Cryostat, baffles, scattered light and noise

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Abstract : *evaluation of the scattered light noise possibly generated by imperfect glass baffles.*

1) Introduction

Installation of small aperture glass baffles in the links between towers in order to protect the vacuum pipes from scattered light raises the question of the influence of these baffles themselves on scattered light if those are imperfect, or imperfectly installed.

Les us recall that scattered light noise (SLN) is a second order process with respect to the scattering rate. Any particular channel of SLN begins by emission of scattered light off a mirror due to local roughness, a more or less complicated path involving specular reflections, and a second scattering on a mirror (possibly the same one). The spurious noise is caused by the phase modulation undergone by the light at each reflection off an object linked to ground and moving by seismic excitation, its modulation being transmitted to the main beam after the last scattering process.

We neglect third order scattering involving a rough surface on the path because firstly that kind of surface is systematically hidden by the baffles, and secondly because even for ordinary surfaces (stainless steel, glass or other), unless a special treatment was done (grating), the scattering rate is low.

We are thus in the present case faced with three extra channels of noise caused by the presence of baffles linked to the ground :

- There exists on a given baffle a zone directly reflecting the light scattered either by the nearby mirror or the far one to the emitter. This may happen if there is some splinter at the baffle surface caused by a shock during manufacture or installation.

- Either the axis of the baffle is imperfectly aligned with the optical axis, or equivalently, its inner edge is imperfectly manufactured, so that there is a zone on it able to reflect scattered light from the far mirror to the nearby one (and conversely) under grazing incidence.

- Dynamical diffraction of the beam by the finite and vibrating aperture of the baffle

We first recall the theoretical tools available after [1] and [2].

2) Theory

If we consider a mirror illuminated by a TEM00 gaussian mode, the light re-emitted has two components : a specularly reflected TEM00 wave, and a scattered wave. The incoming power is shared between the two, according to the roughness of the surface. In Virgo-like mirrors, the power carried by the scattered wave is fortunately very weak. If the rough surface is viewed as a 2D random process, the scattered wave is also a random process, and at some location $\vec{x}$ at a distance D of the mirror, we have after diffraction, a new random complex optical amplitude $s_D(\vec{x})$. It has been shown [1] that the relevant quantity for scattering studies is the coherence function of the speckle at distance D :

$$C(D,\vec{x},\vec{x}\,') = \langle s_D(\vec{x}) s_D(\vec{x}\,')^* \rangle = \frac{\varepsilon}{2\pi D^2}\, p(\theta) \exp\left[-\frac{1}{2}\left(k w_0 \frac{\vec{x}-\vec{x}\,'}{2D} \right)^2 \right] \exp\left[ik \frac{\vec{x}^2 - \vec{x}\,'^2}{2D} \right] \quad (1),$$

where $k \equiv 2\pi/\lambda$, where w_0 is the waist of the TEM00 beam, ε the scattering rate (a few ppm). The scattered light is emitted under all directions (θ,φ) with respect to the optical axis, but we assume an isotropic distribution in φ. $p(\theta)$ is the normalized distribution, in the sense that $\int_0^{\pi/2} p(\theta)\sin\theta d\theta = 1$.

We consider now a reflecting element at distance D_1 from a mirror M_1. There is a source $s_1(\vec{x})$ of scattered light at the surface of M_1. At distance D_1, the diffracted wave is $s_{D_1}(\vec{y})$. It is assumed reflected by a (spurious) mirror denoted by $m(t,\vec{y})$ (the time dependance takes into account the motion of the mirror due to seismic excitation). The reflected wave is then diffracted again along distance D_2 and hits a mirror M_2 on which it gives rise to a new scattered wave $s_2(\vec{z})$. The spurious effect comes from the coupling of $s_2(\vec{z})$ with the main beam $\Phi_0(\vec{z})$. The coupling coefficient is simply the Hermitian product
$\gamma \equiv (\Phi_0, s_1)$. γ is a complex random process, and its variance is (see [1]):

$$\langle \gamma\gamma^* \rangle = \int m(t,\vec{x}) m(t,\vec{x}\,')^* C(D_1,\vec{x},\vec{x}\,') C(D_2 \vec{x},\vec{x}\,')\, d\vec{x}\, d\vec{x}\,' \quad (2)$$

After the last scattering process, the optical amplitude re-emitted by the mirror M_2 is

$$A(t) = A_0(1+\gamma(t))$$

The phase noise being given by:

$$\Delta\phi(t) = \mathrm{Im}\left[\gamma(t)\right]$$

In the case of a moving reflecting element such that its surface has a motion $\delta x(t)$, the reflection operator is of the form $m(t,\vec{x}) = m(\vec{x})\exp\left[i\left(\phi_0 + 2k\delta x(t)\cos\vartheta\right)\right]$ where ϑ is the incidence angle and ϕ_0 an unkown phase. The coupling coefficient is therefore of the form: $\gamma(t) = \gamma\exp\left[i\left(\phi_0 + 2k\delta x(t)\cos\vartheta\right)\right]$ and consequently , $\Delta\phi(t) = \gamma\sin\left[\phi_0 + 2k\delta x(t)\cos\vartheta\right]$.For a small amplitude motion (compared to the wavelength), we have at first order:
$\Delta\phi(t) \simeq 2k\sin\phi_0\cos\vartheta\,\gamma\delta x(t)$, so that the GW amplitude $h(t)$ producing the same phase is

$$h(t) = \frac{\lambda}{4\pi L}\Delta\phi(t) = \gamma\cos\vartheta\sin\phi_0 \frac{\delta x(t)}{\ell}$$

where ℓ is the length of the interferometer's arms. In terms of spectral density, owing to the unknown phase ϕ_0, we get :

$$h(f) = \frac{\langle \gamma\gamma^* \rangle^{1/2}}{\sqrt{2}} \cos\vartheta \frac{\delta x(f)}{\ell} \quad (3),$$

so that the problem amounts to compute $\langle \gamma\gamma^* \rangle$ in our various situations.

3) Back reflecting surface elements

In the case of a reflecting element facing a mirror, we have $D_1 = D_2 = D$. The element's surface will be assumed having an axis of direction (α, β) and a mean curvature radius r_C. It is located at $\vec{x}_0 = (a, 0)$ so that $\vec{x} = (a + X, Y)$ and the reflexion operator is :

$$m(\vec{x}) = \sqrt{R}\exp\left[-2ik\alpha(X\cos\beta + Y\sin\beta) - ik\frac{X^2+Y^2}{r_C}\right] \qquad (4)$$

where R is the reflection coefficient. The element is seen from the mirror under a direction $(\alpha_0 = a/D, \beta_0 = 0)$. We have

$$\langle \gamma\gamma^* \rangle = \frac{\varepsilon^2 p(\theta)^2}{4\pi^2 D^4} R\,\Gamma'\Gamma'',$$

with

$$\Gamma' = \iint dX dX' \exp\left[2ik(\alpha_0 - \alpha\cos\beta)(X - X')\right]\exp\left[ik\left(\frac{1}{D} - \frac{1}{r_C}\right)\left(X^2 - X'^2\right)\right]\exp\left[-\left(\pi w_0 \frac{X - X'}{D\lambda}\right)^2\right]$$

$$\Gamma'' = \iint dY dY' \exp\left[-2ik\alpha\sin\beta(Y - Y')\right]\exp\left[ik\left(\frac{1}{D} - \frac{1}{r_C}\right)\left(Y^2 - Y'^2\right)\right]\exp\left[-\left(\pi w_0 \frac{Y - Y'}{D\lambda}\right)^2\right]$$

where the integrals are extended to the surface of the element. For obtaining an order of magnitude, it is convenient to assume a rectangular shape of the element (we expect the exact shape of marginal importance), and we shall take $-L/2 \le X, X' \le L/2$, $-H/2 \le Y, Y' \le H/2$.
After some algebra, these can be expressed as:

$$\Gamma' = L^2 \times 2\int_0^1 \exp\left[-(u/\sigma')^2\right]\cos(p'u)\,\mathrm{sinc}\left[q'u(1-u)\right](1-u)\,du \qquad (5.a)$$

$$\Gamma'' = H^2 \times 2\int_0^1 \exp\left[-(u/\sigma'')^2\right]\cos(p''u)\,\mathrm{sinc}\left[q''u(1-u)\right](1-u)\,du \qquad (5.b)$$

With the following notation:

$$\sigma' = \frac{\lambda D}{\pi L w_0},\quad p' = 2kL(\alpha_0 - \alpha\cos\beta),\quad q' = kL^2(1/D - 1/r_C)$$

$$\sigma'' = \frac{\lambda D}{\pi H w_0},\quad p'' = 2kH\alpha\sin\beta,\quad q'' = kH^2(1/D - 1/r_C)$$

It is clear that the maximum is reached when the orientation and ROC of the spurious element happens to match the diffused light, i.e. $\alpha = \alpha_0,\quad \beta = 0,\quad r_C = D$, in which case, we have simply (provided σ', σ'' very large) $\Gamma' = L^2, \Gamma'' = H^2$ and thus $\Gamma'\Gamma'' = S^2$ where S is the area of the element. Denoting $\Gamma' = L^2 F', \Gamma'' = H^2 F''$, we get in general $\gamma = \dfrac{\varepsilon p(\theta)}{2\pi D^2} RS\sqrt{F'F''}$.
The distribution at significant angles has been measured, and is of the form $p(\theta) \approx \kappa/\theta^2$ in

the angular region relevant here with $\kappa \sim 0.1$. Now, the angle θ is nothing but $\theta = \alpha_0 = a/D$, so that :

$$\gamma = \frac{\varepsilon\kappa}{2\pi a^2} RS\sqrt{F'F''}$$

And finally

$$h(f) = \frac{\varepsilon\kappa}{2\sqrt{2}\pi a^2} RS\sqrt{F'F''}\frac{\delta x(f)}{\ell} \qquad (6)$$

We have neglected here the factor $\cos\vartheta$ because in the cases of interest, the incidence angle is nearly zero. For mirrors having **10 ppm scattering losses**, $\varepsilon\kappa \sim 10^{-6}$, and on site, $\delta x(f) \sim 10^{-8}\left[\frac{10\text{ Hz}}{f}\right]^2$ m/$\sqrt{\text{Hz}}$, so that in the worst case ($F'F''$=1), we get even with a=0.3 m, R=1, S=10^{-6} m^2, and obviously ℓ=3000 m :

$$h_{\max}(f) \sim 4\times 10^{-24}\ \text{Hz}^{-1/2}\left[\frac{10\text{ Hz}}{f}\right]^2\left[\frac{S}{10^{-6}\text{m}^2}\right] \qquad (7)$$

We have now to study the influence of the form factors F',F''. We can for instance study the situation when the collimation is not perfect. Assume a perfect matching $r_C=D$, a perfect azimuthal orientation $\beta=0$, and an approximate radial orientation $\alpha=\alpha_0+\delta\alpha$. We have two cases :

- The case of a far (assume 3000 m) mirror, then $\sigma' \approx 50$ is large compared to the integration range, we can ignore the Gaussian factor in the integrals (5.a,b), and we find

$$F' \simeq 2\frac{1-\cos(2kL\delta\alpha)}{(2kL\delta\alpha)^2}, \quad F'' \simeq 1$$

so that F' becomes negligible for $|\delta\alpha| > \lambda/\pi L$. For L= 1 mm, this gives an interval for $\delta\alpha$ of width about 0.6 mRd. Moreover, we see that increasing L doesn't increase the noise, it only decreases the range of $\delta\alpha$.

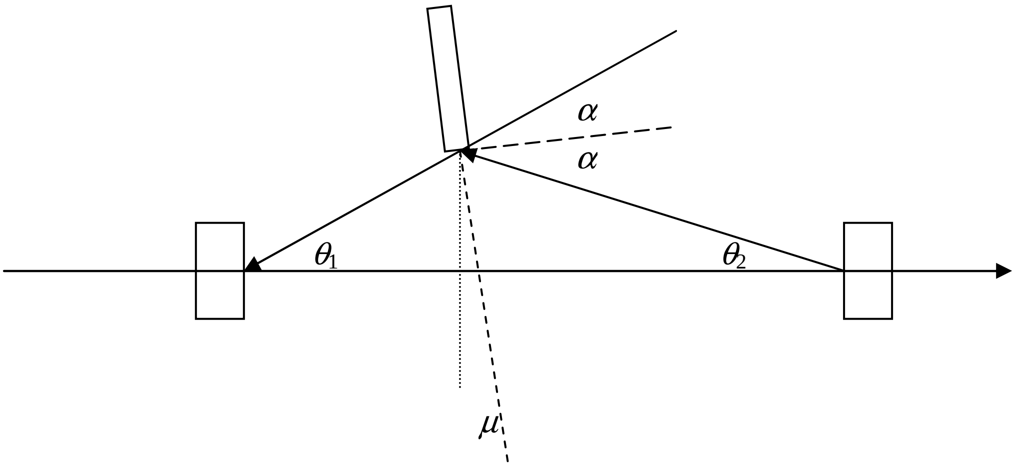
- The case of a close (assume 3 m) mirror, then conversely, $\sigma' \approx 0.05$ is rather small, and the Gaussian factor becomes predominant. The result is

$$F' \simeq \sigma'\sqrt{\pi}\left[1-\left(\frac{2\pi L\delta\alpha\sigma'}{\lambda}\right)^2\right], \quad F'' \simeq \sigma'\sqrt{\pi}$$

Giving a reduction factor of $\sqrt{F'F''} \sim \sigma'\sqrt{\pi} \sim 0.09$, with an angular width of about 6 mRd.

The conclusion is that the maximum value is significant, but unless severe collimation conditions, the noise coming from the far mirror spurious light is negligible, and with less severe collimation conditions, the noise coming from the close mirror spurious light is also negligible.

4) **Grazing reflection off inner edges**



We consider a situation in which, due to misalignment of a baffle, a piece of its inner edge is able to directly reflect the light scattered by mirror M_2 to mirror M_1 (and conversely) (see Fig. above). The axis of the baffle is assumed making an angle μ with the optical axis. In order to retrieve the preceding situation of a mirror under quasi normal incidence, we consider the reflecting surface as a plane mirror, and using the method of images, we replace the grazing incidence by a quasi-normal incidence of a virtual mirror orthogonal to the preceding, such that the virtual incidence is now $\alpha = \mu + \theta_2 = \theta_1 - \mu = (\theta_1+\theta_2)/2$. If now the baffle has not exactly this dangerous attitude, it has an inclination $\mu+\delta\mu$ and the virtual incidence is $\alpha=(\theta_1+\theta_2)/2+\delta\mu$. The virtual mirror may be represented by

$$m(\vec{x}) = \sqrt{R}\exp\left[-2ik\alpha X\right]$$

where we have assumed, without loss of generality the axis of the baffle in the (*xz*) plane, X representing the excursion relative to the center of the virtual mirror. We are back to the preceding problem in a simplified version. The coupling coefficient has a variance:

$$\langle \gamma\gamma^* \rangle = \int m(\vec{x})m(\vec{x}\,')^* C(D_1, \vec{x}, \vec{x}\,')C(D_2\vec{x}, \vec{x}\,')\, d\vec{x}\, d\vec{x}\,'$$

Where D_1 and D_2 are the distances of the baffle to nearby and far mirror respectively.
As in the preceding section, we take $\vec{x} = (a + X, Y)$, $\vec{x}\,' = (a + X', Y')$. We get thus

$$\langle \gamma\gamma^* \rangle = \frac{\varepsilon^2 p(\theta_1) p(\theta_2)}{4\pi^2 D_1^2 D_2^2} R\Gamma'\Gamma''$$

with

$$\Gamma' = \int dX dX' \exp\left[\frac{1}{2}\left(\frac{1}{D_1^2}+\frac{1}{D_2^2}\right)\left(\frac{kw_0}{2}(X-X')\right)^2\right]\exp\left[ik\left(\frac{a}{D_1}+\frac{a}{D_2}-2\alpha\right)(X-X')\right]\exp\left[i\frac{k}{2}\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\left(X^2-X'^2\right)\right]$$

If we take $a/D_1=\theta_1, a/D_2=\theta_2$, we have $a/D_1+a/D_2-2\alpha=2\delta\mu$, so that

$$\Gamma' = \int dX dX' \exp\left[\frac{1}{2}\left(\frac{1}{D_1^2}+\frac{1}{D_2^2}\right)\left(\frac{kw_0}{2}(X-X')\right)^2\right]\exp\left[2ik\delta\mu(X-X')\right]\exp\left[i\frac{k}{2}\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\left(X^2-X'^2\right)\right]$$

and

$$\Gamma'' = \int dY dY' \exp\left[\frac{1}{2}\left(\frac{1}{D_1^2}+\frac{1}{D_2^2}\right)\left(\frac{kw_0}{2}(Y-Y')\right)^2\right]\exp\left[i\frac{k}{2}\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\left(Y^2-Y'^2\right)\right]$$

As in the preceding section, we consider the surface integrals as being extended to the rectangular zone $-L/2 \le X, X' \le L/2$, $-H/2 \le Y, Y' \le H/2$. But (*H*,*L*) must be interpreted as the projection of the actual reflecting surface onto the incoming/reflected beam, so that *L* is not the full width *W* of the inner edge of the baffle, but its projected value $W\sin\mu$.

After some algebra, this is as well $\Gamma' = L^2 F', \Gamma'' = H^2 F''$, with

$$F' = 2\int_0^1 dx\, \mathrm{e}^{-x^2/\sigma'^2}\cos(p'x)\,\mathrm{sinc}\left[q'x(1-x)\right](1-x) \qquad (8.a)$$

$$F'' = 2\int_0^1 dx\, \mathrm{e}^{-x^2/\sigma''^2}\,\mathrm{sinc}\left[q''x(1-x)\right](1-x) \qquad (8.b)$$

And the following notation:

$$\sigma' \equiv \frac{\sqrt{2}\lambda D_1 D_2}{\pi w_0 L\sqrt{D_1^2+D_2^2}}, \quad \sigma'' \equiv \frac{\sqrt{2}\lambda D_1 D_2}{\pi w_0 H\sqrt{D_1^2+D_2^2}}, \; p' \equiv 2kL\delta\mu, \; q' \equiv \frac{\pi L^2}{\lambda}\left(\frac{1}{D_1}+\frac{1}{D_2}\right), \quad q'' \equiv \frac{\pi H^2}{\lambda}\left(\frac{1}{D_1}+\frac{1}{D_2}\right)$$

the LSD of noise is now :

$$h(f) = \frac{\varepsilon\kappa}{2\sqrt{2}\pi a^2} RS\sqrt{F'F''}\sin\left(\frac{\theta_1+\theta_2}{2}\right)\frac{\delta x(f)}{\ell} \qquad (9)$$

An expression quite similar to (6) , except that the modulation factor is $\cos\left(\frac{\pi}{2}-\alpha\right)$, a small quantity due to grazing incidence. In the worst case (*F'F''*=1, *R*=1), we have

$$h_{\max}(f) \sim 4\times10^{-24}\ \mathrm{Hz}^{-1/2}\left[\frac{10\ \mathrm{Hz}}{f}\right]^2\left[\frac{S}{10^{-6}\mathrm{m}^2}\right]\sin\alpha \qquad (7)$$

If we study the form factors *F'*, *F''*, we see that those are dominated by the Gaussian factor, So that we have approximately $F' \simeq \sigma'\sqrt{\pi}\left(1-\sigma'^2 p'^2/4\right), \quad F'' \simeq \sigma'\sqrt{\pi}$

which gives an attenuation factor of $\sqrt{F'F''} \sim \sigma'\sqrt{\pi} \sim 0.12$, and a tolerance of about 5 mRd for $\delta\mu$. On the other hand, we have (we assume the close mirror at 3m) $\alpha \sim 0.05$. Finally, we get the actual LSD of noise as

$$h_{\text{eff}}(f) \sim 2.4\times10^{-26}\ \text{Hz}^{-1/2}\left[\frac{10\ \text{Hz}}{f}\right]^2\left[\frac{S}{10^{-6}\text{m}^2}\right]\left[1\text{-}\left(\frac{\delta\mu}{5\ \text{mRd}}\right)^2\right] \qquad (10)$$

This result shows that the preceding result vanishes if the baffle's obliquity angle is not very close to $\mu=(\theta_1-\theta_2)/2$ $(=\theta_1/2$ in practice). It is thus necessary to test this for the four baffles involved in the design :

- Baffle #1 : a=0.28m, D_1= 0.9m → μ=8.9°
- Baffle #2 : a=0.235m, D_1= 1.5m → μ=4.5°
- Baffle #3 : a=0.3m, D_1= 2.2m → μ=3.9°
- Baffle #4 : a=0.3m, D_1= 5m → μ=1.7°

The three first angles are rather large, and a careful installation should easily avoid such misalignments. It seems thus that only the last baffle could possibly present a danger if it has an obliquity angle $\mu = 1.7° \pm 0.1°$, which has however a low probability.

5) Dynamical diffraction : clipping noise

This question is not linked to scattered light, but has been raised in the past. The baffle has a finite inner radius a, so that there is a coupling coefficient γ between the incoming beam A, and the transmitted one B:

$$\gamma = (A,B) = (A,\mathfrak{D}A) = \int_{\mathfrak{D}} |A(x,y)|^2\ dx\,dy\ ,$$

where the integral is taken over the free aperture of the diaphragm. If we assume an offset δ of the optical axis with respect to the baffle's axis, and if we consider a normalized TEM00 mode $\phi_0(x,y)$ of width w at the location of the baffle, this is simply:

$$\gamma(t) = \int_0^{2\pi} d\varphi \int_0^a rdr\ |\phi_0[x+\delta+\xi(t),\, y+\eta(t)]|^2\ ,$$

where we assume without loss of generality the offset to be in the x direction. The couple $(\xi(t),\eta(t))$ represents the transverse motion of the baffle's axis due to seismic excitation. $\gamma(t)$ being real, it is clear that the effect is an power modulation of the beam (clipping noise). In fact, γ is nothing but $1\text{-}\Delta P/P$. A relative power noise $\Delta P(f)/P$ is related to a phase noise $\Delta\Phi(f)$ by $\Delta\Phi(f) = \frac{1}{2}\sqrt{1-C}\ \frac{\Delta P(f)}{P}$ where C is the contrast of the interferometer. On the other hand, the GW linear spectral density equivalent to the phase noise $\Delta\Phi(f)$ is $h(f) = \frac{\lambda}{2\pi\ell}\Delta\Phi(f)$, so that finally, the GW LSD equivalent to the clipping noise is:

$$h(f) = \frac{\lambda}{4\pi\ell}\sqrt{1-C}\ [1-\gamma(f)]$$

Now, we have

$$\gamma(t) = \frac{2}{\pi w^2}\int_0^{2\pi} d\varphi \int_0^a rdr \exp\left[-\frac{2}{w^2}\left(r^2+\rho^2+2r\rho\cos(\varphi-\psi)\right)\right]$$

where we have used the following notation : $(x = r\cos\varphi, y = r\sin\varphi)$, $(\delta+\xi(t) = \rho\cos\psi, \eta(t) = \rho\sin\psi)$. We can compute the azimuthal integral:

$$\gamma(t) = \frac{4}{w^2}\exp\left[-2\rho^2/w^2\right]\int_0^a rdr \exp\left[-2r^2/w^2\right] I_0\left[4r\rho/w^2\right]$$

where I_0 denotes the modified Bessel function of the 1st kind. If we assume $\rho \ll w$, we have up to 4th order :

$$\gamma(t) = \frac{4}{w^2}\exp\left[-2\rho^2/w^2\right]\int_0^a rdr \exp\left[-2r^2/w^2\right]\left[1+\frac{4r^2\rho^2}{w^4}\right]$$

Or as well :

$$\gamma(t) = 2\,\mathrm{e}^{-2\rho^2/w^2}\int_0^{\sqrt{2}a/w} xdx\,\mathrm{e}^{-x^2}\left[1+\frac{2\rho^2}{w^2}x^2\right] = \mathrm{e}^{-2\rho^2/w^2}\left\{\left(1-\mathrm{e}^{-2a^2/w^2}\right)\left(1+\frac{2\rho^2}{w^2}\right)-\frac{4a^2\rho^2}{w^4}\mathrm{e}^{-2a^2/w^2}\right\}$$

So that

$$\frac{\Delta P(t)}{P} \simeq \frac{4\rho^2 a^2}{w^4}\ \mathrm{e}^{-2a^2/w^2} + \mathcal{O}\left[(\rho/w)^4\right]$$

With $\rho^2 = (\delta+\xi(t))^2+\eta(t)^2 \simeq \delta^2 + 2\delta.\xi(t)$, we get the relative power noise :

$$\frac{\Delta P(f)}{P} \simeq \frac{8a^2\delta}{w^4}\ \mathrm{e}^{-2a^2/w^2}\xi(f) \qquad (11)$$

It is not necessary to finish the calculation : The preceding expression involves (as could have been foreseen from the beginning) e^{-2a^2/w^2} which is so small for *a*~30cm and *w*~3cm that the final result for the clipping noise is obviously negligible whatever are other factors and final details.

Conclusion:

Analysis of the three identified channels shows that the proposed design does not increase significantly the scattered light noise level, if unlikely errors are avoided:

- Baffle having surface defects caused by shocks and directly reflecting a part of the diffused light to the emitter

- Baffle misaligned with a such a high precision, that is transmits directly diffused light between mirrors by a specular reflection on its inclined inner edge.

The third channel (noise due to diffraction of the beam by the finite moving aperture of a baffle) is too weak for consideration.

References:

[1] Vinet, Brisson & Braccini, PRD 54, p.1276 (1996)
[2] Vinet, Brisson, Braccini, Ferrante, Pinard, Bondu & Tournié, PRD 56, p.6085 (1997)