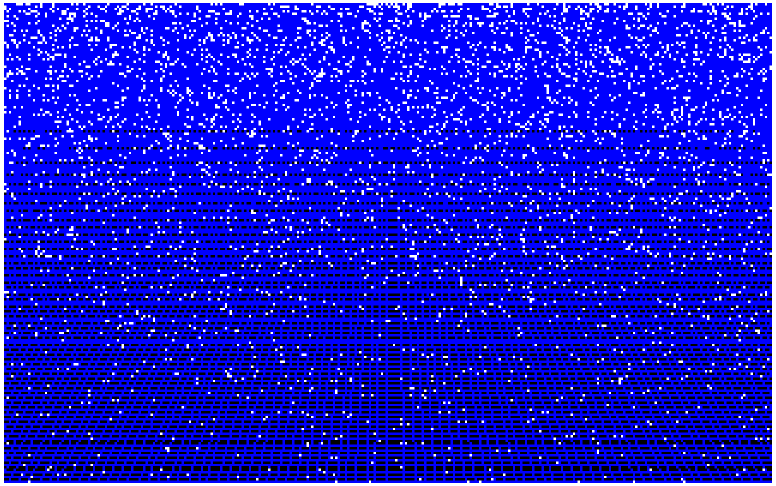
Elementaire Deeltjesfysica

FEW Cursus



Jo van den Brand

8 December, 2009

Structuur der Materie

Inhoud

- Inleiding
  - Deeltjes
  - Interacties
- Relativistische kinematica
  - Lorentz transformaties
  - Viervectoren
  - Energie en impuls
- Symmetrieën
  - Behoudwetten
  - Discrete symmetrieën
- Feynman berekeningen
  - Gouden regel
  - Feynman regels
  - Diagrammen
- Elektrodynamica
  - Dirac vergelijking
  - Werkzame doorsneden
- Quarks en hadronen
  - Elektron-quark interacties
  - Hadron productie in e^+e^-
- Zwakke wisselwerking
  - Muon verval
  - Unificatie

Diracvergelijking in impulsruimte

Diracvergelijking $\boxed{i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0}$

Vlakke-golf oplossingen van de vorm $\underbrace{\psi(\mathbf{r}, t) = ae^{-i/\hbar(Et-\mathbf{p}\cdot\mathbf{r})}u(E, \mathbf{p})}_{\boxed{\psi(x) = ae^{-(i/\hbar)x\cdot p}u(p)}}$

Normering $\rightarrow$ a

Identificeer $\boxed{p \equiv (E/c, \mathbf{p})}$ met energie en impuls

Diracvergelijking in impulsruimte $\boxed{(\gamma^\mu p_\mu - mc)u = 0}$

Als u hieraan voldoet, dan voldoet ψ aan de Diracvergelijking

Vlakke-golf oplossingen Diracvergelijking

We vinden

$$u^{(1)} = N\begin{pmatrix} 1 \\ 0 \\ \dfrac{c(p_z)}{E+mc^2} \\ \dfrac{c(p_x+ip_y)}{E+mc^2} \end{pmatrix}, \qquad u^{(2)} = N\begin{pmatrix} 0 \\ 1 \\ \dfrac{c(p_x-ip_y)}{E+mc^2} \\ \dfrac{c(-p_z)}{E+mc^2} \end{pmatrix},$$

$$E = \sqrt{m^2c^4 + \mathbf{p}^2c^2}$$

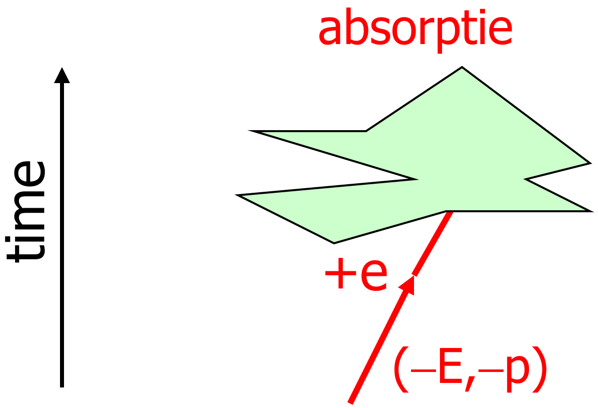
$$u^{(3)} = N\begin{pmatrix} \dfrac{c(p_z)}{E-mc^2} \\ \dfrac{c(p_x+ip_y)}{E-mc^2} \\ 1 \\ 0 \end{pmatrix}, \qquad u^{(4)} = N\begin{pmatrix} \dfrac{c(p_x-ip_y)}{E-mc^2} \\ \dfrac{c(-p_z)}{E-mc^2} \\ 0 \\ 1 \end{pmatrix},$$

$$E = -\sqrt{m^2c^4 + \mathbf{p}^2c^2}$$

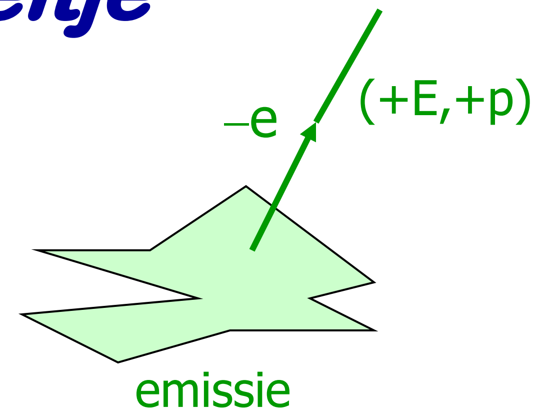
Normering

$$N = \sqrt{(|E| + mc^2)/c}$$

Deeltje ↔ antideeltje



$$\Delta(\text{systeem}) = \begin{cases} -\vec{p} \\ -E \\ +e \end{cases}$$



Er geldt $e^{-iEt} = e^{-i(-E)(-t)}$

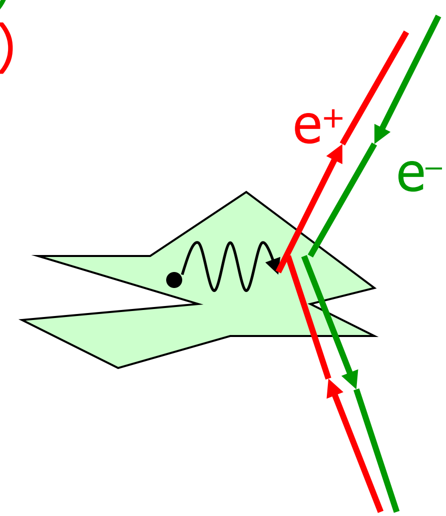
Voor een systeem is er geen verschil tussen:

Emissie: e^- met $p^\mu = (+E, +\vec{p})$

Absorptie: e^+ met $p^\mu = (-E, -\vec{p})$

In termen van de geladen stroom (dichtheid):

$$\begin{aligned} j^\mu_{+(E,\vec{p})}(e^+) &= +e \times |N|^2(+E, +\vec{p}) \\ &= -e \times |N|^2(-E, -\vec{p}) \\ &= j^\mu_{-(E,\vec{p})}(e^-) \end{aligned}$$



Ofwel: de volgende scenario's zijn identiek!

Vlakke-golf oplossingen Diracvergelijking

Interpretatie: ook antideeltjes hebben positieve energie

$$\psi(\mathbf{r}, t) = a e^{i/\hbar(Et - \mathbf{p}\cdot\mathbf{r})} u(-E, -\mathbf{p})$$

$$u^{(3)} = N \begin{pmatrix} \frac{c(p_z)}{E - mc^2} \\ \frac{c(p_x + ip_y)}{E - mc^2} \\ 1 \\ 0 \end{pmatrix}, \qquad u^{(4)} = N \begin{pmatrix} \frac{c(p_x - ip_y)}{E - mc^2} \\ \frac{c(-p_z)}{E - mc^2} \\ 0 \\ 1 \end{pmatrix},$$

met positieve energie

$$E = \sqrt{m^2c^4 + \mathbf{p}^2c^2}$$

$$v^{(1)}(E, \mathbf{p}) = u^{(4)}(-E, -\mathbf{p}) = N \begin{pmatrix} \frac{c(p_x - ip_y)}{E + mc^2} \\ \frac{c(-p_z)}{E + mc^2} \\ 0 \\ 1 \end{pmatrix}$$

$$v^{(2)}(E, \mathbf{p}) = -u^{(3)}(-E, -\mathbf{p}) = -N \begin{pmatrix} \frac{c(p_z)}{E + mc^2} \\ \frac{c(p_x + ip_y)}{E + mc^2} \\ 1 \\ 0 \end{pmatrix},$$

u's voldoen aan $(\gamma^\mu p_\mu - mc)u = 0$

v's aan $(\gamma^\mu p_\mu + mc)v = 0$

Lorentztransformaties van spinoren

Boost het systeem in de *x*-richting $\psi \longrightarrow \psi' = S\psi$

Golffunctie transformeert niet met een Lorentztransformatie, dus niet als een vector

Transformatie *S* is de 4 × 4 matrix

$$S = a_+ + a_-\gamma^0\gamma^1 = \begin{pmatrix} a_+ & a_-\sigma_1 \\ a_-\sigma_1 & a_+ \end{pmatrix}$$

Met $a_\pm = \pm\sqrt{\frac{1}{2}(\gamma \pm 1)}$ en $\gamma = 1/\sqrt{1 - v^2/c^2}$

We zoeken een scalaire grootheid (bij viervectoren bedachten we covariantie)

Definieer *adjoint* spinor $\bar{\psi} \equiv \psi^\dagger\gamma^0 = (\psi_1^* \; \psi_2^* \; -\psi_3^* \; -\psi_4^*)$

$$\boxed{\bar{\psi}\psi = \psi^\dagger\gamma^0\psi = |\psi_1|^2 + |\psi_2|^2 - |\psi_3|^2 - |\psi_4|^2}$$

Relativistisch invariant! $(\bar{\psi}\psi)' = (\psi')^\dagger\gamma^0\psi' = \psi^\dagger S^\dagger\gamma^0 S\psi = \psi^\dagger\gamma^0\psi = \bar{\psi}\psi$

Pariteit

Pariteit is gedefinieerd als ruimtelijke spiegeling door de oorsprong

$$x' \equiv -x; \quad y' \equiv -y; \quad z' \equiv -z; \quad t' \equiv t;$$

Beschouw Dirac spinor $\psi(x,y,z,t)$

Deze spinor is oplossing van de Diracvergelijking

$$i\hbar\gamma^1 \frac{\partial\psi}{\partial x} + i\hbar\gamma^2 \frac{\partial\psi}{\partial y} + i\hbar\gamma^3 \frac{\partial\psi}{\partial z} - mc\,\psi = \frac{i\hbar}{c}\gamma^0 \frac{\partial\psi}{\partial t}$$

Onder pariteit transformatie $\psi'(x',y',z',t') = \hat{P}\psi(x,y,z,t)$

Probeer $\hat{P} = \gamma^0$ en dus $\psi'(x',y',z',t') = \gamma^0\psi(x,y,z,t)$

$$\left(\gamma^0\right)^2 = 1 \quad \Rightarrow \quad \psi(x,y,z,t) = \gamma^0\psi'(x',y',z',t')$$

$$i\hbar\gamma^1\gamma^0 \frac{\partial\psi'}{\partial x} + i\hbar\gamma^2\gamma^0 \frac{\partial\psi'}{\partial y} + i\hbar\gamma^3\gamma^0 \frac{\partial\psi'}{\partial z} - mc\gamma^0\psi' = -\frac{i\hbar}{c}\gamma^0\gamma^0 \frac{\partial\psi'}{\partial t}$$

In termen van het primed systeem

$$-i\hbar\gamma^1\gamma^0 \frac{\partial\psi'}{\partial x'} - i\hbar\gamma^2\gamma^0 \frac{\partial\psi'}{\partial y'} - i\hbar\gamma^3\gamma^0 \frac{\partial\psi'}{\partial z'} - mc\gamma^0\psi' = -\frac{i\hbar}{c}\gamma^0\gamma^0 \frac{\partial\psi'}{\partial t'}$$

Intrinsieke pariteit

$$-i\hbar\gamma^1\gamma^0\frac{\partial\psi'}{\partial x'}-i\hbar\gamma^2\gamma^0\frac{\partial\psi'}{\partial y'}-i\hbar\gamma^3\gamma^0\frac{\partial\psi'}{\partial z'}-mc\gamma^0\psi'=-\frac{i\hbar}{c}\gamma^0\gamma^0\frac{\partial\psi'}{\partial t'}$$

Omdat γ^0 anticommuteert met $\gamma^1,\gamma^2,\gamma^3$

$$+i\hbar\gamma^0\gamma^1\frac{\partial\psi'}{\partial x'}+i\hbar\gamma^0\gamma^2\frac{\partial\psi'}{\partial y'}+i\hbar\gamma^0\gamma^3\frac{\partial\psi'}{\partial z'}-mc\gamma^0\psi'=-\frac{i\hbar}{c}\frac{\partial\psi'}{\partial t'}$$

Vermenigvuldigen met γ^0

$$\boxed{+i\hbar\gamma^1\frac{\partial\psi'}{\partial x'}+i\hbar\gamma^2\frac{\partial\psi'}{\partial y'}+i\hbar\gamma^3\frac{\partial\psi'}{\partial z'}-mc\psi'=-\frac{i\hbar}{c}\gamma^0\frac{\partial\psi'}{\partial t'}}$$

Dit is de Diracvergelijking in de nieuwe coordinaten

Onder een pariteit transformatie verandert de Diracvergelijking niet als de spinoren transformeren als $\psi \to \hat{P}\psi = \pm\gamma^0\psi$

Deeltje in rust $\psi = u_1 e^{-imt};\ \psi = u_2 e^{-imt};\ \psi = v_1 e^{+imt};\ \psi = v_2 e^{+imt}$

$$u_1 = N\begin{pmatrix}1\\0\\0\\0\end{pmatrix};\ u_2 = N\begin{pmatrix}0\\1\\0\\0\end{pmatrix};\ v_1 = N\begin{pmatrix}0\\0\\0\\1\end{pmatrix};\ v_2 = N\begin{pmatrix}0\\0\\1\\0\end{pmatrix};\quad \hat{P}u_1 = \pm\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \pm u_1$$

$$\boxed{\begin{array}{ll}\hat{P}u_1 = \pm u_1 & \hat{P}v_1 = \mp v_1\\ \hat{P}u_2 = \pm u_2 & \hat{P}v_2 = \mp v_2\end{array}}$$

Deeltje en antideeltje in rust hebben tegengestelde intrinsieke pariteit

Spinoren: hoe transformeren ze?

Ergeldt

$$(\bar{\psi}\psi)' = (\psi')^\dagger\gamma^0\psi' = \psi^\dagger(\gamma^0)^\dagger\gamma^0\gamma^0\psi = \psi^\dagger\gamma^0\psi = \bar{\psi}\psi$$

Invariant onder pariteit transformatie: een echte scalar

Definieer

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Gedrag onder pariteit

$$(\bar{\psi}\gamma^5\psi)' = (\psi')^\dagger\gamma^0\gamma^5\psi' = \psi^\dagger\gamma^0\gamma^0\gamma^5\gamma^0\psi = \psi^\dagger\gamma^5\gamma^0\psi$$

$$\gamma^5\gamma^0 = i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^0 = (-1)^3\gamma^0(i\gamma^0\gamma^1\gamma^2\gamma^3) = -\gamma^0\gamma^5$$

$$\Rightarrow \quad (\bar{\psi}\gamma^5\psi)' = -\psi^\dagger\gamma^0\gamma^5\psi = -(\bar{\psi}\gamma^5\psi)$$

pseudoscalar

Transformatiegedrag van lineaire combinaties $\psi_i^*\psi_j$

(1)	$\bar{\psi}\psi$ = scalar	(one component)
(2)	$\bar{\psi}\gamma^5\psi$ = pseudoscalar	(one component)
(3)	$\bar{\psi}\gamma^\mu\psi$ = vector	(four components)
(4)	$\bar{\psi}\gamma^\mu\gamma^5\psi$ = pseudovector	(four components)
(5)	$\bar{\psi}\sigma^{\mu\nu}\psi$ = antisymmetric tensor	(six components)

$$\sigma^{\mu\nu} \equiv \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Basis van alle 4 × 4 matrices $1,\ \gamma^5,\ \gamma^\mu,\ \gamma^\mu\gamma^5,$ en $\sigma^{\mu\nu}$

Ladingsconjugatie

Ladingsconjugatie operator *C* voert spinor ψ over in de ladingsgeconjugeerde spinor ψ_C

Er geldt $\boxed{\psi_c = i\gamma^2\psi^*}$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

Als we deze operator laten werken op

$$u^{(1)} = N\begin{pmatrix} 1 \\ 0 \\ \dfrac{c(p_z)}{E+mc^2} \\ \dfrac{c(p_x+ip_y)}{E+mc^2} \end{pmatrix}, \qquad u^{(2)} = N\begin{pmatrix} 0 \\ 1 \\ \dfrac{c(p_x-ip_y)}{E+mc^2} \\ \dfrac{c(-p_z)}{E+mc^2} \end{pmatrix},$$

vinden we de geconjugeerde toestanden

$$v^{(1)}(E,\mathbf{p}) = u^{(4)}(-E,-\mathbf{p}) = N\begin{pmatrix} \dfrac{c(p_x-ip_y)}{E+mc^2} \\ \dfrac{c(-p_z)}{E+mc^2} \\ 0 \\ 1 \end{pmatrix}$$

$$v^{(2)}(E,\mathbf{p}) = -u^{(3)}(-E,-\mathbf{p}) = -N\begin{pmatrix} \dfrac{c(p_z)}{E+mc^2} \\ \dfrac{c(p_x+ip_y)}{E+mc^2} \\ 1 \\ 0 \end{pmatrix},$$

Spinoren: normalisatie, orthogonaliteit en compleetheid

Normalisatie

$$\begin{cases} \bar{u}u = u^\dagger \gamma^0 u = u^\dagger \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} u = +2m \\ \bar{v}v = v^\dagger \gamma^0 v = v^\dagger \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} v = -2m \end{cases}$$

Orthogonaliteit

$$\begin{cases} \bar{u}^{(1)} u^{(2)} = u^{(1)\dagger} \gamma^0 u^{(2)} = 0 \\ \bar{v}^{(1)} v^{(2)} = v^{(1)\dagger} \gamma^0 v^{(2)} = 0 \end{cases}$$

$$\not{a} \equiv a^\mu \gamma_\mu$$

Compleetheid

$$\begin{cases} \sum_{s=1,2} u^{(s)} \bar{u}^{(s)} = u^{(1)} \bar{u}^{(1)} + u^{(2)} \bar{u}^{(2)} = \gamma^\mu p_\mu + m \equiv \not{p} + m \\ \sum_{s=1,2} v^{(s)} \bar{v}^{(s)} = v^{(1)} \bar{v}^{(1)} + v^{(2)} \bar{v}^{(2)} = \gamma^\mu p_\mu - m \equiv \not{p} - m \end{cases}$$

Ijkinvariantie

Maxwellvergelijkigen en Lorentzinvariantie

Maxwell
$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \qquad \vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} = 0 \qquad \vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = \frac{4\pi}{c}\vec{j}$$

Stel $h/2\pi=c=1$ en introduceer potentiaal $A^\mu=(V,A)$ en stroom $j^\mu=(\rho,j)$:

$$\begin{cases} \vec{B} = \vec{\nabla}\times\vec{A} \longrightarrow \vec{\nabla}\cdot\vec{B} = 0 \\ \boxed{\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}V} \longrightarrow \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{\nabla}\times\vec{A}}{\partial t} = -\frac{\partial\vec{B}}{\partial t} \end{cases}$$

Er geldt $\partial_\mu\partial^\mu A^\nu - \partial^\nu\partial_\mu A^\mu = 4\pi j^\nu$

Compacter met $F^{\mu\nu}\equiv\partial^\mu A^\nu-\partial^\nu A^\mu$

$$F^{\mu\nu} = \begin{pmatrix} 0 & \boxed{-E_x} & -E_y & -E_z \\ +E_x & 0 & -B_z & +B_y \\ +E_y & +B_z & 0 & -B_x \\ +E_z & -B_y & +B_x & 0 \end{pmatrix}$$

$$\boxed{\partial_\mu F^{\mu\nu} = 4\pi j^\nu \quad \begin{cases} \nu=0: & \vec{\nabla}\cdot\vec{E} = 4\pi j^0 = 4\pi\rho \\ \nu=k: & \vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} = 4\pi\vec{j} \end{cases}}$$

Ijkinvariantie (gauge invariance)

Vrijheid in de keuze van ijkveld A^μ (gauge field)

$$\begin{cases} A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu \lambda = (V + \frac{\partial \lambda}{\partial t}, -\vec{A} + \vec{\nabla}\lambda) \\ A^\mu \longrightarrow A'^\mu = A^\mu + \partial^\mu \lambda = (V + \frac{\partial \lambda}{\partial t}, +\vec{A} - \vec{\nabla}\lambda) \end{cases} \qquad \forall \lambda$$

Omdat
$$\begin{cases} \vec{B} \rightarrow \vec{B}' = \vec{\nabla} \times \vec{A}' = \vec{\nabla} \times \vec{A} - \vec{\nabla} \times \vec{\nabla}\lambda = \vec{\nabla} \times \vec{A} = \vec{B} \\ \vec{E} \rightarrow \vec{E}' = -\vec{\nabla}V' - \frac{\partial \vec{A}'}{\partial t} = -\vec{\nabla}(V + \frac{\partial \lambda}{\partial t}) - \frac{\partial}{\partial t}(\vec{A} - \vec{\nabla}\lambda) = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t} = \vec{E} \end{cases}$$

Gebruik vrijheid om A^μ te vereenvoudigen *(covariant)*

Lorentz conditie $\quad \partial_\mu A^\mu = 0 \quad \longrightarrow \quad \partial_\nu \partial^\nu A^\mu \equiv \Box A^\mu = 4\pi j^\mu$

A^μ is nog steeds niet uniek; omdat

$$A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu \lambda \qquad \Box \lambda \equiv \partial_\mu \partial^\mu \lambda = 0$$

We kunnen bijvoorbeeld de oplossing kiezen *(niet covariant)*

$$A^0 = 0 \quad \text{oftewel:} \quad \vec{\nabla} \cdot \vec{A} = 0$$

Coulomb conditie

Foton golffunctie

In vacuüm $j^\mu=0$ en daarom $\quad 0 = \partial_\nu \partial^\nu A^\mu \equiv \Box A^\mu$

Met als oplossingen (vlakke golven)

$$A^\mu(x) = N\epsilon^\mu e^{-ip\cdot x} \quad \text{met:} \quad p^2 = p_\mu p^\mu = 0 \quad \Rightarrow \quad E = |\vec{p}|$$

Gauge keuze: Lorentz conditie $\quad \partial_\mu A^\mu = 0 \ \Rightarrow \ \epsilon^\mu p_\mu = 0$

Coulomb conditie $\quad \begin{cases} A^0 = 0 \\ \vec{\nabla}\cdot\vec{A} = 0 \end{cases} \Rightarrow \begin{cases} \epsilon^0 = 0 \\ \vec{\epsilon}\cdot\vec{p} = 0 \end{cases}$

In plaats van 4 vrijheidsgraden (ε^μ) slechts 2

$$\vec{\varepsilon}_+ \to \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} \\ i/\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} (-\cos\varphi + i\sin\varphi)/\sqrt{2} \\ (-\sin\varphi + i\cos\varphi)/\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} e^{-i\varphi}\left(-1/\sqrt{2}\right) \\ e^{-i\varphi}\left(i/\sqrt{2}\right) \\ 0 \end{pmatrix}$$

transversaal

$$\begin{cases} \vec{\epsilon}_1 = (1,0,0) \\ \vec{\epsilon}_2 = (0,1,0) \end{cases}$$

circulair

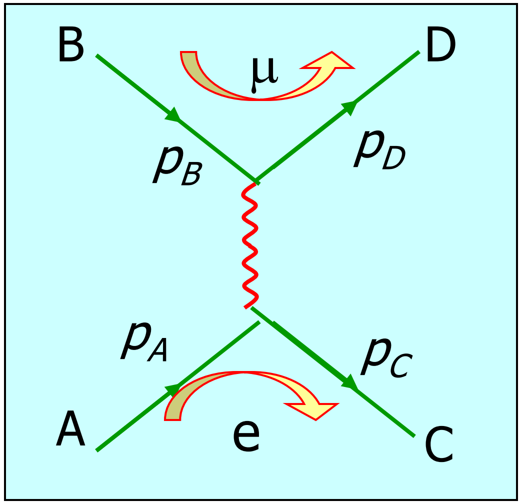
$$\begin{cases} \vec{\epsilon}_+ = \dfrac{-\vec{\epsilon}_1 + i\vec{\epsilon}_2}{\sqrt{2}} = \dfrac{(-1,+i,0)}{\sqrt{2}} \\ \vec{\epsilon}_- = \dfrac{+\vec{\epsilon}_1 - i\vec{\epsilon}_2}{\sqrt{2}} = \dfrac{(+1,-i,0)}{\sqrt{2}} \end{cases}$$

rotatie φ rond de *z*-axis: $\quad \vec{\epsilon}_\pm \to e^{\mp i\phi} \times \vec{\epsilon}_\pm \quad \Rightarrow \begin{cases} \vec{\epsilon}_+ & \text{Voor} \ m = +1 \\ \vec{\epsilon}_- & \text{Voor} \ m = -1 \end{cases}$

Interactie van spin-½ deeltjes met het fotonveld

Quantumelektrodynamica

Relatie tussen stroom en vectorveld



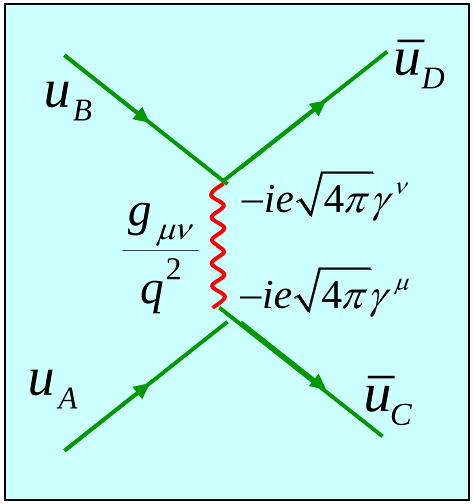
Beschouw elektron-muon verstrooiing

$$AC \text{ overgangsstroom} \qquad j_\mu^{CA} = -e\bar{u}_C\gamma_\mu u_A \times e^{i(p_C-p_A)\cdot x}$$
$$BD \text{ overgangsstroom} \qquad j_\mu^{DB} = -e\bar{u}_D\gamma_\mu u_B \times e^{i(p_D-p_B)\cdot x}$$

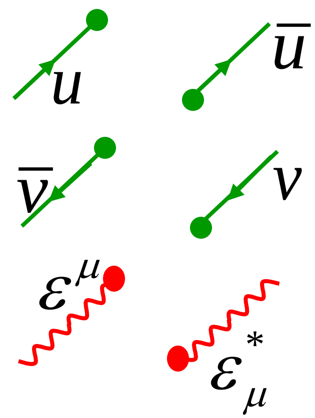
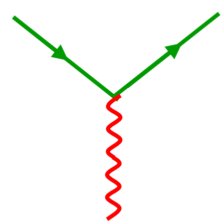
$$A^\mu \text{ t.g.v. } j_{DB}^\mu \qquad \Box A^\mu = 4\pi j_{DB}^\mu \Rightarrow A^\mu = \frac{-4\pi}{q^2} \times j_{DB}^\mu$$

En de overgangsamplitude wordt dus ($q=p_A-p_C=p_D-p_B$)

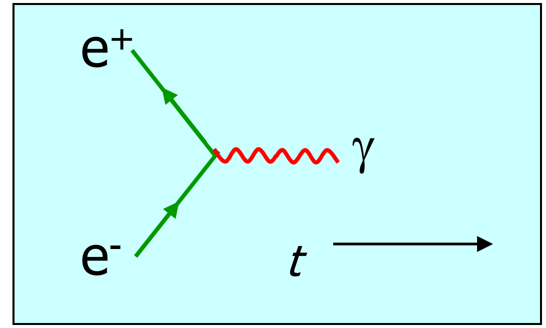
$$\begin{aligned}
T_{fi} &= -i\int j_\mu^{CA} \times \frac{-4\pi}{q^2} j_{DB}^\mu d^4x \\
&= -i\int [-e\bar{u}_C\gamma_\mu u_A \times e^{i(p_C-p_A)\cdot x}] \times \frac{-4\pi}{q^2} \times [-e\bar{u}_D\gamma^\mu u_B \times e^{i(p_D-p_B)\cdot x}] d^4x \\
&= -i[-e\bar{u}_C\gamma_\mu u_A] \times \frac{-4\pi}{q^2} \times [-e\bar{u}_D\gamma^\mu u_B] \times (2\pi)^4\delta^4(p_A+p_B-p_C-p_D)
\end{aligned}$$



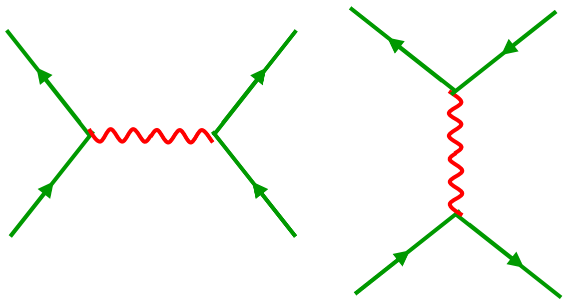
Feynman regels voor QED (S=1/2)

Externe lijnen	Deeltje Anti-deeltje Foton		inkomend: u uitgaand: $\bar{u}$ inkomend: $\bar{v}$ uitgaand: v inkomend: ϵ_μ uitgaand: ϵ'^{*}_{μ}
Vertices			$-i(\sqrt{4\pi}e)\gamma_\mu$
Propagatoren	Deeltje		$\frac{i\left(\gamma_{\mu}q^{\mu}+m\right)}{q^2-m^2}$
	Foton		$\frac{-ig_{\mu\nu}}{q^2}$

QED

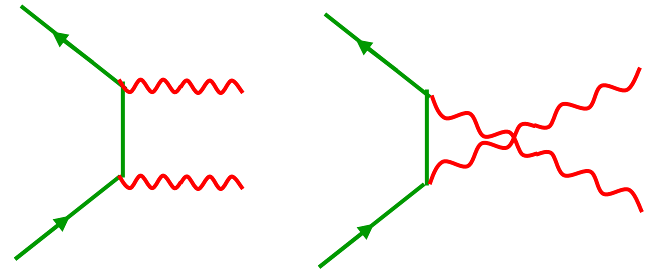


Fundamentele interacties:



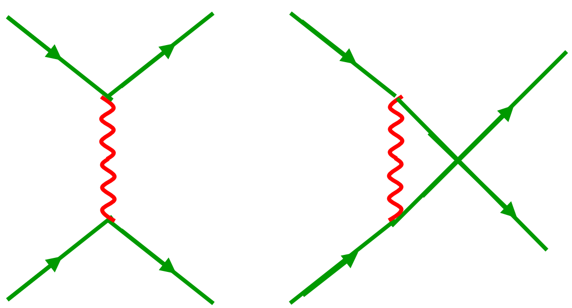
Bhabha verstrooiing

$e^+e^- \rightarrow e^+e^-$



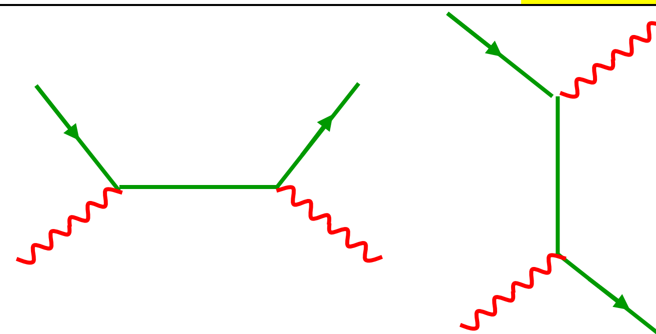
paar annihilatie/creatie

$e^-e^+ \rightarrow \gamma\gamma$



Möller verstrooiing

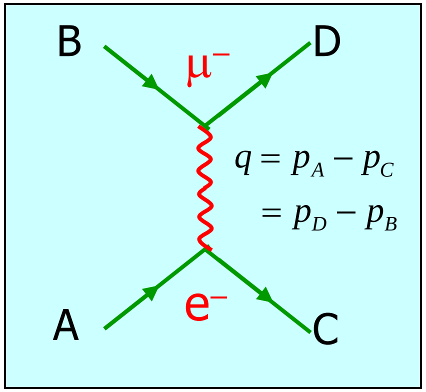
$e^-e^- \rightarrow e^-e^-$



Compton verstrooiing

$e^-\gamma \rightarrow e^-\gamma$

Relativistische spin



Relativistische spin $\qquad e^- \mu^- \rightarrow e^- \mu^- \quad$ (1 diagram)

$$-i\mathcal{M} = [ie\sqrt{4\pi}\bar{u}_C\gamma^\mu u_A] \times \frac{-ig_{\mu\nu}}{(p_A-p_C)^2} \times [ie\sqrt{4\pi}\bar{u}_D\gamma^\nu u_B] \Rightarrow$$

$$\mathcal{M} = -4\pi e^2[\bar{u}_C\gamma^\mu u_A] \times \frac{1}{q^2} \times [\bar{u}_D\gamma_\mu u_B] \qquad q = p_A - p_C$$

Voor $|M|^2$ volgt dus

$$|\mathcal{M}|^2 = \left(\frac{4\pi e^2}{q^2}\right)^2 \times [\bar{u}_C\gamma^\mu u_A][\bar{u}_C\gamma^\nu u_A]^* \times [\bar{u}_D\gamma_\mu u_B][\bar{u}_D\gamma_\nu u_B]^* \Rightarrow$$

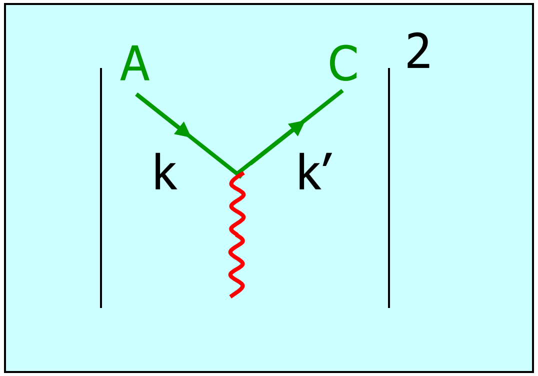
$$|\bar{\mathcal{M}}|^2 = \left(\frac{4\pi e^2}{q^2}\right)^2 \times \frac{1}{2}\sum_{spin}[\bar{u}_C\gamma^\mu u_A][\bar{u}_C\gamma^\nu u_A]^* \times \frac{1}{2}\sum_{spin}[\bar{u}_D\gamma_\mu u_B][\bar{u}_D\gamma_\nu u_B]^*$$

$$\equiv \left(\frac{4\pi e^2}{q^2}\right)^2 \times L^{\mu\nu}_{elektron} \times L^{muon}_{\mu\nu}$$

lepton tensor

$$\boxed{\begin{aligned} L^{\mu\nu}_{elektron} &= \frac{1}{2}\sum_{spin}[\bar{u}_C\gamma^\mu u_A][\bar{u}_C\gamma^\nu u_A]^* = \frac{1}{2}\sum_{spin}[\bar{u}_C\gamma^\mu u_A][\bar{u}_C\gamma^\nu u_A]^\dagger \\ &= \frac{1}{2}\sum_{spin}[\bar{u}_C\gamma^\mu u_A][u_A^\dagger\gamma^{\nu\dagger}\bar{u}_C^\dagger] = \frac{1}{2}\sum_{spin}[\bar{u}_C\gamma^\mu u_A][\bar{u}_A\gamma^0\gamma^{\nu\dagger}\gamma^0 u_C] \\ &= \frac{1}{2}\sum_{spin}[\bar{u}_C\gamma^\mu u_A][\bar{u}_A\gamma^\nu u_C] \end{aligned}}$$

De elektron lepton tensor



$$
\begin{aligned}
L^{\mu\nu}_{elektron} &= \frac{1}{2}\sum_{spin}[\bar{u}_C\gamma^\mu u_A][\bar{u}_A\gamma^\nu u_C] \rightarrow \frac{1}{2}\sum_{spin}[\bar{u}(k')\gamma^\mu u(k)][\bar{u}(k)\gamma^\nu u(k')] \\
&= \frac{1}{2}\sum_{spin}[\bar{u}(k')_{\alpha'}\gamma^\mu_{\alpha'\alpha}u(k)_\alpha][\bar{u}(k)_\beta\gamma^\nu_{\beta\beta'}u(k')_{\beta'}] \\
&= \frac{1}{2}\left[\underbrace{\sum_s u(k)^{(s)}_\alpha\bar{u}(k)^{(s)}_\beta}_{(\not{k}+m)_{\alpha\beta}}\right]\times\left[\underbrace{\sum_{s'} u(k')^{(s')}_{\beta'}\bar{u}(k')^{(s')}_{\alpha'}}_{(\not{k}'+m)_{\beta'\alpha'}}\right]\times\gamma^\mu_{\alpha'\alpha}\gamma^\nu_{\beta\beta'}
\end{aligned}
$$

$$\not{k} \equiv k^\mu\gamma_\mu$$

$$
\begin{aligned}
&= \frac{1}{2}\times(\not{k}'+m)_{\beta'\alpha'}\times\gamma^\mu_{\alpha'\alpha}(\not{k}+m)_{\alpha\beta}\times\gamma^\nu_{\beta\beta'} \\
&= \frac{1}{2}\mathrm{Tr}\,[(\not{k}'+m)\gamma^\mu(\not{k}+m)\gamma^\nu]
\end{aligned}
$$

Hoewel je van deze uitdrukking op het eerste gezicht niet vrolijk wordt, geldt wel

1) Geen spinoren meer: via compleetheid relaties
2) De spoorberekening is rechtoe-rechtaan: m.b.v. enkele regels

\`Casimir's trick'

Sporen met γ-matrices

Gebruik altijd:

$$\boxed{\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}}$$

producten van γ matrices

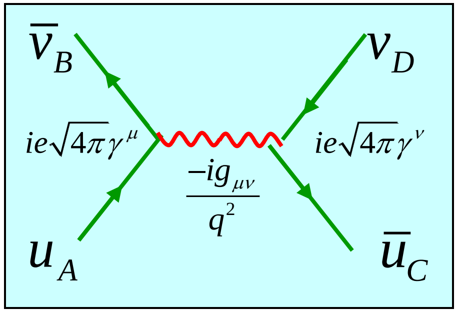
1. $\gamma^\mu\gamma_\mu = 4$
2. $\gamma^\mu \not{a}\gamma_\mu = \gamma^\mu\gamma^\alpha a_\alpha\gamma_\mu = -4\gamma^\alpha a_\alpha + 2g^{\mu\alpha}a_\alpha\gamma_\mu = -2 \not{a}$
3. $\gamma^\mu \not{a} \not{b}\gamma_\mu = 4a\cdot b$
4. $\gamma^\mu \not{a} \not{b} \not{c}\gamma_\mu = -2 \not{c} \not{b} \not{a}$

sporen van producten van γ matrices

1. $\mathrm{Tr}\ 1 = 4$
2. Tr van *oneven* aantal γ matrices identiek 0
3. $\mathrm{Tr}\ \not{a} \not{b} = \mathrm{Tr}\ \gamma^\alpha\gamma^\beta a_\alpha b_\beta = -\mathrm{Tr}\ \not{a} \not{b} + \mathrm{Tr}\ 2a\cdot b \Rightarrow$
   $\mathrm{Tr}\ \not{a} \not{b} = \mathrm{Tr}\ a\cdot b = 4a\cdot b$
4. $\mathrm{Tr}\ \not{a} \not{b} \not{c} \not{d} = 4[(a\cdot b)(c\cdot d) - (a\cdot c)(b\cdot d) + (a\cdot d)(b\cdot c)]$
5. $\mathrm{Tr}\ \gamma^5 = \mathrm{Tr}\ i\gamma^0\gamma^1\gamma^2\gamma^3 = 0$
6. $\mathrm{Tr}\ \gamma^5 \not{a} \not{b} = 0$
7. $\mathrm{Tr}\ \gamma^5 \not{a} \not{b} \not{c} \not{d} = 4i\epsilon_{\alpha\beta\gamma\delta} \times a^\alpha b^\beta c^\gamma d^\delta$

$\left.\begin{array}{l}6.\\7.\end{array}\right\}$ Zwakke wisselwerking

Berekening $e^-e^+ \rightarrow \mu^-\mu^+$



Feynman regels geven

$$-i\mathcal{M} = [ie\sqrt{4\pi}\bar{v}_B\gamma^\mu u_A] \times \frac{-ig_{\mu\nu}}{(p_A+p_B)^2} \times [ie\sqrt{4\pi}\bar{u}_C\gamma^\nu v_D] \Rightarrow$$

$$\mathcal{M} = -4\pi e^2[\bar{v}_B\gamma^\mu u_A] \times \frac{1}{q^2} \times [\bar{u}_C\gamma_\mu v_D] \qquad q = p_A + p_B$$

De spin algebra geeft een spoor

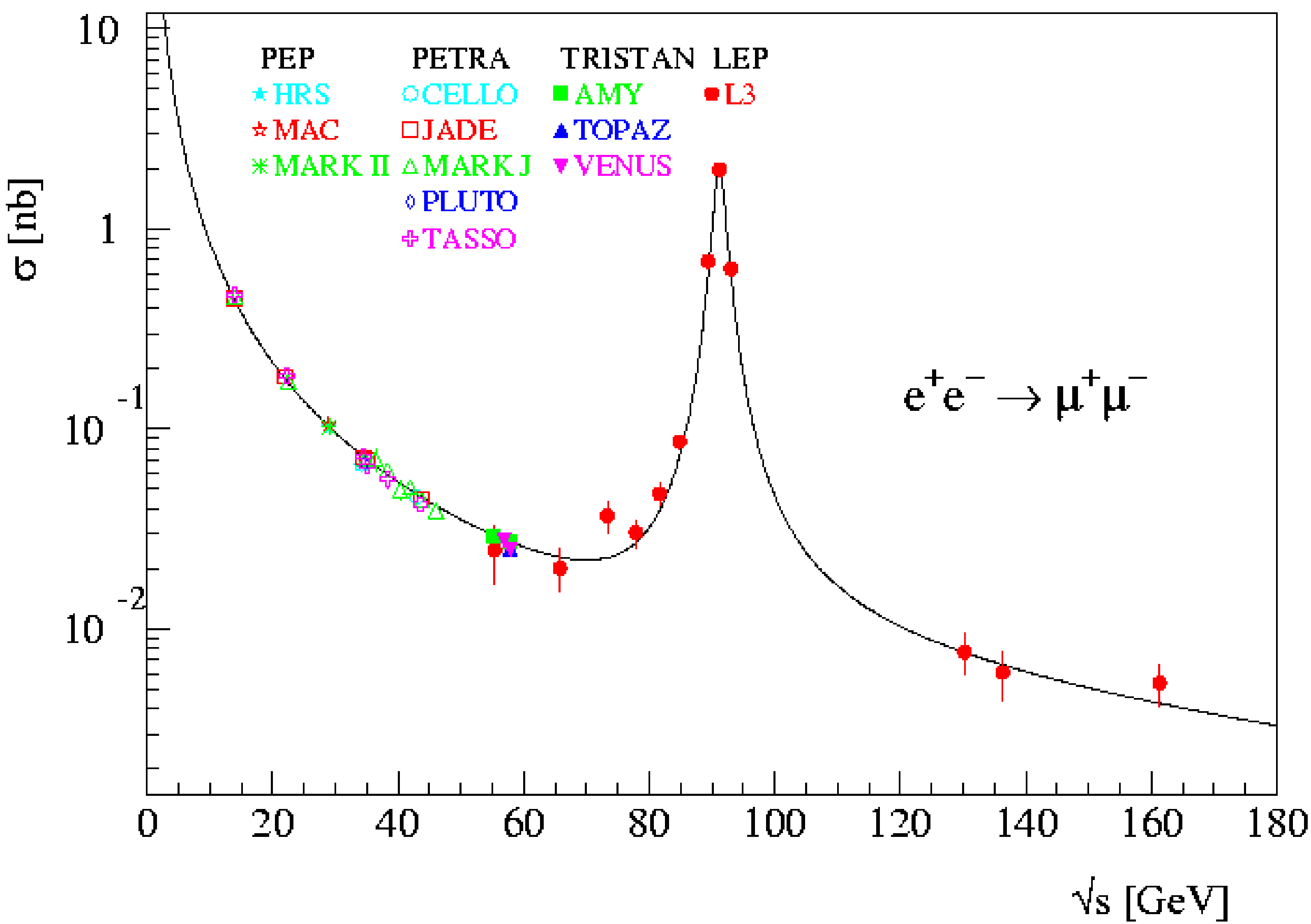
$$\begin{cases} P_A \to k \\ P_B \to p \\ P_C \to k' \\ P_D \to p' \end{cases}$$

$$\begin{aligned}
|\bar{\mathcal{M}}|^2 &= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times \frac{1}{4}\sum[\bar{v}(p)\gamma^\mu u(k)] \times [\bar{u}(k')\gamma_\mu v(p')] \times [\bar{v}(p')\gamma_\nu u(k')] \times [\bar{u}(k)\gamma^\nu v(p)] \\
&= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times \frac{1}{4} \times (\mathrm{Tr}\,[(\not{p}-M)\gamma^\mu(\not{k}+m)\gamma^\nu]) \times (\mathrm{Tr}\,[(\not{p}'-M)\gamma_\nu(\not{k}'+m)\gamma_\mu]) \\
&= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times \frac{1}{4} \times (\mathrm{Tr}\,[\not{p}\gamma^\mu\ \not{k}\gamma^\nu] - 4mMg^{\mu\nu}) \times (\mathrm{Tr}\,[\not{p}'\gamma_\nu\ \not{k}'\gamma_\mu] - 4mMg_{\mu\nu}) \\
&= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times 4\,(p^\mu k^\nu + p^\nu k^\mu - [p\cdot k + mM]g^{\mu\nu}) \times \left(p'_\mu k'_\nu + p'_\nu k'_\mu - [p'\cdot k' + mM]g_{\mu\nu}\right) \\
&= \left(\frac{4\pi e^2}{(k+p)^2}\right)^2 \times 4\,(2\underbrace{(p\cdot p')(k\cdot k')}_{t^2/4} + 2\underbrace{(p\cdot k')(p'\cdot k)}_{u^2/4} + \ldots\ldots)
\end{aligned}$$

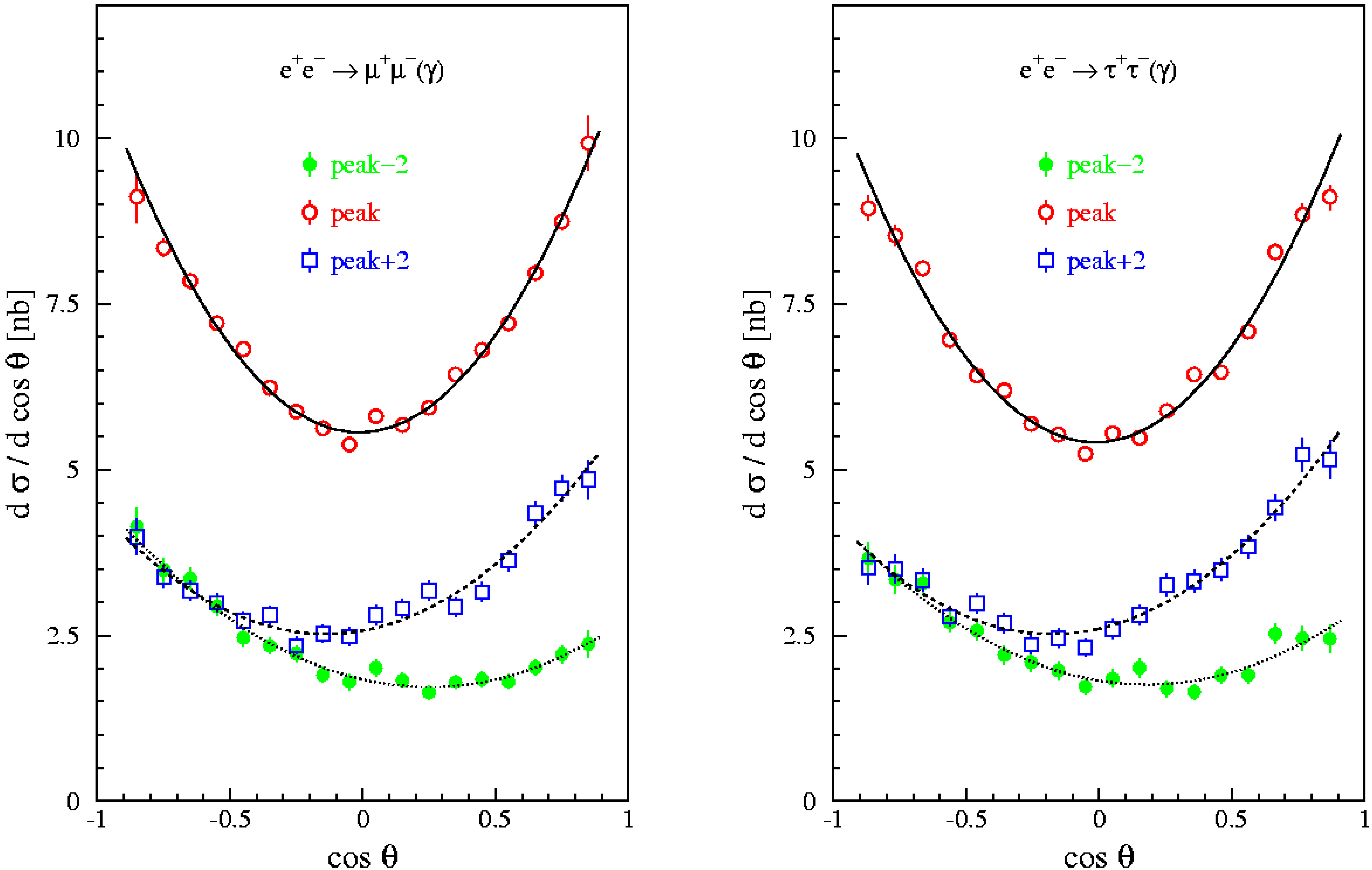
En dit wordt in de extreem relativistische

$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \rightarrow \mu^+ + \mu^-) = \frac{\pi\alpha^2}{8E^2}(1+\cos^2\theta)$$

Werkzame doorsnede $e^-e^+ \rightarrow \mu^-\mu^+$



Hoekverdelingen: $e^-e^+ \to \mu^-\mu^+$ en $\tau^-\tau^+$



En nu heb je ook $e^-e^+ \rightarrow \bar{q}q$ gedaan!

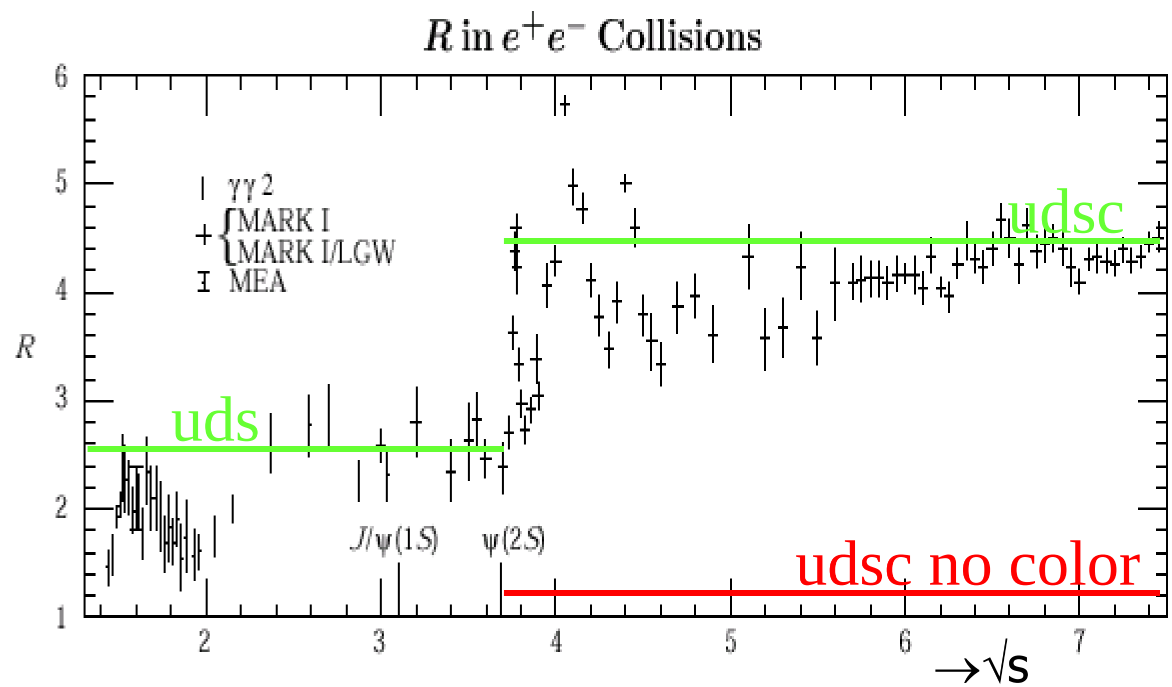
Met drie kleuren

u, c, t $\quad |q| = \frac{2}{3}: \quad \frac{d\sigma}{d\Omega} = \frac{4}{9} \times \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right) \Rightarrow \sigma_{tot} = \frac{16\pi\alpha^2}{27s} \qquad \sum_{colour} \frac{16\pi\alpha^2}{27s} \rightarrow \frac{16\pi\alpha^2}{9s}$

d, s, b $\quad |q| = \frac{1}{3}: \quad \frac{d\sigma}{d\Omega} = \frac{1}{9} \times \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right) \Rightarrow \sigma_{tot} = \frac{4\pi\alpha^2}{27s} \qquad \sum_{colour} \frac{4\pi\alpha^2}{27s} \rightarrow \frac{4\pi\alpha^2}{9s}$

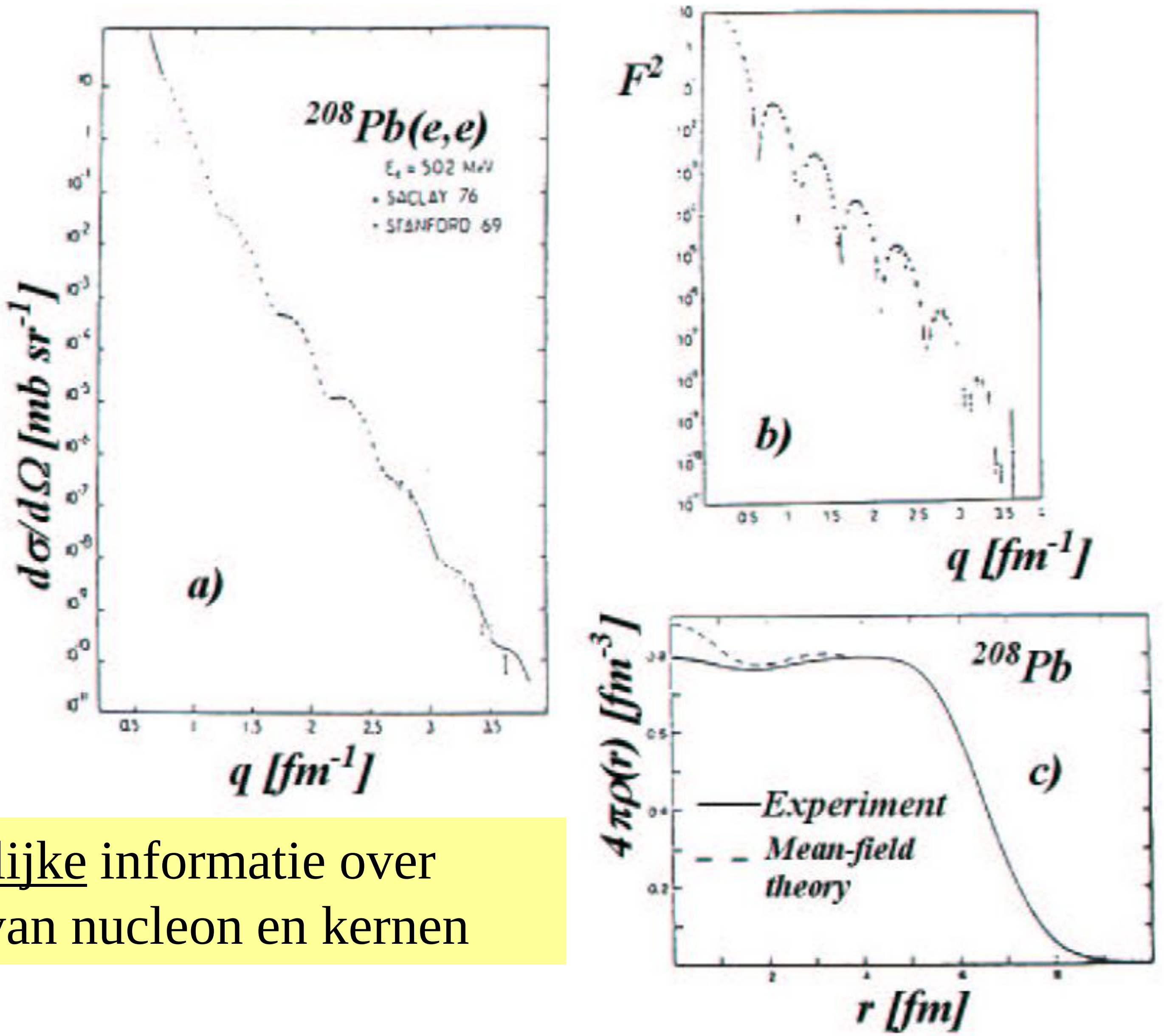
$$R \equiv \frac{\sigma_{qq}}{\sigma_{\mu\mu}} = \frac{\sum_{quarks} e^+e^- \rightarrow q\bar{q}}{e^+e^- \rightarrow \mu^+\mu^-}$$

$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{3s}$$



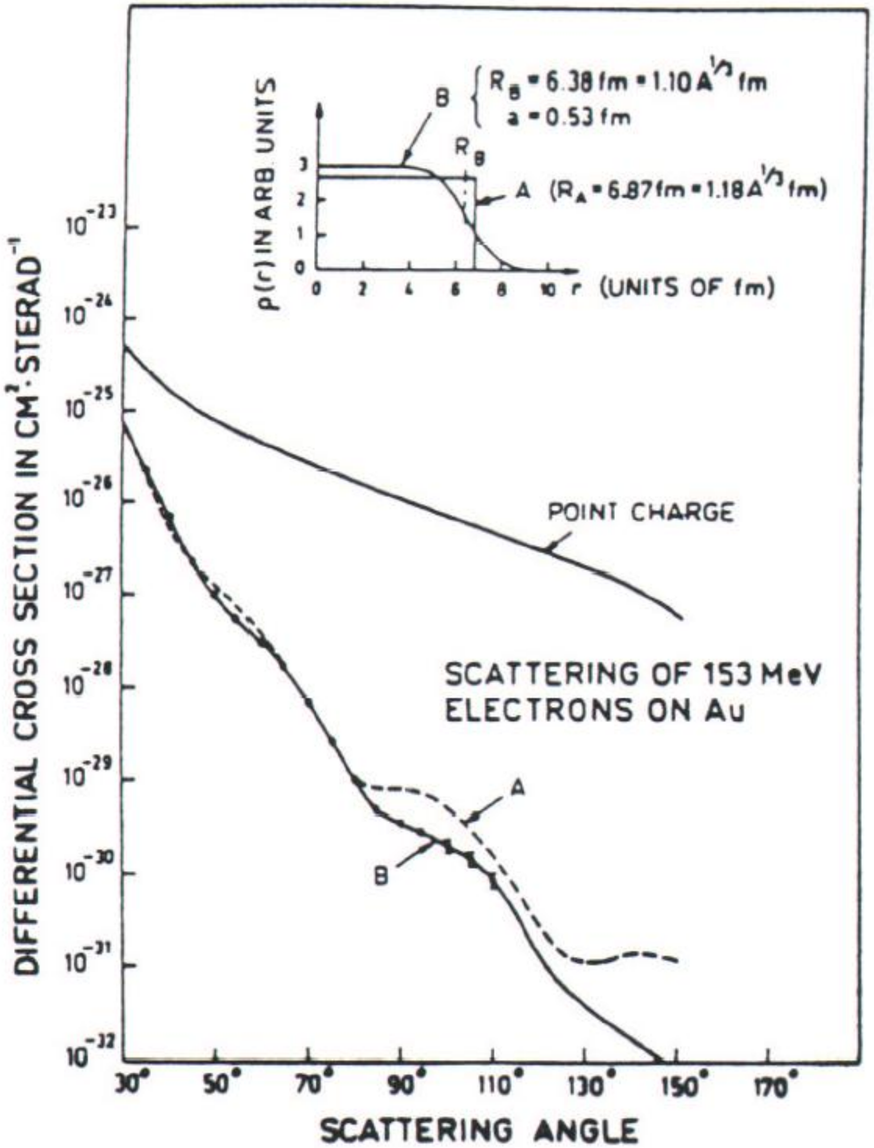
Elastische elektronen verstrooiïng - Voorbeelden

Elektronen aan lood:
- 502 MeV
- ^{208}Pb spinloos
- 12 decaden



<u>Model-onafhankelijke</u> informatie over ladingsverdeling van nucleon en kernen

Elastische elektronen verstrooiïng - Voorbeelden



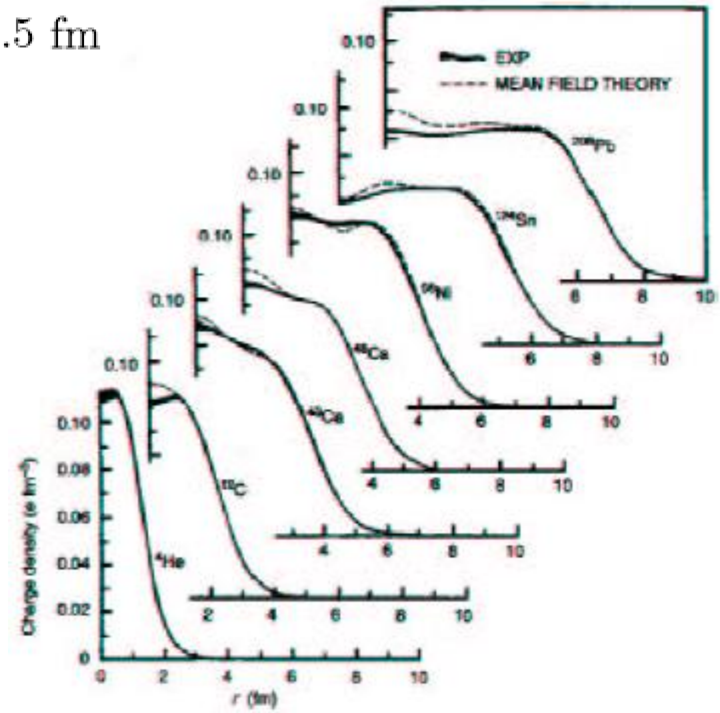
Elektron-goud verstrooiing
- energie: 153 MeV

ladingsverdeling:

$$\boxed{\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-R}{a}}}}$$

$R \simeq 1.2 \text{ fm } \sqrt[3]{A} - 0.48 \text{ fm, voor } A \gg 1$

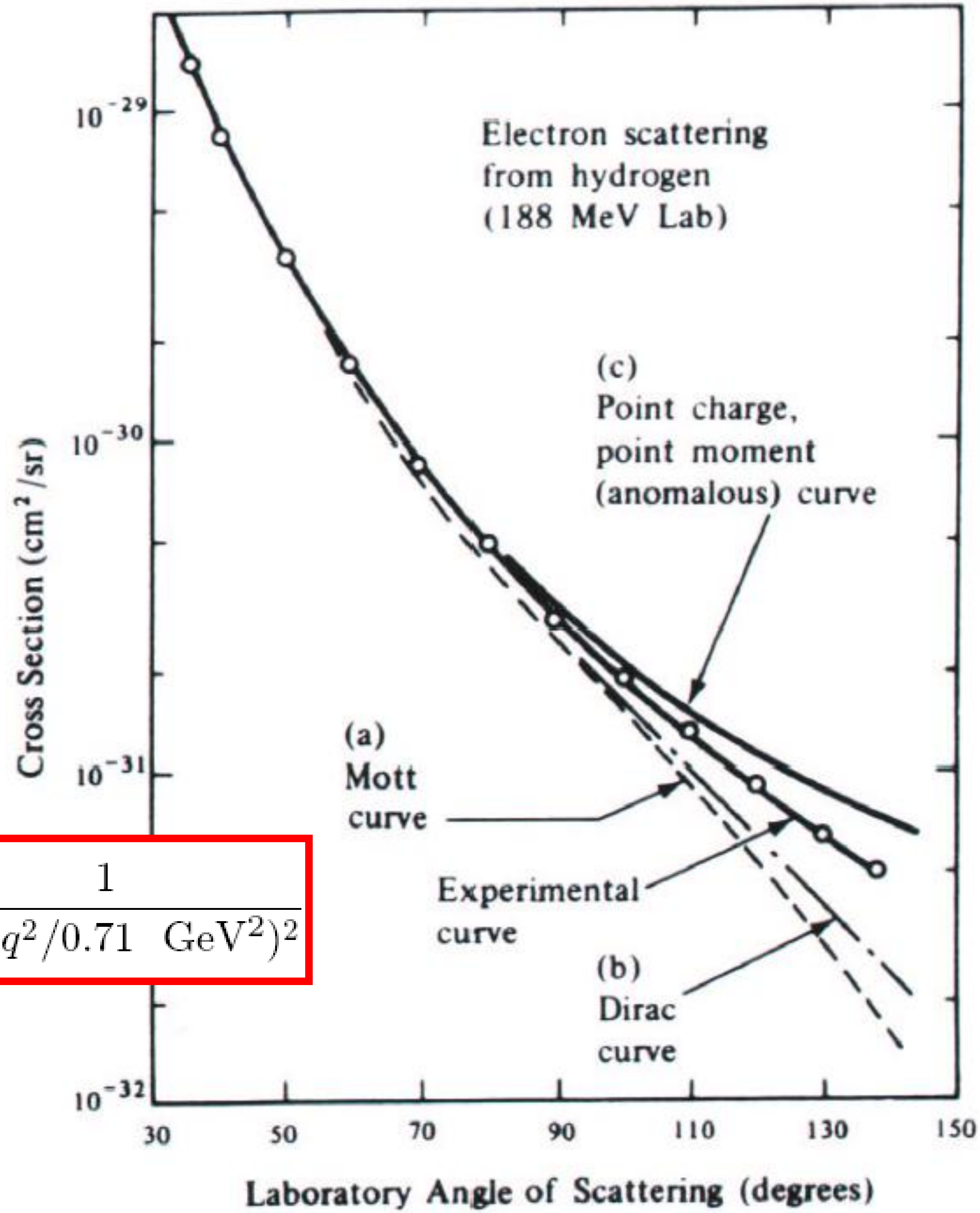
$a \simeq 0.5 \text{ fm}$



Ladingsdichtheid is constant!

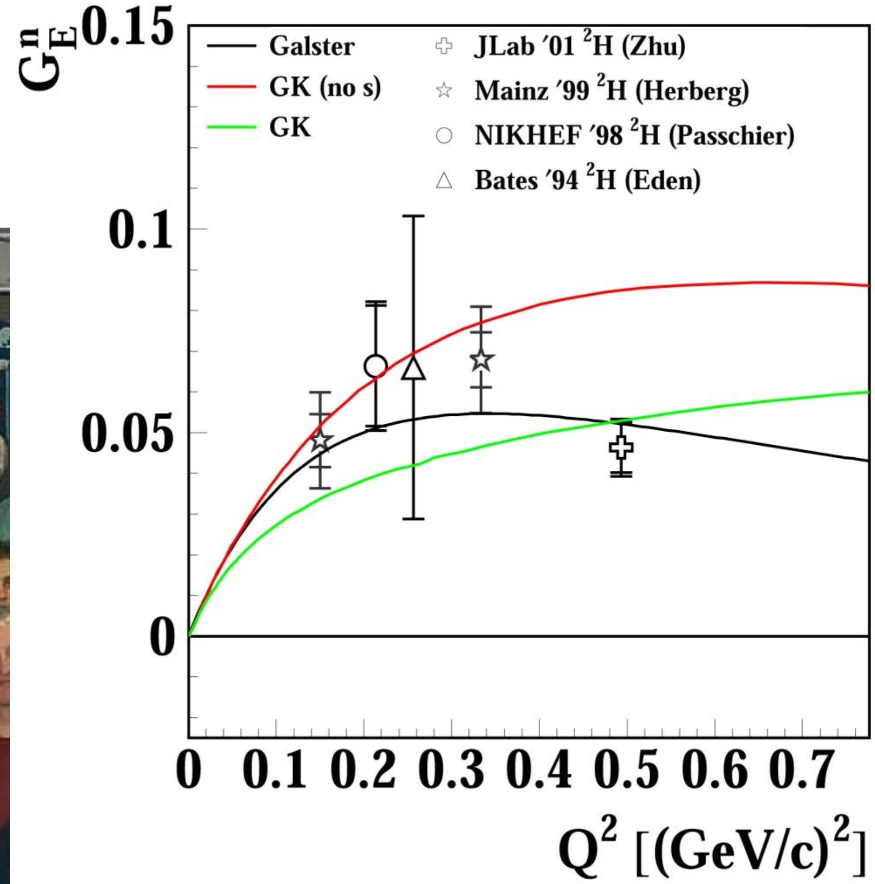
Elastische elektron-proton verstrooiïng

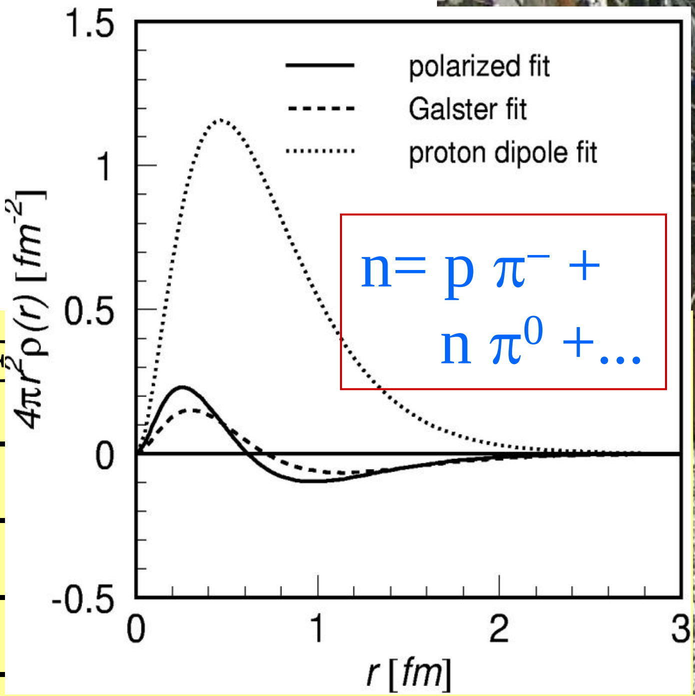
Proton structuur
- niet puntvormig
- geen Dirac deeltje (g=2)
- straal is 0.8 fm
- exponentiele vormfactor



$$G_E^p(q^2) = \frac{G_M^p(q^2)}{|\mu_p|} = \frac{G_M^n(q^2)}{|\mu_n|} \simeq \frac{1}{(1 - q^2/0.71 \ \ \mathrm{GeV}^2)^2}$$

Ladingsverdeling van het neutron





E

-

-

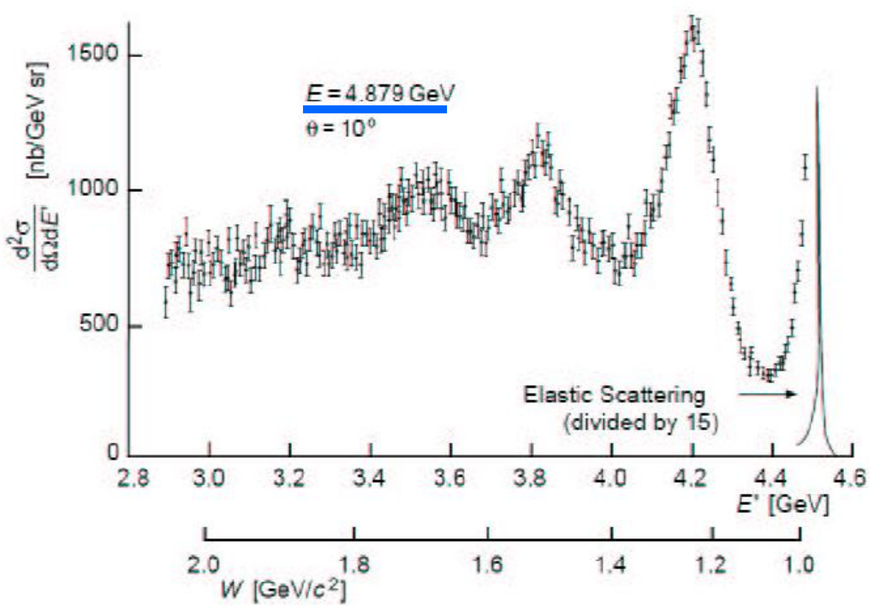
-

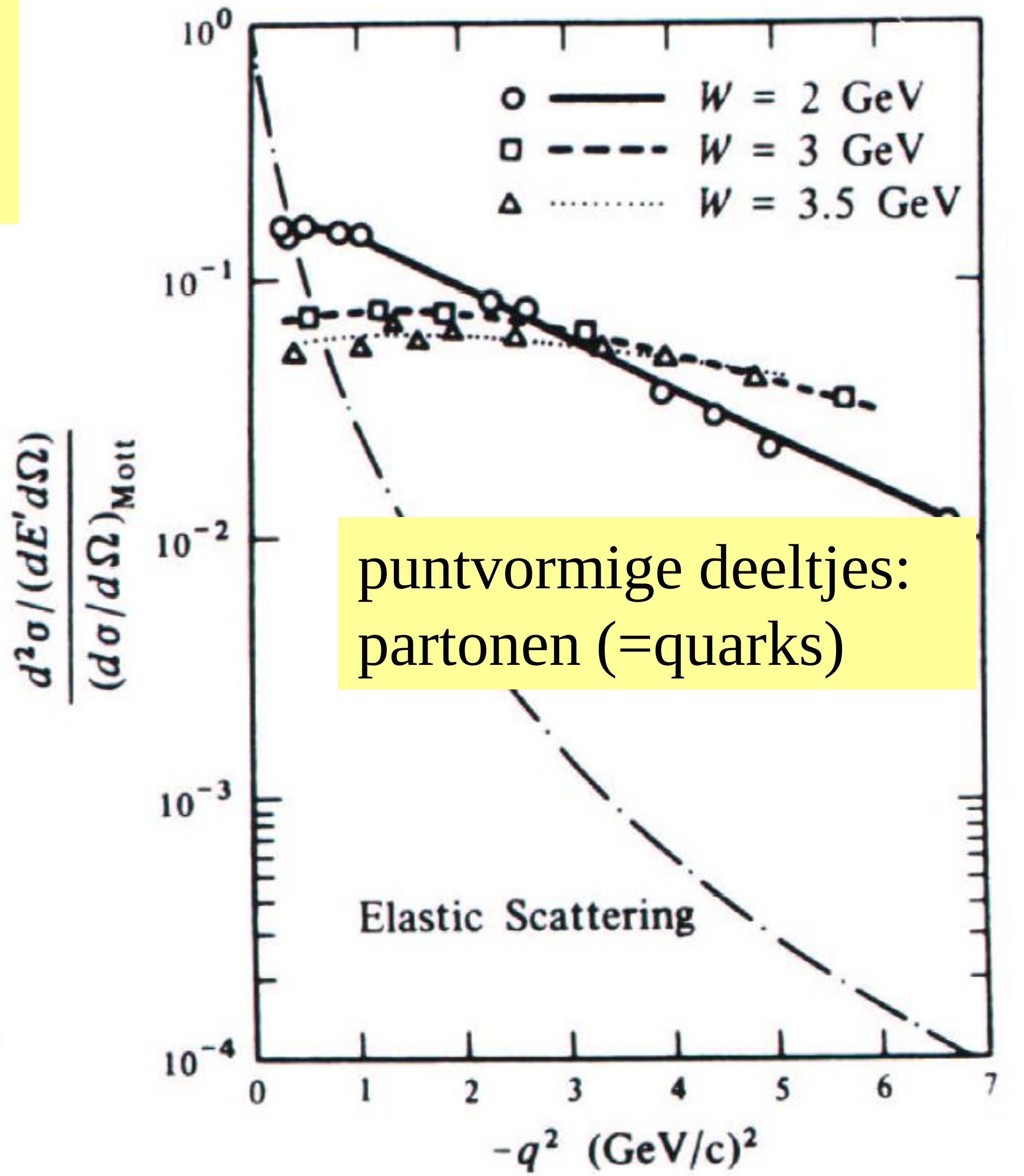
-

- elektron-neutron coincidentie meting

Diep-inelastische verstrooiïng

DIS definitie:
- Vierimpuls $Q^2 > 1$ (GeV/c)2
- Invariante massa $W > 2$ GeV

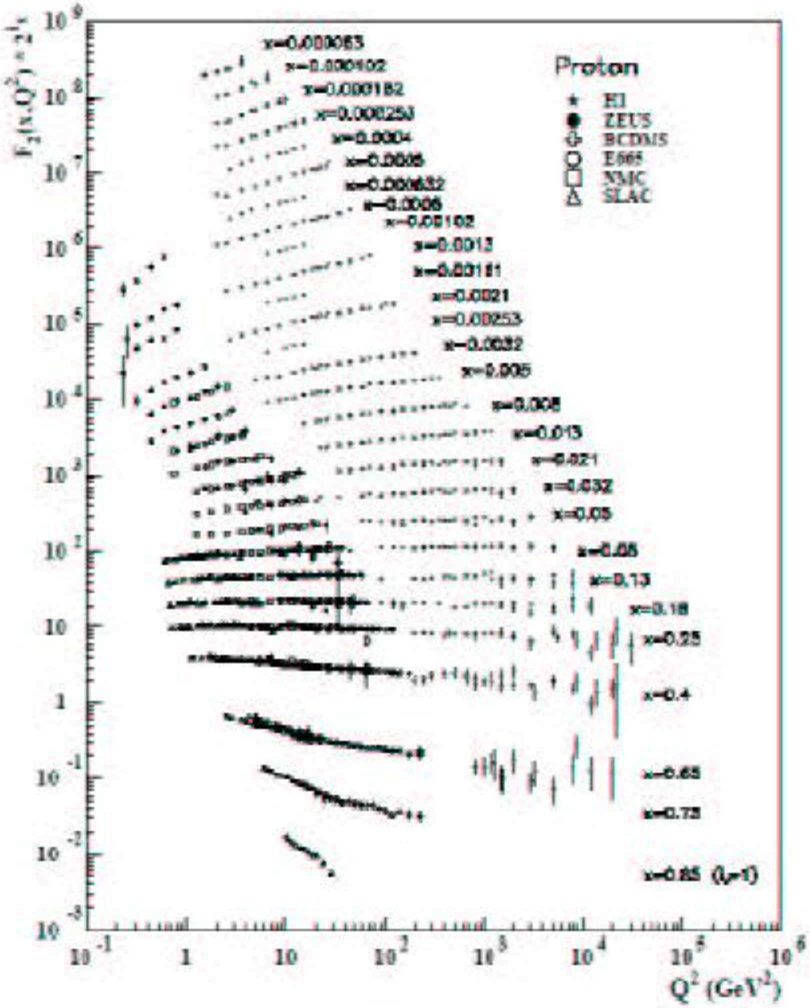


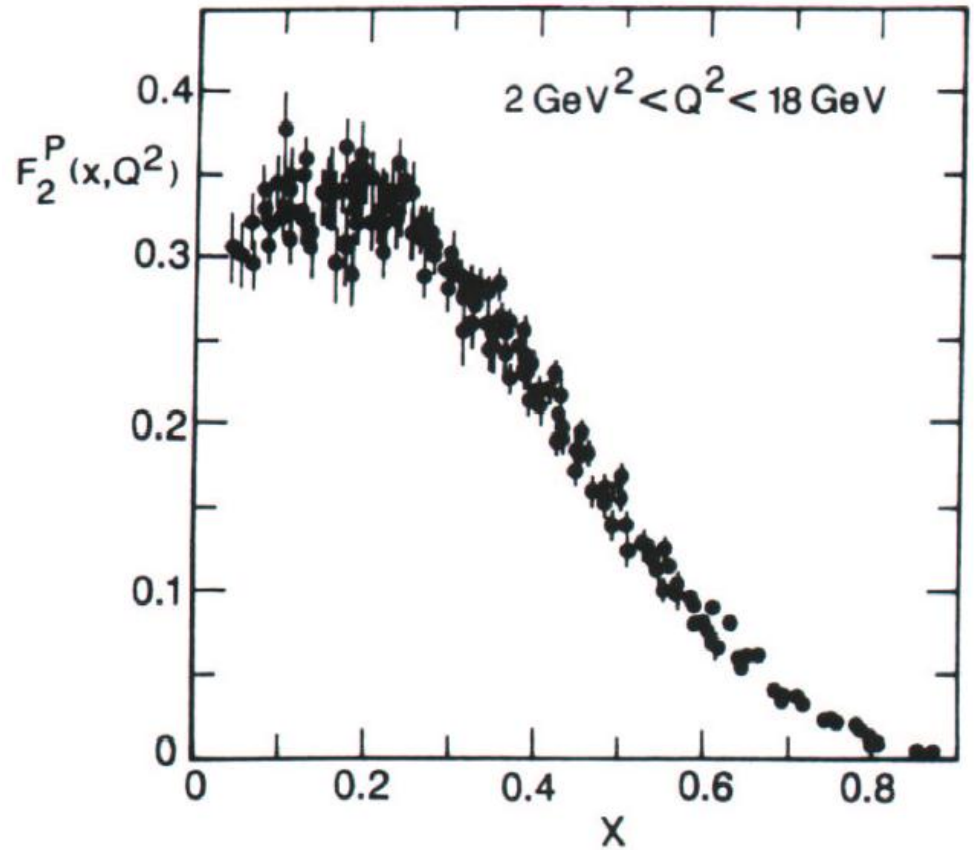


puntvormige deeltjes:
partonen (=quarks)

DIS – Bjørken schaling

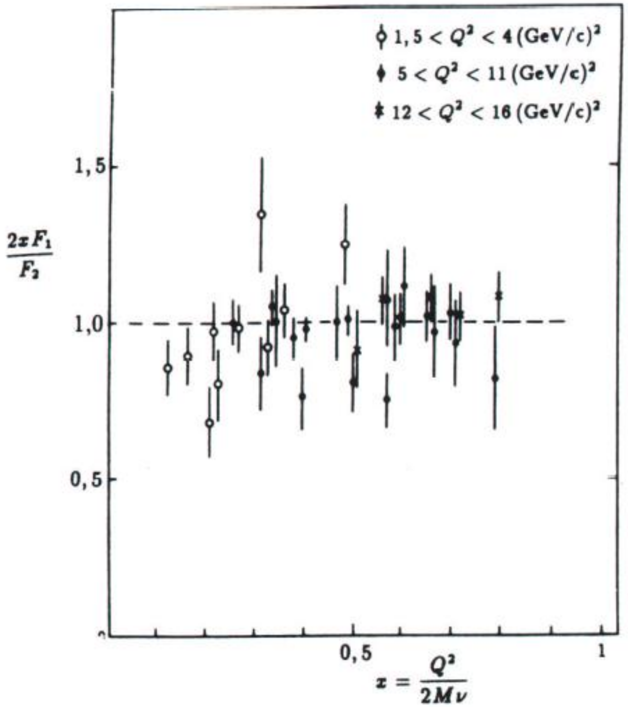
<u>Schaling</u>:
structuurfuncties enkel functie van *x*





$$F_2(x) = x \sum_f e_f^2 (q_f(x) + \bar{q}_f(x))$$

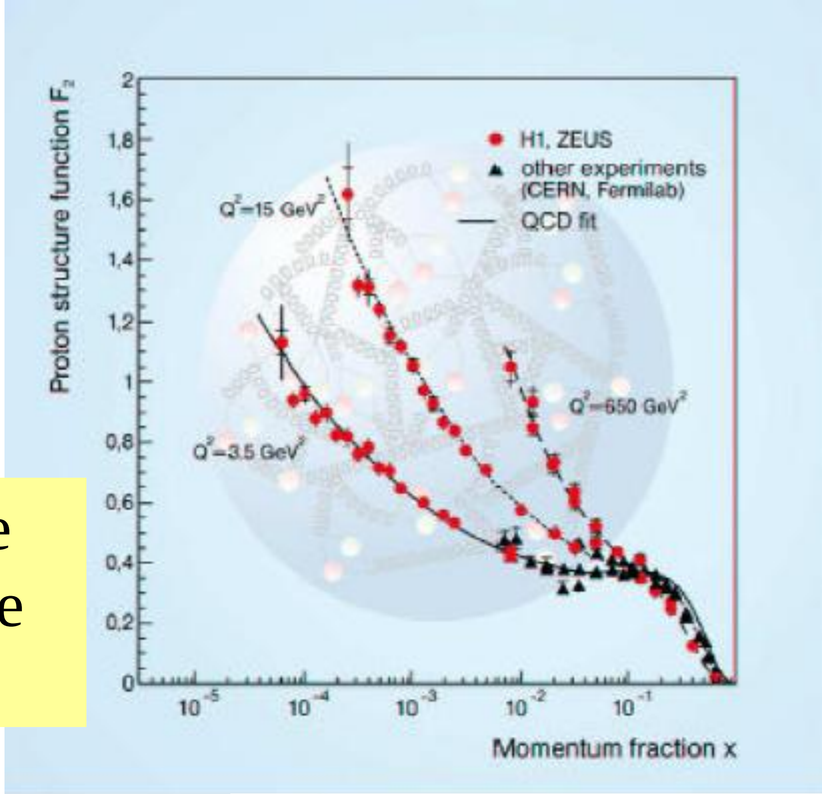
DIS – Bjørken schaling



$$\boxed{2xF_1(x) = F_2(x)}$$

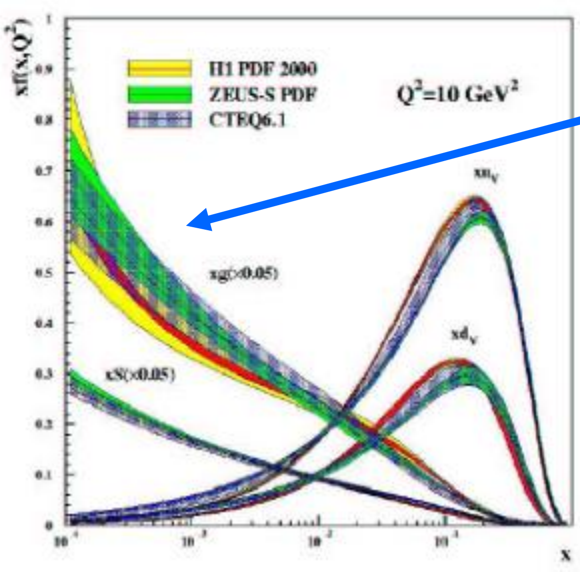
Callan-Gross relatie

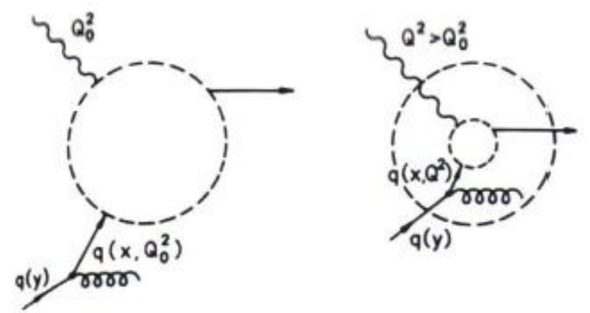
Quarks
spin *1/2*



Decompositie:

Gluon bijdrage
van Q^2 evolutie
van F_2





Elektron-positron annihilatie

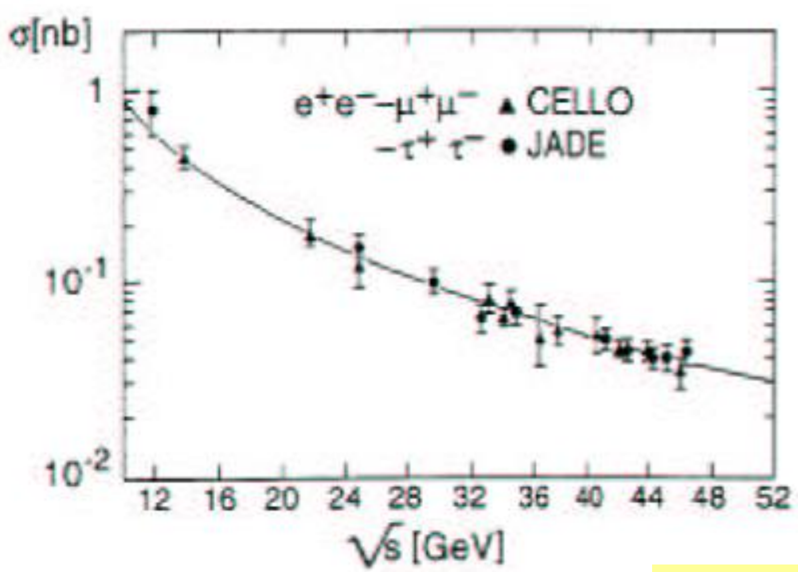
Muon productie

$e^+ + e^- \to \mu^+ + \mu^-$

$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \to \mu^+ + \mu^-) = \frac{\pi\alpha^2}{8E^2}(1 + \underbrace{\cos^2\theta}_{\text{spinfactor}})$$

integreer

$$\sigma(e^+ + e^- \to \mu^+ + \mu^-) = \sigma(e^+ + e^- \to \tau^+ + \tau^-) = \frac{21.7}{E^2}\ \text{nanobarns}$$



Elektron, muon en tau zijn identiek, afgezien van massa en levensduur

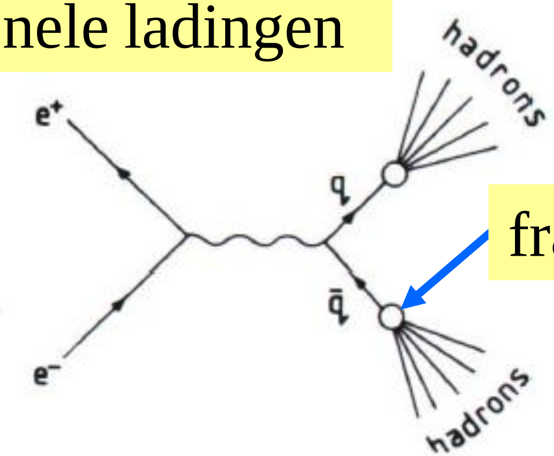
fractionele ladingen

kleur

Twee-jet productie

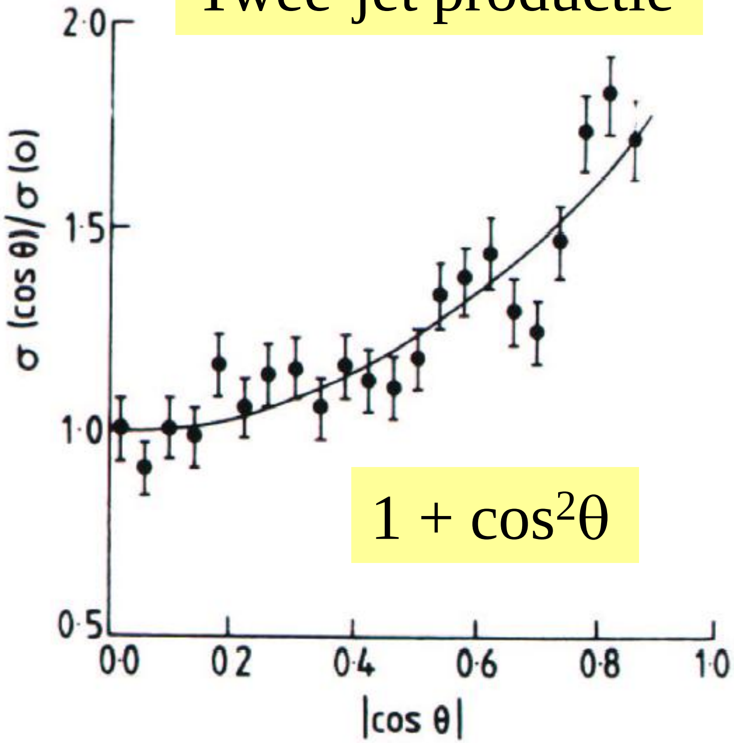
$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \to q + \bar{q}) = \frac{3\pi e_q^2\alpha^2}{8E^2}(1 + \cos^2\theta)$$

fragmentatie



Elektron-positron annihilatie

Twee-jet productie



$1 + \cos^2\theta$

$$\boxed{R \equiv \frac{\sigma(e^+e^- \to \ \text{hadronen})}{\sigma(e^+e^- \to \ \mu^+\mu^-)}}$$

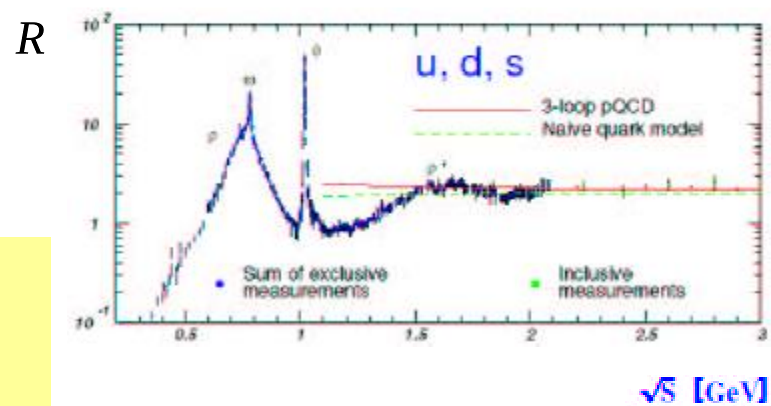
$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \to q + \bar{q}) = \frac{3\pi e_a^2\alpha^2}{8E^2}(1+\cos^2\theta)$$

$$\frac{d\sigma}{d\cos\theta}(e^+ + e^- \to \mu^+ + \mu^-) = \frac{\pi\alpha^2}{8E^2}(1+\cos^2\theta)$$

Voor E < 3.5 GeV

$$R = 3\left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = 2$$

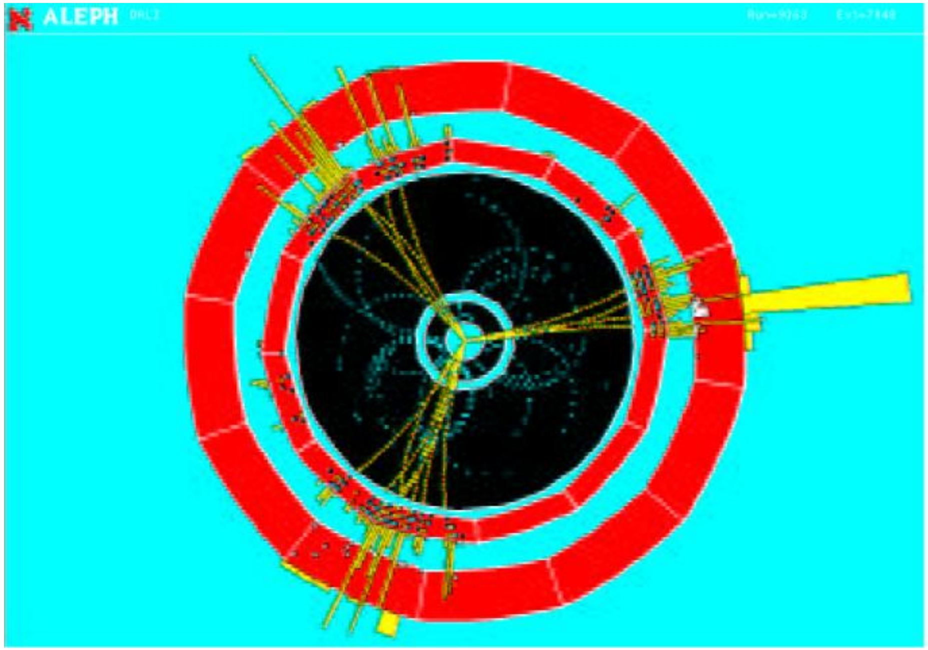
Quarks hebben spin 1/2

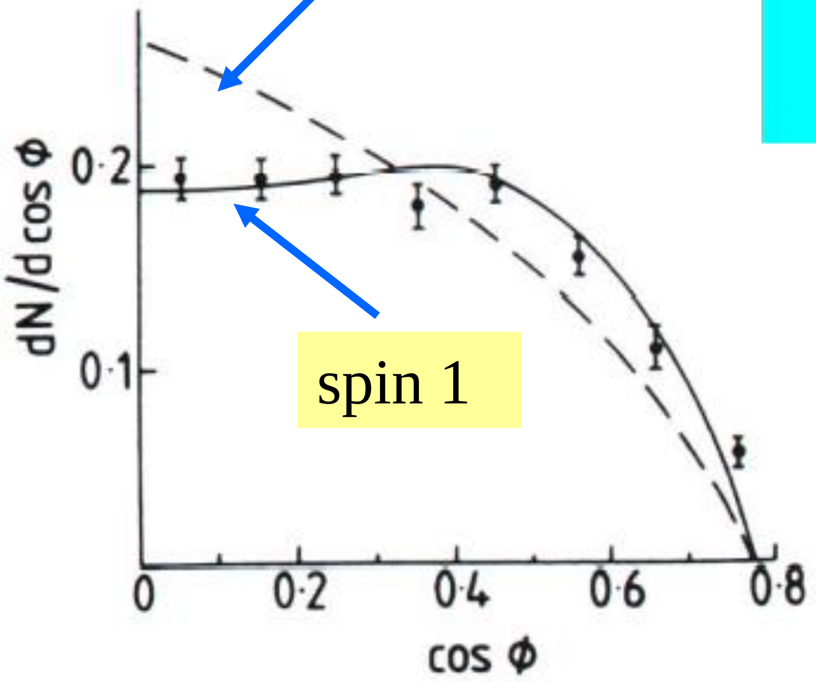


Quarks hebben kleur en fractionele lading

Elektron-positron annihilatie

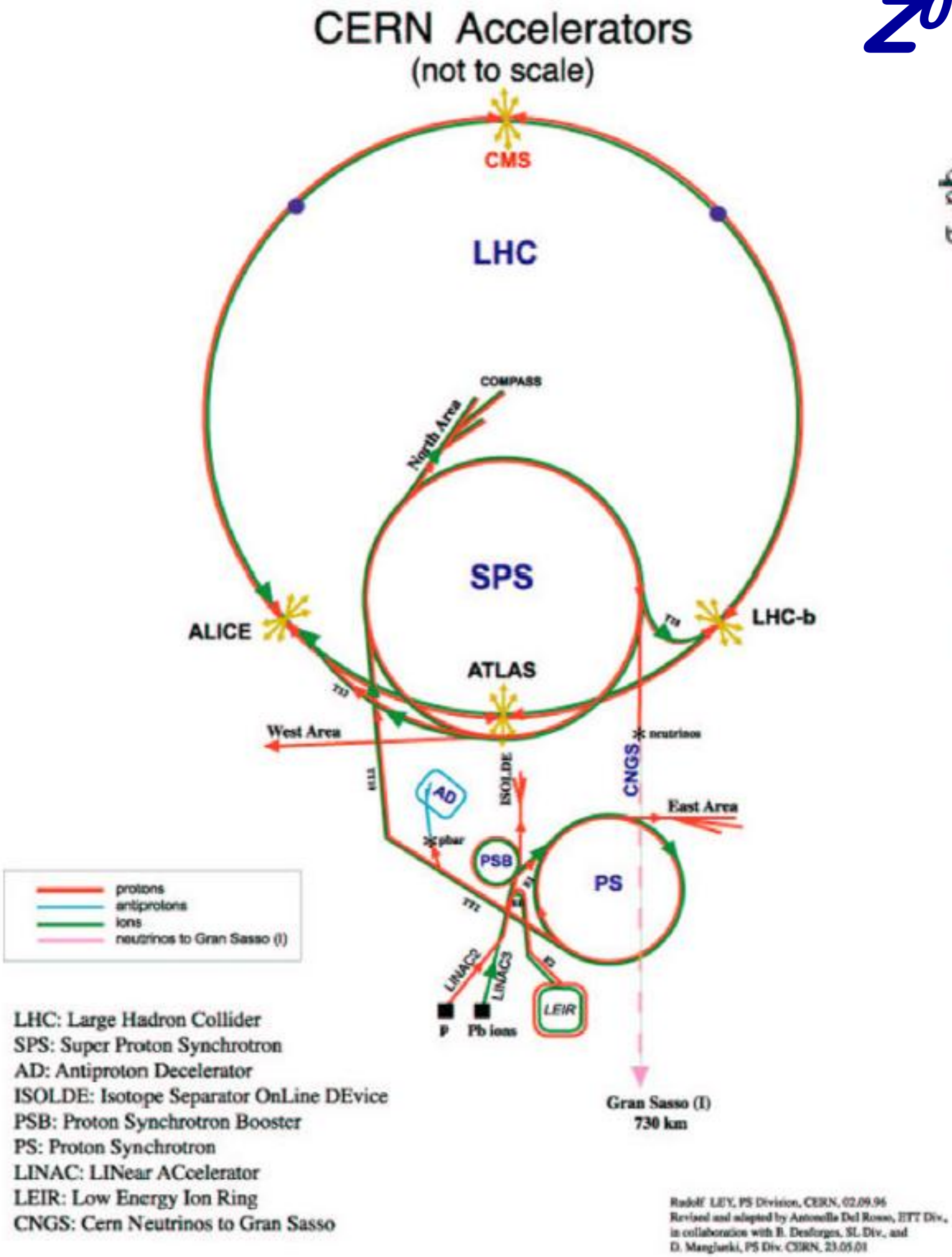
Drie-jet productie

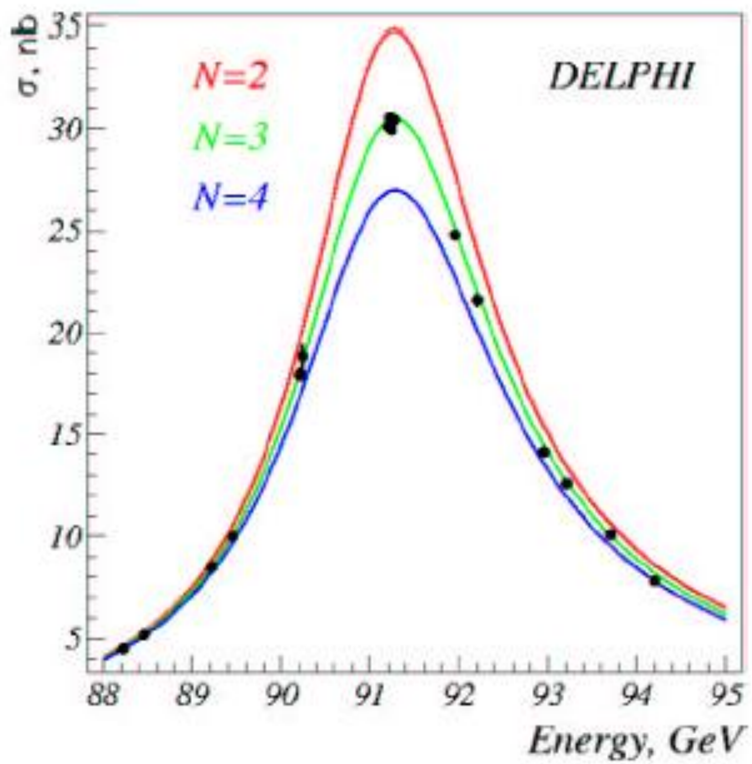




Gluonen hebben spin 1

Z^0-resonantie





e^+e^- verstrooiing

Drie generaties!