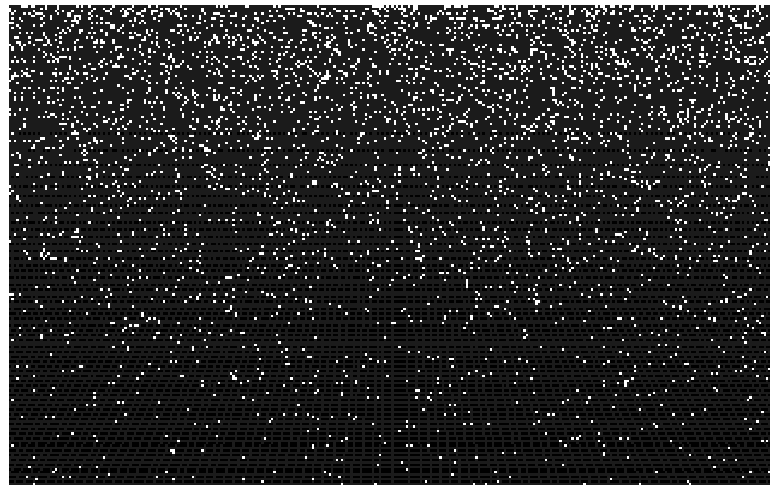


Elementaire Deeltjesfysica

FEW Cursus - verstrooiing



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7 December, 2006
Structuur der Materie

Elektronen verstrooiing

Meten van
ladingsverdeling

Voor resolutie geldt

$$\Delta x \geq \frac{\lambda}{2} \quad p_{\min} = \frac{h}{\lambda} \geq \frac{h}{2\Delta x}$$



$$E_{\min} = \sqrt{m^2 c^4 + p_{\min}^2 c^2} \approx p_{\min} c \geq \pi \frac{\hbar c}{\Delta x} \approx \frac{620 \text{ MeV} \cdot \text{fm}}{\Delta x}$$

Werkzame doorsnede

$$\frac{d\sigma}{d\Omega} = \frac{m^2 |M_{fi}|^2}{(2\pi\hbar^2)^2}$$

Opgave: Geef afleiding!

Eerste Born benadering
(geen spin / terugstoot)

$$\psi_i = e^{i\vec{k}\cdot\vec{x}} \quad \psi_f = e^{i\vec{k}'\cdot\vec{x}} \quad \vec{q}/\hbar = \vec{k} - \vec{k}'$$

$$M_{fi} = \langle \psi_f | V | \psi_i \rangle = \iiint V(\vec{x}) e^{\frac{i\vec{q}\cdot\vec{x}}{\hbar}} d\vec{r}$$

Sferische symmetrie

$$V(\vec{x}) = V(x)$$



$$M_{fi} = \frac{4\pi\hbar}{|\vec{q}|} \int_0^\infty V(x) \sin\left(\frac{|\vec{q}|x}{\hbar}\right) x dx$$

Opgave: Geef afleiding!

Elastische elektronen verstrooiing

Afgeschermd
Coulombpotentialiaal

$$V(x) = \frac{Ze^2}{4\pi\epsilon_0} \frac{e^{-x/a}}{x}$$

$a \approx$ atoomstraal

Integraal levert

Opgave: Geef afleiding!

$$M_{fi} = 4\pi \frac{Ze^2}{4\pi\epsilon_0} \frac{\hbar^2}{|\vec{q}|^2 + (\hbar/a)^2}$$

$$|\vec{q}| \gg \frac{\hbar}{a} \simeq 2 \times 10^{-3} \frac{\text{MeV}}{c}$$

$$M_{fi} = 4\pi \frac{Ze^2}{4\pi\epsilon_0} \frac{\hbar^2}{|\vec{q}|^2} = 4\pi Z\alpha\hbar c \frac{\hbar^2}{|\vec{q}|^2}$$

Overgedragen impuls

$$|\vec{q}| = 2|\vec{p}| \sin \frac{\theta}{2}$$

met in COM

$$|\vec{p}| = |\vec{p}_i| = |\vec{p}_f|$$

Rutherford verstrooiing

$$\frac{d\sigma}{d\Omega} = \frac{m^2 |M_{fi}|^2}{(2\pi\hbar^2)^2}$$

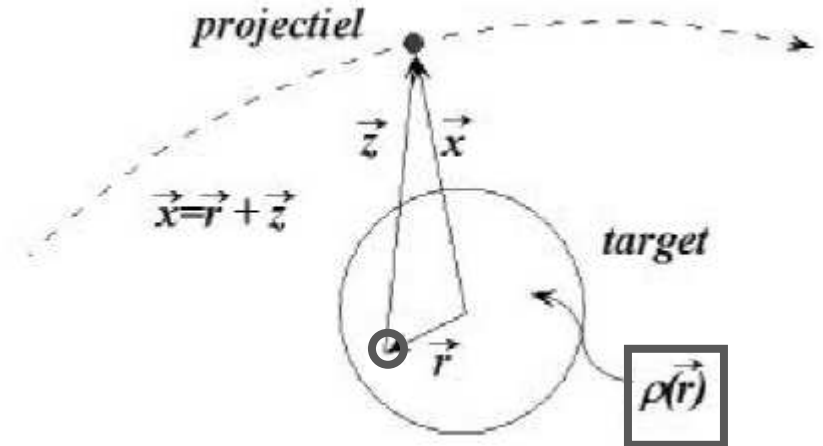
$$\frac{\vec{q}^2}{2m} = 4 \frac{\vec{p}^2}{2m} \sin^2(\theta/2) = 4T \sin^2(\theta/2)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z\alpha\hbar c}{4T \sin^2(\theta/2)} \right)^2$$

Opgave: Geef afleiding!

Elektronen verstrooiing

Uitgebreide sferisch symmetrische ladingsverdeling



potentiaal $V(\vec{x}) = Z\alpha\hbar c \iiint \rho(\vec{r}) \frac{e^{-z/a}}{z} d\vec{r}$.

matricelement $M_{fi} = \langle \psi_f | V | \psi_i \rangle = \iiint V(\vec{x}) e^{\frac{i\vec{q}\cdot\vec{x}}{\hbar}} d\vec{r}$,

met $d\vec{r} \Rightarrow d\vec{x}$

$$\begin{aligned} M_{fi} &= Z\alpha\hbar c \iiint \iiint e^{i\vec{q}\cdot\vec{x}/\hbar} \rho(\vec{r}) \frac{e^{-z/a}}{z} d\vec{r} d\vec{x} \\ &= Z\alpha\hbar c \iiint \frac{e^{-z/a}}{z} e^{i\vec{q}\cdot\vec{z}/\hbar} dz \iiint \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}/\hbar} d\vec{r} \\ &= M_{fi}(\text{punt}) F(\vec{q}), \end{aligned}$$

Opgave: Geef afleiding!

Form factor

$$F(\vec{q})$$

ladingsverdeling

$$\rho(\vec{r}) = \frac{1}{(2\pi\hbar)^3} \iiint F(\vec{q}) e^{-i\vec{q}\cdot\vec{r}/\hbar} d\vec{q}$$

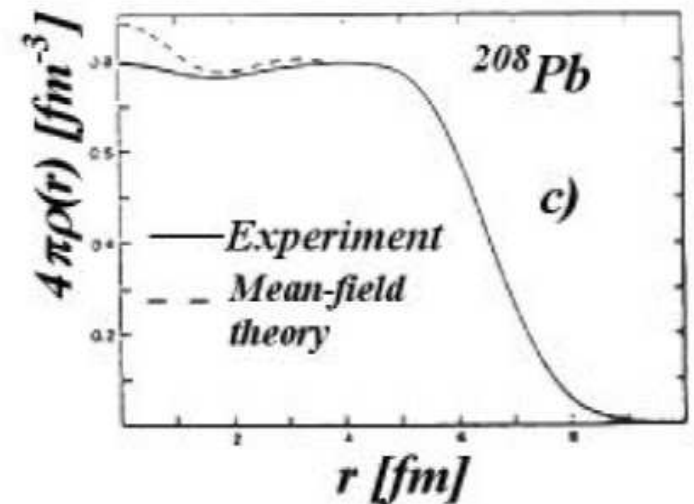
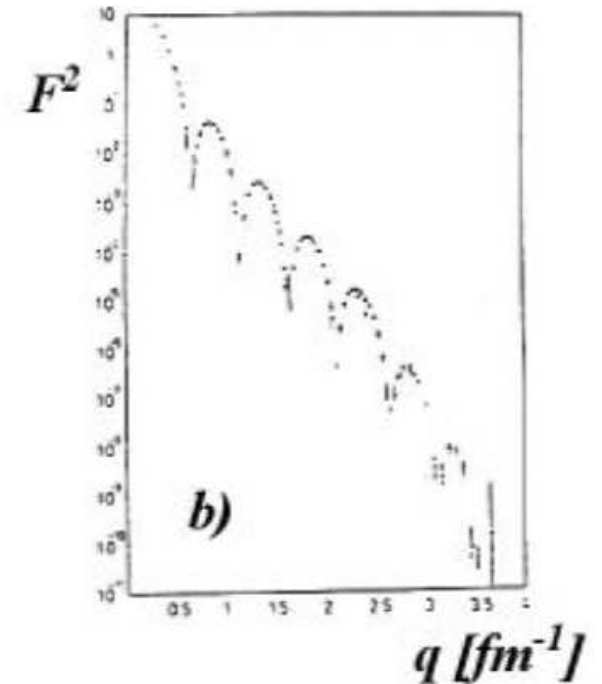
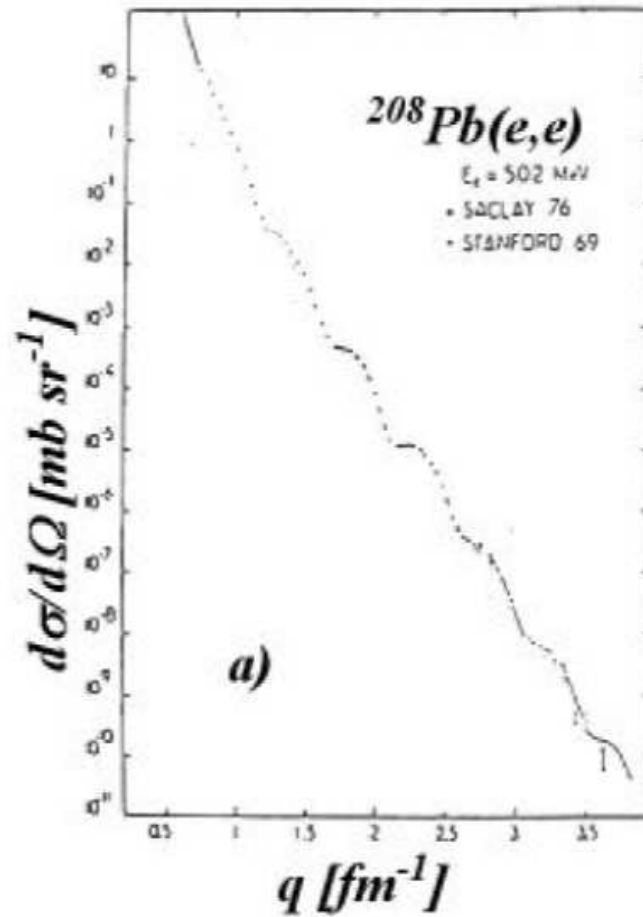
Opgave: Vergelijk met Fraunhofer diffractie aan een object.

Elastische elektronen verstrooiing - Voorbeelden

Elektronen

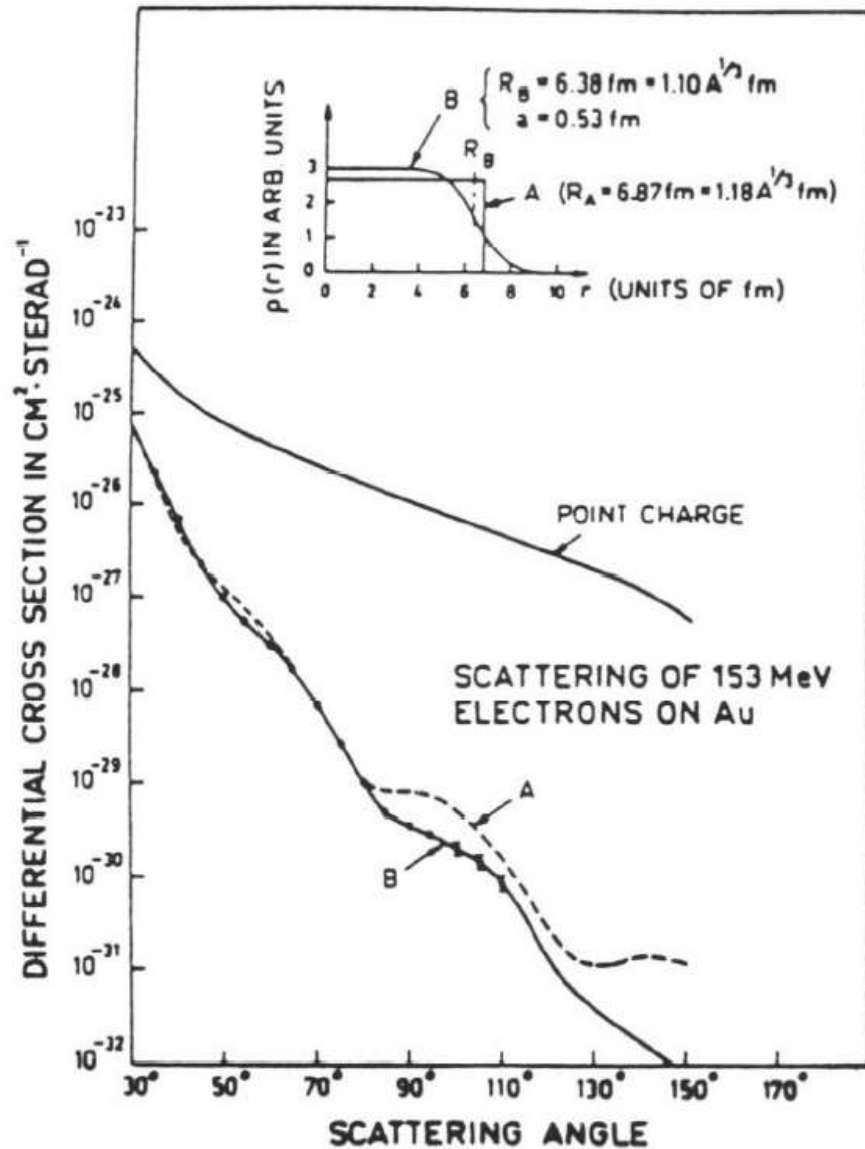
aan lood:

- 502 MeV
- ^{208}Pb spinloos
- 12 decaden



Model-onafhankelijke informatie over ladingsverdeling van nucleon en kernen

Elastische elektronen verstrooiing - Voorbeelden

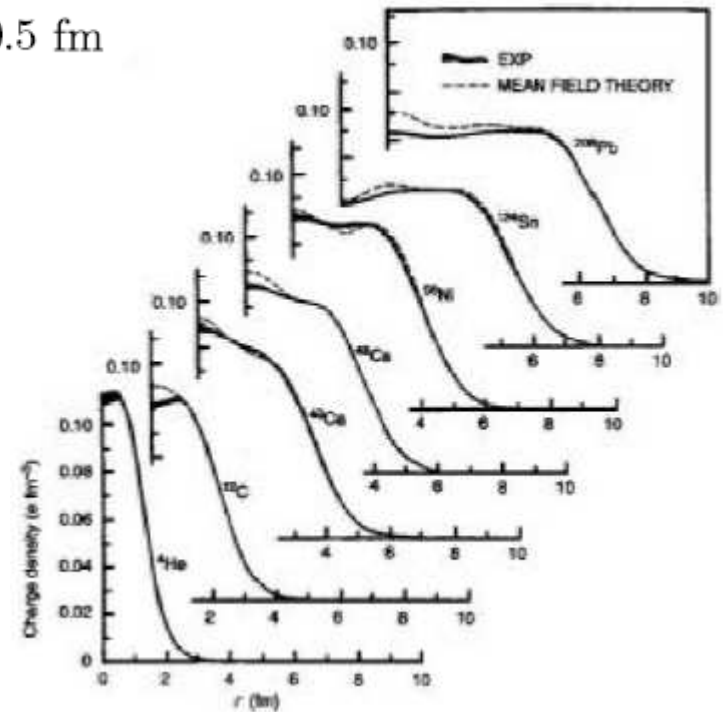


Elektron-goud verstrooiing
- energie: 153 MeV

ladingsverdeling:

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-R}{a}}}$$

$R \simeq 1.2 \text{ fm } \sqrt[3]{A} - 0.48 \text{ fm}$, voor $A \gg 1$
 $a \simeq 0.5 \text{ fm}$



Ladingsdichtheid is constant!

Multipoolexpansie van vormfactor

Vormfactor

$$M_{fi} = Z\alpha\hbar c \iiint e^{i\vec{q}\cdot\vec{x}/\hbar} \rho(\vec{r}) \frac{e^{-z/a}}{z} d\vec{r} d\vec{x}$$

$$= Z\alpha\hbar c \iiint \frac{e^{-z/a}}{z} e^{i\vec{q}\cdot\vec{z}/\hbar} d\vec{z} \left[\iiint \rho(\vec{r}) e^{i\vec{r}\cdot\vec{z}/\hbar} d\vec{r} \right]$$

$$= M_{fi}(\text{punt}) F(\vec{q}),$$

Multipoolexpansie levert

$$\begin{aligned} F(\vec{q}^2) &= \int \rho(\vec{r}) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i|\vec{q}||r|\cos\theta}{\hbar} \right)^n d\vec{r} \quad \text{met } \theta = \angle(\vec{r}, \vec{q}) \\ &= \int_0^{\infty} \int_{-1}^{+1} \int_0^{2\pi} \rho(r) \left[1 - \frac{1}{2} \left(\frac{|\vec{q}|r}{\hbar} \right)^2 \cos^2\theta + \dots \right] d\phi d\cos\theta r^2 dr \\ &= \underbrace{4\pi \int_0^{\infty} \rho(r) r^2 dr}_{\text{lading}} - \frac{1}{6} \frac{\vec{q}^2}{\hbar^2} \underbrace{4\pi \int_0^{\infty} \rho(r) r^4 dr}_{\text{kwadratische ladingstraal } \langle r^2 \rangle} + \dots \end{aligned}$$

lading

kwadratische ladingstraal $\langle r^2 \rangle$

$$\langle r^2 \rangle = 4\pi \int_0^{\infty} r^2 \cdot \rho(r) r^2 dr$$

en dus

$$F(\vec{q}^2) = 1 - \frac{1}{6} \frac{\vec{q}^2 \langle r^2 \rangle}{\hbar^2} + \dots \quad \Rightarrow \quad \boxed{\langle r^2 \rangle = -6\hbar^2 \frac{dF(\vec{q}^2)}{d\vec{q}^2} \Big|_{\vec{q}^2=0}}$$

Elastische elektron-nucleon verstrooiing

Vierimpuls overdracht

$$Q^2 \equiv -q^2 \approx \vec{q}^2 = (\vec{p}_i - \vec{p}_f)^2 \approx 2p^2(1 - \cos \theta) = 4p^2 \sin^2(\theta/2)$$

en $|d\Omega| = |2\pi d(\cos \theta)| = \frac{2\pi}{2p^2} dQ^2$

Werkzame doorsnede

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2\hbar^2}{Q^4} F^2(Q^2)$$

enkel Coulomb

Mott werkzame doorsnede

$$\left(\frac{d\sigma}{dQ^2}\right)_{\text{Mott}} = \frac{4\pi\alpha^2\hbar^2}{Q^4} [1 - \beta^2 \sin^2(\theta/2)]$$

B-veld door
stroom nucleon

Rosenbluth werkzame doorsnede

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left\{ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2(\theta/2) \right\} \quad \tau \equiv -q^2/4M^2$$

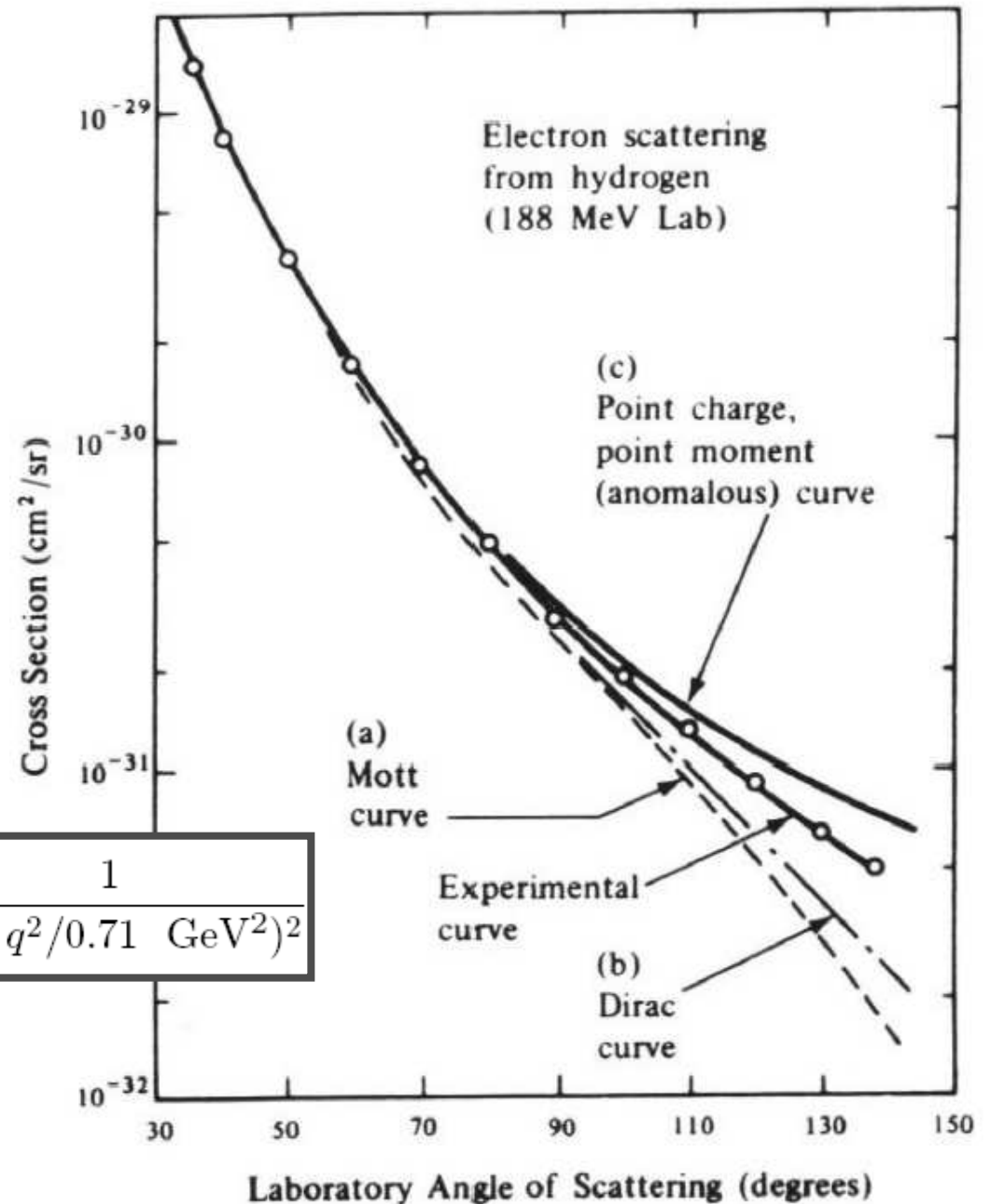
G_E en G_M zijn elektrische en magnetisch vormfactoren (inclusief magnetisch moment van het nucleon)

Elastische elektron-proton verstrooiing

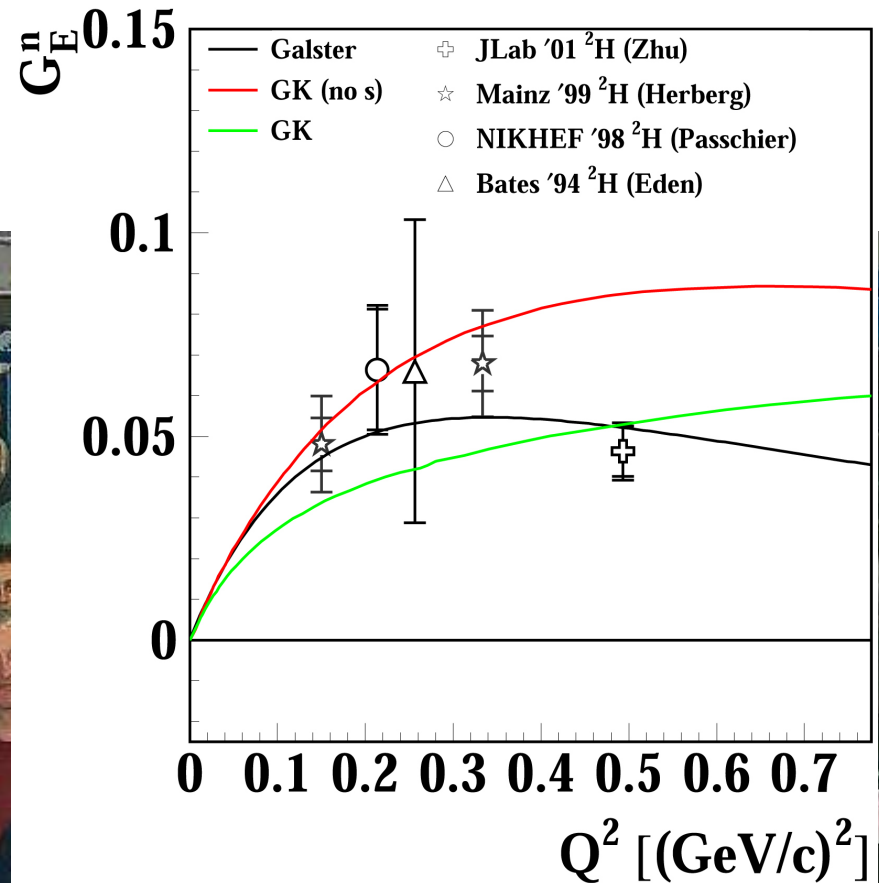
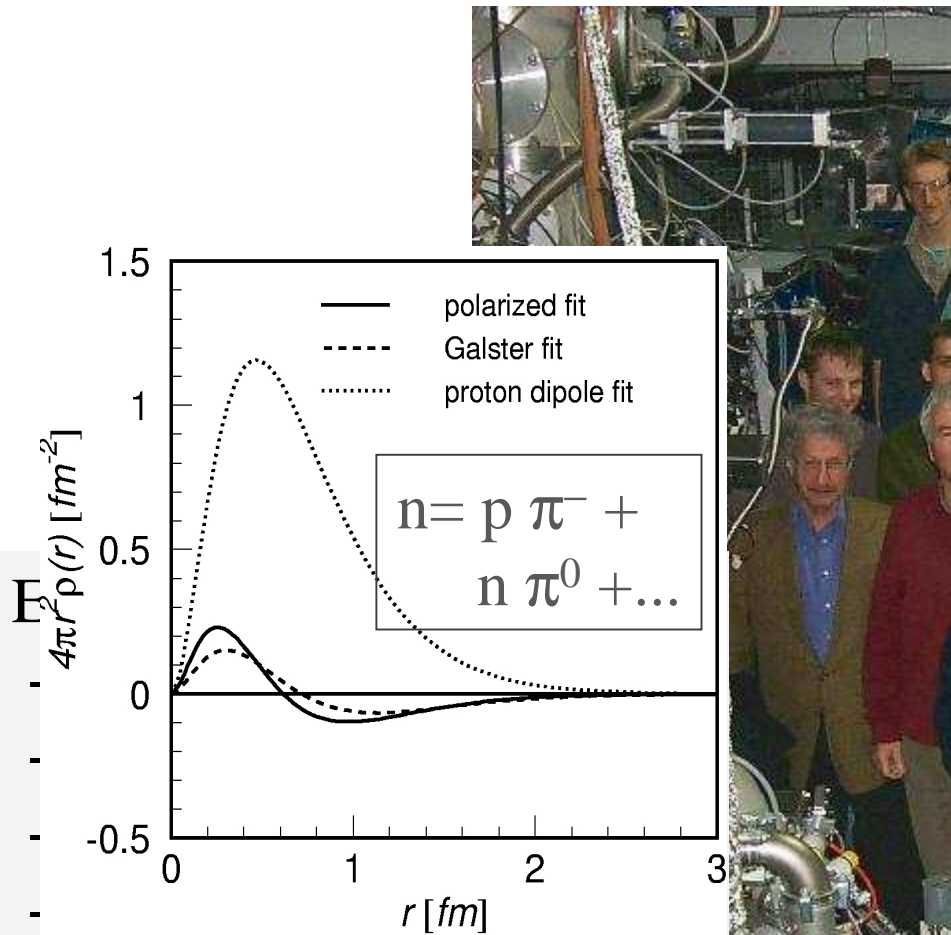
Proton structuur

- niet puntvormig
- geen Dirac deeltje ($g=2$)
- straal is 0.8 fm
- exponentiele vormfactor

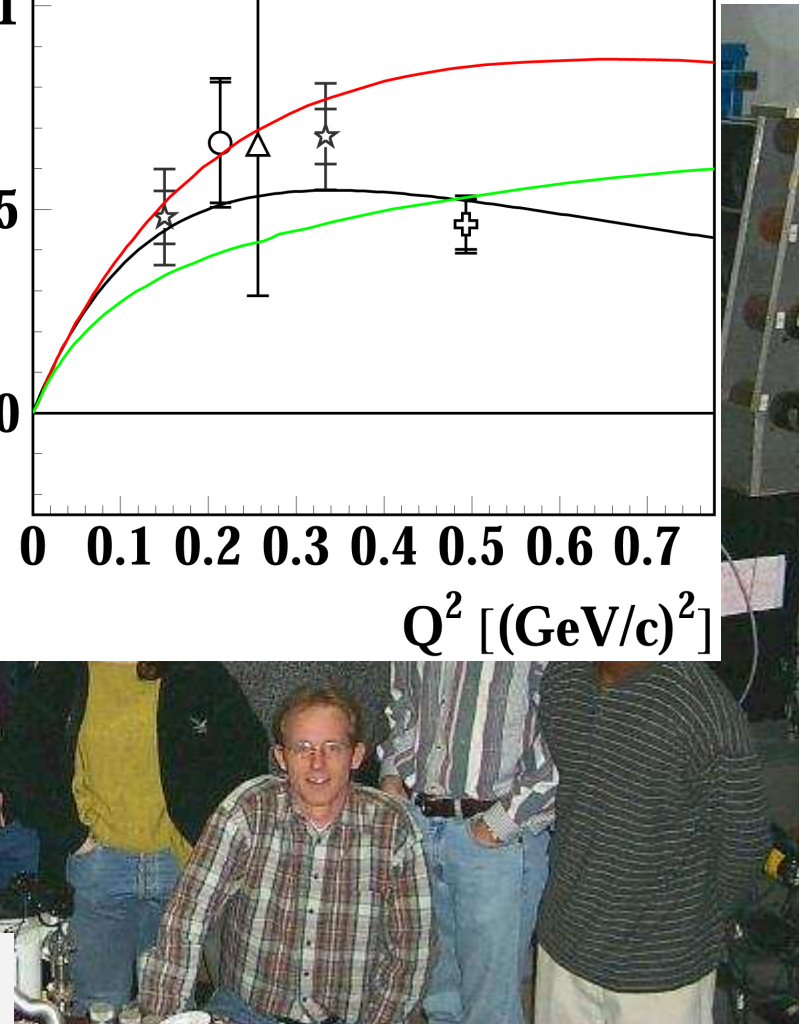
$$G_E^p(q^2) = \frac{G_M^p(q^2)}{|\mu_p|} = \frac{G_M^n(q^2)}{|\mu_n|} \simeq \frac{1}{(1 - q^2/0.71 \text{ GeV}^2)^2}$$



Ladingsverdeling van het neutron



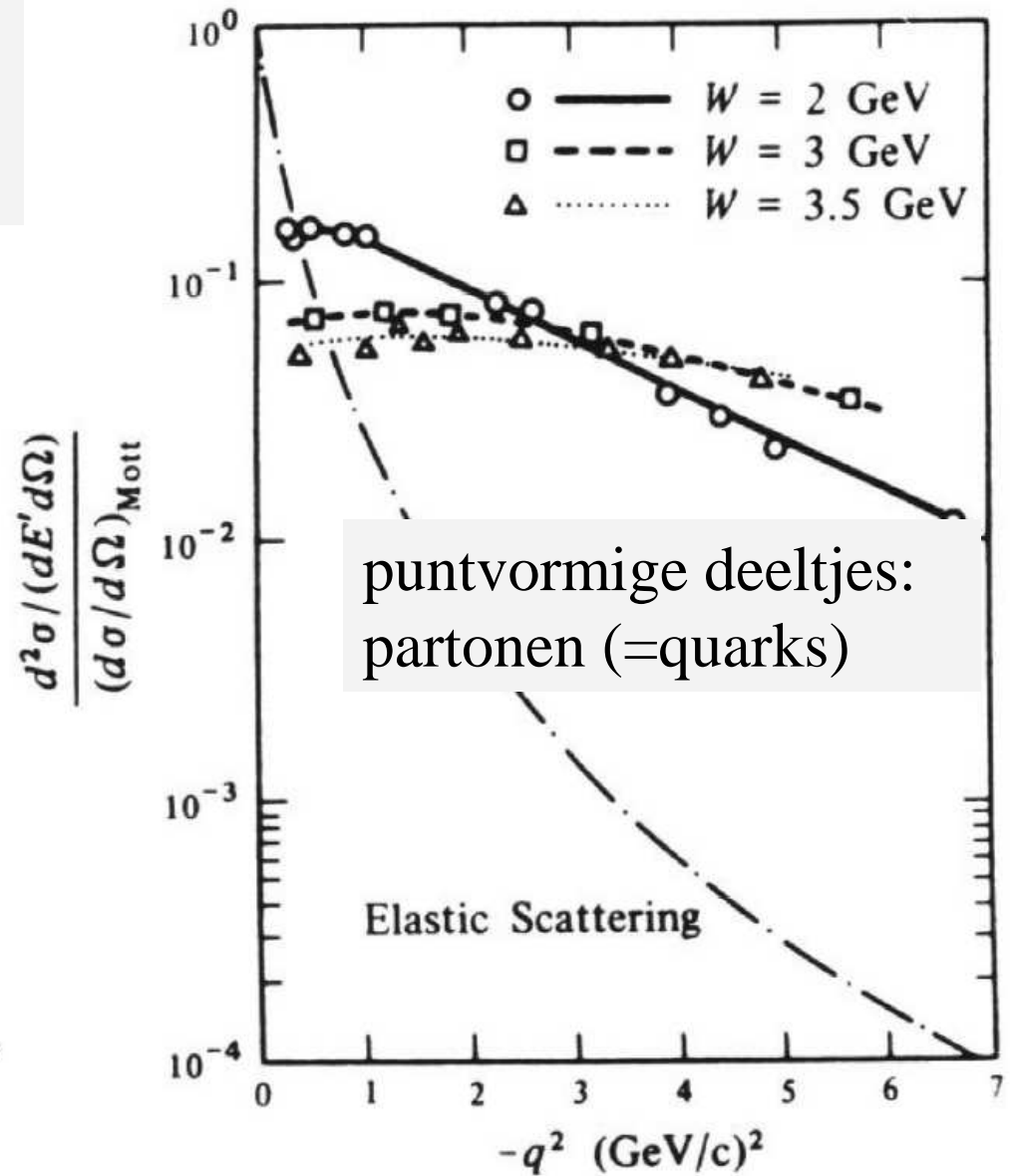
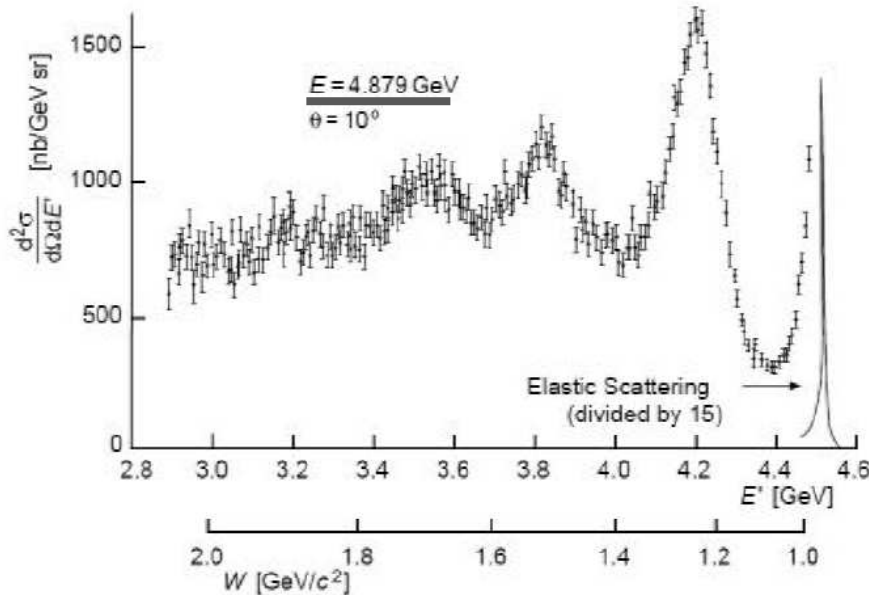
- elektron-neutron coincidentie meting



Diep-inelastische verstrooiing

DIS definitie:

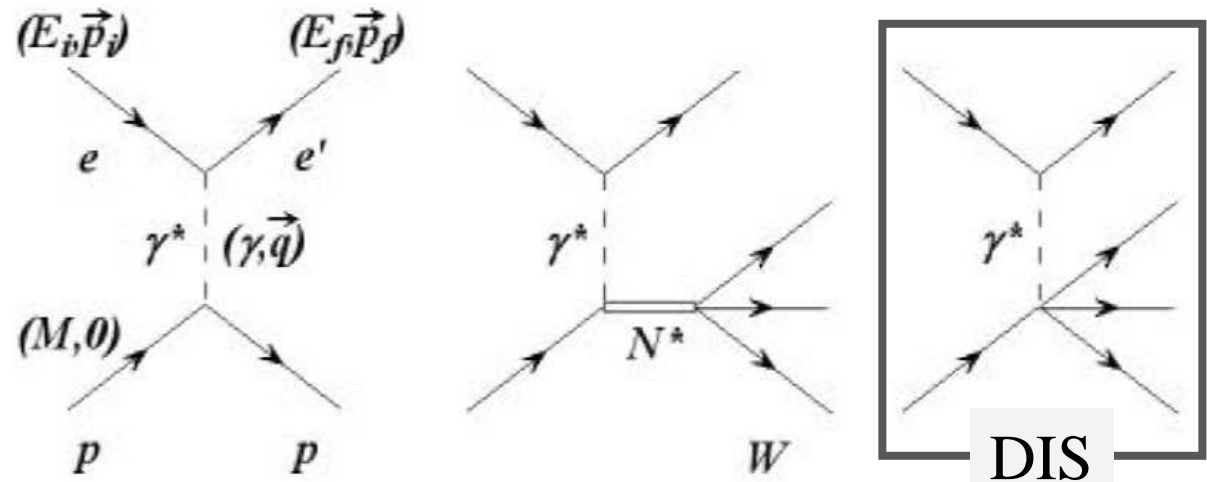
- Vierimpuls $Q^2 > 1$ (GeV/c)²
- Invariante massa $W > 2$ GeV



Diep-inelastische verstrooiing

Werkzame doorsnede:

- elastisch
- resonantie
- DIS



Variabelen

$$\begin{aligned}\vec{q} &= \vec{p}_i - \vec{p}_f \\ \nu &= Pq/M = E_i - E_f \\ Q^2 &= -q^2 = \vec{q}^2 - \left(\frac{\nu}{c}\right)^2 \quad (\neq 0)\end{aligned}$$

Opgave: Wat is Q^2

Invariante massa

$$W^2 c^4 = E_q^2 - \vec{q}^2 c^2$$



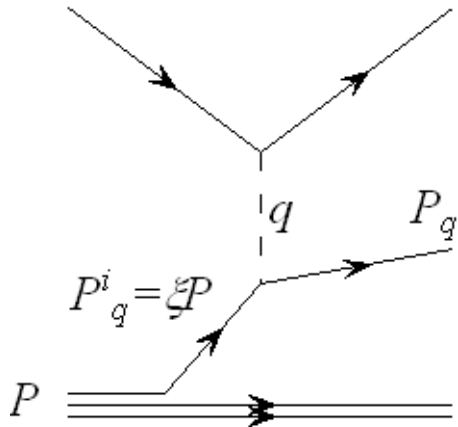
$$Q^2 = 2M\nu - (W^2 - M^2)c^2$$

Werkzame doorsnede

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2\hbar^2}{Q^4} \frac{E_f}{E_i\nu} \left[F_2(Q^2, \nu) \cos^2 \frac{\theta}{2} + 2\frac{\nu}{Mc^2} F_1(Q^2, \nu) \sin^2 \frac{\theta}{2} \right]$$

Structuurfuncties F_1 en F_2

DIS – Bjørken schaling



Infinite momentum frame

$$q + P_q^i = P_q$$

$$q^2 + 2\xi P + \xi^2 P^2 = P_q^2$$

$$\xi = -q^2 / 2qP = \underline{Q^2 / 2Mv}$$

in LAB

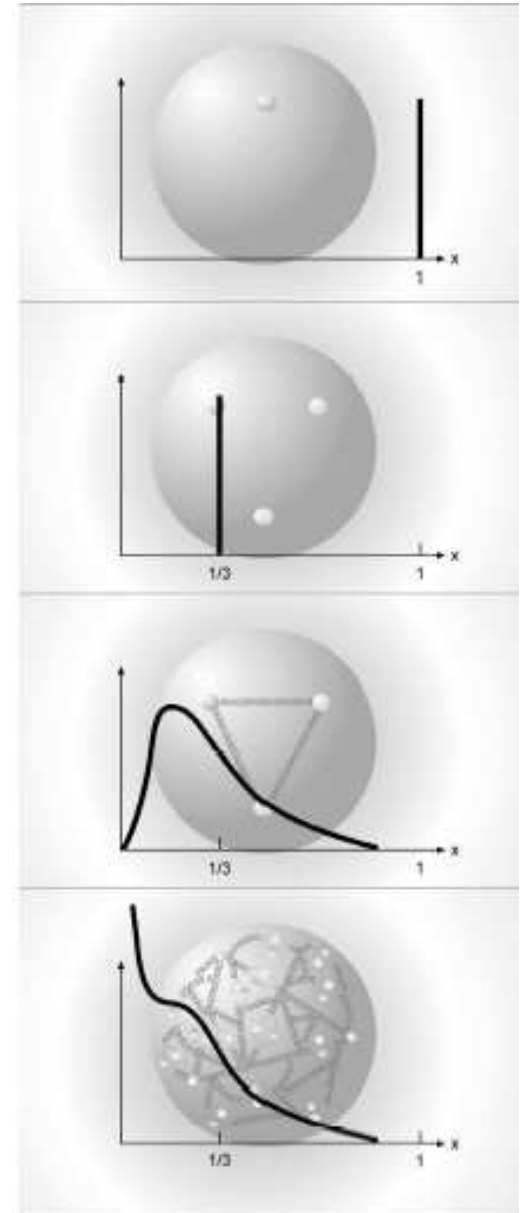
Bjørken x variabele
Lorentz invariant

We verwachten

- een piek bij $x = 1/3$
- Fermi impuls

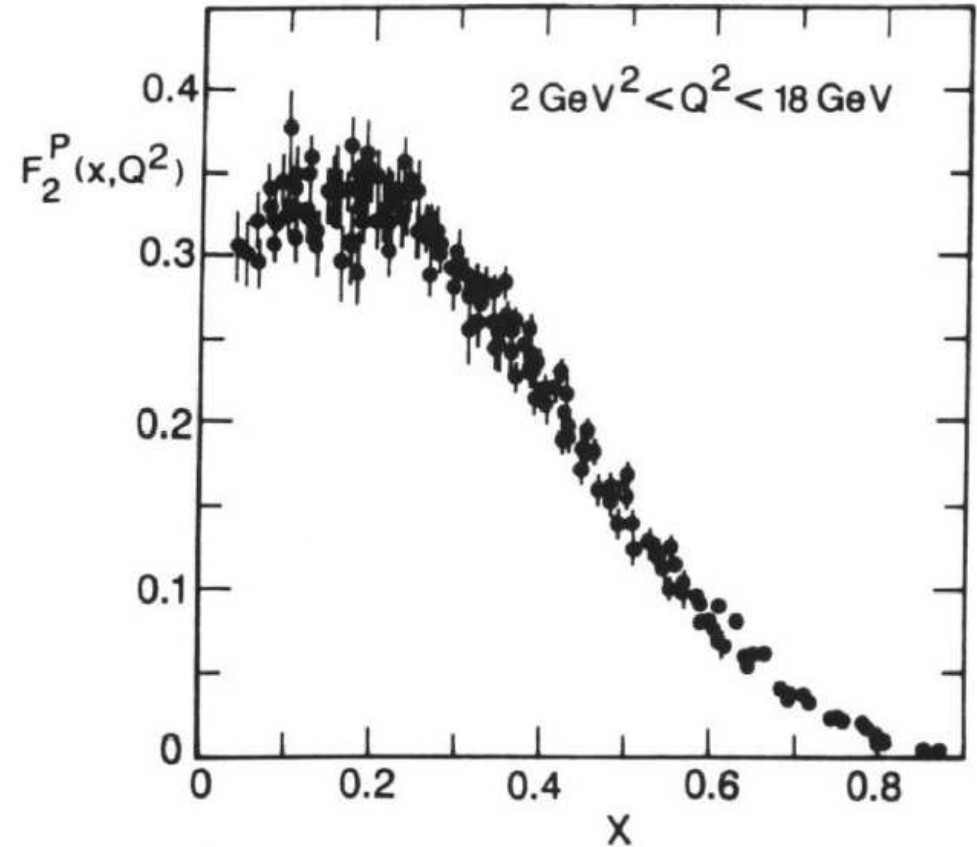
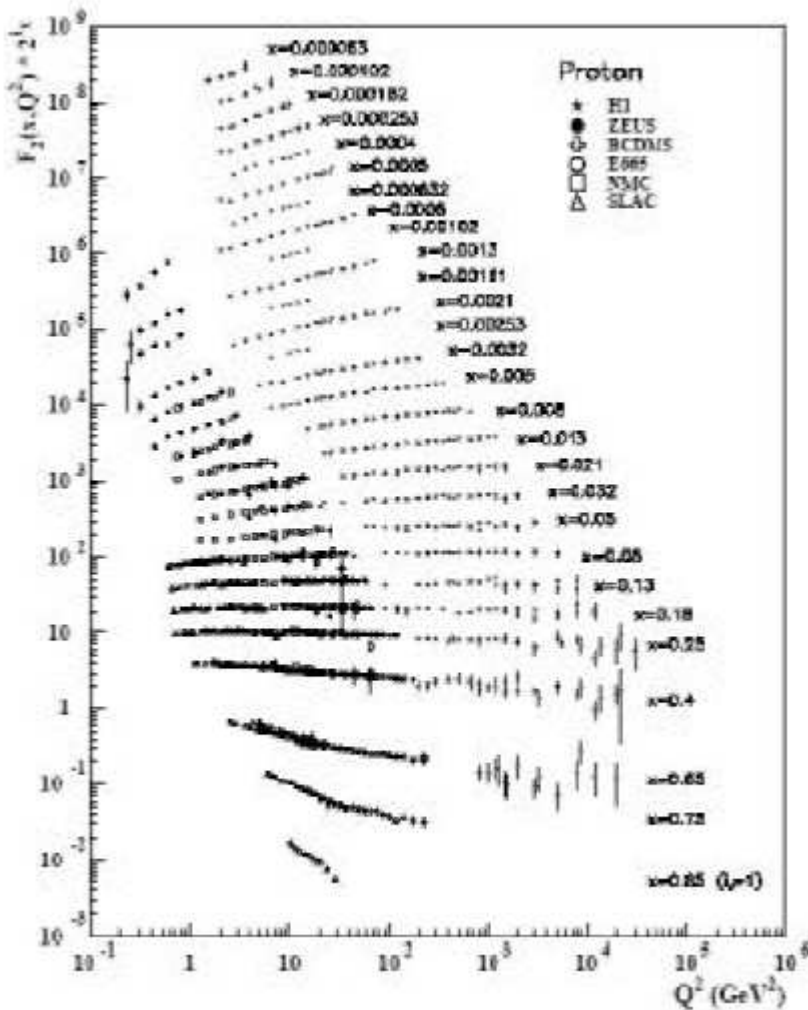
$$x = \frac{m_{\text{quark}}}{M} \simeq \frac{1}{3}$$

$$p_F \simeq \frac{\hbar}{R} \simeq 250 \text{ MeV}/c$$



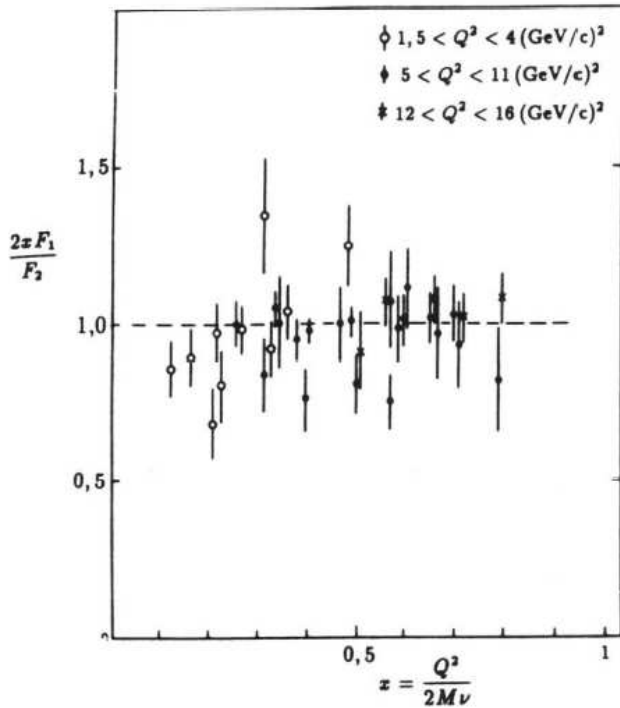
DIS – Bjørken schaling

Schaling:
structuurfuncties enkel
functie van x



$$F_2(x) = x \sum_f e_f^2 (q_f(x) + \bar{q}_f(x))$$

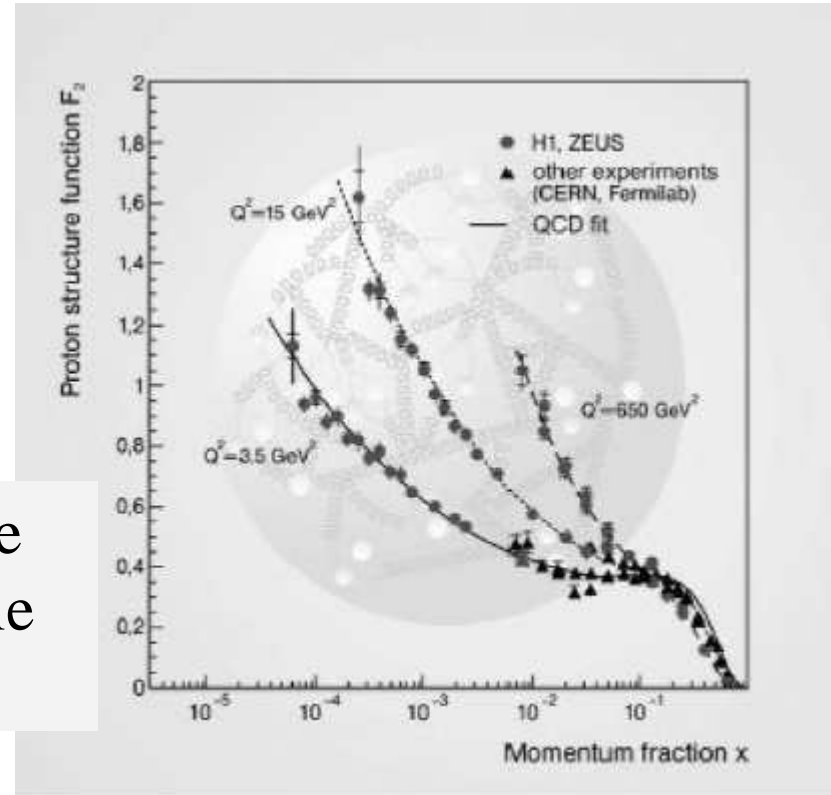
DIS – Bjørken schaling



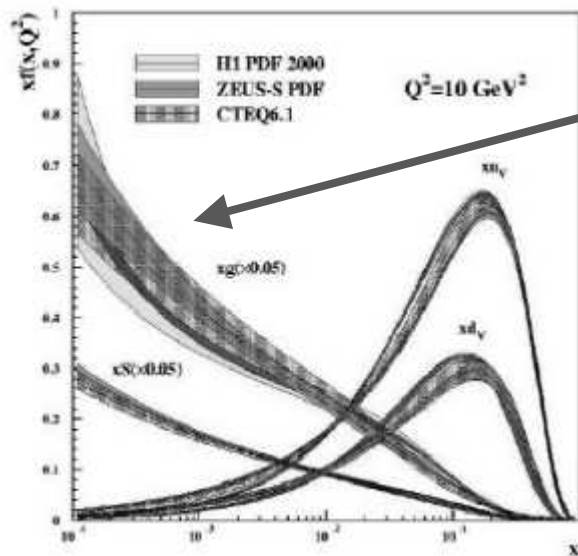
$$2xF_1(x) = F_2(x)$$

Callan-Gross relatie

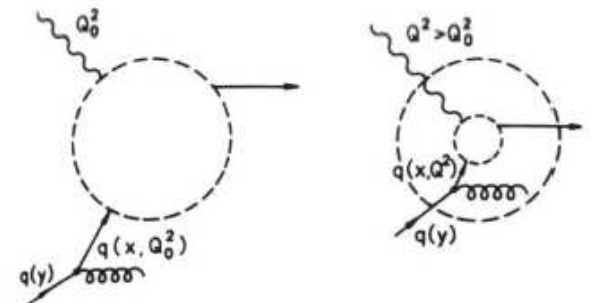
Quarks
spin $1/2$



Decompositie:



Gluon bijdrage
van Q^2 evolutie
van F_2



Elektron-positron annihilatie

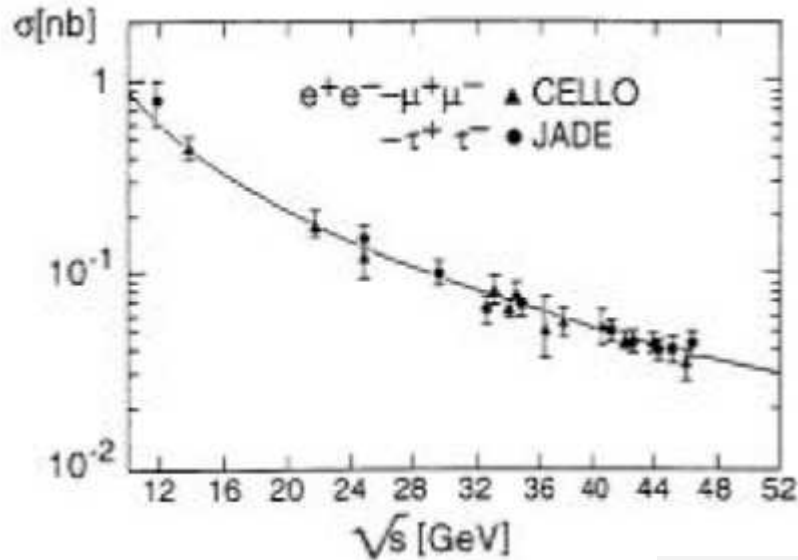
Muon productie

$$e^+ + e^- \rightarrow \mu^+ + \mu^-$$

$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \rightarrow \mu^+ + \mu^-) = \frac{\pi\alpha^2}{8E^2} (1 + \underbrace{\cos^2\theta}_{\text{spinfactor}})$$

integreer

$$\sigma(e^+ + e^- \rightarrow \mu^+ + \mu^-) = \sigma(e^+ + e^- \rightarrow \tau^+ + \tau^-) = \frac{21.7}{E^2} \text{ nanobarns}$$



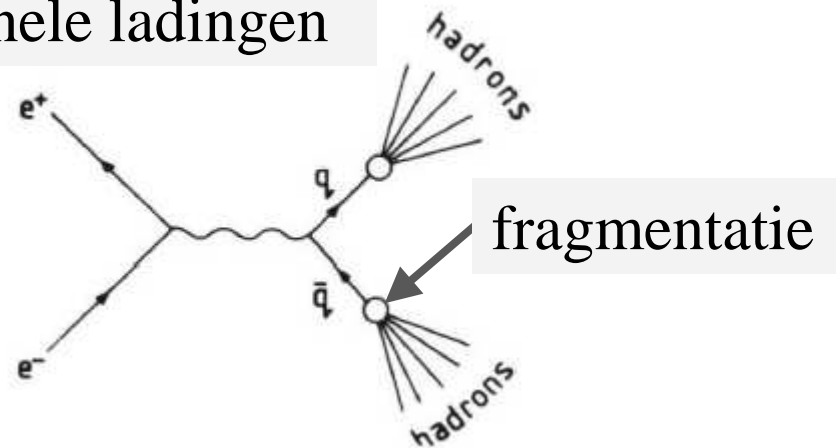
Elektron, muon en tau zijn identiek, afgezien van massa en levensduur

Twee-jet productie

$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \rightarrow q + \bar{q}) = \frac{3 \pi e_q^2 \alpha^2}{8E^2} (1 + \cos^2\theta)$$

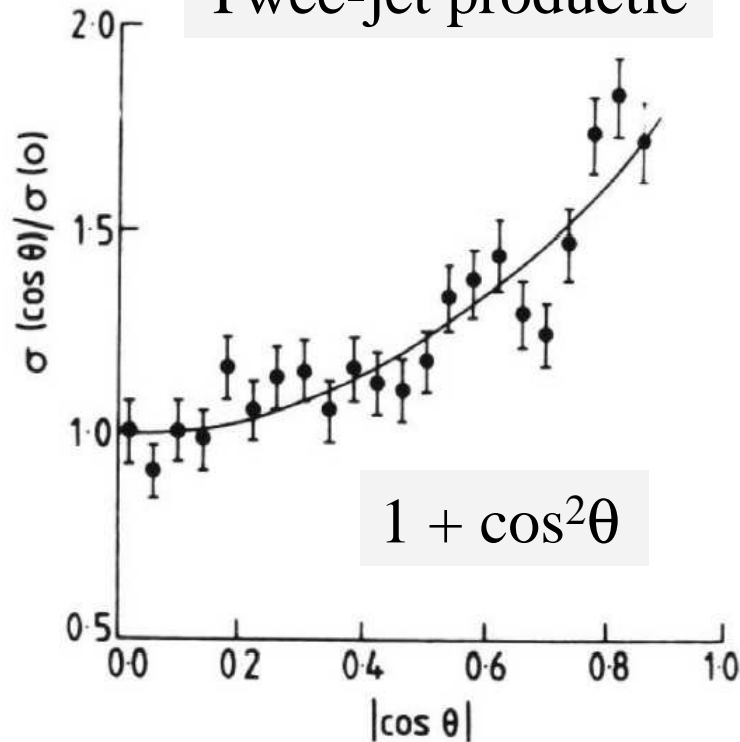
kleur

fractionele ladingen



Elektron-positron annihilatie

Twee-jet productie



$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadronen})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \rightarrow q + \bar{q}) = \frac{3\pi e_a^2 \alpha^2}{8E^2} (1 + \cos^2\theta)$$

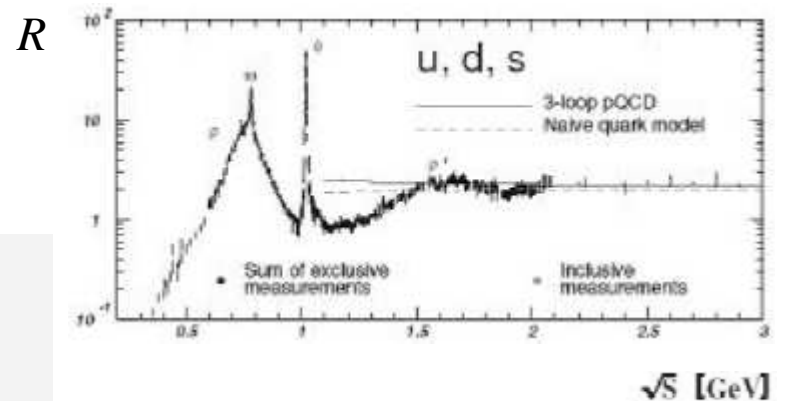
$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \rightarrow \mu^+ + \mu^-) = \frac{\pi\alpha^2}{8E^2} (1 + \cos^2\theta)$$

Voor $E < 3.5$ GeV

$$R = 3 \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] = 2$$

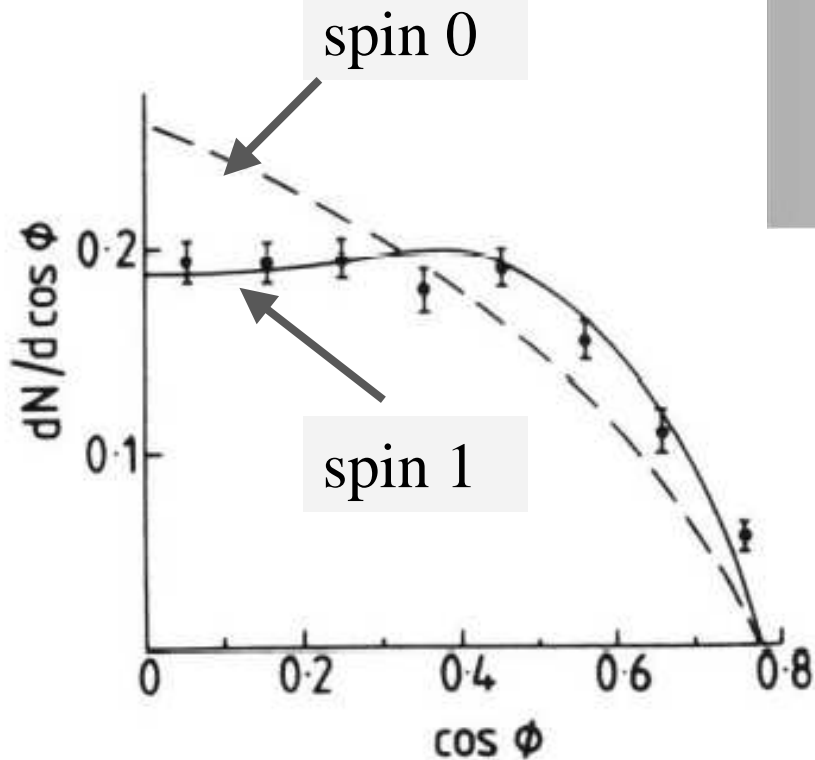
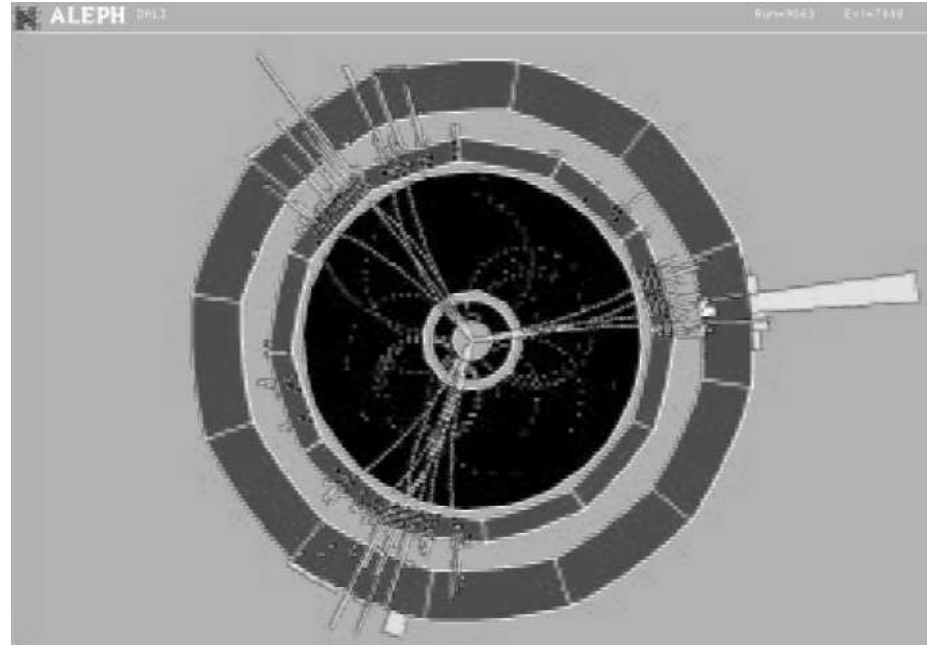
Quarks hebben spin 1/2

Quarks hebben kleur en fractionele lading



Elektron-positron annihilatie

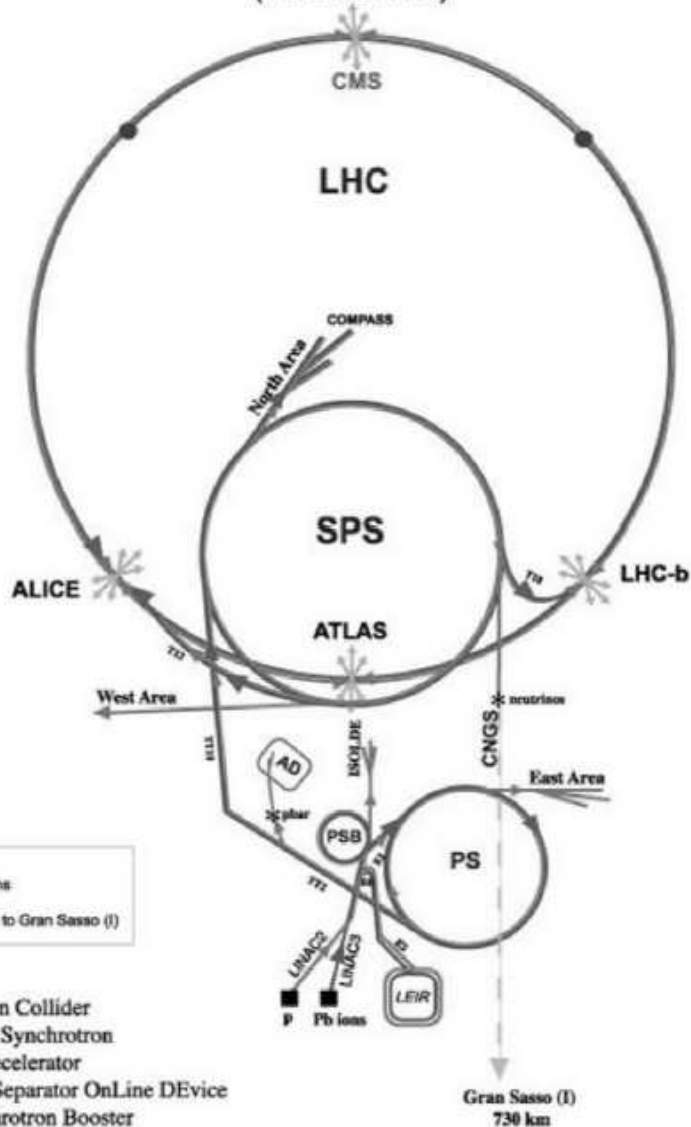
Drie-jet productie



Gluons hebben spin 1

Z^0 -resonantie

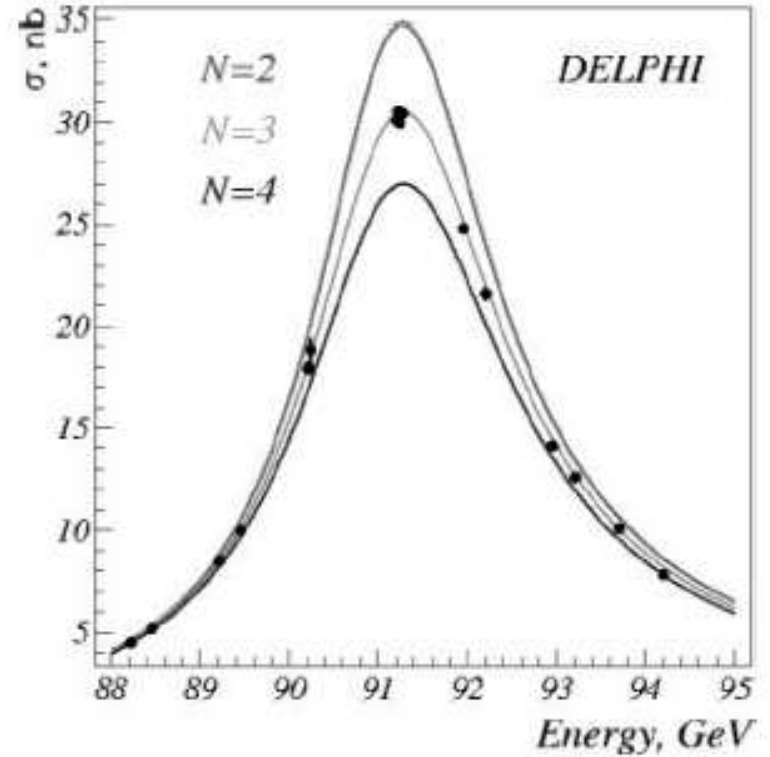
CERN Accelerators
(not to scale)



— protons
 - - - antiprotons
 ... ions
 * neutrinos to Gran Sasso (I)

LHC: Large Hadron Collider
 SPS: Super Proton Synchrotron
 AD: Antiproton Decelerator
 ISOLDE: Isotope Separator OnLine DEvice
 PSB: Proton Synchrotron Booster
 PS: Proton Synchrotron
 LINAC: LInear ACcelerator
 LEIR: Low Energy Ion Ring
 CNGS: Cern Neutrinos to Gran Sasso

Redef. LEY, PS Division, CERN, 02/09/96
 Revised and adapted by Antonella Del Ross, ITT Div.,
 in collaboration with B. Desforges, SL Div., and
 D. Majumari, PS Div, CERN, 23.05.01



e^+e^- verstrooiing

Drie generaties!

