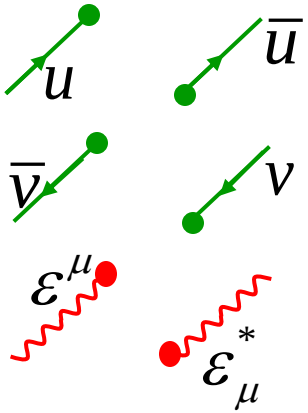
Feynman regels voor QED (S=1/2)

Externe lijnen

Deeltje

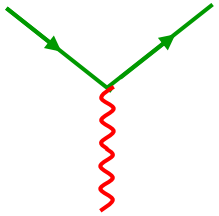
Anti-deeltje

Foton



inkomend:	u
uitgaand:	$\bar{u}$
inkomend:	$\bar{v}$
uitgaand:	v
inkomend:	ϵ_{μ}
uitgaand:	ϵ'^{*}_{μ}

Vertices



$$-i(\sqrt{4\pi}e)\gamma_{\mu}$$

Propagatoren

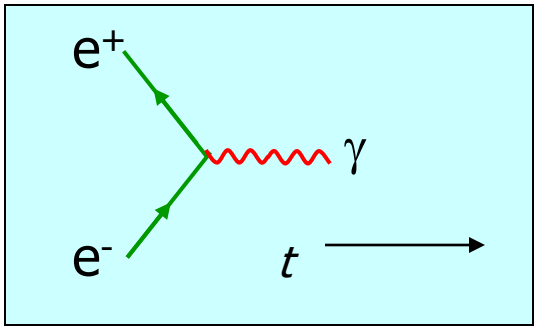
Deeltje

$$\frac{i\left(\gamma_{\mu}q^{\mu}+m\right)}{q^{2}-m^{2}}$$

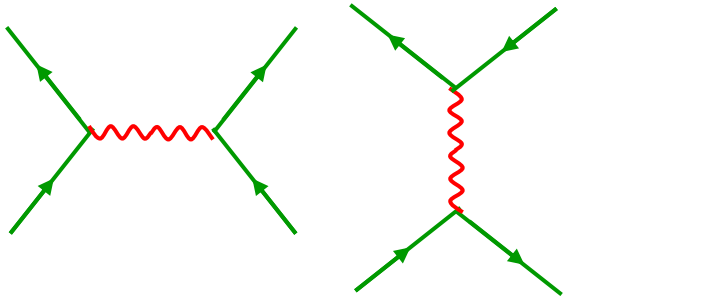
Foton

$$\frac{-ig_{\mu\nu}}{q^{2}}$$

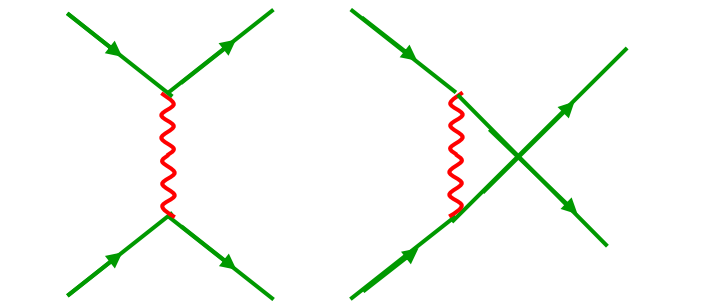
QED



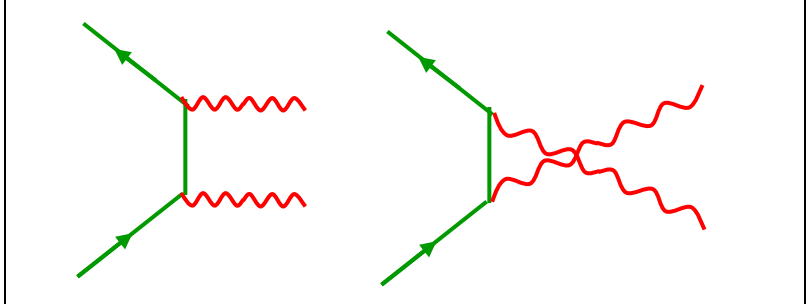
Fundamentele interacties:



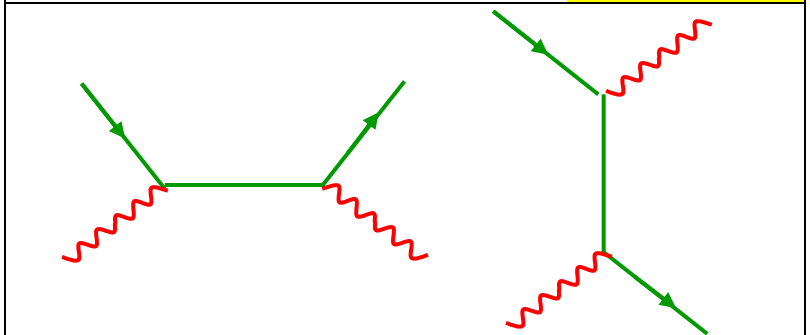
Bhabha verstrooiing $e^+e^- \rightarrow e^+e^-$



Möller verstrooiing $e^-e^- \rightarrow e^-e^-$

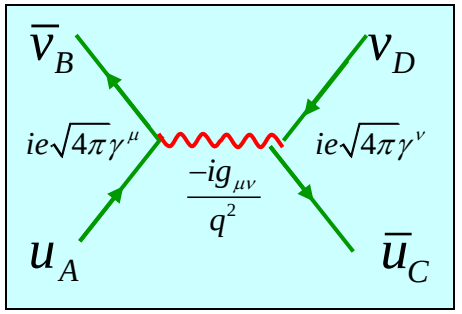


paar annihilatie/creatie $e^-e^+ \rightarrow \gamma\gamma$



Compton verstrooiing $e^-\gamma \rightarrow e^-\gamma$

Berekening $e^-e^+ \to \mu^-\mu^+$



Feynman regels geven

$$-i\mathcal{M} = [ie\sqrt{4\pi}\bar{v}_B\gamma^\mu u_A] \times \frac{-ig_{\mu\nu}}{(p_A+p_B)^2} \times [ie\sqrt{4\pi}\bar{u}_C\gamma^\nu v_D] \Rightarrow$$

$$\mathcal{M} = -4\pi e^2[\bar{v}_B\gamma^\mu u_A] \times \frac{1}{q^2} \times [\bar{u}_C\gamma_\mu v_D] \qquad q = p_A + p_B$$

De spin algebra geeft een spoor

$$\begin{cases} P_A \to k \\ P_B \to p \\ P_C \to k' \\ P_D \to p' \end{cases}$$

$$\begin{aligned}
|\bar{\mathcal{M}}|^2 &= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times \frac{1}{4}\sum[\bar{v}(p)\gamma^\mu u(k)] \times [\bar{u}(k')\gamma_\mu v(p')] \times [\bar{v}(p')\gamma_\nu u(k')] \times [\bar{u}(k)\gamma^\nu v(p)] \\
&= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times \frac{1}{4} \times (\mathrm{Tr}\,[(\not{p} - M)\gamma^\mu(\not{k} + m)\gamma^\nu]) \times (\mathrm{Tr}\,[(\not{p}' - M)\gamma_\nu(\not{k}' + m)\gamma_\mu]) \\
&= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times \frac{1}{4} \times (\mathrm{Tr}\,[\not{p}\gamma^\mu\ \not{k}\gamma^\nu] - 4mMg^{\mu\nu}) \times (\mathrm{Tr}\,[\not{p}'\gamma_\nu\ \not{k}'\gamma_\mu] - 4mMg_{\mu\nu}) \\
&= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times 4\,(p^\mu k^\nu + p^\nu k^\mu - [p\cdot k + mM]g^{\mu\nu}) \times \left(p'_\mu k'_\nu + p'_\nu k'_\mu - [p'\cdot k' + mM]g_{\mu\nu}\right) \\
&= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times 4\,(2(p\cdot p')(k\cdot k') + 2(p\cdot k')(p'\cdot k) + \ldots\ldots)
\end{aligned}$$

$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \to \mu^+ + \mu^-) = \frac{\pi\alpha^2}{8E^2}(1+\cos^2\theta)$$

Elektron-positron annihilatie

Muon productie

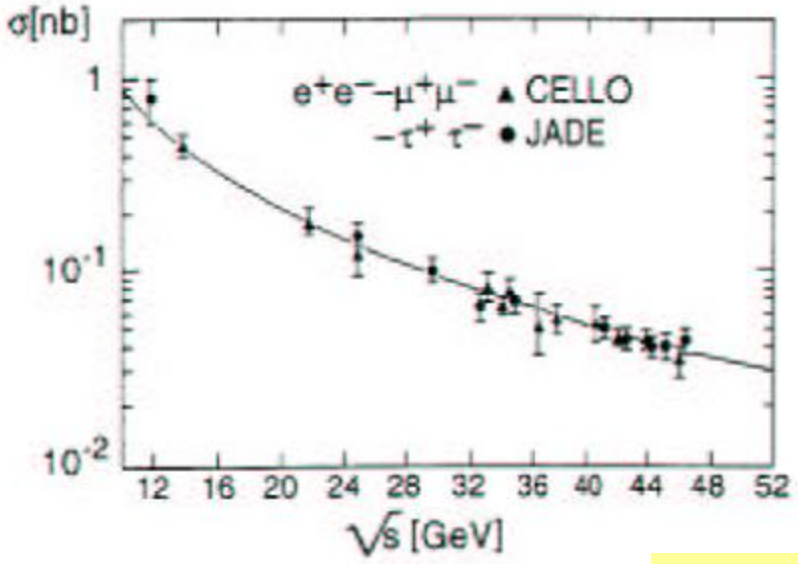
$e^+ + e^- \rightarrow \mu^+ + \mu^-$

$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \rightarrow \mu^+ + \mu^-) = \frac{\pi\alpha^2}{8E^2}(1 + \cos^2\theta)$$

spinfactor

integreer

$$\sigma(e^+ + e^- \rightarrow \mu^+ + \mu^-) = \sigma(e^+ + e^- \rightarrow \tau^+ + \tau^-) = \frac{21.7}{E^2} \text{ nanobarns}$$



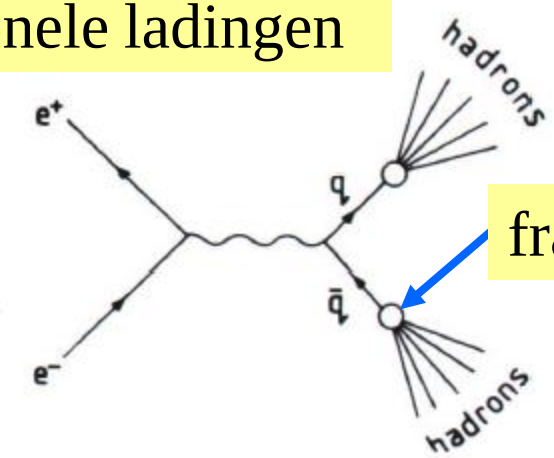
Elektron, muon en tau zijn identiek, afgezien van massa en levensduur

fractionele ladingen

kleur

Twee-jet productie

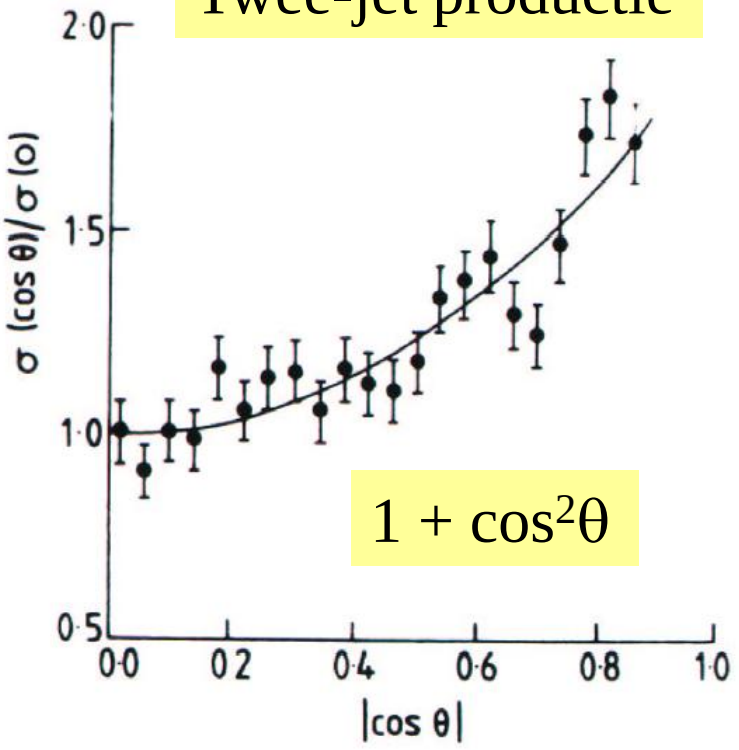
$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \rightarrow q + \bar{q}) = \frac{3\pi e_q^2 \alpha^2}{8E^2}(1 + \cos^2\theta)$$



fragmentatie

Elektron-positron annihilatie

Twee-jet productie



$1 + \cos^2\theta$

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{ hadronen})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \rightarrow q + \bar{q}) = \frac{3\pi e_a^2 \alpha^2}{8E^2}(1 + \cos^2\theta)$$

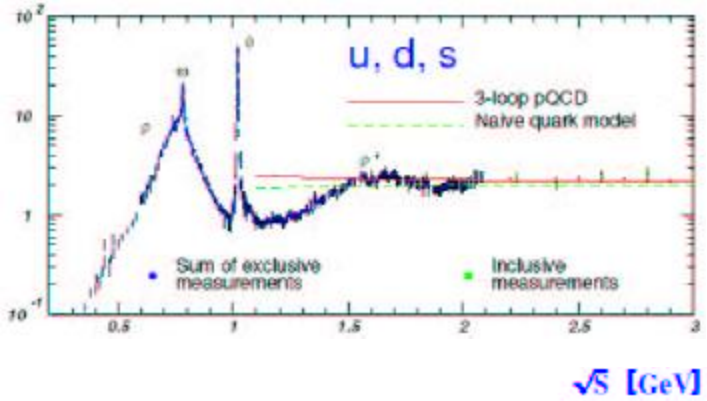
$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \rightarrow \mu^+ + \mu^-) = \frac{\pi\alpha^2}{8E^2}(1 + \cos^2\theta)$$

Voor $E < 3.5$ GeV

$$R = 3\left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = 2$$

Quarks hebben spin 1/2

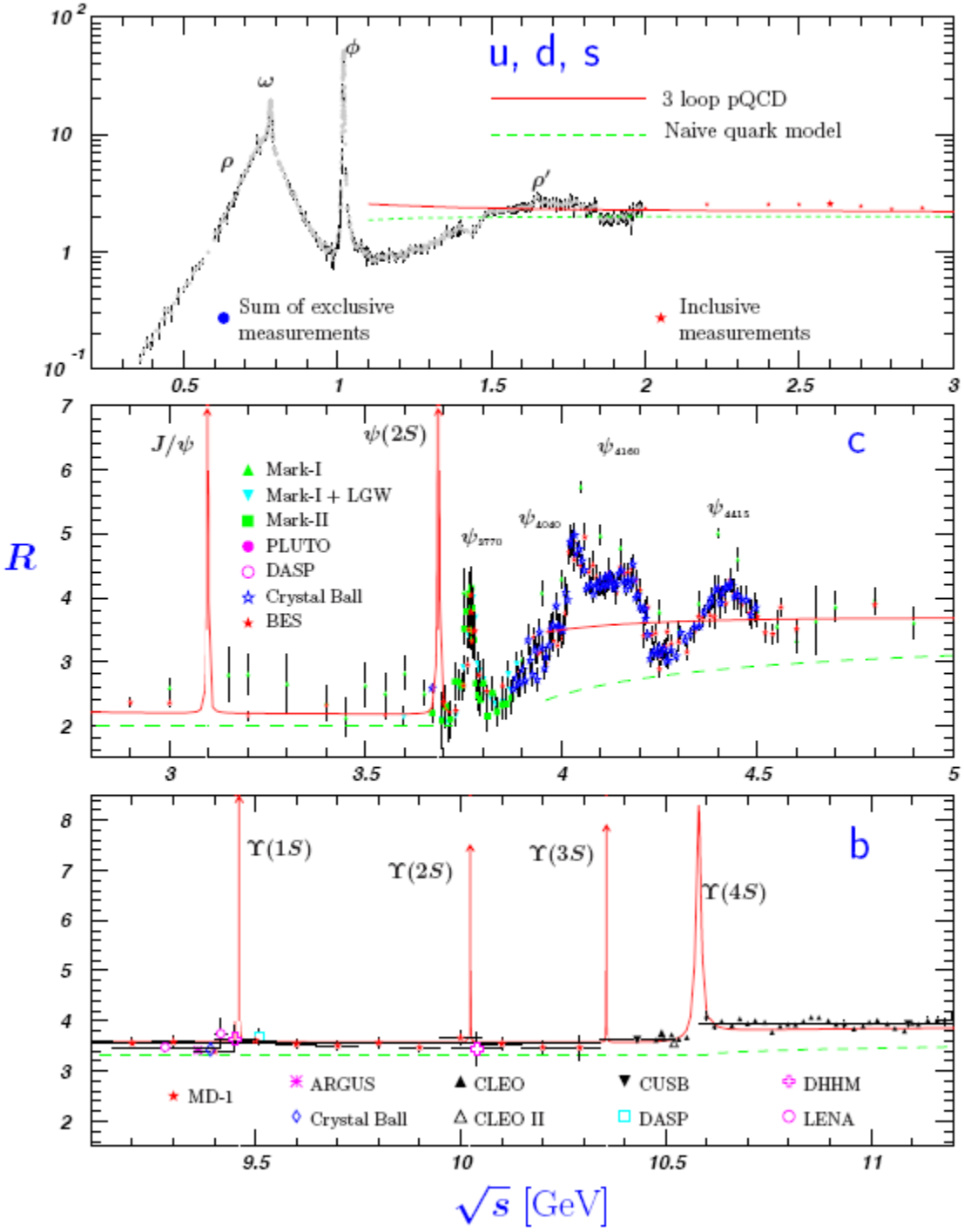
R



Quarks hebben kleur en fractionele lading

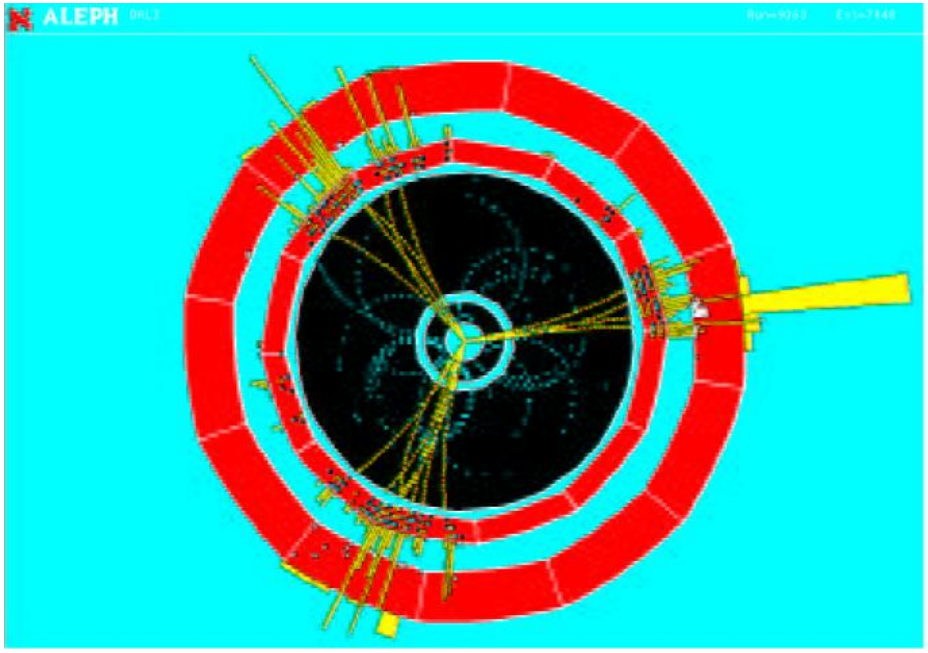
Elektron-positron annihilatie

$$R \equiv \frac{\sigma_{qq}}{\sigma_{\mu\mu}} = \frac{\sum_{quarks} e^+e^- \to q\bar{q}}{e^+e^- \to \mu^+\mu^-}$$



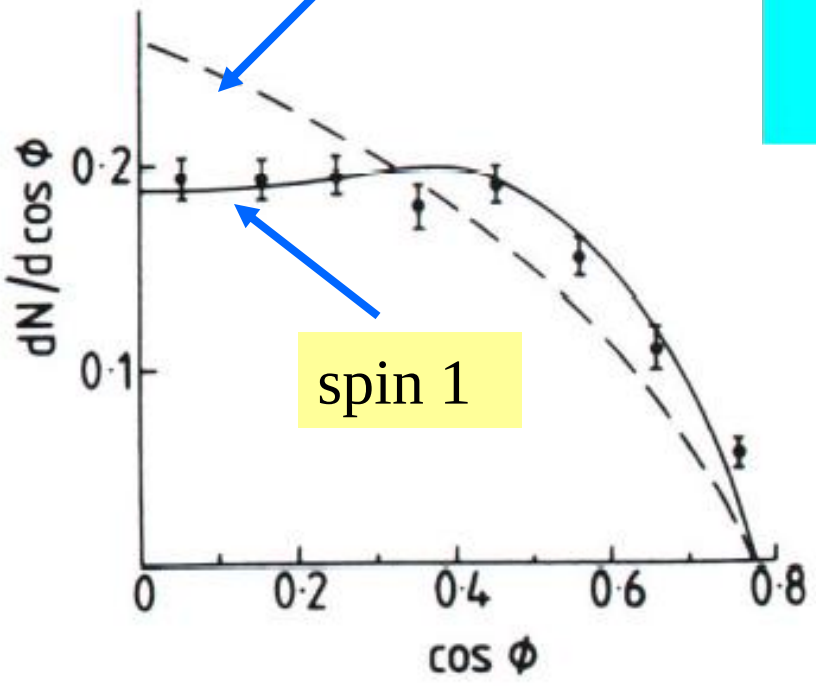
Elektron-positron annihilatie

Drie-jet productie



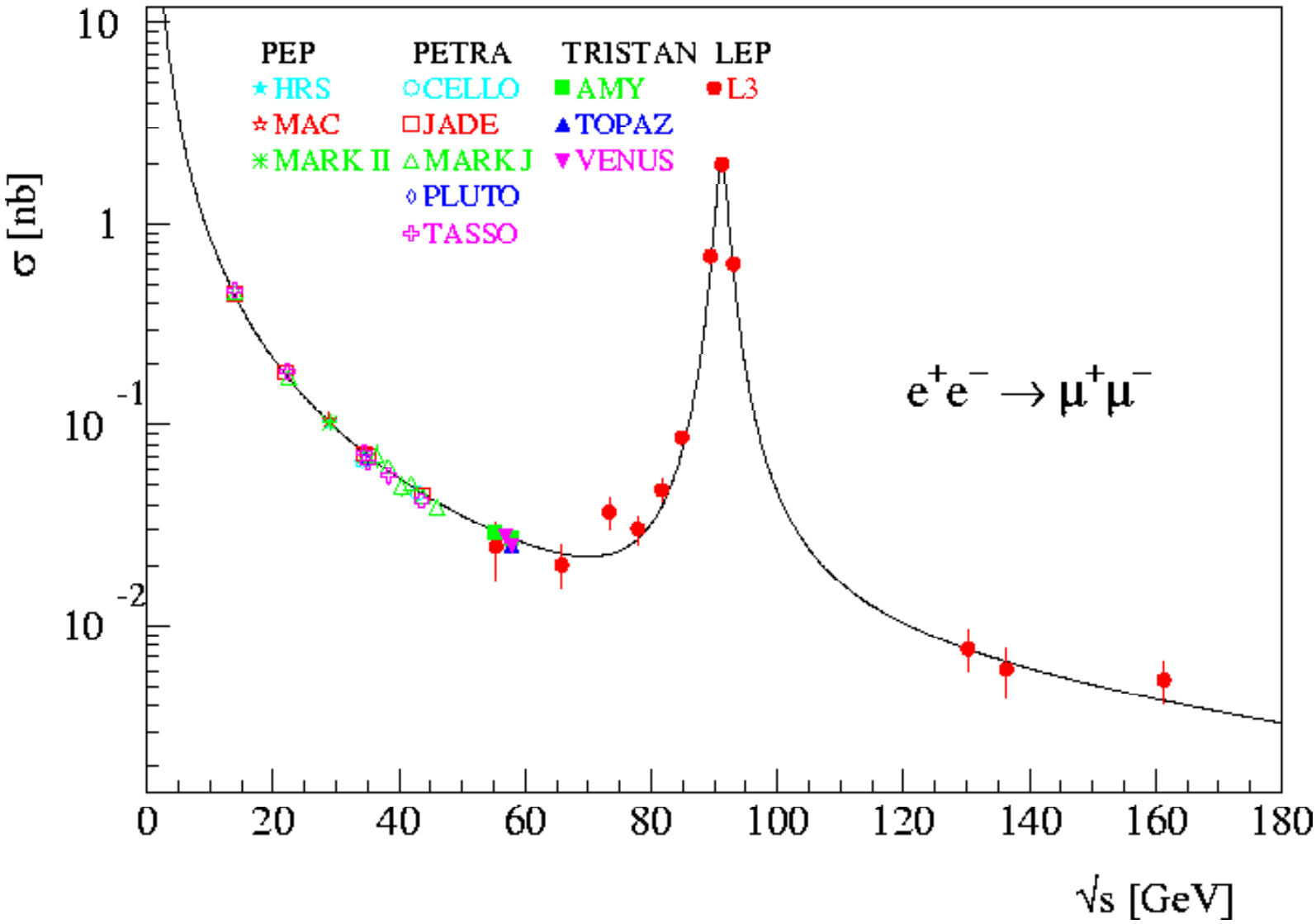
spin 0

spin 1

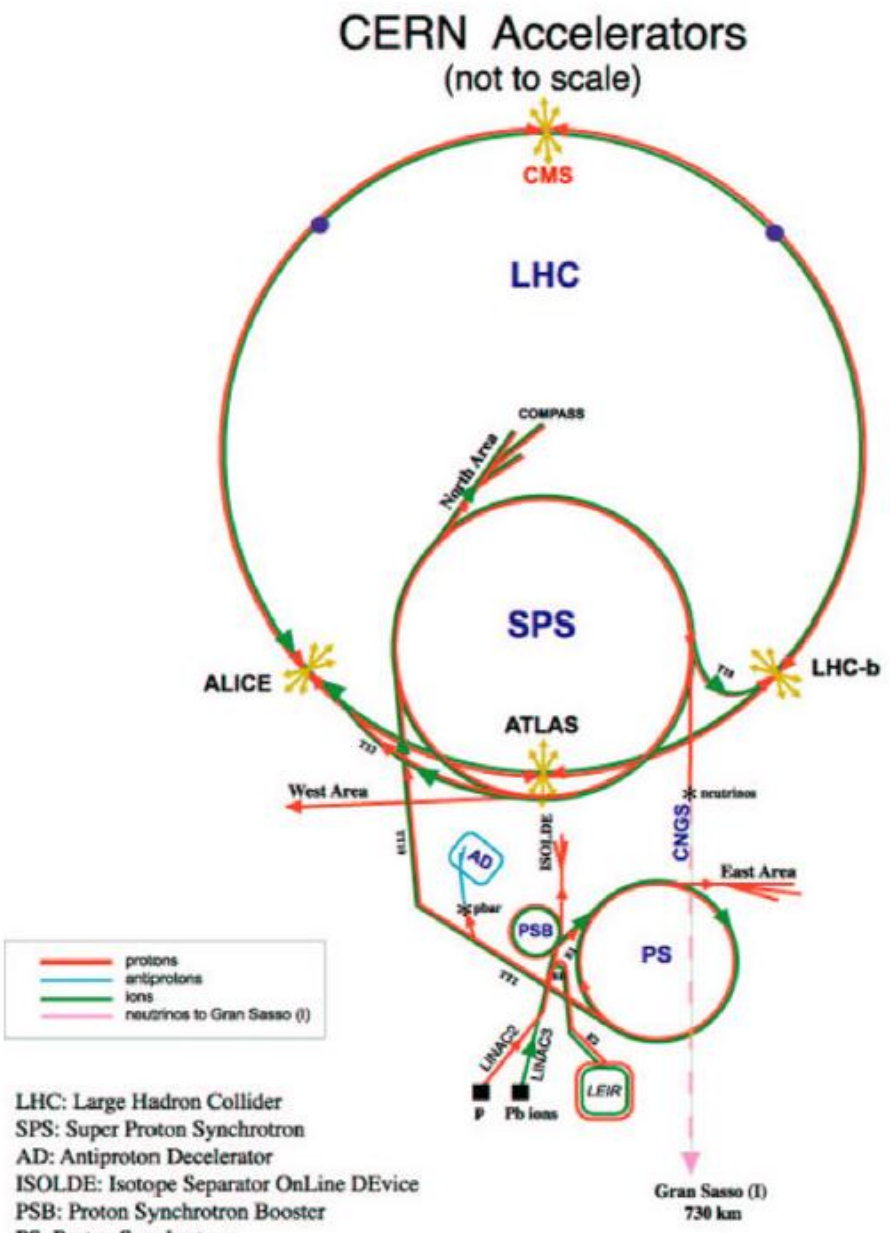


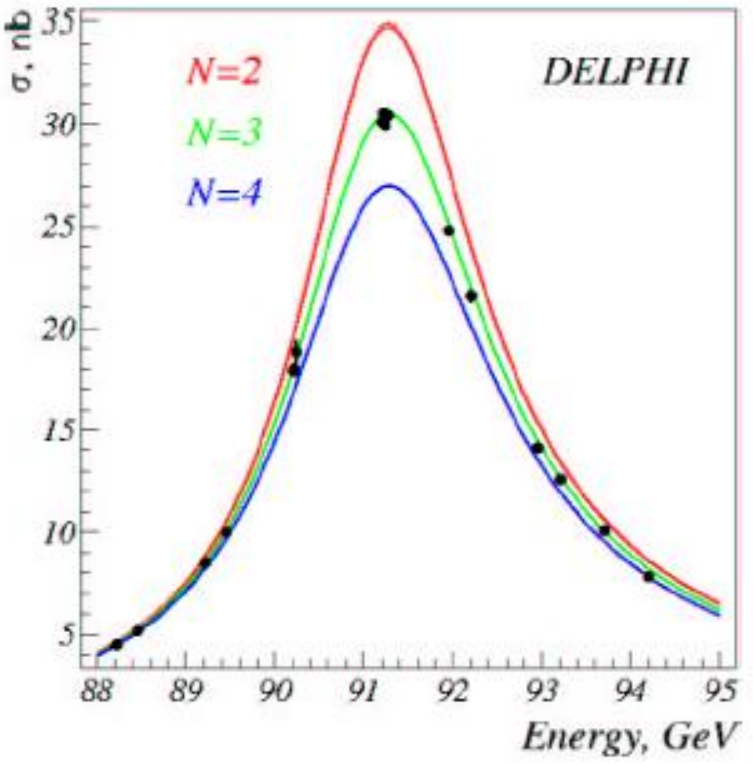
Gluonen hebben spin 1

Werkzame doorsnede $e^-e^+ \rightarrow \mu^-\mu^+$



Z^0-resonantie





e^+e^- verstrooiing

Drie generaties!