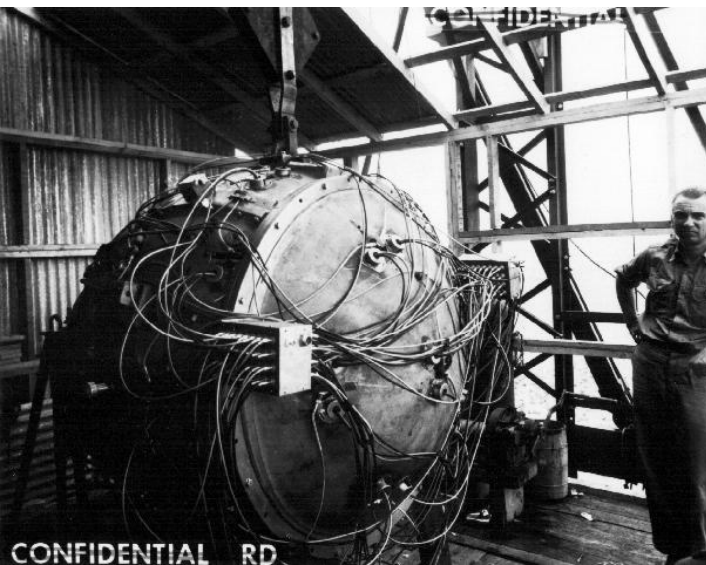
# Het begin

- Enrico Fermi
- Chicago, Dec. 2, 1942
- Criticality reached



# *Het begin*

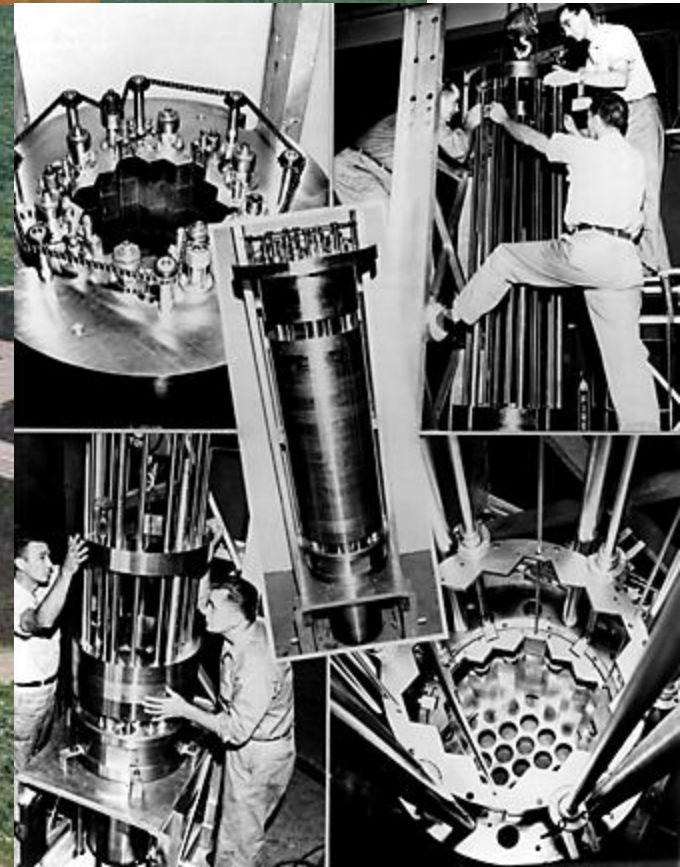
- Manhattan project
- Plutonium productie
- Reactor B in Hanford
- Trinity: the gadget
- Nagasaki bom



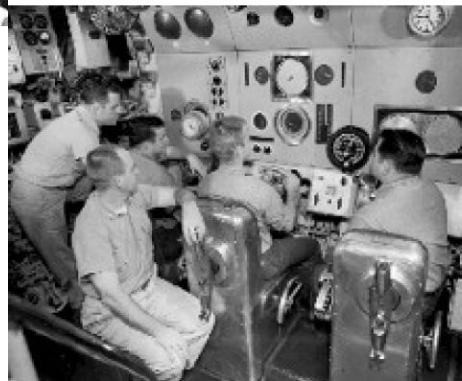
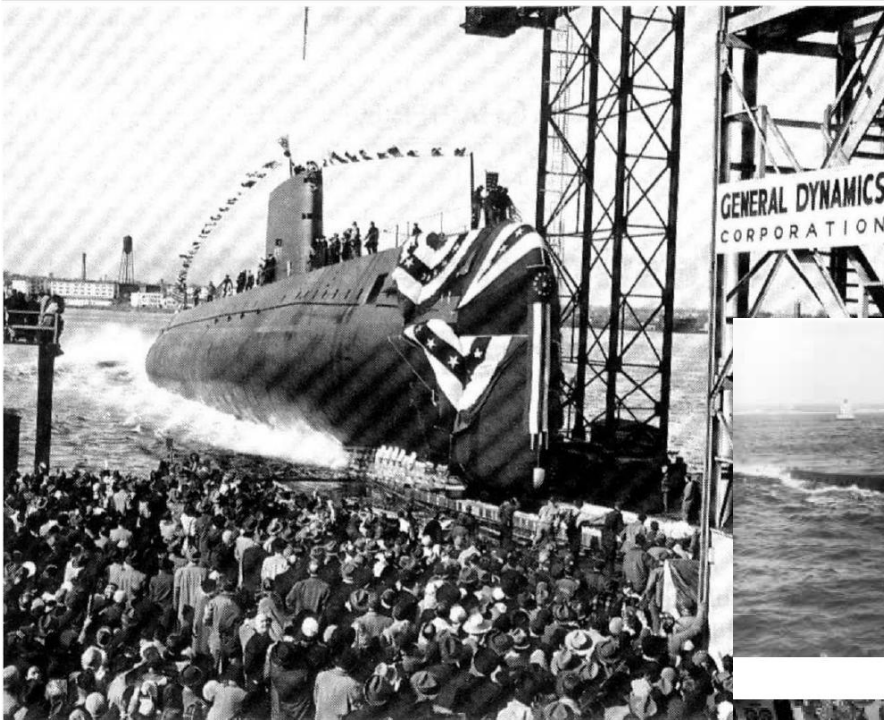
# EBR – 1 in Idaho (1951)



*Electricity was first generated here  
from Atomic Energy on Dec. 20, 1951.  
On Dec. 21, 1951—all of the electrical  
power in this building was supplied from  
Atomic Energy ~*



# Nautilus (1954)



| SHIP'S POSITION                         |                                          |                                                                     |                                 |
|-----------------------------------------|------------------------------------------|---------------------------------------------------------------------|---------------------------------|
| U. S. S. NAUTILUS                       |                                          |                                                                     |                                 |
| TO: COMMANDING OFFICER                  |                                          |                                                                     |                                 |
| (Time of day) 1915U                     |                                          | DATE 3 August 1958                                                  |                                 |
| LATITUDE 90° 00.0' N Indefinite         |                                          | LONGITUDE —                                                         |                                 |
| BY (Indicate by check in box)           |                                          |                                                                     |                                 |
| <input checked="" type="checkbox"/> N6A | <input checked="" type="checkbox"/> MK19 | <input type="checkbox"/> RADAR                                      | <input type="checkbox"/> VISUAL |
| SPEED —                                 |                                          | DISTANCE MADE GOOD SINCE (time) (miles) Honolulu 4844               |                                 |
| DISTANCE TO North Pole                  |                                          | MILES Zero                                                          |                                 |
| TABLE ERROR 180 MK19                    | ERROR 3E MK23                            | VARIATION 170 E                                                     |                                 |
| MAGNETIC COMPASS HEADING (EACH 090)     |                                          |                                                                     |                                 |
| <input type="checkbox"/> STD            | <input type="checkbox"/> STEER           | <input checked="" type="checkbox"/> REMOTE                          | <input type="checkbox"/> OTHER  |
| 244                                     |                                          | 359                                                                 |                                 |
| DEVIATION 126E                          |                                          | 100A TABLE DEVIATION 30W                                            |                                 |
| DGI (Indicate by check in box)          |                                          | <input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF |                                 |
| REMARKS                                 |                                          |                                                                     |                                 |
| N6A DR<br>$\sigma = 0$<br>$\mu = 0$     |                                          | N6A<br>$\eta_x = 0$<br>$\eta_y = 0$<br>$\eta_z = 1$                 |                                 |
| RESPECTFULLY SUBMITTED (Signature)      |                                          |                                                                     |                                 |
| LT Shepard M. Jenks, USN                |                                          |                                                                     |                                 |

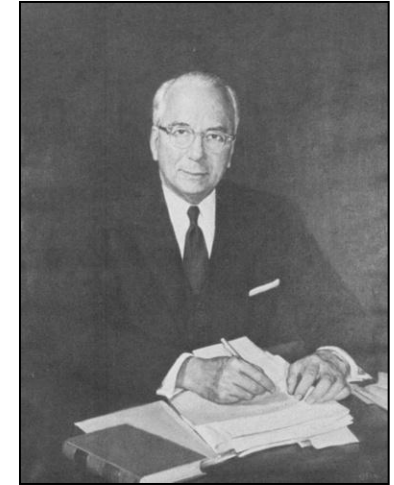


# Kernenergie

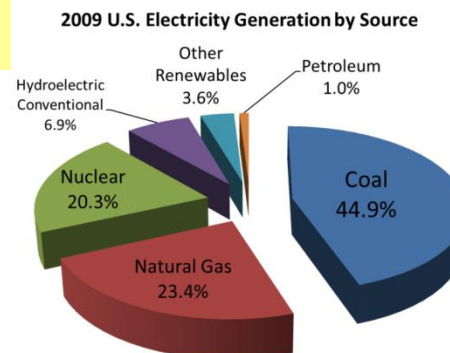
## Kernenergie vandaag:

- Levert 16% van de elektriciteit in de wereld
  - 20% in USA
  - 77% in Frankrijk
  - 54% België
  - 26% Duitsland
  - 46% Zweden
  - 4% Nederland
- 69% van de non-carbon elektriciteit in USA
- Ongeveer 441 reactoren in de wereld
  - 147 in EU (200+ in Europe)
  - 104 in USA
    - ☐ Geen gebouwd in USA na 1970s
    - ☐ Kleine budgetten voor R&D

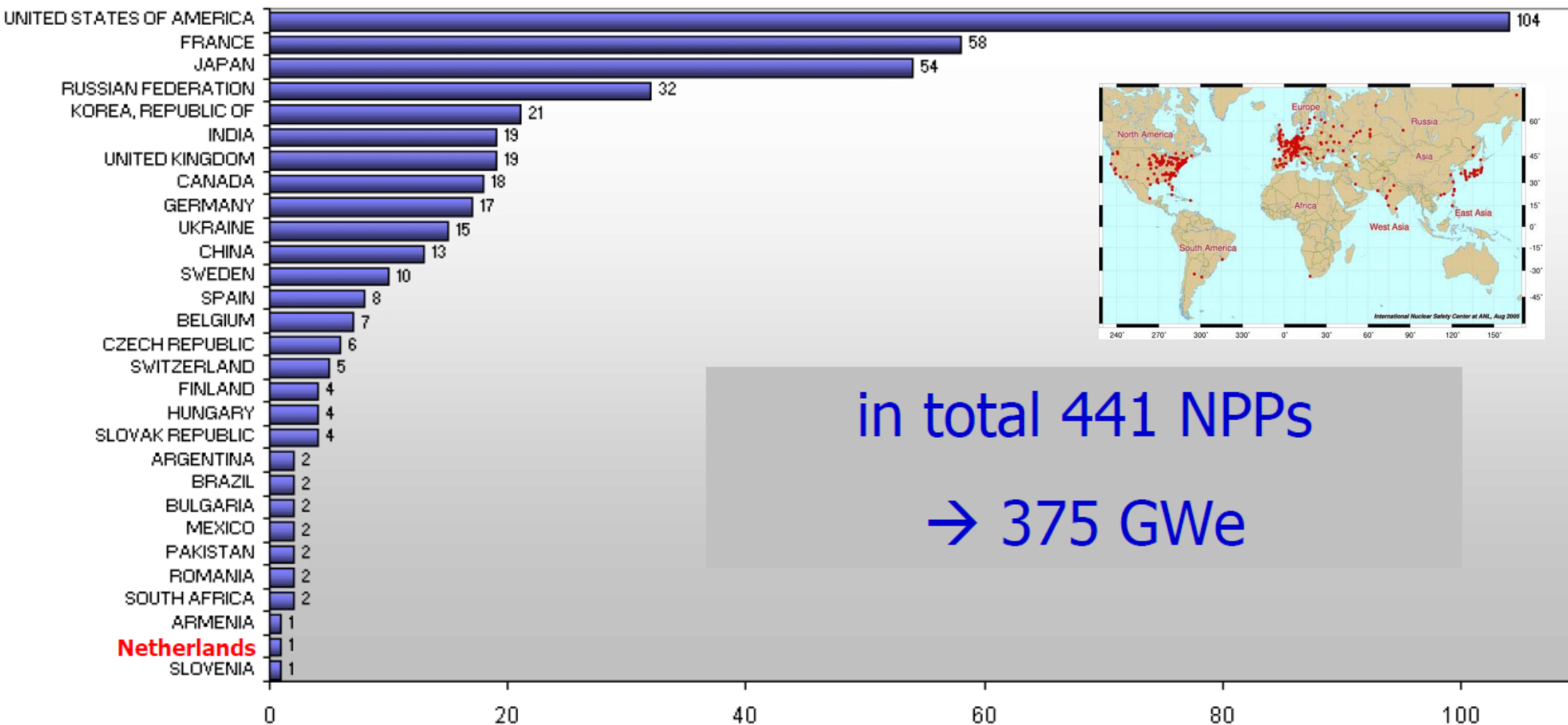
Lewis Strauss, Chairman of the U.S. Atomic Energy Commission (1954)



*"It is not too much to expect that our children will enjoy in their homes [nuclear generated] electrical energy too cheap to meter."*



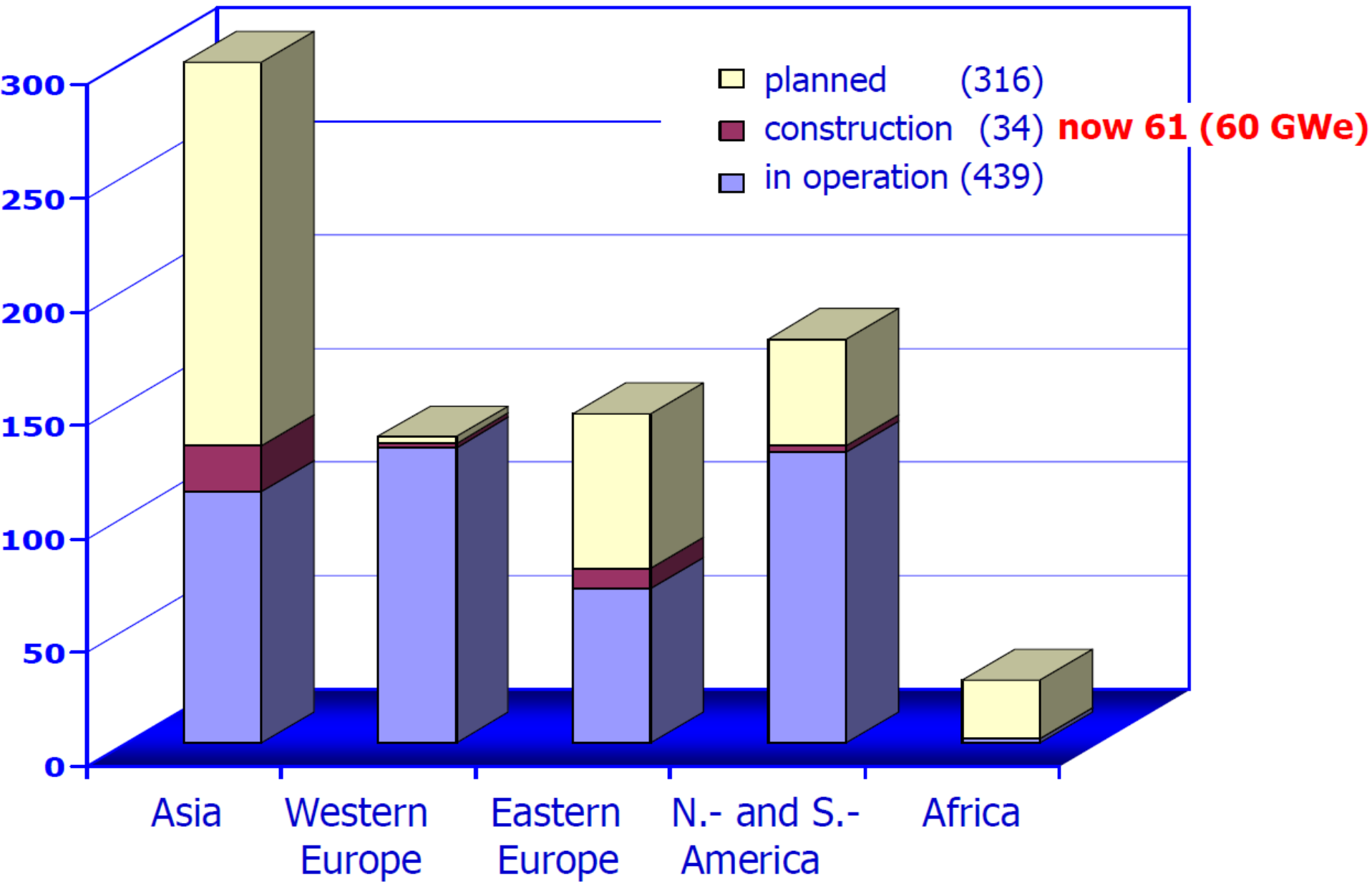
# Nuclear Power Plants in operation



Alle reactoren in de USA zijn gebouwd in ongeveer 25 jaar

# Status nuclear power plans

January 2008





# European Pressurized Water Reactor

## 4500 MWth

EPR NUCLEAR ISLAND

reactor building

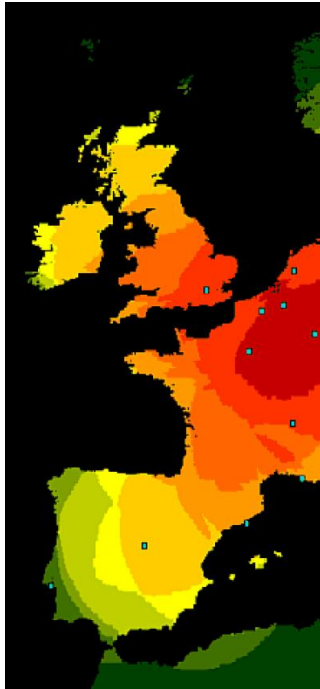
turbine building

reactor  
vessel



4 safety buildings  
4 x 100%

# Kernenergie en Nederland



## Small volumes of material needed

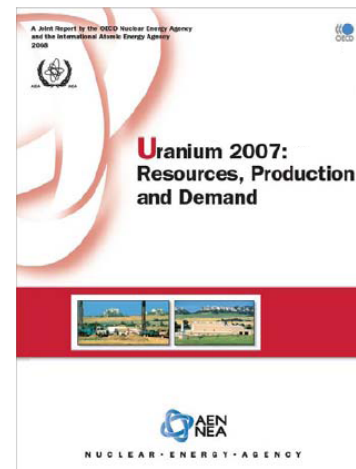
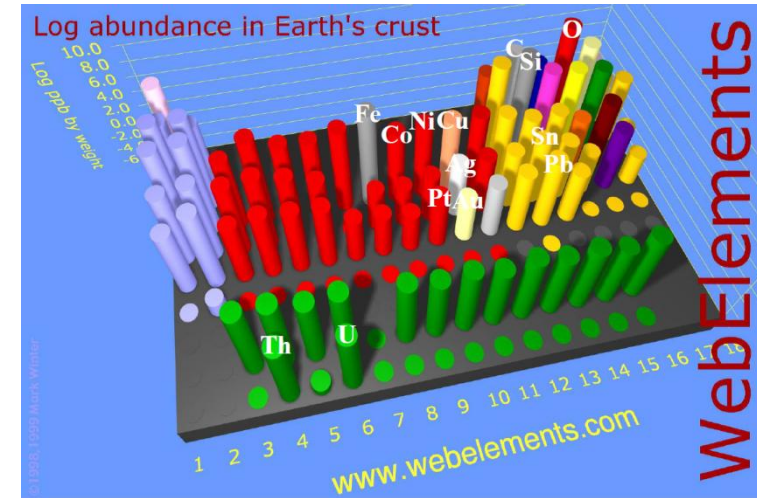
- strategic stock possible
- low amounts of waste

- all electricity in the Netherlands nuclear:  
0.4 gram uranium fissioned (=waste)  
per family per year
- in a human life: a volume of 1 billiard ball
- 'Borssele' produces 1.3 m<sup>3</sup> highly radioactive waste per year, but 'prevents' the emission of 2 billion kilograms CO<sub>2</sub> per year
- a radioactive material emits radiation →  
it clears itself (the more radioactive, the quicker)

# Beschikbaarheid uranium

## Uranium resources:

- The earth's crust contains 40 x as much uranium as silver; as much uranium as tin
- Cheap uranium (up to 130\$ per kg): 5.5 million tons; enough for 80 years (0.1 ct/kWh)
- For the double price:  
10 times as much; enough for 800 years  
using fast reactors: 80,000 years
- Uranium as byproduct from phosphate deposits (22 Mt recoverable)
- Uranium from seawater (450\$ per kg): 4 billion tons; enough for 6,000,000 years



# *International Nuclear Event Scale*



International Atomic Energy Agency



| Level 7: Major accident                   | Chernobyl                            | Large off-site impact                           |
|-------------------------------------------|--------------------------------------|-------------------------------------------------|
| Level 6: Serious accident                 | Mayak                                | Significant off-site release                    |
| Level 5: Accident with wider consequences | Windscale, Three mile island         | Severe reactor damage, limited off-site release |
| Level 4: Accident with local consequences | Sellafield, Saint-Laurent, Tokaimura | Public exposure (near limits), fatal exposure   |
| Level 3: Serious incident                 | Thorp Sellafield, Paks               | Public exposure (below limits), near accident   |
| Level 2: Incident                         | Asco, Forsmark                       | No off-site impact, overexposure of worker      |
| Level 1: Anomaly                          | Tricastin                            | Anomaly (water leak, contamination)             |

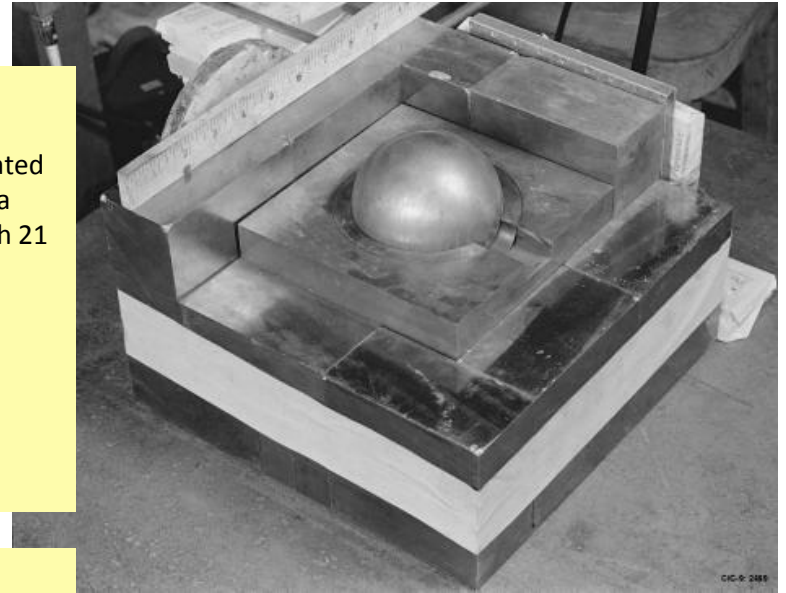
# First nuclear accidents

## Harry K. Daghljan, Jr., (1921 – September 15, 1945)

Physicist of Armenian descent with the Manhattan Project who accidentally irradiated himself on August 21, 1945 during a critical mass experiment at the remote Omega Site facility at Los Alamos National Laboratory in New Mexico, resulting in his death 21 days later.

Daghljan was irradiated as a result of a criticality accident that occurred when he accidentally dropped a small tungsten carbide brick onto a 6.2 kg delta phase plutonium bomb core.

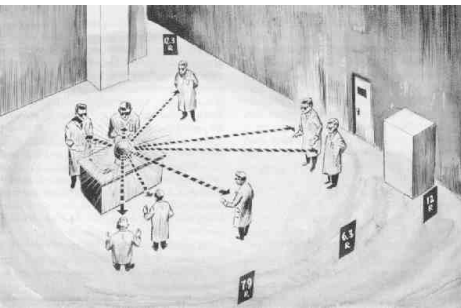
This core was later nicknamed the "Demon core"



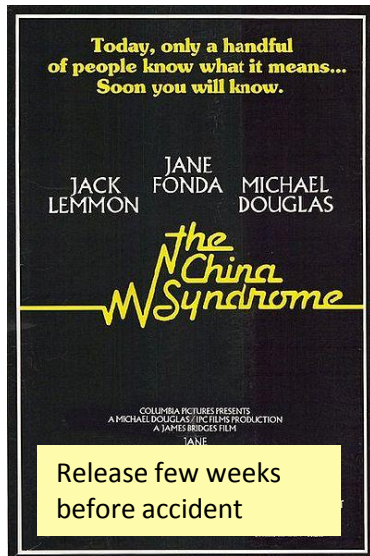
## Louis Alexander Slotin (December 1, 1910 – May 30, 1946)

Canadian physicist and chemist who took part in the Manhattan Project. Performed experiments with uranium and plutonium cores to determine their critical mass values. After World War II, Slotin continued his research at Los Alamos National Laboratory.

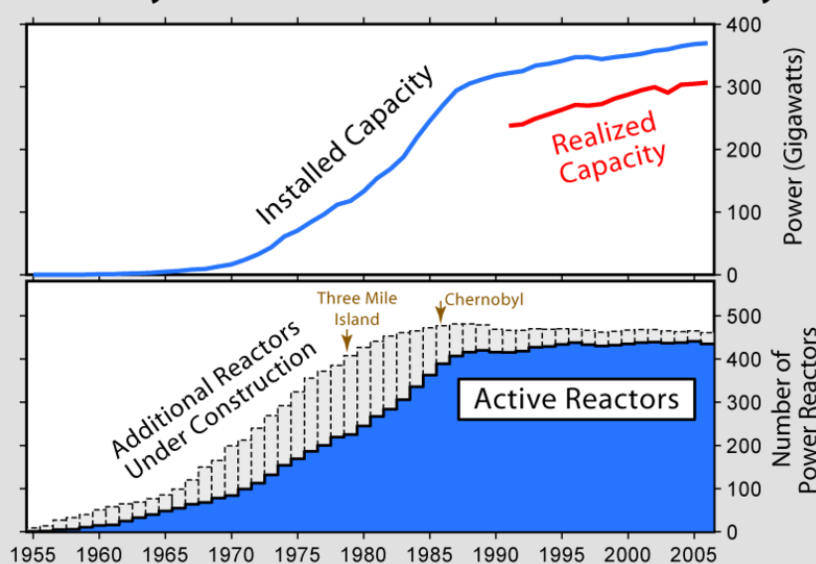
On May 21, 1946, Slotin accidentally began a fission reaction, which released a burst of hard radiation. He was rushed to hospital, and died nine days later.



# Three Mile Island – TMI-2



## History of the Global Nuclear Power Industry



### TMI-2: PWR (Babcock & Wilcox)

March 28, 1979. Biggest nuclear accident in USA. Pump of secondary non-nuclear cooling fails. Turbine and reactor are shutdown (normal procedure).

Temperature and pressure in reactor rise (normal). Relief valve of pressurizer (PORV) opens.

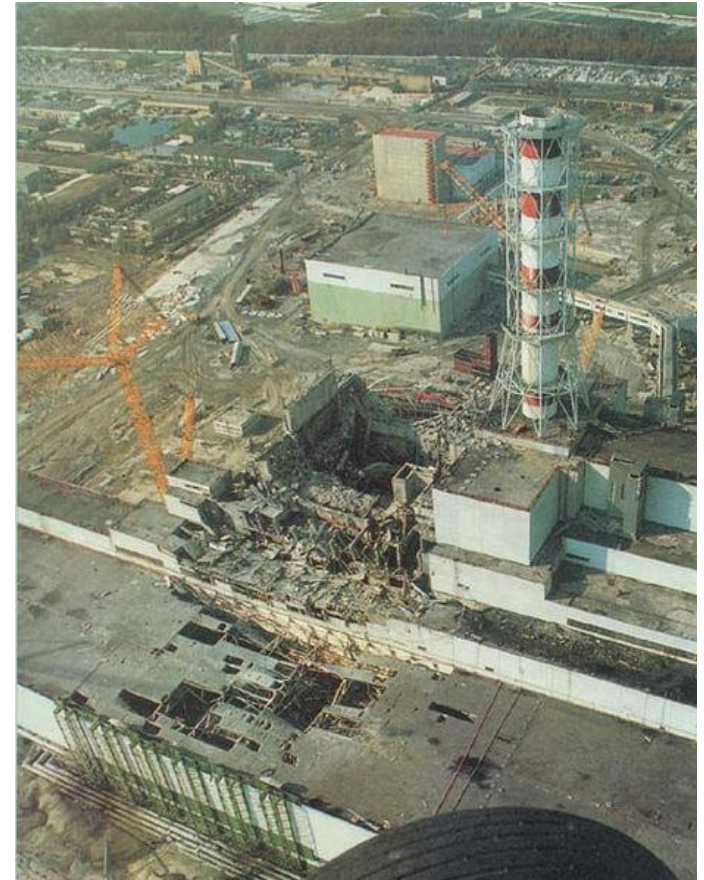
PORV should close, but fails to do so (not noticed by operators). Pressure keeps dropping, cooling water pours out of PORV. Reactor core overheats.

Backup system failed since after tests prior to accident people forgot to open valves (human error). Half of the core melted. All contained. Radioactive noble gases (~43 kCi krypton) were vented (<20 Ci of I-131).

Average dose to people within ten miles was 8 mrem. Nobody received more than 100 mrem (power plant workers norm: < 5 rem per year. Estimate of additional cancers <~ 1).

# Tsjernobyl

- Grootste kernramp in de geschiedenis
  - 26 april 1986
  - Level 7 op International Nuclear Event Scale
- De ramp
  - Test met kernreactor nummer 4
  - Schakel generator uit en kijk of er voldoende vermogen is om de koelinstallatie 60 seconde te laten werken totdat de noodaggregaten aanslaan
  - Reactorvermogen onbedoeld naar 30 MW
  - Hierdoor Xenon vergiftiging
  - Alle regelstaven uit en vermogen naar 200 MW
  - Voor de test was minimaal 600 MW nodig
  - Test toch voortgezet: waterpompen ingeschakeld
  - Door extra n-absorptie zakte vermogen verder
  - 20 van de 26 handbediende veiligheidsstaven uit
  - Turbine uit: vermogen steeg exponentieel
  - Noodstop uitgevoerd, maar dat duurt 19 seconden
  - Brandstofstaven braken, controlestaven klem
  - Reactor bereikt 30 GW, staven smelten
  - Stoomontploffing: 2000 ton dak van reactor
  - Grafiet moderator vat vlam



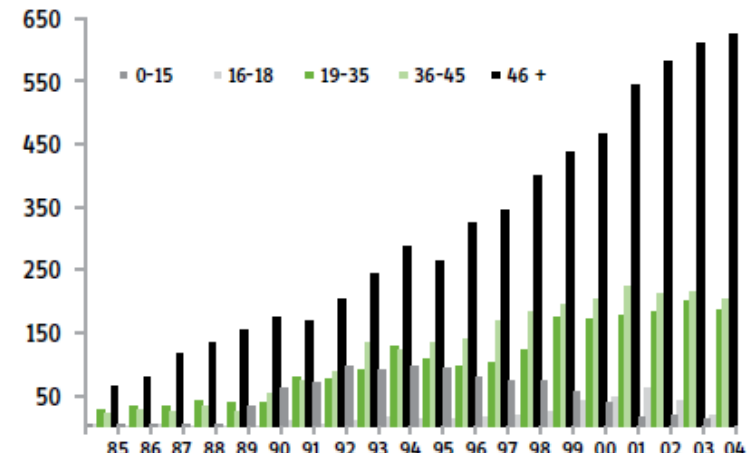
# Tsjernobyl

- Consequenties

- 42 werkers gedood door straling binnen weken
- 600.000 burgers en militaire 'liquidators' blootgesteld aan hoge stralingsniveaus: decontaminatie reactor, site, straten en constructie sarcofaag
- Radioactieve besmetting van 3000 km<sup>2</sup> oppervlak door cesium-37 (halfwaardetijd gamma-emitter 30 jaar)
- Groeiende epidemie van schildklierkanker door besmetting met jodium
- Andere kankersoorten worden verwacht, maar zijn niet detecteerbaar vanwege de hoge achtergrond van kanker door andere oorzaken. Een theoretische studie stelt op basis van Hiroshima en Nagasaki overlevenden dat 4000 extra kankerdoden voor de 600.000 liquidators, 5000 voor de 6 miljoen mensen die in besmette gebieden (> 37 kBq/m<sup>2</sup> voor cesium-137), en ongeveer 7000 voor de 500 miljoen Europeanen. Totaal 16.000 (6700 – 38.000 voor 95% confidence level)

- Gemiddeld

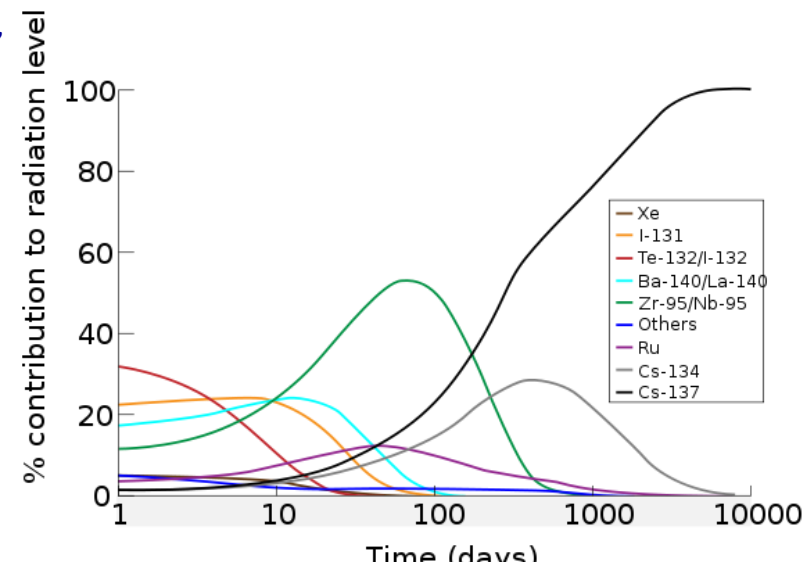
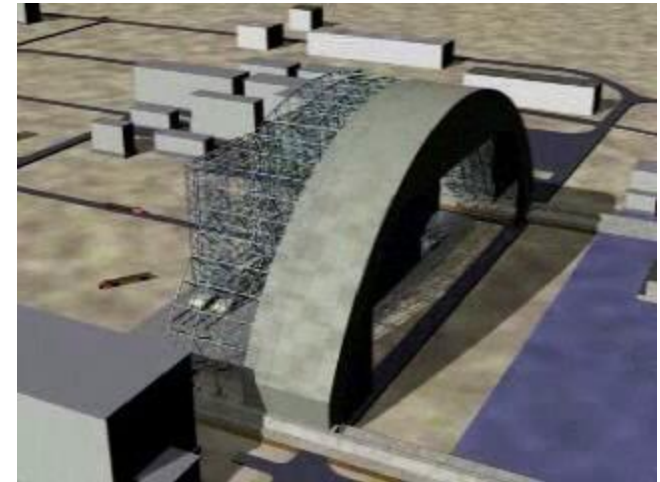
- Eind 2008: 10.000 GWe-jaar kernreactor ervaring
- Dus minder dan 2 doden per GWe-jaar; dat is minder dan bij fossiele brandstoffen
- Trauma groot: 200.000 mensen verplicht verhuisd



# Tsjernobyl

- Economische aspecten

- Schattingen variëren van \$ 6.7 miljard tot \$235 en \$148 door overheden van Belarus en Ukraine
- Sociale uitkeringen (Tsjernobyl gerelateerd) aan 7 miljoen mensen in 3 landen)
- Verplaatsing populatie
- Verlies van assets: 784.320 hectare landbouwgrond en 694.200 hectare bos. Merendeel is nu weer in gebruik
- Belarus: 20% nationaal budget in 1992, 5% in 2001
- Betaald door 18% extra belasting voor non-agricultural firms in 1994
- Chernobyl Shelter Fund: \$1.2 miljard voor de grootste bewegende structuur die ooit gebouwd is (span 270 m, hoogte 100 m en lengte 150 m; 2024 ton massa)
- Potentiele kosten van een brand in spent-fuel pools in de USA worden op honderden miljarden geschat

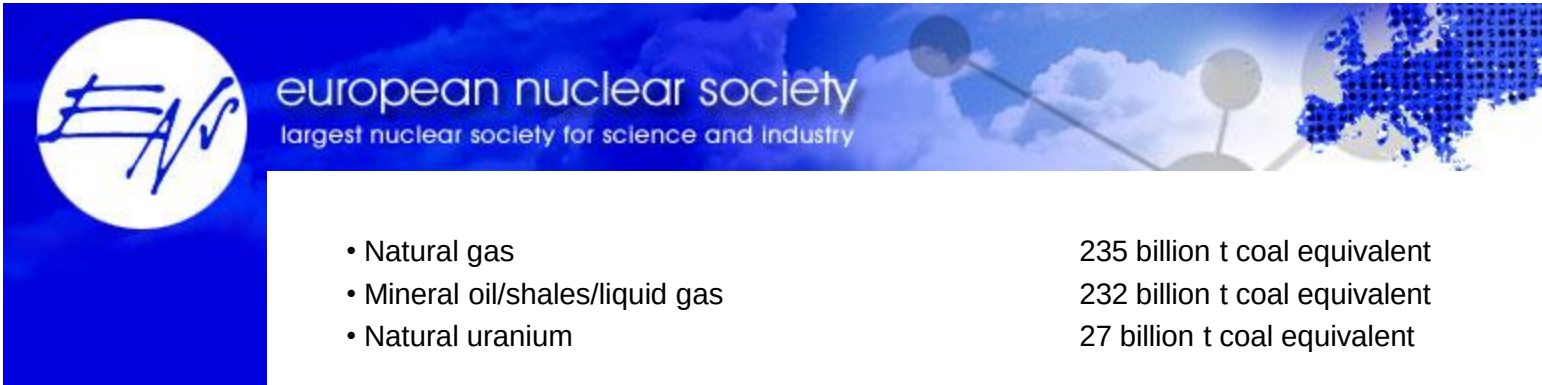


# ***Nuclear power – October 2008***

| Reactor type               | In operation |                  | Under construction |                  |
|----------------------------|--------------|------------------|--------------------|------------------|
|                            | Number       | net capacity MWe | Number             | net capacity MWe |
| PWR                        | 265          | 243,295          | 27                 | 24,195           |
| BWR                        | 94           | 85,287           | 3                  | 3,925            |
| AGR, GGR                   | 18           | 9,034            | -                  | -                |
| CANDU/D <sub>2</sub> O-PWR | 44           | 22,390           | 4                  | 1,298            |
| RBMK                       | 16           | 11,404           | 1                  | 925              |
| SNR                        | 2            | 690              | 2                  | 1,220            |
| <b>total</b>               | <b>439</b>   | <b>372,100</b>   | <b>34</b>          | <b>31,563</b>    |

The world total annual energy consumptions amount to 14 billion coal equivalent

# *Energy reserves – 2006*

The banner features the European Nuclear Society logo on the left, which consists of a white circle containing a stylized blue 'E' and 'N'. To the right of the logo, the text 'european nuclear society' is written in white, with 'largest nuclear society for science and industry' in smaller white text below it. The background of the banner is a blue sky with white clouds. On the right side of the banner, there is a stylized graphic of a person holding a telescope, with a globe of the European Union map visible in the background.

european nuclear society  
largest nuclear society for science and industry

- Natural gas
- Mineral oil/shales/liquid gas
- Natural uranium
- Coal (all forms)

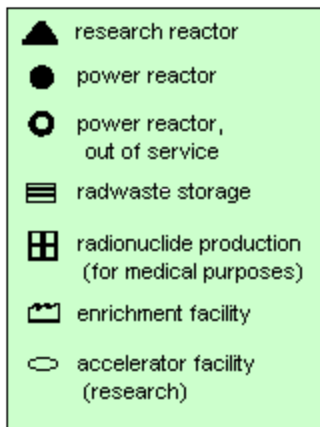
235 billion t coal equivalent

232 billion t coal equivalent

27 billion t coal equivalent

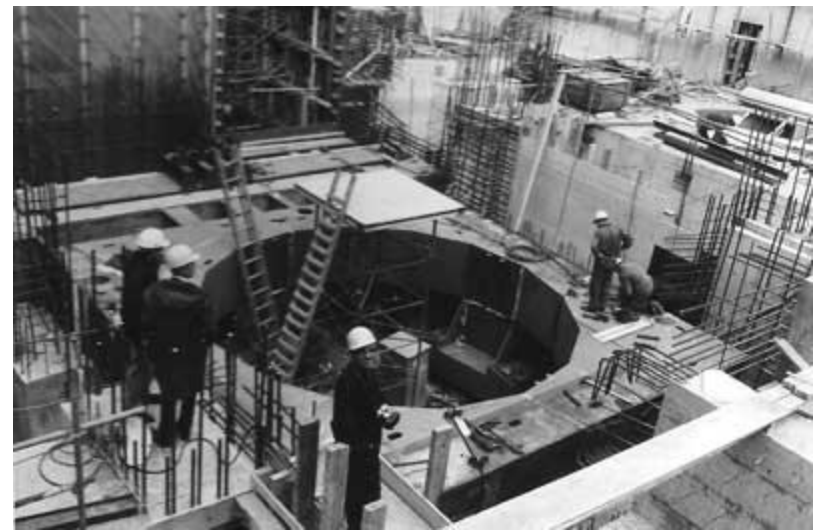
726 billion t coal equivalent.

# *Nuclear installations in The Netherlands*



# Borssele PWR

- 1969 PZEM bestelt reactor bij Siemens/KWU
- 25 Oktober 1973 levering
  - Na succesvolle eerste testen
  - Overheid geeft permanente bedrijfsvergunning
- 1979 – 1984 Upgrade veiligheid
  - Reserve koelwatersysteem
  - Na Harrisburg, Maart 1979
- 1990 EPZ wordt eigenaar
- 11 Juli 1994 EZ stelt dat bedrijf wordt verlengd tot 2007
  - Upgrade project 'Modifications'
  - 450 miljoen gulden
  - Er dient voldoende terugverdiendtijd te zijn
- Mei - Juni 2003 Balkenende-2
  - Sluiting uiterlijk in 2013
- 16 Juni 2006 Borssele Covenant
  - Bedrijf mogelijk tot 2034
  - Nuclear Energy Act Licence: elke 10 jaar safety check
  - Essent en Delta investeren 250 miljoen € duurzaam
  - Overheid idem dito



# ***Borssele PWR***

- PWR met 485 MWe
- Brandstof
  - MOX
  - Uranium van Kazakhstan
- Kernafval
  - Borssele produceert 12 ton per jaar
  - Areva NC doet reprocessing
  - Restafval moet teruggenomen worden en wordt opgeslagen door COVRA
  - Voldoende opslagcapaciteit voor 100 jaar
  - Transporten naar La Hague
  - Eerste in Juni 2011; 10 in 2012 – 2015
  - Reprocessed uranium wordt verrijkt in Rusland met uranium van duikboten; 25% blijft in Rusland
- 2009 Delta memorandum voor 2e centrale
  - Kosten 4 – 5 miljard euro
  - Verzoek tot vergunning in 2012
  - Start constructie in 2013, bedrijf in 2018



# *Kernsplijting*

Opslag van radioactief materiaal staat ter discussie

Ongelukken hebben grote gevolgen (Chernobyl, Fukushima)

Decommissioning moet beschouwd worden

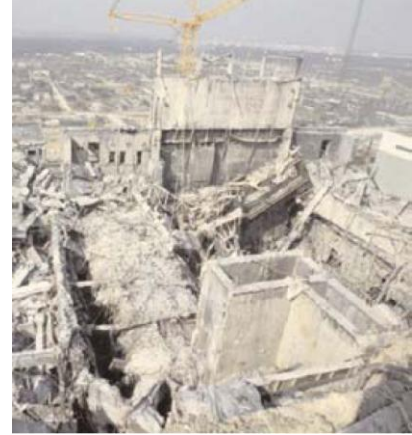
Snelle broedreactoren: genereren hun eigen brandstof (plutonium)

Proliferatie, diefstal van plutonium moet voorkomen worden

Manhattan project in WOII

Uranium en plutonium bommen (1945)

Nuclear weapons test ban treaty (1963)  
verbiedt testen van kernwapens in  
atmosfeer (fall-out is gevaarlijk in verband  
met consumptie)



Oppenheimer &  
Groves



Nagasaki

## ***Extra slides***

# Voorbeeld: $UO_2$ PWR

Druk four factors uit in termen van verrijking en verhouding moderator / fuel

Er geldt  $\bar{\sigma}_{aT}^f = \tilde{e} \bar{\sigma}_{aT}^{fi} + (1 - \tilde{e}) \bar{\sigma}_{aT}^{fe}$   $\longrightarrow$   $\eta_T = \eta_T^{fi} / \left[ 1 + (1 - \tilde{e}) \bar{\sigma}_{aT}^{fe} / \tilde{e} \bar{\sigma}_{aT}^{fi} \right]$

Resonance escape probability is functie van  $\tilde{e}$  en  $V_m N_m / V_f N_f$

Omdat  $N_{fe} = (1 - \tilde{e}) N_f$   $\longrightarrow$   $p = \exp \left( - \frac{(1 - \tilde{e})}{(V_m N_m / V_f N_f)} \frac{I}{\xi \sigma_s^m} \right)$ , met  $\Sigma_s^m = N_s \sigma_s^m$

Thermal utilization factor  $f = \frac{1}{1 + \zeta (V_m N_m / V_f N_f) (\bar{\sigma}_{aT}^m / \bar{\sigma}_{aT}^f)}$

Fast fission factor  $\epsilon = 1 + \frac{(1 - \tilde{e}) \nu^{fe} \bar{\sigma}_{fF}^{fe}}{\tilde{e} \nu^{fi} \bar{\sigma}_{fT}^{fi}}$

Invloed van toename in  $V_m N_m / V_f N_f$

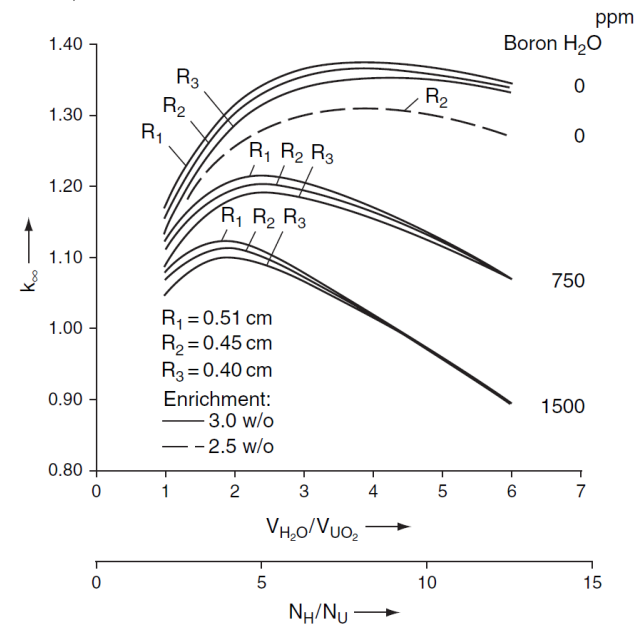
Toename resonance escape probability

Afname thermal utilization (absorptie in moderator)

Er is dus een optimale verhouding!

Grotere rod diameter geeft hogere multiplication

Negatieve feedback met temperatuur (stabiliteit)



# ***Diffusie van neutronen***

# Diffusie van neutronen

Tot nu toe hebben we globale neutronendiffusie met  $P_{NL}$  gekarakteriseerd

Diffusievergelijking nodig

Verband tussen reactorafmetingen, vorm en criticality

Ruimtelijke flux distributies in power reactoren

Aannamen

Een energie-groep model

Neutron flux en werkzame doorsneden zijn al gemiddeld over energie

Diffusievergelijking en randvoorwaarden opstellen

Eenvoudige 1D gevallen

Eindige cilindersymmetrische reactor core

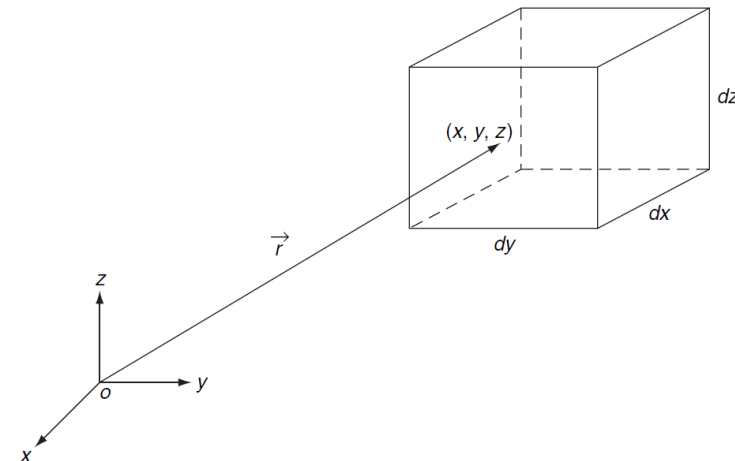
Ruimtelijke neutronenbalans (steady state conditie)

Volume element  $dV = dxdydz$  op punt  $\vec{r} = (x, y, z)$

Er geldt

$$\left( \begin{array}{c} \text{neutrons leaking} \\ \text{out of } dxdydz/s \end{array} \right) + \left( \begin{array}{c} \text{neutrons absorbed} \\ \text{in } dxdydz/s \end{array} \right)$$
$$= \left( \begin{array}{c} \text{source neutrons} \\ \text{emitted in } dxdydz/s \end{array} \right) + \left( \begin{array}{c} \text{fission neutrons} \\ \text{produced in } dxdydz/s \end{array} \right)$$

Neutronenstroom  $J_x(x, y, z)$  is het netto aantal neutronen/cm<sup>2</sup>/s door het y-z vlak in de positieve x richting op punt (x,y,z)



# Diffusievergelijking

Aantal neutronen dat door het voorvlak naar binnen stroomt

$$J_x(x + \frac{1}{2}dx, y, z)dydz$$

En door het achtervlak naar buiten  $-J_x(x - \frac{1}{2}dx, y, z)dydz$

Evenzo voor de andere vlakken

Netto neutronenlek per seconde uit de kubus

$$\begin{aligned} \left( \begin{array}{l} \text{neutrons leaking} \\ \text{out of } dxdydz/s \end{array} \right) &= [J_x(x + \frac{1}{2}dx, y, z) - J_x(x - \frac{1}{2}dx, y, z)]dydz \\ &\quad + [J_y(x, y + \frac{1}{2}dy, z) - J_y(x, y - \frac{1}{2}dy, z)]dxdz \\ &\quad + [J_z(x, y, z + \frac{1}{2}dz) - J_z(x, y, z - \frac{1}{2}dz)]dxdy. \end{aligned}$$

Gebruik definitie van partiële afgeleide

$$\lim_{dx \rightarrow 0} [J_x(x + \frac{1}{2}dx, y, z) - J_x(x - \frac{1}{2}dx, y, z)]/dx \equiv \frac{\partial}{\partial x} J_x(x, y, z)$$

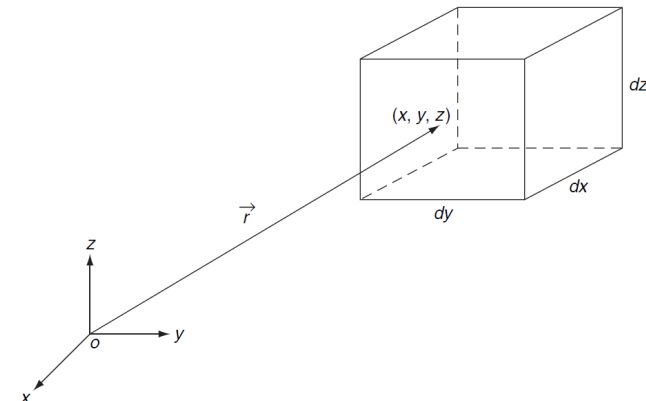
We vinden dan  $\left( \begin{array}{l} \text{neutrons leaking} \\ \text{out of } dxdydz/s \end{array} \right) = \left[ \frac{\partial}{\partial x} J_x(x, y, z) + \frac{\partial}{\partial y} J_y(x, y, z) + \frac{\partial}{\partial z} J_z(x, y, z) \right] dxdydz.$

Verder geldt  $\left( \begin{array}{l} \text{neutrons absorbed} \\ \text{in } dxdydz/s \end{array} \right) = \Sigma_a(x, y, z)\phi(x, y, z)dxdydz,$

$$\left( \begin{array}{l} \text{source neutrons} \\ \text{emitted in } dxdydz/s \end{array} \right) = s'''(x, y, z)dxdydz,$$

and

$$\left( \begin{array}{l} \text{fission neutrons} \\ \text{produced in } dxdydz/s \end{array} \right) = \nu \Sigma_f(x, y, z)\phi(x, y, z)dxdydz.$$



# Diffusievergelijking

Invullen in van gevonden uitdrukkingen in

$$\begin{aligned} & \left( \begin{array}{l} \text{neutrons leaking} \\ \text{out of } dxdydz/s \end{array} \right) + \left( \begin{array}{l} \text{neutrons absorbed} \\ \text{in } dxdydz/s \end{array} \right) \\ &= \left( \begin{array}{l} \text{source neutrons} \\ \text{emitted in } dxdydz/s \end{array} \right) + \left( \begin{array}{l} \text{fission neutrons} \\ \text{produced in } dxdydz/s \end{array} \right) \end{aligned}$$

Levert 
$$\frac{\partial}{\partial x} J_x(\vec{r}) + \frac{\partial}{\partial y} J_y(\vec{r}) + \frac{\partial}{\partial z} J_z(\vec{r}) + \Sigma_a(\vec{r})\phi(\vec{r}) = s'''(\vec{r}) + \nu\Sigma_f(\vec{r})\phi(\vec{r})$$

Schrijf neutronenstroom in vectorvorm 
$$\vec{J}(\vec{r}) = \hat{i}J_x(\vec{r}) + \hat{j}J_y(\vec{r}) + \hat{k}J_z(\vec{r})$$

Definitie van gradiënt 
$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

We vinden dan de *balansvergelijking* 
$$\vec{\nabla} \cdot \vec{J}(\vec{r}) + \Sigma_a(\vec{r})\phi(\vec{r}) = s'''(\vec{r}) + \nu\Sigma_f(\vec{r})\phi(\vec{r})$$

Diffusiebenadering: relatie tussen stroom en flux 
$$\vec{J}(\vec{r}) = -D(\vec{r})\vec{\nabla}\phi(\vec{r})$$
 **Wet van Fick**

Neutron diffusievergelijking



Diffusiecoëfficiënt

$$-\vec{\nabla} \cdot D(\vec{r})\vec{\nabla}\phi(\vec{r}) + \Sigma_a(\vec{r})\phi(\vec{r}) = s'''(\vec{r}) + \nu\Sigma_f(\vec{r})\phi(\vec{r})$$

Er geldt  $D = 1/3\Sigma_{tr}$  met transport cross section

$$\Sigma_{tr} = \Sigma_t - \bar{\mu}\Sigma_s$$



Gemiddelde verstrooiingshoek  
(isotroop: 0)

# Neutronenverdeling

Diffusievergelijking in cilindrische coördinaten

Tijdsonafhankelijk (zonder bron)  $\vec{\nabla} \cdot D\vec{\nabla}\phi + \nu\Sigma_f\phi - \Sigma_a\phi = 0$

Enkel oplossing voor kritische reactor (anders tijdafhankelijke oplossingen)

Neem aan dat je het aantal neutronen per splijting kunt variëren, dan

$$\vec{\nabla} \cdot D\vec{\nabla}\phi + (\nu_o/\nu)\nu\Sigma_f\phi - \Sigma_a\phi = 0$$

Neem aan dat met  $\nu_o$  de reactor kritisch is ( $k = 1$ ), met echt aantal  $\nu$

Dan geldt  $\nu_o/\nu = 1/k$  en dus

$$\vec{\nabla} \cdot D\vec{\nabla}\phi + \frac{1}{k}\nu\Sigma_f\phi - \Sigma_a\phi = 0$$

Dit is een eigenwaardenvergelijking: eigenwaarde  $k$ , eigenfunctie  $\phi$

Er geldt  $D = \text{constant}$ , en  $k_\infty = \nu\Sigma_f/\Sigma_a$  en  $L^2 = D/\Sigma_a$

Dan geldt  $\nabla^2\phi + \frac{k_\infty/k - 1}{L^2}\phi = 0 \quad \Rightarrow \quad -\frac{\nabla^2\phi}{\phi} = \frac{k_\infty/k - 1}{L^2}$

Er moet nu gelden  $\nabla^2\phi/\phi = -B^2 \quad \Rightarrow \quad k = \frac{k_\infty}{1 + L^2B^2} \quad \Rightarrow \quad P_{NL} = \frac{1}{1 + L^2B^2}$

Buckling  $B$  volgt uit Helmholtz vergelijking

$$\nabla^2\phi + B^2\phi = 0$$

# Eindige cilindrische core

Cilindrische reactor (extrapolated straal en hoogte)

Dan geldt  $\nabla^2 \phi + B^2 \phi = 0 \longrightarrow \frac{1}{r} \frac{\partial}{\partial r} r \frac{d}{dr} \phi + \frac{\partial^2}{\partial z^2} \phi + B^2 \phi = 0$

met  $0 < \phi(r, z) < \infty, \quad 0 \leq r \leq \tilde{R}, \quad -\tilde{H}/2 \leq z \leq \tilde{H}/2$

Separabele oplossing  $\phi(r, z) = \psi(r)\chi(z)$

Invullen  $\underbrace{\frac{1}{\psi r} \frac{d}{dr} r \frac{d}{dr} \psi}_{-B_r^2} + \underbrace{\frac{1}{\chi} \frac{d^2}{dz^2} \chi}_{-B_z^2} + B^2 = 0 \longrightarrow B_r^2 + B_z^2 = B^2$

We vinden  $\frac{1}{r} \frac{d}{dr} r \frac{d}{dr} \psi + B_r^2 \psi = 0, \quad 0 \leq r \leq \tilde{R},$

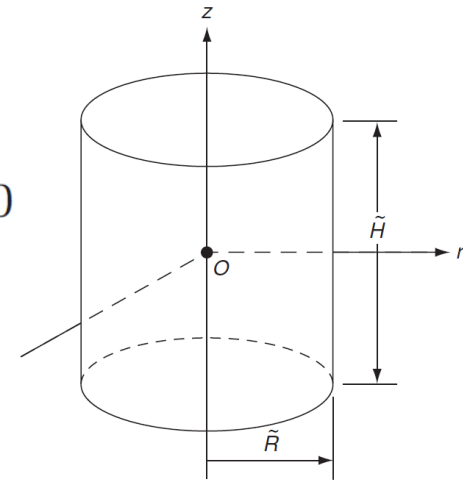
$$\frac{d^2}{dz^2} \chi + B_z^2 \chi = 0, \quad -\tilde{H}/2 \leq z \leq \tilde{H}/2$$

Probeer  $\chi(z) = \cancel{C_1 \sin(B_z z)} + C_2 \cos(B_z z), \quad -\tilde{H}/2 \leq z \leq \tilde{H}/2,$

Randvoorwaarden  $\chi(\pm\tilde{H}/2) = 0 \longrightarrow C_1 = 0 \longrightarrow \cos(\pm B_z \tilde{H}/2) = 0$

$\longrightarrow B_z \tilde{H}/2 = \pi/2, 3\pi/2, 5\pi/2, \dots \longrightarrow B_z = \pi/\tilde{H}$

Positieve flux



# Radiële oplossing

We hadden  $\frac{1}{r} \frac{d}{dr} r \frac{d}{dr} \psi + B_r^2 \psi = 0, \quad 0 \leq r \leq \tilde{R},$

Bessel functies

$$\psi(r) = C'_1 J_0(B_r r) + \cancel{C'_2 Y_0(B_r r)}, \quad 0 \leq r \leq \tilde{R}$$

Merk op  $Y_0(0) \rightarrow -\infty \rightarrow C'_2 = 0$

Verder  $B_r \tilde{R} = 2.405 \rightarrow B_r = 2.405/\tilde{R}$

Buckling

$$\left. \begin{array}{l} B_z = \pi/\tilde{H} \\ B_r^2 + B_z^2 = B^2 \\ B_r = 2.405/\tilde{R} \end{array} \right\} B^2 = (2.405/\tilde{R})^2 + (\pi/\tilde{H})^2$$

Fluxverdeling

$$\chi(z) = \cancel{C_1 \sin(B_z z)} + C_2 \cos(B_z z), \quad -\tilde{H}/2 \leq z \leq \tilde{H}/2,$$

$$\psi(r) = C'_1 J_0(B_r r) + \cancel{C'_2 Y_0(B_r r)}, \quad 0 \leq r \leq \tilde{R}$$

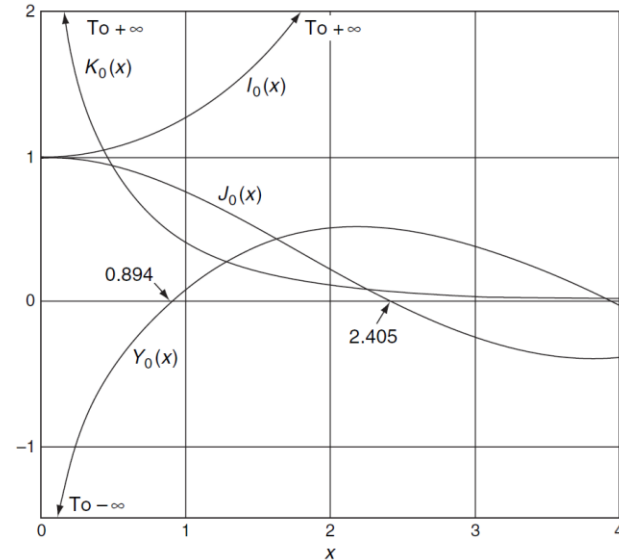
$$\phi(r, z) = \psi(r) \chi(z)$$



$$\phi(r, z) = C J_0(2.405r/\tilde{R}) \cos(\pi z/\tilde{H})$$



$$C = C'_1 C_2$$



$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2) y = 0$$

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m! \Gamma(m + \alpha + 1)} \left(\frac{1}{2}x\right)^{2m + \alpha}$$

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt.$$

$$Y_\alpha(x) = \frac{J_\alpha(x) \cos(\alpha\pi) - J_{-\alpha}(x)}{\sin(\alpha\pi)}.$$

# Reactor vermogen

Energie per kernsplijting  $\gamma = 3.1 \times 10^{-11}$  J/fission    #splijtingen / cm<sup>3</sup> / s     $\Sigma_f \phi$

Reactor vermogen  $P = \gamma \int \Sigma_f \phi dV \longrightarrow P = \gamma 2\pi \int_0^{\tilde{R}} \int_{-\tilde{H}/2}^{\tilde{H}/2} \Sigma_f \phi dz r dr$

Flux invullen  $\phi(r, z) = C J_0(2.405r/\tilde{R}) \cos(\pi z/\tilde{H})$

$\longrightarrow P = \gamma \Sigma_f 2\pi C \int_0^{\tilde{R}} J_0(2.405r/\tilde{R}) r dr \int_{-\tilde{H}/2}^{\tilde{H}/2} \cos(\pi z/\tilde{H}) dz$

Herschrijven met  $V = \pi \tilde{R}^2 \tilde{H}$

$$P = \gamma \Sigma_f V C \left[ \frac{2}{\tilde{R}^2} \int_0^{\tilde{R}} J_0(2.405r/\tilde{R}) r dr \right] \left[ \frac{1}{\tilde{H}} \int_{-\tilde{H}/2}^{\tilde{H}/2} \cos(\pi z/\tilde{H}) dz \right]$$

Verander variabele  $\xi = 2.405r/\tilde{R}$  en gebruik Bessel functie relatie

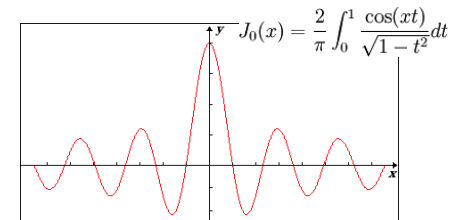
$\longrightarrow \frac{2}{\tilde{R}^2} \int_0^{\tilde{R}} J_0(2.405r/\tilde{R}) r dr = \frac{2}{2.405^2} \int_0^{2.405} J_0(\xi) \xi d\xi = \frac{2}{2.405} J_1(2.405)$

Evenzo, met  $\varsigma = \pi z/\tilde{H} \longrightarrow \frac{1}{\tilde{H}} \int_{-\tilde{H}/2}^{\tilde{H}/2} \cos(\pi z/\tilde{H}) dz = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos(\varsigma) d\varsigma = \frac{2}{\pi}$

$$P = \gamma \Sigma_f V C \frac{2J_1(2.405)}{2.405} \frac{2}{\pi} = 0.275 \gamma \Sigma_f V C$$

$\longrightarrow C = 3.63 \frac{P}{\gamma \Sigma_f V}$

$$\phi(r, z) = 3.63 \frac{P}{\gamma \Sigma_f V} J_0(2.405r/R) \cos(\pi z/H)$$



# Neutron leakage

Two group approximation: neutronenmigratie in slowdown en thermisch gebied

Definieer snelle en thermische flux  $\phi_1$  and  $\phi_2$

|          |                                                               |                                                                                                        |                                     |               |                                 |
|----------|---------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|-------------------------------------|---------------|---------------------------------|
| <b>F</b> | $n \rightarrow (1) \rightarrow$<br>$\varepsilon p f \eta_T n$ | $\rightarrow \varepsilon n \rightarrow$<br>$\downarrow$                                                | $\rightarrow$                       | $\rightarrow$ | $\rightarrow$ Fast Diffusion    |
| <b>I</b> | $\uparrow$<br>(4)<br>$\uparrow$                               | $\varepsilon(1-p)n \leftarrow$<br>$\leftarrow$                                                         | $\downarrow$<br>(2)<br>$\downarrow$ |               |                                 |
| <b>T</b> | $\uparrow$<br>$\varepsilon p f n$                             | $\leftarrow \leftarrow (3) \leftarrow$<br>$\leftarrow \varepsilon p n \rightarrow \varepsilon p(1-f)n$ | $\rightarrow$                       | $\rightarrow$ | $\rightarrow$ Thermal Diffusion |
|          | Fuel                                                          | Moderator                                                                                              |                                     |               | Leakage                         |

Diffusievergelijking voor snelle neutronen

$$-\vec{\nabla} \cdot D_1 \vec{\nabla} \phi_1 + \Sigma_r \phi_1 = \frac{1}{k} \varepsilon \eta_T f \Sigma_a \phi_2$$

$$D_1 = 1/3 \Sigma_{tr1}$$

Fast leakage

Verlies door slowing down

# snelle neutronen geproduceerd / cm<sup>3</sup> / s

Fast fission

Thermal utilization: absorbed in fuel

Diffusievergelijking voor thermische neutronen

$$-\vec{\nabla} \cdot D_2 \vec{\nabla} \phi_2 + \Sigma_a \phi_2 = p \Sigma_r \phi_1$$

$$D_2 = 1/3 \Sigma_{tr2}$$

Thermische leakage

Bronterm thermische neutronen

Bereken diffusiecoëfficiënten en removal werkzame doorsnede  $\Sigma_r$

# Two group approximation

Deel door  $\Sigma_r$  en  $\Sigma_a$  en definieer  $L_1 = \sqrt{D_1/\Sigma_r}$   $L_2 = \sqrt{D_2/\Sigma_a}$

$$\left. \begin{aligned} -\vec{\nabla} \cdot D_1 \vec{\nabla} \phi_1 + \Sigma_r \phi_1 &= \frac{1}{k} \varepsilon \eta_T f \Sigma_a \phi_2 \\ -\vec{\nabla} \cdot D_2 \vec{\nabla} \phi_2 + \Sigma_a \phi_2 &= p \Sigma_r \phi_1 \end{aligned} \right\} \quad \begin{aligned} -L_1^2 \nabla^2 \phi_1 + \phi_1 &= \frac{1}{k} \varepsilon \eta_T f \frac{\Sigma_a}{\Sigma_r} \phi_2 \\ -L_2^2 \nabla^2 \phi_2 + \phi_2 &= p \frac{\Sigma_r}{\Sigma_a} \phi_1 \end{aligned}$$

Beschouw uniforme reactor met zero flux randvoorwaarden. Dan  
weer

$$\nabla^2 \phi_1 + B^2 \phi_1 = 0 \quad \text{en} \quad \nabla^2 \phi_2 + B^2 \phi_2 = 0$$

Gebruik dit om de Laplace operatoren te elimineren

  $\phi_1 = \frac{1}{1 + L_1^2 B^2} \frac{1}{k} \varepsilon \eta_T f \frac{\Sigma_a}{\Sigma_r} \phi_2 \quad \text{en} \quad \phi_2 = \frac{1}{1 + L_2^2 B^2} p \frac{\Sigma_r}{\Sigma_a} \phi_1$

Combineren levert  $k = \frac{1}{1 + L_1^2 B^2} \frac{1}{1 + L_2^2 B^2} k_\infty \quad \text{met} \quad k_\infty = p \varepsilon f \eta_T$

We vinden  $P_{NL} = \frac{1}{1 + L_1^2 B^2} \frac{1}{1 + L_2^2 B^2}$

Bepaal diffusielengten uit transport, resonantie en absorptie werkzame doorsneden

# Migratielengte

Er geldt  $P_{NL} = \frac{1}{1 + L_1^2 B^2} \frac{1}{1 + L_2^2 B^2} \longrightarrow P_{NL} = \frac{1}{1 + (L_1^2 + L_2^2) B^2 + \cancel{L_1^2 L_2^2 B^4}}$

Voor grote reactor is  $B^2$  klein en kan  $B^4$  verwaarloosd worden

We vinden dan  $P_{NL} = \frac{1}{1 + M^2 B^2}$

↑  
migratielengte

$$M = \sqrt{L_1^2 + L_2^2}$$

Grootste correctie voor thermische  
diffusielengte in geval van H<sub>2</sub>O gemodereerde  
power reactoren

Dit komt door de grote absorptie werkzame  
doorsnede van waterstof

Snelle reactoren (diffusie en migratielengte  
zijn hetzelfde):

SFR:  $M = 19.2$  cm

GCFR:  $M = 25.5$  cm

Representative Diffusion Properties for Moderators and Thermal Reactors

| Type             | Description                                  | $L_1 = \sqrt{\tau}$ (cm)<br>Fast Diffusion<br>Length | $L_2$ (cm)<br>Thermal<br>Diffusion<br>Length | $M$ (cm)<br>Migration<br>Length |
|------------------|----------------------------------------------|------------------------------------------------------|----------------------------------------------|---------------------------------|
| H <sub>2</sub> O | Light Water                                  | 5.10                                                 | 2.85                                         | 5.84                            |
| PWR              | Pressurized-H <sub>2</sub> O<br>Reactor      | 7.36                                                 | 1.96                                         | 7.62                            |
| BWR              | Boiling-H <sub>2</sub> O<br>reactor          | 7.16                                                 | 1.97                                         | 7.43                            |
| D <sub>2</sub> O | Heavy water                                  | 11.5                                                 | 173                                          | 174                             |
| PHWR             | CANDU-D <sub>2</sub> O<br>reactor            | 11.6                                                 | 15.6                                         | 19.4                            |
| C                | Graphite                                     | 19.5                                                 | 59.0                                         | 62.0                            |
| HTGR             | Graphite-<br>moderated He-<br>cooled reactor | 17.1                                                 | 10.6                                         | 20.2                            |

# ***Neutron diffusion***

Study the preceding part on diffusion theory yourself

# Samenvatting diffusie van neutronen

Buckling  $B$  volgt uit Helmholtz vergelijking

$$\nabla^2 \phi + B^2 \phi = 0$$

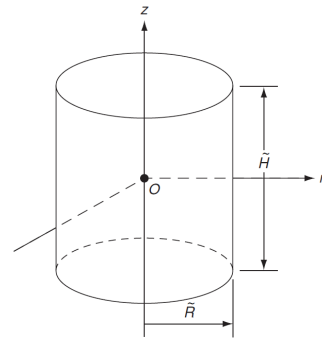
Er geldt  $P_{NL} = \frac{1}{1 + L_1^2 B^2} \frac{1}{1 + L_2^2 B^2} \rightarrow P_{NL} = \frac{1}{1 + (L_1^2 + L_2^2) B^2 + \cancel{L_1^2 L_2^2 B^4}}$

Voor grote reactor is  $B^2$  klein en kan  $B^4$  verwaarloosd worden

We vinden dan  $P_{NL} = \frac{1}{1 + M^2 B^2}$   $B^2 = (2.405/\tilde{R})^2 + (\pi/\tilde{H})^2$

↑  
migratielengte

$$M = \sqrt{L_1^2 + L_2^2}$$



Grootste correctie voor thermische  
diffusielengete in geval van  $H_2O$  gemodereerde  
power reactoren

Dit komt door de grote absorptie werkzame  
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| H <sub>2</sub> O | Light Water                                  | 5.10                                                 | 2.85                                         | 5.84                            |
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| PHWR             | CANDU-D <sub>2</sub> O<br>reactor            | 11.6                                                 | 15.6                                         | 19.4                            |
| C                | Graphite                                     | 19.5                                                 | 59.0                                         | 62.0                            |
| HTGR             | Graphite-<br>moderated He-<br>cooled reactor | 17.1                                                 | 10.6                                         | 20.2                            |

# Leakage en ontwerp

Er geldt  $k = \frac{1}{1 + M^2 B^2} k_\infty$

Stel we hebben een cilindrische reactor met  $\tilde{H} = 2\tilde{R}$

De buckling volgt uit  $B^2 = (2.405/\tilde{R})^2 + (\pi/\tilde{H})^2 \longrightarrow B^2 = 33.0/\tilde{H}^2$

Aldus  $k = \frac{1}{1 + 33.0(M/\tilde{H})^2} k_\infty$

Leakage van neutronen wordt primair bepaald door  $\tilde{H}/M$   
 Karakteristieke dimensie in eenheden van migratielengten

Ontwerp van reactor core:

Kies vermogen  $P$

Bepaal structuur van de core lattice

Kies brandstof, moderator, koelmiddel en andere materialen

Bepaal volume ratio's en geometrische configuraties (straal fuel rods, etc.)

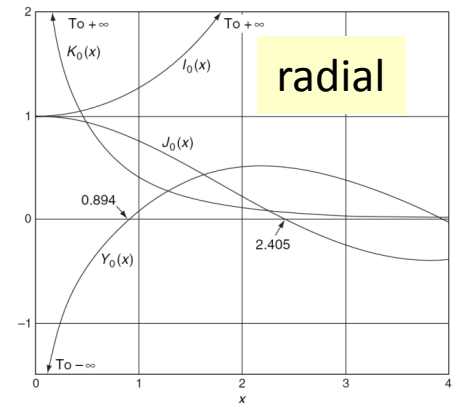
Kies lattice parameters, zodat voor gegeven enrichment  $k_\infty$  bijna optimaal is en de powerdichtheid van fuel naar koelmiddel maximaal

Nu ligt de migratielengte  $M$  vast

Lattice design en maximum/gemiddelde flux bepaalt power density

Vermogen en power density bepalen core volume

Fuel enrichment wordt aangepast om de juiste  $k_\infty$  te krijgen



Oppervlakteflux  $M/R$

Fractioneel volume rand  $M/R$

Fractioneel verlies  $(M/R)^2$

Critical  $(M/R)^2 \sim (k_\infty - 1)/k_\infty$

$$M = \sqrt{N} \lambda = \sqrt{\frac{\Sigma_s}{\Sigma_a}} \frac{1}{\Sigma_s} = \frac{1}{\sqrt{\Sigma_s \Sigma_a}}$$

# ***Energietransport***

# Energietransport

In het voorgaande hebben we tijd- en ruimteverdelingen van neutronen in een reactor besproken

In een kritische reactor is flux evenredig met vermogen

$$\phi(r, z) = 3.63 \frac{P}{\gamma \Sigma_f V} J_0(2.405r/R) \cos(\pi z/H)$$

Bij hoog vermogen

Thermische limiet bepaalt maximum vermogen (oververhitting fuel)

Dichtheden veranderen (reactivity feedback effecten)

$$\left. \begin{array}{l} \text{Core averaged power density} \quad \bar{P}''' = P/V \\ \text{Power peaking factor} \quad F_q = P'''_{\max} / \bar{P}''' \end{array} \right\} P = \frac{P'''_{\max}}{F_q} V$$

Constructie kosten nemen sterk toe met volume  $V \longrightarrow$  optimaliseer  $P'''_{\max} / F_q$

Maximale  $P'''_{\max}$  wordt bepaald door materiaaleigenschappen

Minimale peaking factor wordt bepaald door reactorfysica

Niet-uniforme verdelingen van fuel enrichment

Plaatsing van control rods and andere neutron poisons

Gekozen core volume bepaalt

Core-averaged fuel enrichment

Non-leakage probabilities

# Core eigenschappen

3000 MW(t) Power Reactor Approximate Core Properties

|                                                         | <i>PWR</i><br><i>Pressurized-</i><br><i>H<sub>2</sub>O</i><br><i>Reactor</i> | <i>BWR</i><br><i>Boiling-</i><br><i>H<sub>2</sub>O</i><br><i>Reactor</i> | <i>PHWR</i><br><i>CANDU-</i><br><i>D<sub>2</sub>O Reactor</i> | <i>HTGR</i><br><i>C-Moderated</i><br><i>Reactor</i> | <i>SFR Na-Cooled</i><br><i>Fast Reactor</i> | <i>GCFR He-cooled</i><br><i>Fast Reactor</i> |
|---------------------------------------------------------|------------------------------------------------------------------------------|--------------------------------------------------------------------------|---------------------------------------------------------------|-----------------------------------------------------|---------------------------------------------|----------------------------------------------|
| $\bar{P}'''$ (MW/m <sup>3</sup> ) average power density | 102                                                                          | 56                                                                       | 7.7                                                           | 6.6                                                 | 217                                         | 115                                          |
| $\bar{q}'$ (kW/m) average linear heat rate              | 17.5                                                                         | 20.7                                                                     | 24.7                                                          | 3.7                                                 | 22.9                                        | 17                                           |
| $V$ (m <sup>3</sup> ) core volume                       | 29.4                                                                         | 53.7                                                                     | 390                                                           | 455                                                 | 13.8                                        | 26.1                                         |
| $H/M$ height and diameter in migration lengths          | 43.9                                                                         | 55.0                                                                     | 40.8                                                          | 68.8                                                | 13.5                                        | 12.6                                         |
| $N$ number of fuel pins                                 | 51,244                                                                       | 35,474                                                                   | 15,344                                                        | 97,303                                              | 50,365                                      | 54,903                                       |
| $P_{NL}$ nonleakage probability                         | 0.956                                                                        | 0.972                                                                    | 0.950                                                         | 0.982                                               | 0.676                                       | 0.644                                        |

Source: Data courtesy of W. S. Yang, Argonne National Laboratory.

# Eindige cilindrische core

Vermogensdichtheid [ W / cm<sup>3</sup> ]  $P'''(\vec{r}) = \gamma \underbrace{\Sigma_f(\vec{r}) \phi(\vec{r})}_{\text{\# fission / cm}^3 \text{ / s}}$

In een kritische reactor is flux evenredig met vermogen

# Ws / fission

Voor cilindrische reactor  $P'''(r, z) = \bar{P}''' f_r(r) f_z(z)$

Core averaged power density  $\bar{P}''' = \frac{1}{V} \int P'''(\vec{r}) dV$

Cilindrische geometrie  $\frac{dV}{V} = \frac{2\pi r dr dz}{\pi R^2 H}$   $\bar{P}''' = \bar{P}''' \frac{2}{R^2} \int_0^R f_r(r) r dr \frac{1}{H} \int_{-H/2}^{H/2} f_z(z) dz$

Normering  $\frac{2}{R^2} \int_0^R f_r(r) r dr = 1$   $\frac{1}{H} \int_{-H/2}^{H/2} f_z(z) dz = 1$

Power peaking factor  $F_q = F_r F_z$  met radiale en axiale peaking

$$F_r = f_r(r)_{\max}$$

$$F_z = f_z(z)_{\max}$$

Local peaking factor  $F_l$

Fuel element manufacturing tolerances

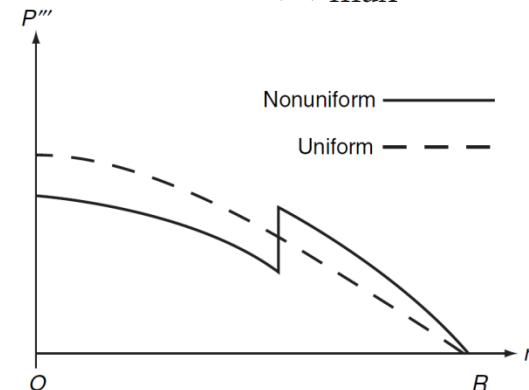
Local control and instrumentation perturbations

$$F_q = F_r F_z F_l$$

Flatten power distribution (reduceer peaking)

Meerdere radiële zone's met verschillende fuel enrichment

Partially inserted control-rod banks



# Voorbeeld: uniforme cilindrische core

Flux in uniforme core  $J_0(2.405r/R) \cos(\pi z/H)$

Power density distributions  $f_r(r) = C_r J_0(2.405r/R)$   $f_z(z) = C_z \cos(\pi z/H)$

Normalisatie coëfficiënten volgen uit

$$\frac{2}{R^2} \int_0^R f_r(r) r dr = 1$$

$$\longrightarrow C_r \frac{2}{R^2} \int_0^R J_0(2.405r/R) r dr = 1$$

$$\frac{1}{H} \int_{-H/2}^{H/2} f_z(z) dz = 1$$

$$C_z \frac{1}{H} \int_{-H/2}^{H/2} \cos(\pi z/H) dz = 1$$

Deze integralen hebben we al eens uitgerekend. Er geldt

$$f_r(r) = 2.32 J_0(2.405r/R)$$

$$f_z(z) = 1.57 \cos(\pi z/H)$$

Zowel Bessel functie als cosinus hebben maximum waarde 1

$$\text{Peaking factoren: } \left. \begin{array}{l} F_r = 2.32 \\ F_z = 1.57 \end{array} \right\} F_q = 3.63 F_1$$

# Warmtetransport

Fuel – coolant model: goed voor thermische en fast reactors

Thermal power per unit length van fuel element  $q'(r, z)$  (linear heat rate in W/cm)

Surface heat flux  $q''(r, z)$  in W/cm<sup>2</sup>

Voor cilindrisch element met straal  $a$  geldt  $q''(r, z) = \frac{1}{2\pi a} q'(r, z)$

Oppervlak van lattice cell met 1 fuel rod  $A_{cell}$

Thermische power geproduceerd per unit core volume is  $P'''(r, z) = q'(r, z) / A_{cell}$

Voor cilindrische reactor  $P'''(r, z) = \bar{P}''' f_r(r) f_z(z)$

Combineren geeft  $q'(r, z) = A_{cell} \bar{P}''' f_r(r) f_z(z) \longrightarrow q'(r, z) = \frac{A_{cell}}{V} P f_r(r) f_z(z)$

Aanname: reactor met  $N$  identieke cellen  $V = \pi R^2 H = N A_{cell} H$

Dan geldt  $q'(r, z) = \frac{1}{NH} P f_r(r) f_z(z)$

Totale lengte fuel rods

Thermische weerstand  
(1/warmte geleiding)

Temperatuurverschil tussen fuel en coolant

$$\underbrace{T_{fe}(r, z)}_{\text{Gemiddeld over } \pi a^2 \text{ van fuel rod}} - \underbrace{T_c(r, z)}_{\text{Gemiddeld over koelkanaal}} = R'_{fe} q'(r, z)$$

Gemiddeld over  $\pi a^2$  van fuel rod

Gemiddeld over koelkanaal

Er geldt  $T_{fe}(r, z) - T_c(r, z) = R_f P f_r(r) f_z(z)$

Thermische weerstand  
reactor core

$$R_f = \frac{1}{NH} R'_{fe}$$

$$\longrightarrow \bar{T}_f - \bar{T}_c = R_f P$$

Gemiddeld over volume

$$\frac{2}{R^2} \int_0^R f_r(r) r dr = 1$$

$$\frac{1}{H} \int_{-H/2}^{H/2} f_z(z) dz = 1$$

# Warmtetransport

Warmtebalans voor een roostercel

Massa flow rate  $W_{ch}$  in [ kg/s ]

We hadden  $q'(r, z) = \frac{1}{NH} P f_r(r) f_z(z)$

Dit levert voor uitgaand koelwater

$$\underbrace{W_{ch} c_p [T_o(r) - T_i]}_{\text{Opwarming koelmiddel}} = \int_{-H/2}^{H/2} q'(r, z') dz'$$

Opwarming  
koelmiddel



Warmte geproduceerd  
in fuel element

$$T_o(r) = \frac{1}{W c_p} \frac{P}{NH} f_r(r) \int_{-H/2}^{H/2} f_z(z) dz + T_i$$

Reactorkern massa flow  $W = N W_{ch}$  door de  $N$  identieke koelkanalen

Combineren met  $\frac{1}{H} \int_{-H/2}^{H/2} f_z(z) dz = 1 \quad \longrightarrow \quad T_o(r) = \frac{1}{W c_p} P f_r(r) + T_i$

De gemiddelde temperatuur van het uitgaande koelwater vinden we door integratie over de doorsnede van de kern

$$2\pi \bar{r} d\bar{r} / R^2$$

Met  $\frac{2}{R^2} \int_0^R f_r(r) r dr = 1$  vinden we  $\bar{T}_o = \frac{1}{W c_p} P + T_i$

Gemiddelde koelwatertemperatuur  $\bar{T}_c = \frac{1}{2} (\bar{T}_o + T_i)$

$$\bar{T}_c = \frac{1}{2 W c_p} P + T_i$$

We hadden  $\bar{T}_f - \bar{T}_c = R_f P$

$$\longrightarrow \bar{T}_f = \left( R_f + \frac{1}{2 W c_p} \right) P + T_i$$

Gemiddelde temperatuur van fuel en koelmiddel is later nodig om reactivity feedback te modelleren.

# Warmtetransport

Maximum koelwatertemperatuur  $T_{0|_{\max}} = \frac{1}{Wc_p} PF_r + T_i$

↑  
Radiale peaking factor

Maximaal temperatuurverschil uit  $T_{fe}(r, z) - T_c(r, z) = R_f P f_r(r) f_z(z)$   
 $\longrightarrow [T_{fe}(r, z) - T_c(r, z)]|_{\max} = R_f P F_r F_z$

↑  
Hiervoor moeten we  $T_c$  weten!

Combineren met  $\bar{T}_f = \left( R_f + \frac{1}{2Wc_p} \right) P + T_i$

$\longrightarrow T_f|_{\max} = \left( R_f F_r F_z + \frac{1}{2Wc_p} F_r \right) P + T_i$

Voor vloeistof gekoelde reactoren geldt

Maximum fuel temperatuur

Gemiddelde koelwater temperatuur

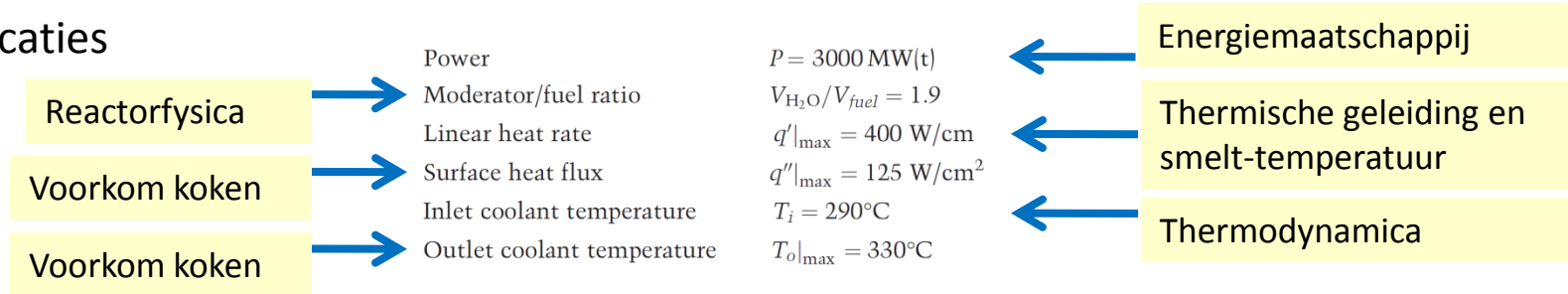
$$\frac{\bar{T}_f - \bar{T}_c}{\bar{T}_c - T_i} = 2Wc_p R_f \gg 1$$

Thermische weerstand  $R_f \gg 1/Wc_p$  gebruikt  $T_{fe}$  gemiddeld over fuel rod

Hoogste temperatuur in fuel rod (center line) geeft limiet op linear heat rate

# Voorbeeld: PWR

## Specificaties



## Dit bepaalt

- Fuel radius
- Lattice pitch
- Core volume and dimensions
- Core-averaged power density
- Number of fuel elements
- Coolant mass flow rate
- Mean coolant velocity.

Fuel radius:  $q''(r, z) = \frac{1}{2\pi a} q'(r, z) \longrightarrow a = \frac{q'_{\text{max}}}{2\pi q''_{\text{max}}} = \frac{400}{2\pi \cdot 125} = 0.509 \text{ cm}$

Lattice (vierkant) pitch:  $V_{\text{H}_2\text{O}}/V_{\text{fuel}} = (p^2 - \pi a^2)/\pi a^2$   
 $\longrightarrow p = \sqrt{\pi(V_{\text{H}_2\text{O}}/V_{\text{fuel}} + 1)} a = \sqrt{2.9\pi} 0.509 = 1.536 \text{ cm}$

Core volume:  $q'(r, z) = \frac{A_{\text{cell}}}{V} P \underbrace{f_r(r) f_z(z)}_{\substack{F_r = 2.32 \\ F_z = 1.57}} \longrightarrow V = p^2 P F_r F_z / q'_{\text{max}} = 1.536^2 \cdot 3000 \cdot 10^6 \cdot 2.32 \cdot 1.57 / 400$   
 $= 6.445 \cdot 10^7 \text{ cm}^3 = 64.45 \text{ m}^3$

$A_{\text{cell}} = p^2$

# Voorbeeld: PWR

## Specificaties

|                            |                                                |                                |
|----------------------------|------------------------------------------------|--------------------------------|
| Power                      | $P = 3000 \text{ MW(t)}$                       | a. Fuel radius                 |
| Moderator/fuel ratio       | $V_{\text{H}_2\text{O}}/V_{\text{fuel}} = 1.9$ | b. Lattice pitch               |
| Linear heat rate           | $q'_{\text{max}} = 400 \text{ W/cm}$           | c. Core volume and dimensions  |
| Surface heat flux          | $q''_{\text{max}} = 125 \text{ W/cm}^2$        | d. Core-averaged power density |
| Inlet coolant temperature  | $T_i = 290^\circ\text{C}$                      | e. Number of fuel elements     |
| Outlet coolant temperature | $T_o _{\text{max}} = 330^\circ\text{C}$        | f. Coolant mass flow rate      |
|                            |                                                | g. Mean coolant velocity.      |

$$H/D = 1: V = \pi(H/2)^2 H \longrightarrow H = (4V/\pi)^{1/3} = (4 \cdot 6.445 \cdot 10^7 / \pi)^{1/3} = 434 \text{ cm} = 4.34 \text{ m}$$

$$\text{Vermogensdichtheid: } \bar{P}''' = P/V = 3000 \cdot 10^6 / 6.445 \cdot 10^7 = 46.5 \text{ W/cm}^3 = 46.5 \text{ MW/m}^3$$

$$\# \text{ brandstofelementen: } N = \frac{\pi R^2}{A_{\text{cell}}} = \frac{\pi(H/2)^2}{p^2} = \frac{\pi(434/2)^2}{1.536^2} = 62,702$$

$$\text{Vloeistofdebiet: } T_o|_{\text{max}} = \frac{1}{W c_p} P F_r + T_i \longrightarrow W = \frac{1}{c_p} \frac{P F_r}{(T_o|_{\text{max}} - T_i)} = \frac{1}{6.4 \cdot 10^3} \frac{3000 \cdot 10^6 \cdot 2.32}{(330 - 290)} \\ = 27.2 \cdot 10^3 \text{ kg/s} = 27.2 \cdot 10^6 \text{ g/s.}$$

$$\text{Snelheid koelvloeistof: } W = \rho A_{\text{flow}} \bar{v}$$

Dichtheid (300 °C: 0.676 g/ml)

$$A_{\text{flow}} = N \cdot (p^2 - \pi a^2)$$

$$\bar{v} = \frac{W}{\rho N(p^2 - \pi a^2)} = \frac{27.2 \cdot 10^6}{0.676 \cdot 62,702 \cdot (1.536^2 - \pi 0.509^2)} \\ = 415 \text{ cm/s} = 4.15 \text{ m/s.}$$

Overige parameters: verrijkingsfactor, control poisson, control rods (die nemen volume in). Een iteratief engineering proces.

# Thermische transients

Steady state condities  $P = (\bar{T}_f - \bar{T}_c)/R_f$

Uitval koelinstallatie  $M_f c_f \frac{d}{dt} \bar{T}_f(t) = P(t)$

Combineer beide situaties in *lumped-parameter model*

$$M_f c_f \frac{d}{dt} \bar{T}_f(t) = P(t) - \frac{1}{R_f} [\bar{T}_f(t) - \bar{T}_c(t)]$$

= 0 in steady state

= 0 indien geen koeling

Schrijf als  $\frac{d}{dt} \bar{T}_f(t) = \underbrace{\frac{1}{M_f c_f} P(t)}_{\text{Adiabatic heatup rate}} - \underbrace{\frac{1}{\tau} [\bar{T}_f(t) - \bar{T}_c(t)]}_{\text{Core thermal time constant}}$

Adiabatic heatup rate

Core thermal time constant

$$\tau = M_f c_f R_f$$

Tijd nodig voor warmteoverdracht van  
fuel naar koelmiddel (paar secs)

Voorbeeld: reactor shutdown

$$\left. \begin{aligned} \frac{d}{dt} \bar{T}_f(t) &= -\frac{1}{\tau} [\bar{T}_f(t) - \bar{T}_c], \quad t > 0 \\ \text{Randvoorwaarde} \quad \bar{T}_f(0) &= R_f P_o + \bar{T}_c \end{aligned} \right\} \bar{T}_f(t) = \bar{T}_c + R_f P_o \exp(-t/\tau)$$

# Diffusielengte

Afstand die een neutron aflegt van geboorte op  $r = 0$  tot absorptie

Er geldt  $\bar{r^2} = \frac{\int r^2 \Sigma_a \phi(r) dV}{\int \Sigma_a \phi(r) dV}$

Met  $\phi(r) = \frac{s_p}{4\pi D r} \exp(-r/L)$

$$\bar{r^2} = \frac{\int_0^\infty r^2 \Sigma_a \frac{s_p}{4\pi D r} \exp(-r/L) 4\pi r^2 dr}{\int_0^\infty \Sigma_a \frac{s_p}{4\pi D r} \exp(-r/L) 4\pi r^2 dr} = \frac{\int_0^\infty r^3 \exp(-r/L) dr}{\int_0^\infty r \exp(-r/L) dr}$$

Uitrekenen levert  $\bar{r^2} = 6L^2 \rightarrow L = \frac{1}{\sqrt{6}} \sqrt{\bar{r^2}} = 0.408 \sqrt{\bar{r^2}}$

Diffusielengte is evenredig met rms diffusieafstand van geboorte tot absorptie

Vrije weglengte  $\lambda = 1/\Sigma$

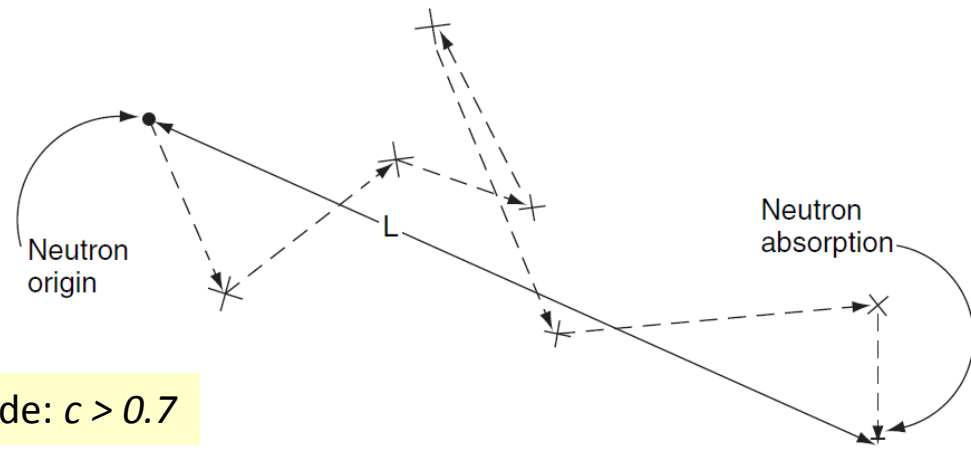
Isotrope verstrooiing  $D = 1/(3\Sigma_t) = \lambda/3$

Met  $c = \Sigma_s/\Sigma_t$  en  $\Sigma_t = \Sigma_s + \Sigma_a$

$\rightarrow \Sigma_a = (1 - c)\Sigma_t = (1 - c)/\lambda$

Dus  $L = \lambda / \sqrt{3(1 - c)}$

Voorwaarde:  $c > 0.7$



# Voorbeeld: kritische bolvormige reactor

Flux neemt toe met toenemende  $k_\infty$

Dit komt door de noemer in

$$\phi(r) = \frac{s_o'''}{(k_\infty - 1)\Sigma_a} \left[ \frac{\tilde{R} \sin\left(L^{-1} \sqrt{k_\infty - 1} r\right)}{r \sin\left(\underbrace{L^{-1} \sqrt{k_\infty - 1} \tilde{R}}_{\pi \rightarrow \text{flux oneindig}}\right)} - 1 \right]$$

Als de flux oneindig wordt is de bol critical

We verwachten dat de uitdrukking voor de flux singulier wordt

Criticality condition voor eindige reactor  $k = P_{NL} k_\infty = 1$

Voor de sferische reactor geldt  $L^{-1} \sqrt{k_\infty - 1} \tilde{R} = \pi$

➡  $1 = \frac{k_\infty}{1 + (\pi L / \tilde{R})^2}$

De nonleakage probability is dus

$$P_{NL} = \frac{1}{1 + (\pi L / \tilde{R})^2}$$

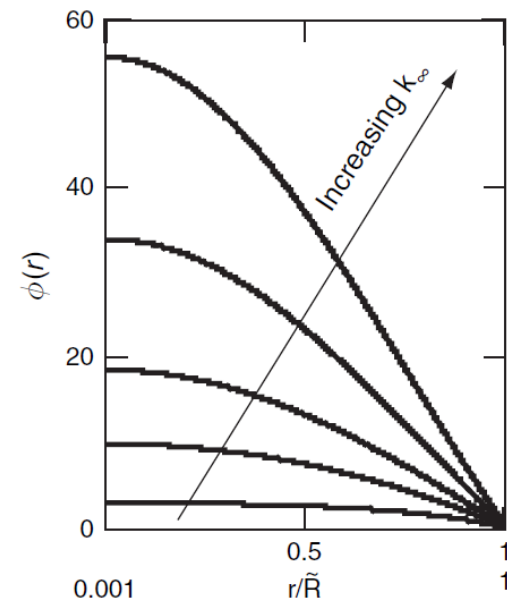
Merk op: dus geldt  $\Gamma = (\pi L / \tilde{R})^2$

Zoals verwacht neemt nonleakage toe met  $\tilde{R}/L$  de ge-extrapoleerde reactorstraal gemeten in diffusielengten

Material buckling term  $B_m = L^{-1} \sqrt{k_\infty - 1}$

Geometric buckling term  $B_g = \pi / \tilde{R}$

Criticality  $B_g = B_m$



# Resonance escape probability

We hadden 
$$p = \frac{\text{\# neutronen die thermische energie bereiken}}{\text{\# snelle neutronen die met slow down beginnen}}$$

Alle snelle neutronen die downward scatteren worden geabsorbeerd  
 In I-range door resonante capture door fuel  
 In T-range door fuel en moderator

Er geldt 
$$p = \frac{V_f \int_T \Sigma_a^f(E) \phi_f(E) dE + V_m \int_T \Sigma_a^m(E) \phi_m(E) dE}{V_f \left[ \int_T \Sigma_a^f(E) \phi_f(E) dE + \int_I \Sigma_a^f(E) \phi_f(E) dE \right] + V_m \int_T \Sigma_a^m(E) \phi_m(E) dE}$$

Schrijf als 
$$p = 1 - \frac{V_f \int_I \Sigma_a^f(E) \phi_f(E) dE}{V_f \left[ \int_T \Sigma_a^f(E) \phi_f(E) dE + \int_I \Sigma_a^f(E) \phi_f(E) dE \right] + V_m \int_T \Sigma_a^m(E) \phi_m(E) dE}$$

= Totale absorptie =  $Vq$  met  $q$  de slowing down dichtheid

Twee volume model  $q = \frac{V_f}{V} q_f + \frac{V_m}{V} q_m \rightarrow Vq \approx V_m q_m$  Verwaarloos slowdown in fuel

Dan geldt 
$$p = 1 - \frac{V_f}{V_m q_m} \int_I \Sigma_a^{fe}(E) \phi_f(E) dE$$
 Capture fertile materiaal dominant  
 $\Sigma_a^f(E) \rightarrow \Sigma_a^{fe}(E)$

# Resonance escape probability

We hadden 
$$p = 1 - \frac{V_f}{V_m q_m} \int_I \Sigma_a^{fe}(E) \phi_f(E) dE$$

In I-range zijn moderatoren zuivere verstrooiers

Er is dan een relatie tussen flux en slowing down density

Als  $\Sigma_s^m(E) = \text{constant}$ , dan is de flux  $1/E$

Er geldt  $q_m = \xi^m \Sigma_s^m E \phi_m(E)$

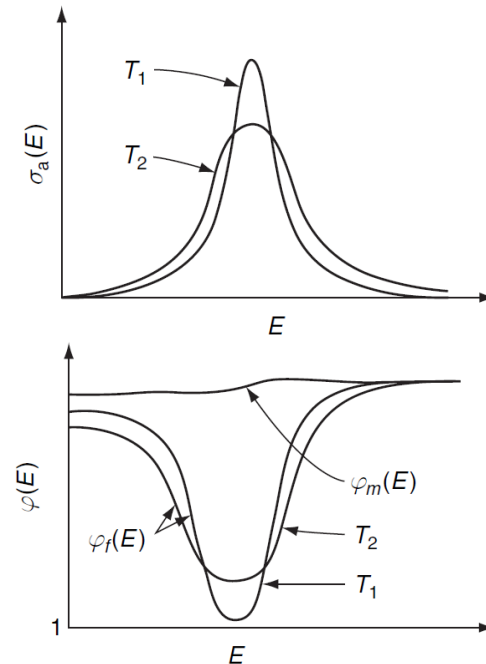
We vinden 
$$p = 1 - \frac{V_f}{V_m \xi^m \Sigma_s^m E \phi_m(E)} \int_I \Sigma_a^{fe}(E') \phi_f(E') dE'$$

Herschrijf als 
$$p = 1 - \frac{V_f}{V_m \xi^m \Sigma_s^m} I, \text{ met } I = \int_I \frac{\sigma_a^{fe}(E) \phi_f(E)}{E \phi(E)} dE$$

Voor 1 resonantie 
$$p_i = \exp\left(-\frac{V_f N_{fe}}{V_m \xi^m \Sigma_s^m} I_i\right)$$

Voor  $T$  resonanties 
$$p = p_1 p_2 p_3 \cdots p_i \cdots p_{T-1} p_T$$

Dan geldt 
$$p = \exp\left(-\frac{V_f N_{fe}}{V_m \xi^m \Sigma_s^m} I\right), \text{ met } I = \sum_{i=1}^T I_i$$



Resonance Integrals for Fuel Rods

$$I = 2.95 + 25.8 \sqrt{4/\rho D} \text{ for U metal}$$

$$I = 4.45 + 26.6 \sqrt{4/\rho D} \text{ for UO}_2$$

Fuel rods  $0.2 < D < 3.5$  cm  
 Integraal  $I$  (absorptie) neemt af  
 als  $D$  toeneemt!

Self shielding depresses  $\phi_f(E) / \phi_m(E)$