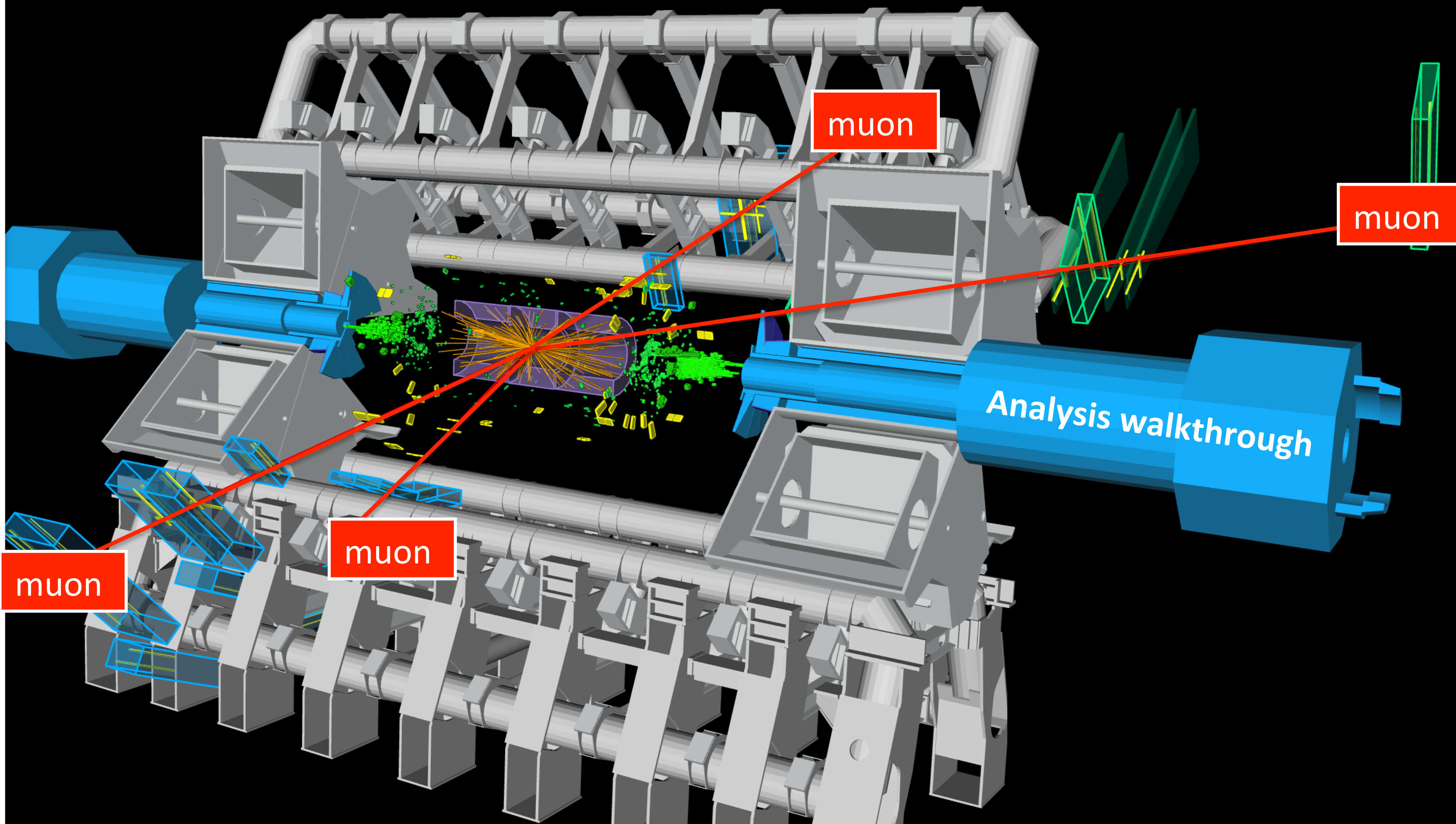
Terascale Statistics School, January 2018 (DESY)



Statistics exercises

Tuesday *3.5 hours (14:00-15:30 & 16:00-18:00)*

Wednesday *3.5 hours (11:20-12:50 & 15:50-18:00)*

Ivo van Vulpen (UvA/Nikhef)

Who am I …

@IvovanVulpen

Lecturer at University of Amsterdam
programming, particle physics, Higgs physics

Researcher at Nikhef (Amsterdam, NL)
ATLAS experiment (top & Higgs physics)

Why am I here …

make you struggle & get uncomfortable

Their job: theory, concepts, tools, …

Glen Cowan

Lydia Brenner

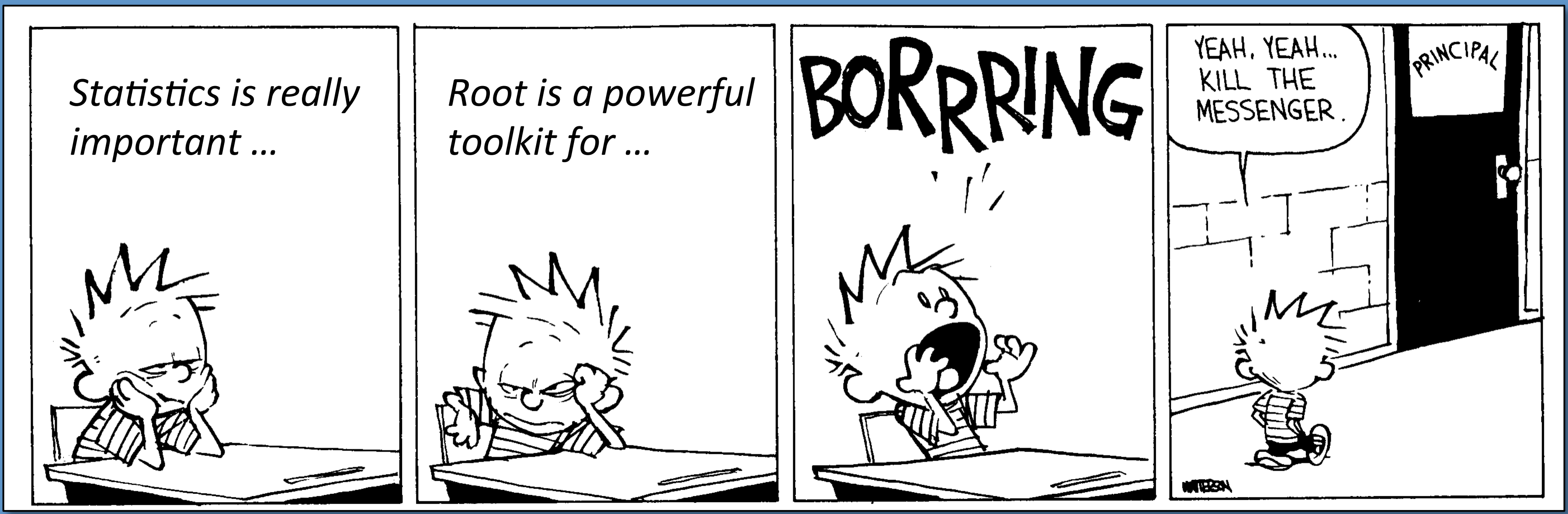
Wouter Verkerke

Kevin Kroeninger

My job: hands-on exercises (intro, DIY)

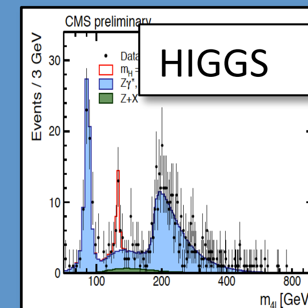
- Make sure *everybody* knows the basics
- Have you *do* things. Guide you through a few 'easy' exericses. DIY … to have you appreciate standard tools like RooFit etc."

A short lecture on statistics



Enters at every step and defines validity/power of you analysis

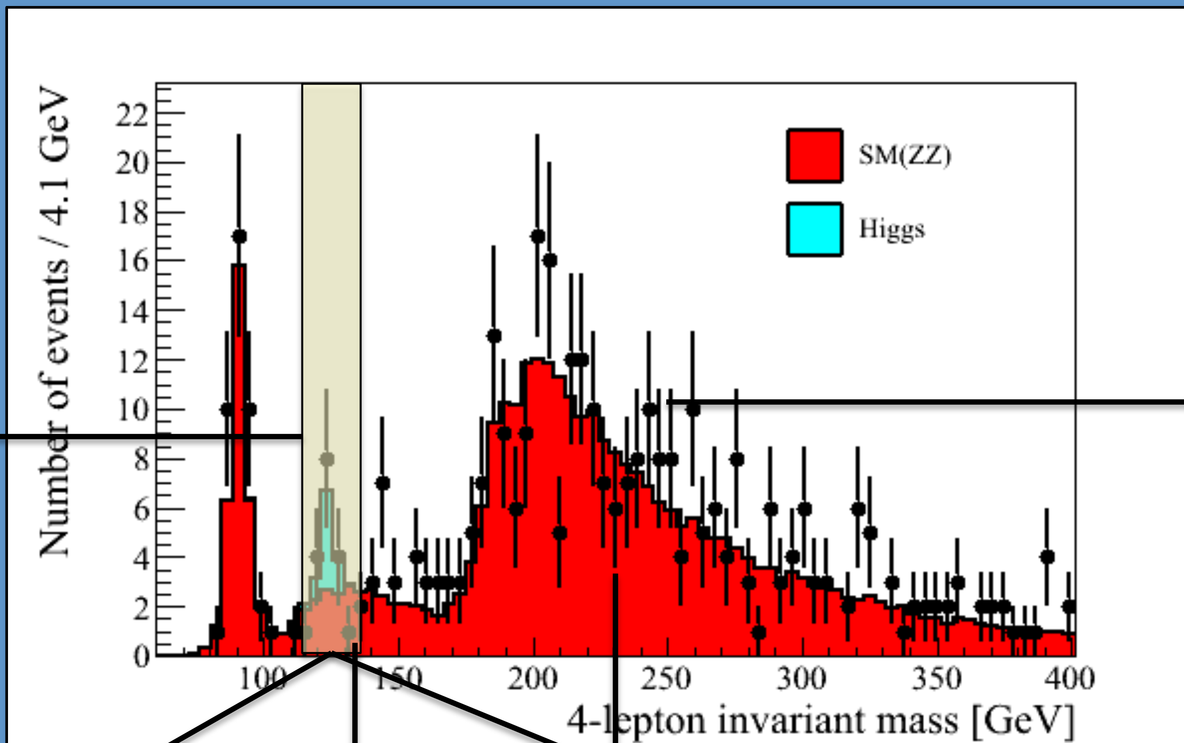
Statistics is everywhere!



- Many mysteries, folklore, buzz-words, bluffing etc., but you **need** to master it to quantify the results of any analysis. Do **not** just follow ‘what everybody else does‘ or your supervisor tells you.

- RooFit, Roostats, TMVA, Machine Learning, TensorFlow, BDT’s are excellent and very powerful tools. Make sure you understand the basics so you know it’s consequences for your result and what you ask it to do.

Data-set for exercises: 4 lepton mass



Cross-section measurement

Exclusions

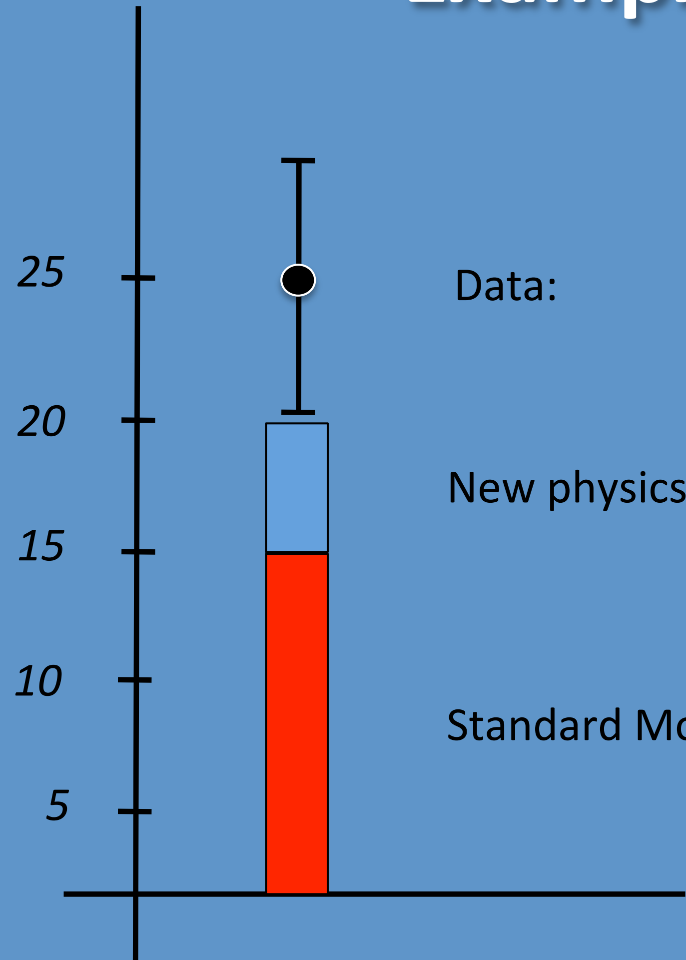
Significance optimization

Test statistic (Toy-MC)

Mass measurement

Data-driven background estimate
(likelihood fit using side bands)

Example 1: significance



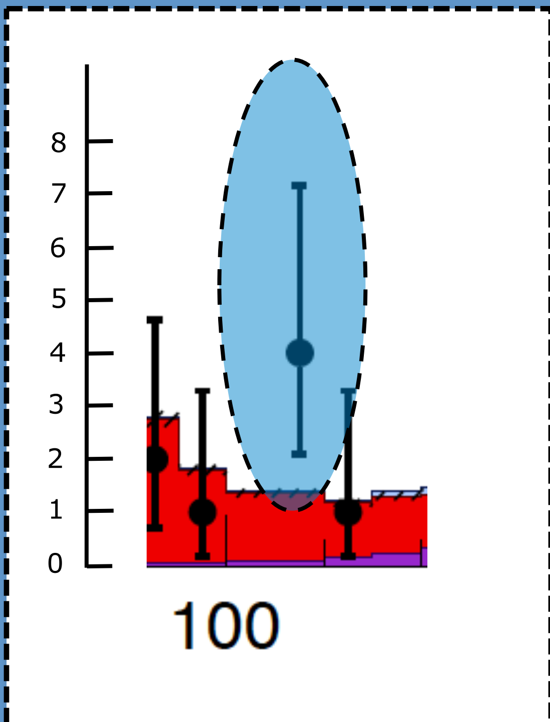
Data: $N_{data} = 25$

New physics: $N_{sig} = 5$

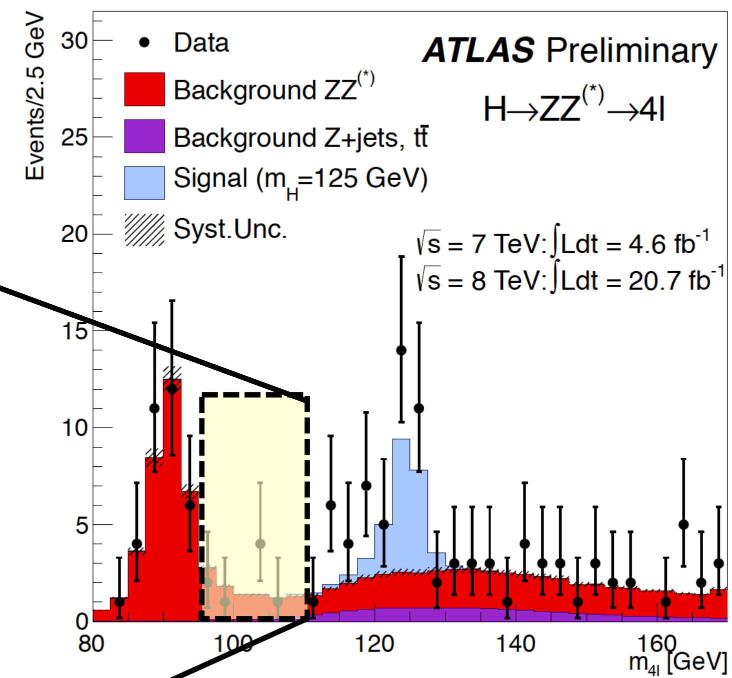
Standard Model: $N_{bgr} = 15$

What is the significance of the excess ?

Example 2: Poisson errors in LHC plots



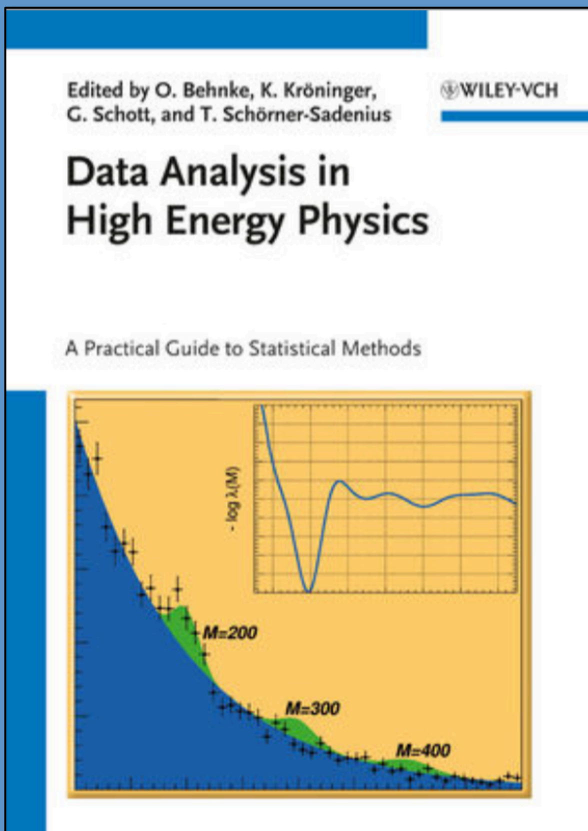
ATLAS H→ 4 lepton peak



I can present 5 options; you tell me which one you prefer.

Why do we put uncertainties on data points?

Example 2: Poisson errors in LHC plots



Exercise 4: Details: Poisson errors on data-points

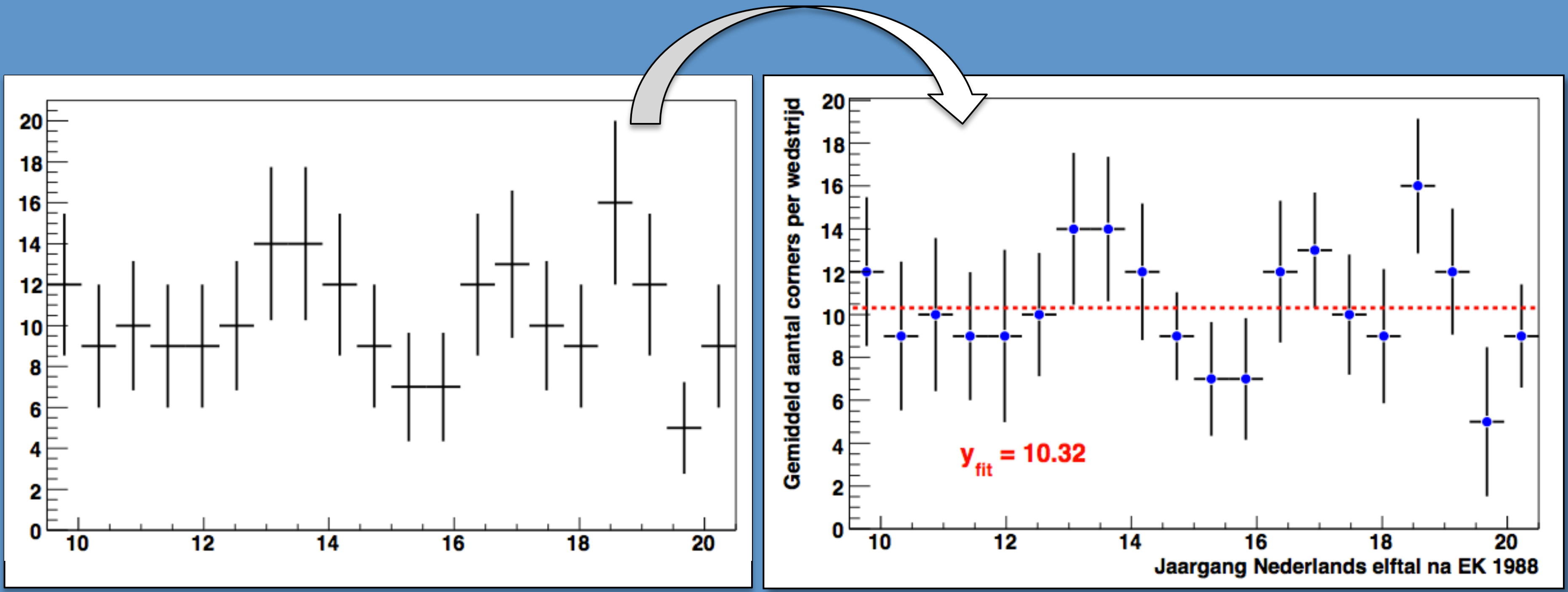
The subtleties of uncertainty regions enter at many stages in the analyses. Although it is for example custom practise to assign a $\sqrt{n}$ uncertainty to an event count of n events, this is not the most natural way to summarize the measurement. While $\sqrt{\lambda_b}$ is a measure for the expected spread in the number of observed events from a Poisson process with a well known mean λ_b, the uncertainty band on an observed event yield is expected to reflect information on what we infer about the underlying model parameter. Although there are many ways to define such a confidence level region summarizing the measurement, the uncertainty interval assigned to data points in Figure 11.1 (and RooFit's default), is the region $(\mu_{\mathrm{low}}, \mu_{\mathrm{up}})$ defined by:

Go through various options

d) Irritate and confuse people at your institute by discussing this over coffee.

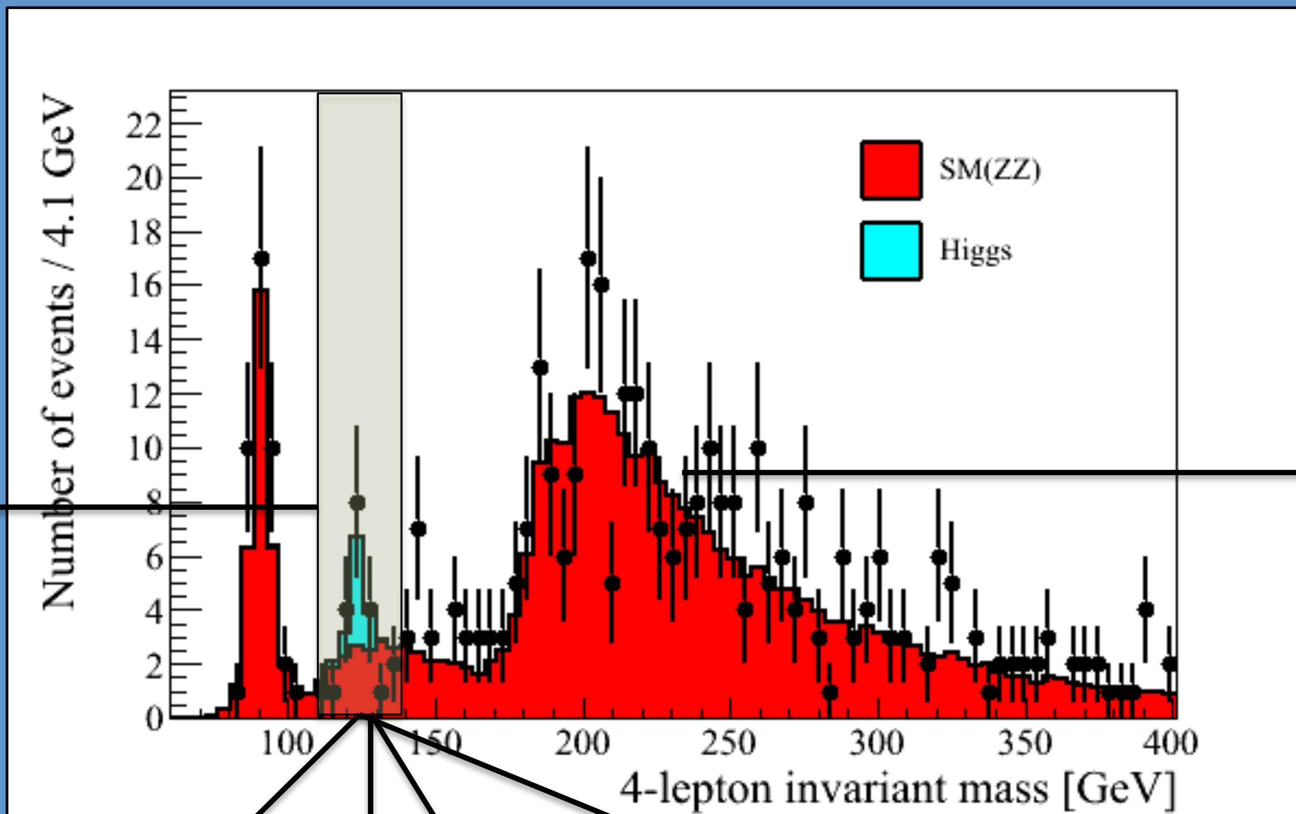
Example: Likelihood fit

Can everybody do this ?



Hands-on exercises

Data-set for the exercises: 4 lepton mass



Cross-section measurement

Exclusions

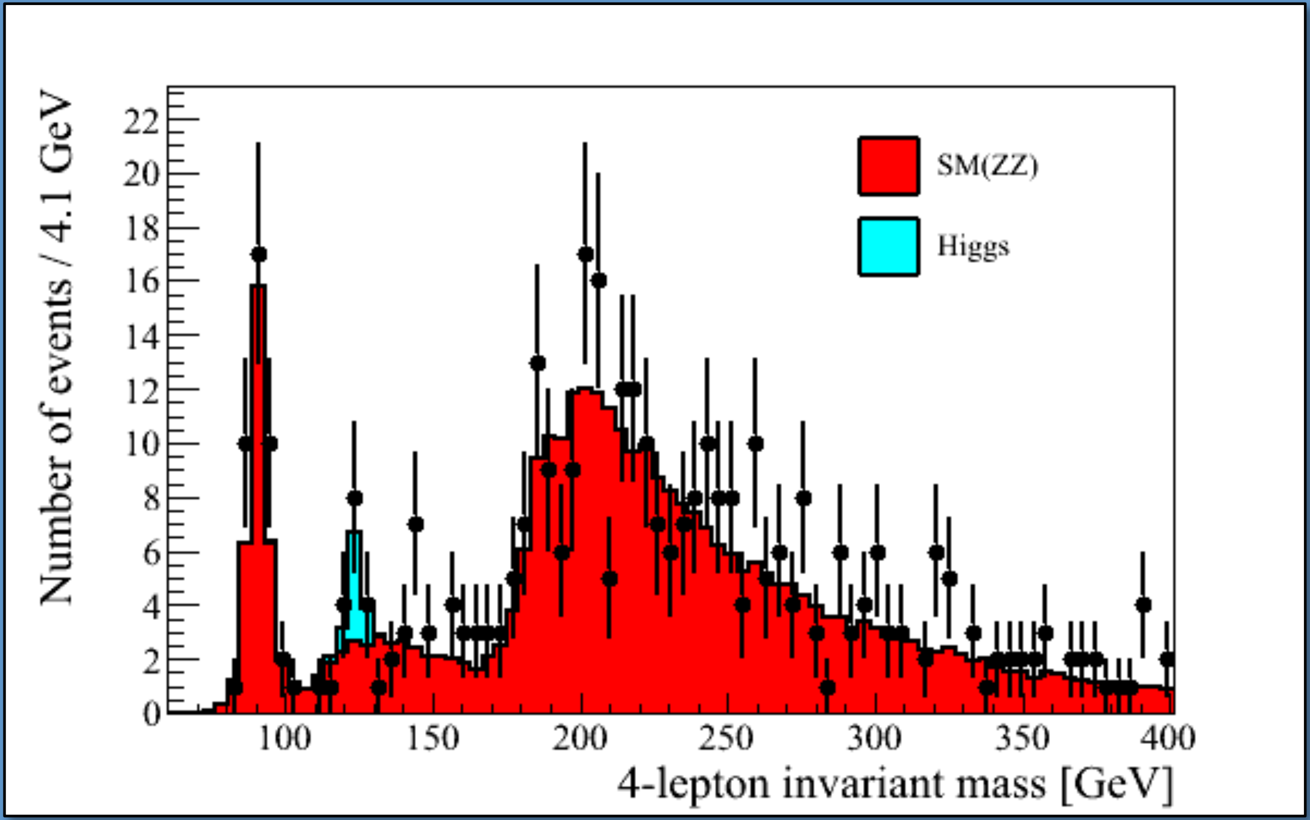
Significance optimization

Data-driven background estimate (likelihood fit using side bands)

Test statistic (Toy-MC)

Mass measurement

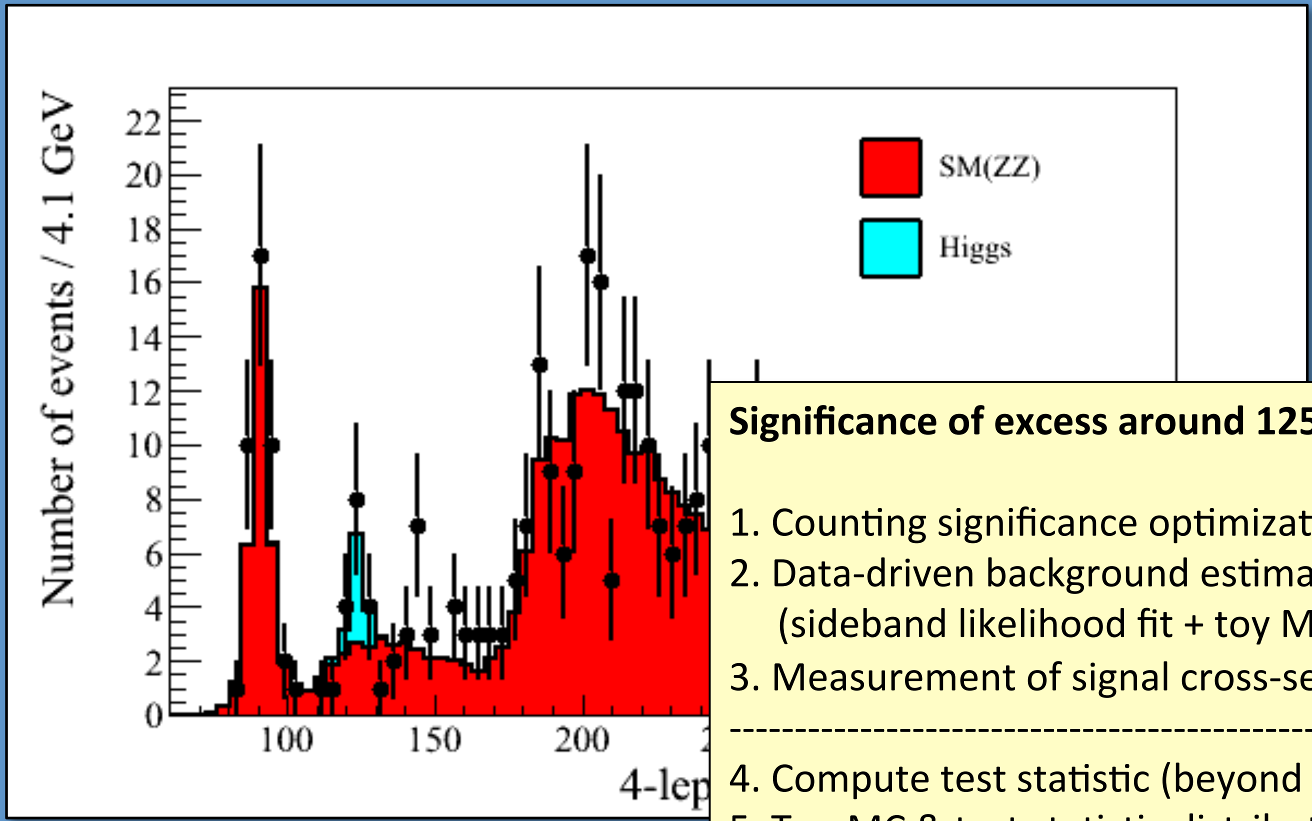
Data-set for the exercises: 4 lepton mass



Note: - Original histograms have 200 MeV bins

- This is fake data

Data-set for the exercises: 4 lepton mass



Significance of excess around 125 GeV

1. Counting significance optimization
2. Data-driven background estimate
   (sideband likelihood fit + toy MC Poisson)
3. Measurement of signal cross-section

----

4. Compute test statistic (beyond counting)
5. Toy-MC & test statistic distribution
6. Exclusion

----

7. Look elsewhere effect
8. Complex/correlated measurements

Basic material for the exercises:

1) Download tarball: **DesyCode2018.tgz**

2) Unpack everything: **tar –vzxf DesyCode2018.tgz**

a) Histograms_fake.root
4 histograms with the 4 lepton invariant mass (H125, H200, ZZ, data)

b) DESY_skeleton.C
Some skeleton code (different levels, as minimal as possible)

c) Rootlogon.C
Some standard Root blabla

Note: - skeleton is as empty as possible (on purpose)
- slides and exercise sheet from the school-website

DESY_skeleton.C

```
//======================
void MassPlot(int Irebin){
//======================
  // ------------------------------------------
  // Goal: produce SM+Higgs+data plot
  //       Note: rebinning is only for plotting
  // ------------------------------------------

  //-----------------------------------
  //-- Standard stuff and prepare canvas
  //-----------------------------------
  gROOT->Clear();
  gROOT->Delete();

  //-- Prepare canvas and plot histograms
  TCanvas * canvas1 = new TCanvas( "canvas1","Standard Canvas",600,400);
  canvas1->SetLeftMargin(0.125);
  canvas1->SetBottomMargin(0.125);
  canvas1->cd();

  //------------------------------------------------------------------
  //-- [1] Prepare histograms
  //--     o Get histograms from the files (signal, background and data)
  //--     o Make cumulative histograms (for signal and background)
  //------------------------------------------------------------------

  //-- Get histograms from the files (higgs, zz and data)
  TH1D *h_sig, *h_bgr, *h_data;
  h_sig  = GetMassDistribution(125);
  h_bgr  = GetMassDistribution(1);
  h_data = GetMassDistribution(2);

  //----------------------------------
  //-- [2] Plot histograms and make gif
  //--     o rebin histograms
  //--     o prepare cumulative histogram
  //--     o make plot + opsmuk + gif
  //----------------------------------

  //-- Rebin histograms (only for plotting)
  h_sig->Rebin(Irebin);
  h_bgr->Rebin(Irebin);
  h_data->Rebin(Irebin);

  //-- Prepare cumulative histogram for signal + background
  TH1D *h_sig_plus_bgr = (TH1D* ) h_bgr->Clone("h_sig_plus_bgr");
  h_sig_plus_bgr->Reset();
  for (int i_bin = 1; i_bin < h_bgr->GetNbinsX(); i_bin++){
       h_sig_plus_bgr->SetBinContent( i_bin, h_sig->GetBinContent(i_bin) + h_bgr->GetBinContent(i_bin));
       printf("  REBINNED HISTOGRAM:  bin %d, Ndata = %d\n",i_bin,(int)h_data->GetBinContent(i_bin));
  }

 //-- prepare histograms and plot them on canvas
 double Data_max = h_data->GetBinContent(h_data->GetMaximumBin());
 double Ymax_plot = 1.10* (Data_max + TMath::Sqrt(Data_max));
 h_sig_plus_bgr->SetFillColor(7);
 h_sig_plus_bgr->SetAxisRange(0.,Ymax_plot,"Y");
 h_sig_plus_bgr->SetAxisRange(0.,400.,"X");
 h_bgr->SetFillColor(2);
 h_sig_plus_bgr->Draw("hist");
 h_bgr->Draw("same");
 h_bgr->Draw("axis same");
 h_data->Draw("e same");

 //-- some nice axes and add legend
 AddText( 0.900, 0.035, "4-lepton invariant mass [GeV]",0.060, 0.,"right");                             // X-axis
 AddText( 0.040, 0.900, Form("Number of events / %3.1f GeV",h_bgr->GetBinWidth(1)) ,0.060,90.,"right"); // Y-axis
 TLegend *leg1 = new TLegend(0.65,0.65,0.90,0.85);
 leg1->SetBorderSize(0); leg1->SetFillColor(0);
 TLegendEntry *leg1a = leg1->AddEntry(h_bgr,          " SM(ZZ)", "f");  leg1a->SetTextSize(0.04);
 TLegendEntry *leg1b = leg1->AddEntry(h_sig_plus_bgr, " Higgs" , "f");  leg1b->SetTextSize(0.04);
 leg1->Draw();

 //-- prepare gif
 canvas1->Print(Form("./MassPlot_rebin%d.gif",Irebin));

 return;

  //==============
} // end MassPlot()
  //==============
```

Define canvas

Get histograms from root-file

Rebin histograms

Print bin content

Make cumulative histogram

Histogram characteristics & plot

Add text

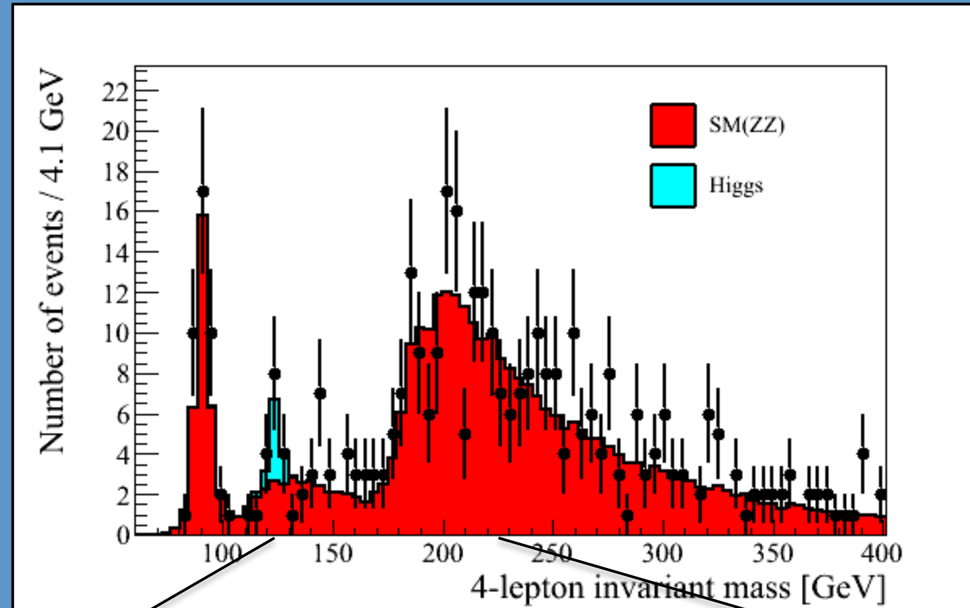
Save plot as gif in your directory

Create the 4-lepton mass plot

```
root> .L DESY_skeleton.C++
root> MassPlot(20)
```

↓ Rebin-factor

hist: h_bgr, h_sig, h_data



Summary in signal mass region

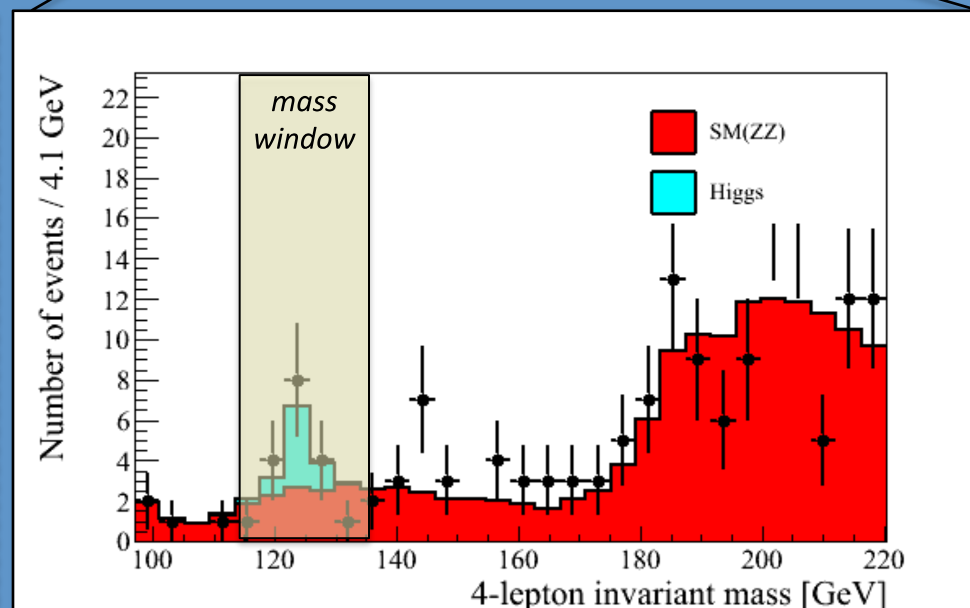
(using 200 MeV bin and 10 GeV window)

Ndata = 16

Nbgr = 6.42

Nsig = 5.96

Exercises: significance



Information required for exercises

significance

10-slide mini lecture on significance:

- discovery and exclusion -

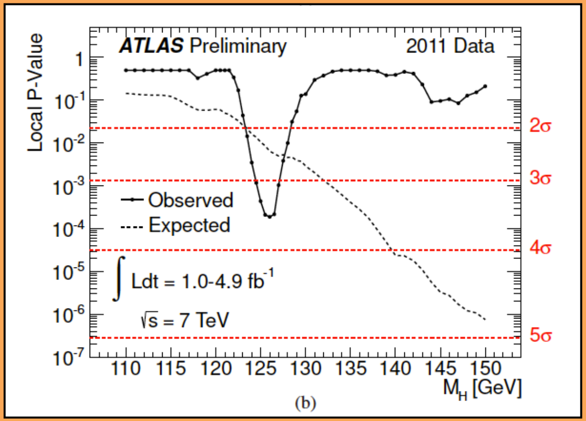
fitting

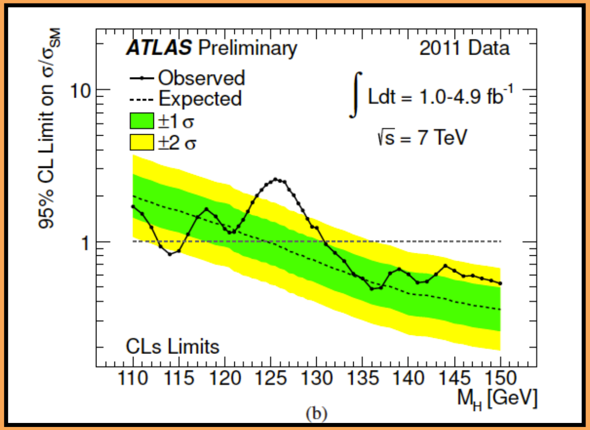
10-slide mini lecture on fitting

- Likelihood fits and uncertainties -

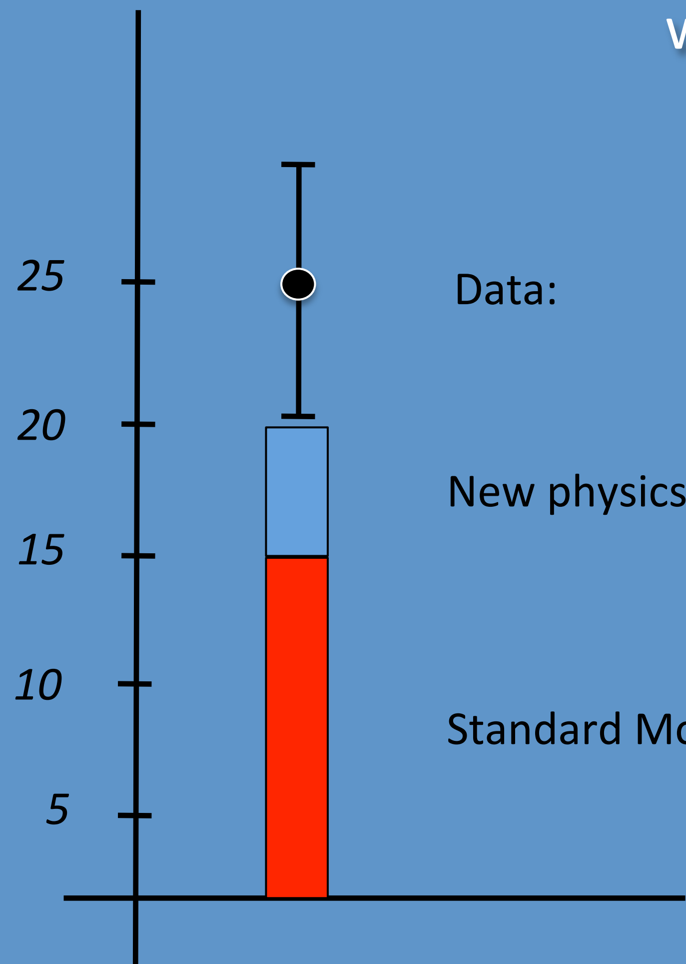
10-slide mini lecture on significance:

- discovery and exclusion -





General remark : what is the significance ?



Data: $N_{data} = 25$

New physics: $N_{sig} = 5$

Standard Model: $N_{bgr} = 15$

Significance for N events: probability to observe N events (or even more) under the background-only hypothesis

Observed significance:

$$\int_{25}^{\infty} \text{Poisson}(N \mid 15)\, dN = 0.0112$$ ← *p-value*

$$= 2.28 \text{ sigma}$$ ← *significance*

Expected significance:

$$\int_{20}^{\infty} \text{Poisson}(N \mid 15)\, dN = 0.1248$$

$$= 1.15 \text{ sigma}$$

Discovery if p-value < 2.87×10^{-7}

→ 39 events

Poisson distribution

The Poisson distribution

Binomial with n→∞, p →0 and np=λ

$$P(n \mid \lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$$

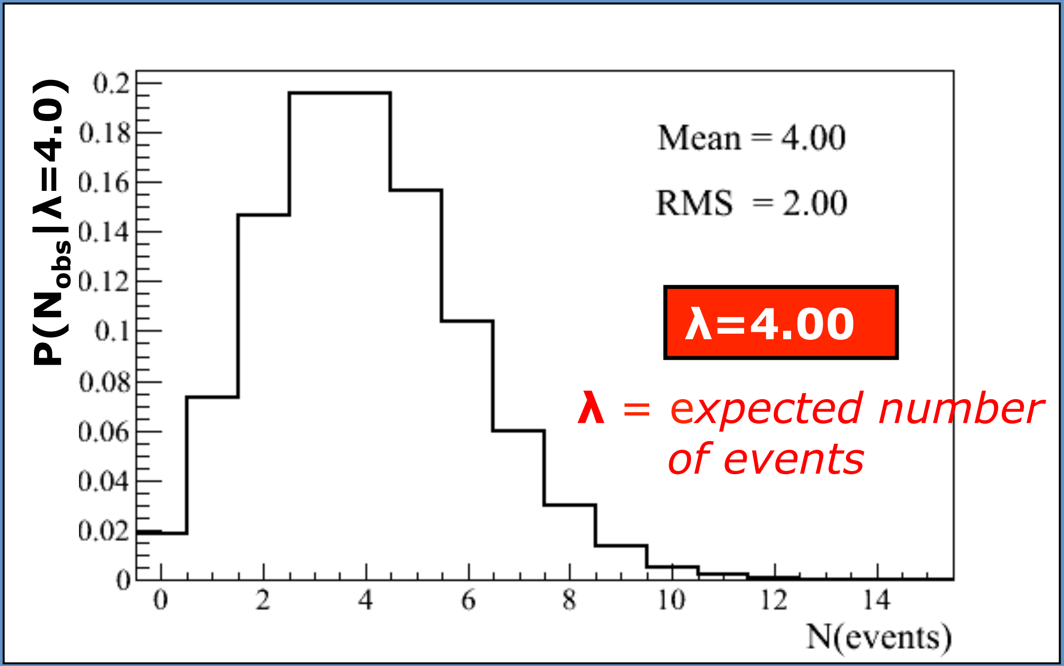
Poisson distribution

Probability to observe n events when λ are expected

$$P(0 \mid 4.0) = 0.01832$$
$$P(2 \mid 4.0) = 0.14653 \quad !$$
$$P(3 \mid 4.0) = 0.19537$$
$$P(4 \mid 4.0) = 0.19537$$
$$P(6 \mid 4.0) = 0.10420 \quad !$$

#observed — **varying**

λ hypothesis — **fixed**



the famous √N

Properties Poisson distribution

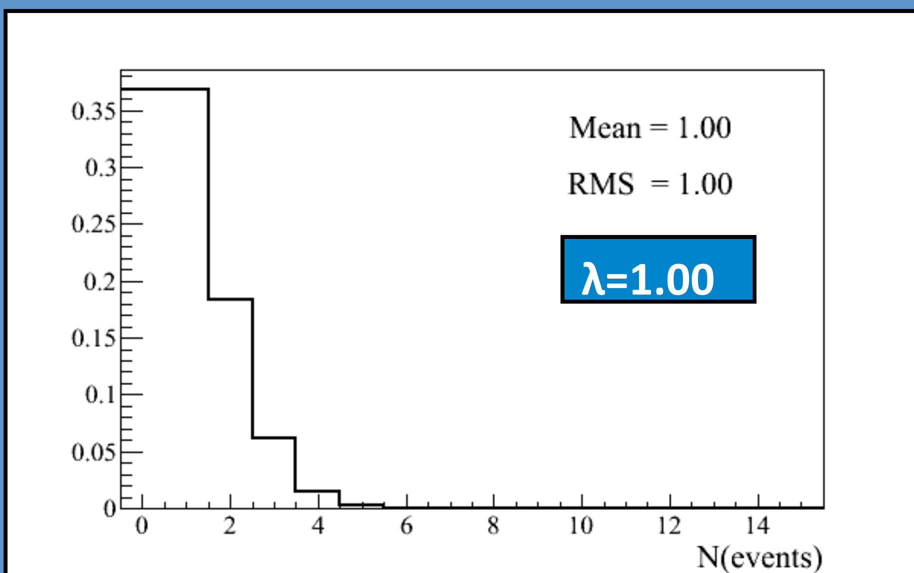
(1) Mean: $\langle n \rangle = \lambda$

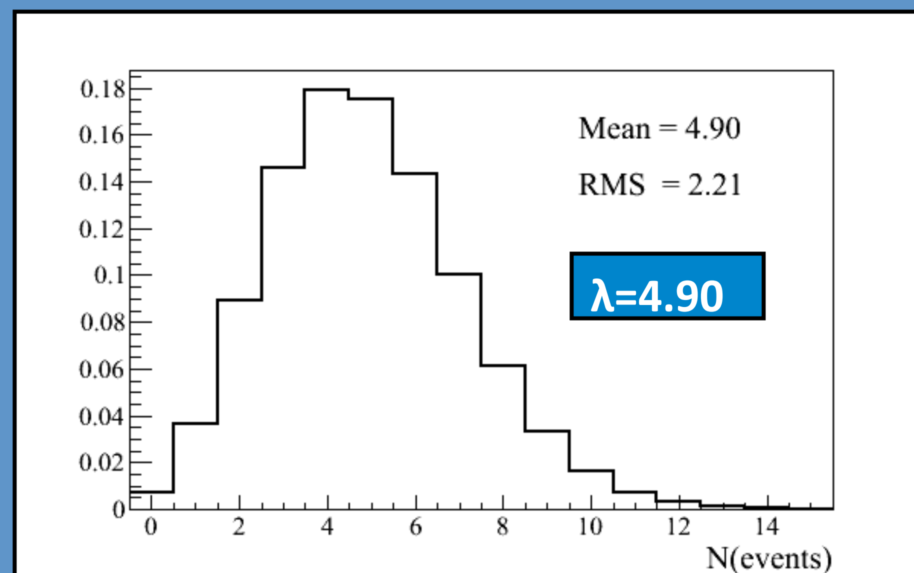
(2) Variance: $\langle (n - \langle n \rangle)^2 \rangle = \lambda$

(3) Most likely: first integer ≤ λ

Usual way to represent the error on a data-point

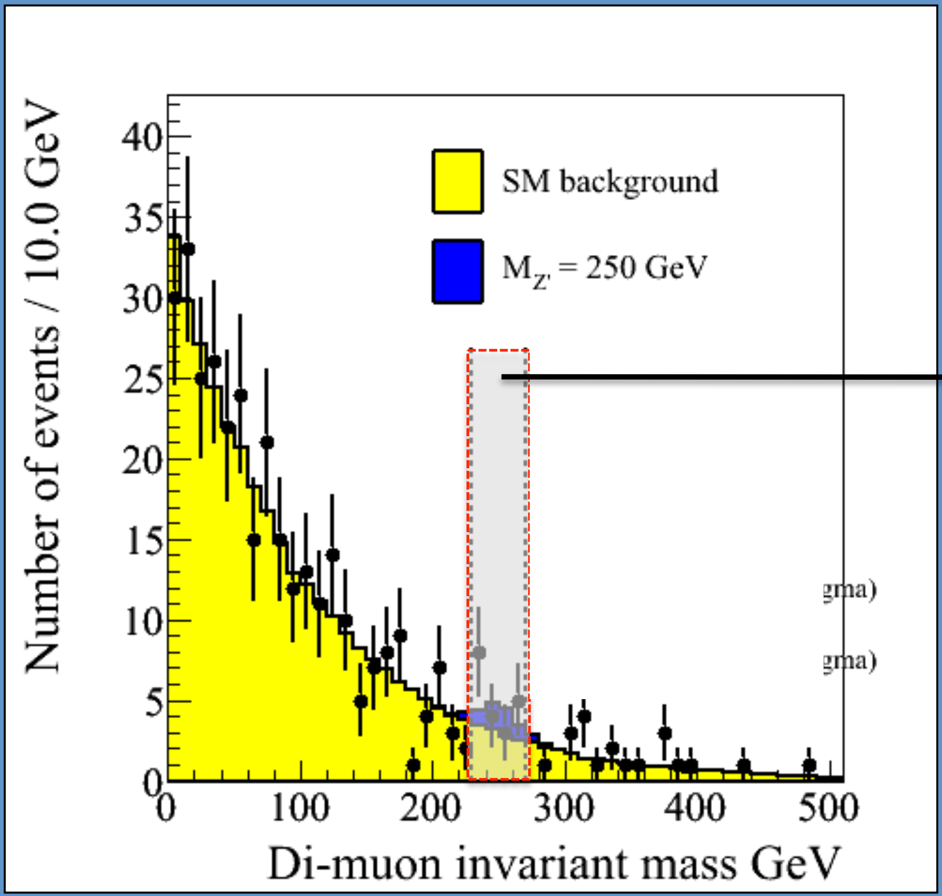
Not default in Root





Significance example

Counting events in a mass window



SM	**10**
Higgs	5
Data	12

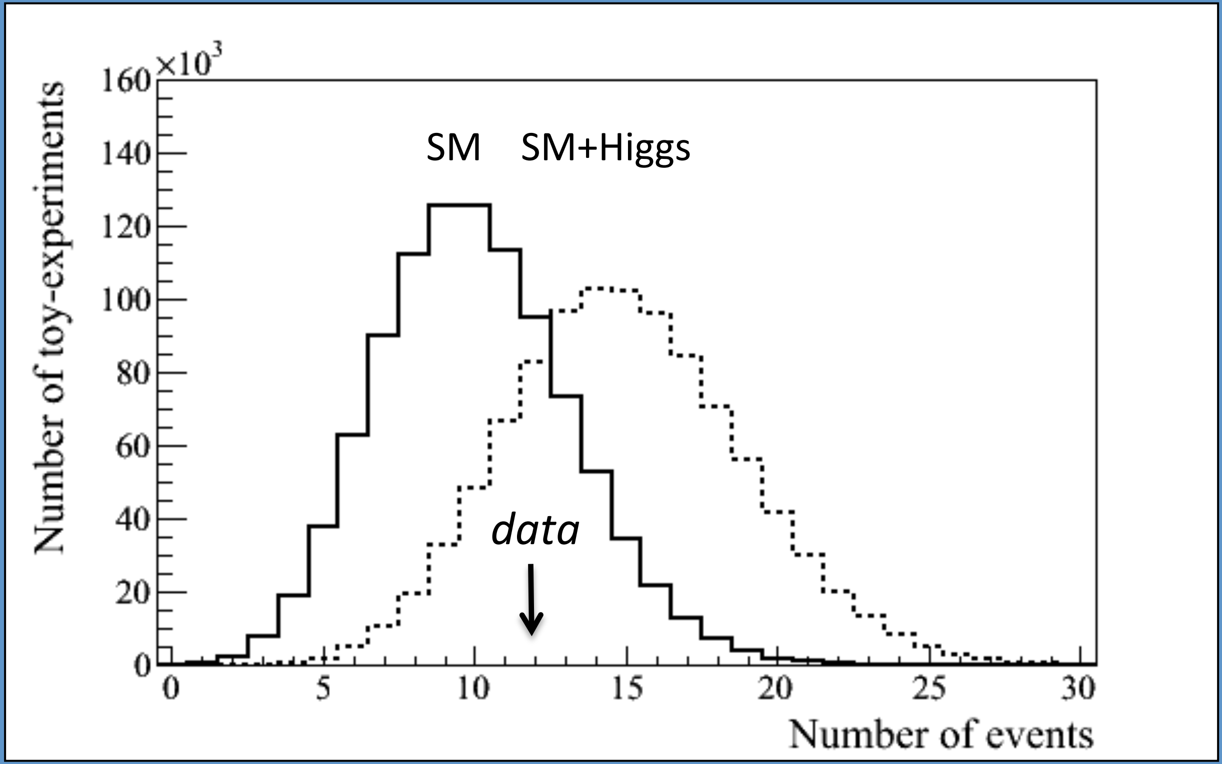
Ok, now what ?

Next slides:
- *discovery*
- *exclusion*

Poisson distribution

SM	10
Higgs	5
Data	12

Ok, now what ?



Significance for N events: probability to observe N events (or even more) under the background-only hypothesis

Interpretation

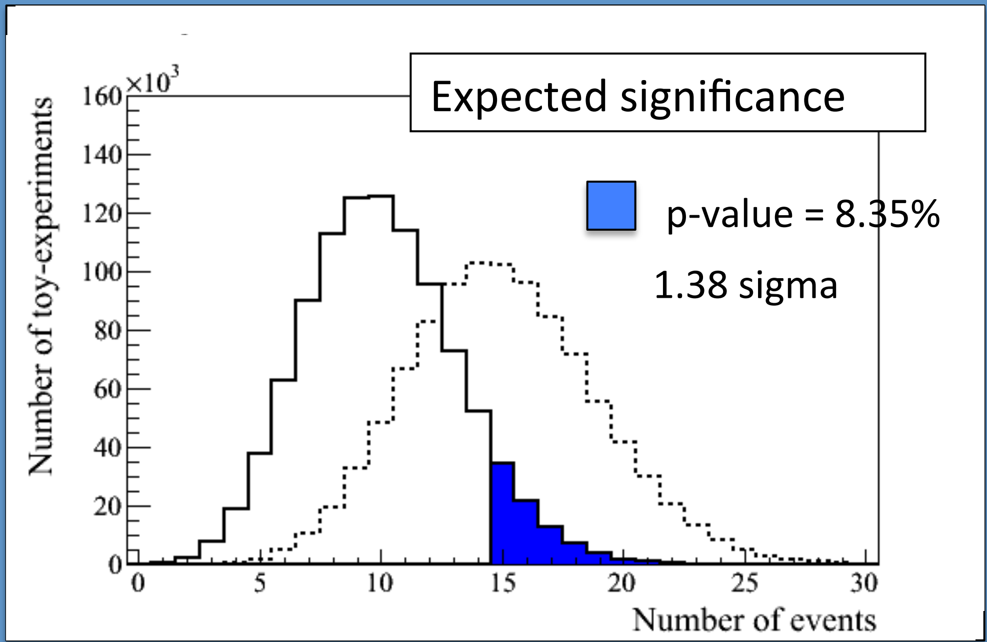
optimistic: discovery

Incompatibility with SM-hypothesis

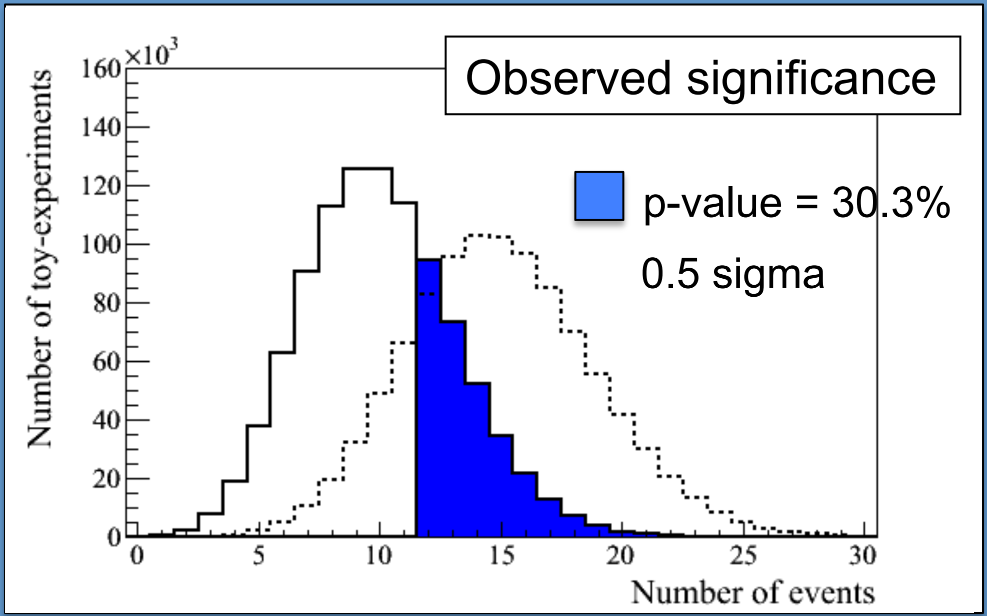
p-value: incompatibility with SM-only hypothesis

SM	**10**
Higgs	5
Data	12

1) What is the ***expected*** *significance ?*

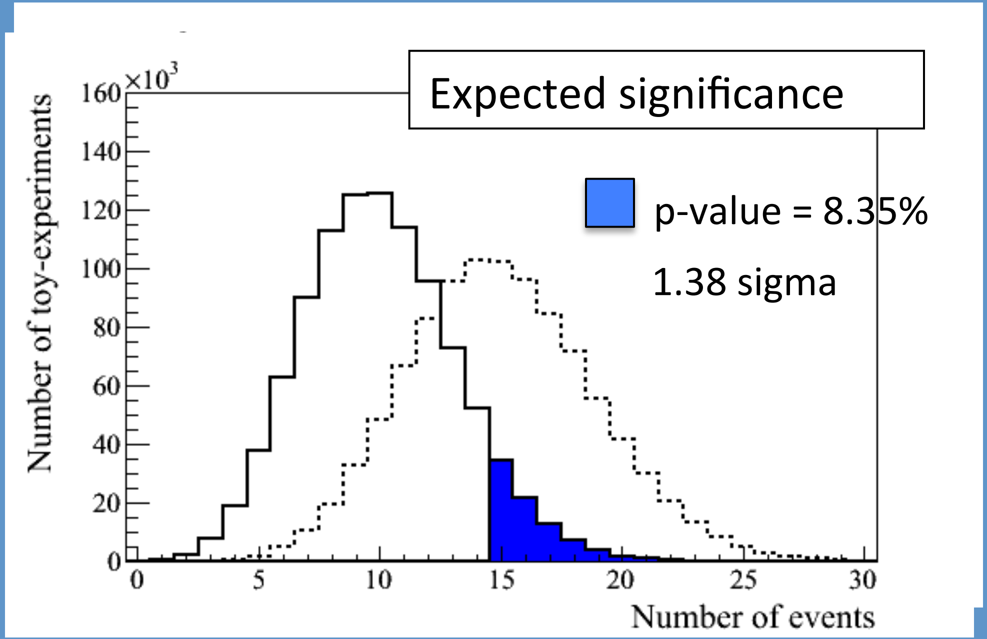


2) What is the ***observed*** *significance ?*



p-value: incompatibility with SM-only hypothesis

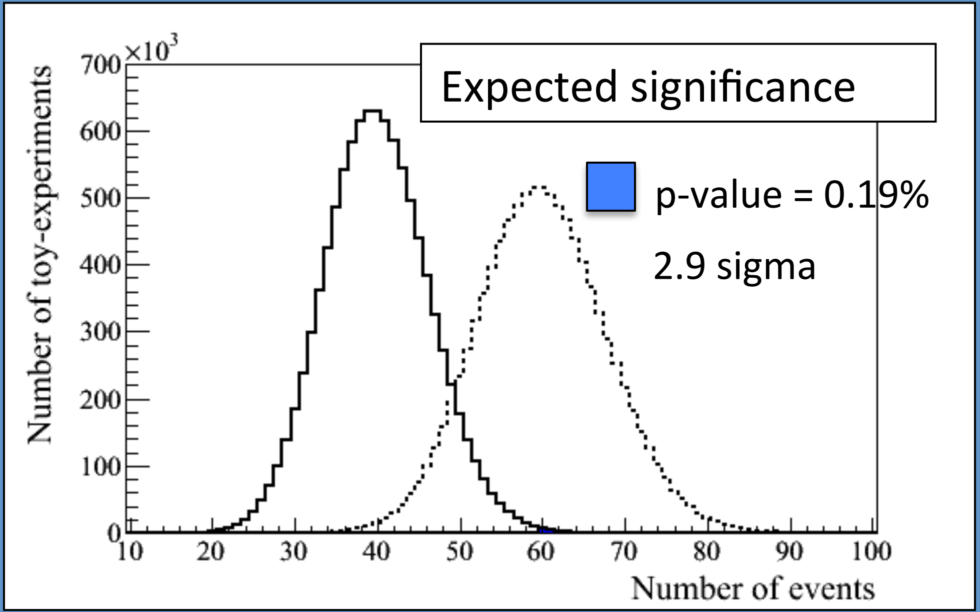
SM	**10**
Higgs	5



3) At what Lumi do you expect to be able to claim a discovery ?

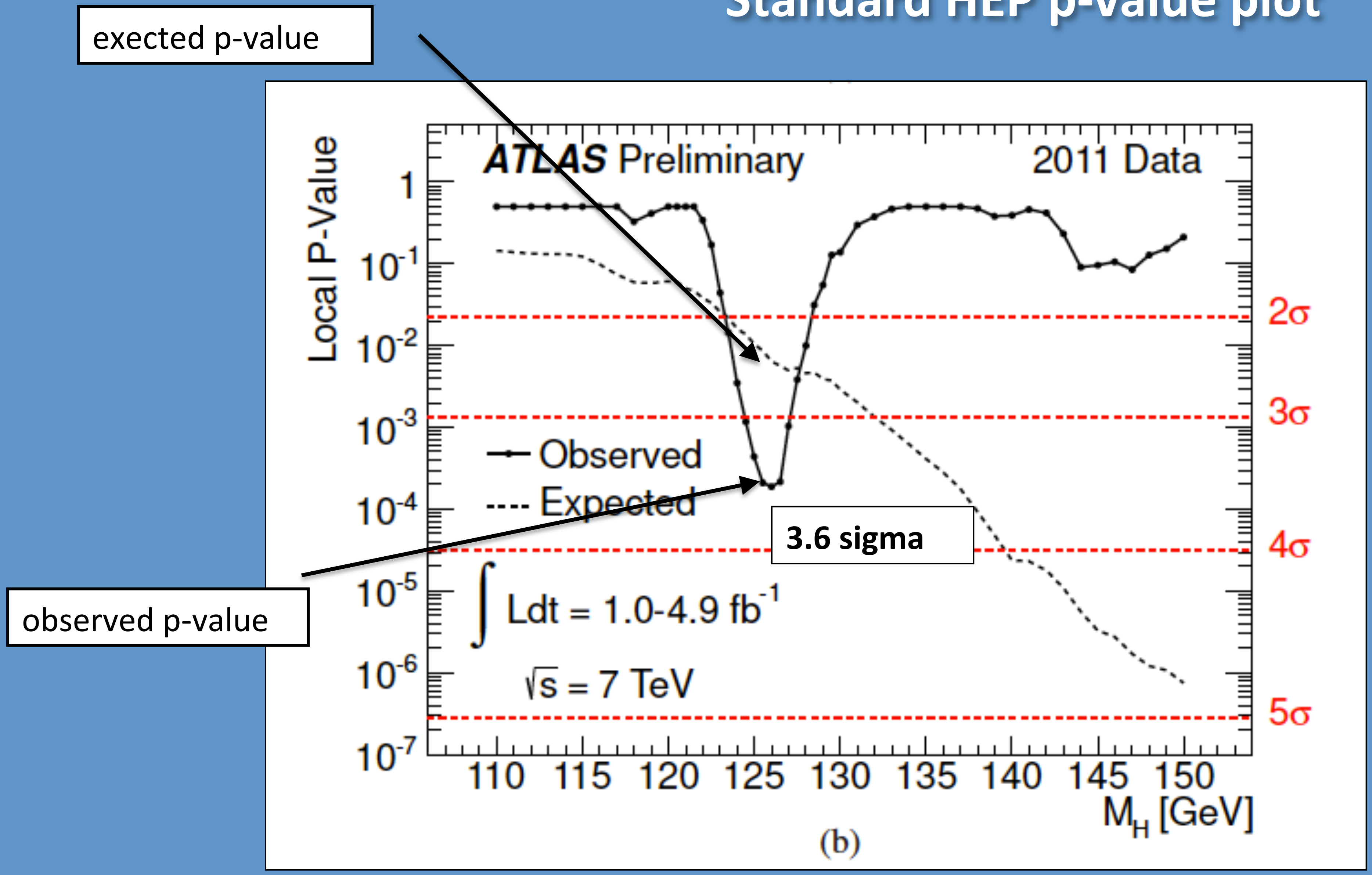
3 TIMES MORE LUMINOSITY

SM	**30**
Higgs	15



Discovery if p-value < 2.87×10^{-7}

Standard HEP p-value plot



Interpretation

pessimistic: exclusion

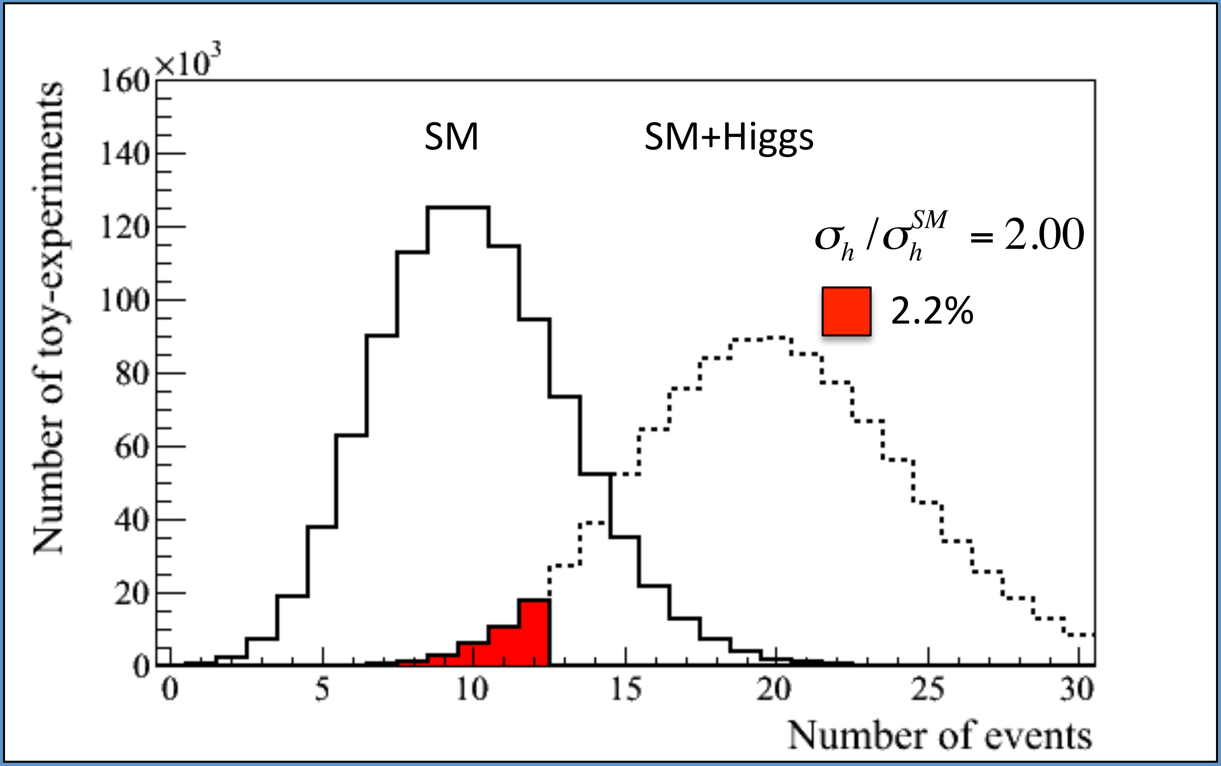
Incompatibility with New Physics-hypothesis

Excluding a signal:
Incompatibility with s+b hypothesis

SM	10
Higgs	5
Data	12

Can we exclude the SM+Higgs hypothesis ?

What σ_h/σ_h^{SM} can we exclude ?



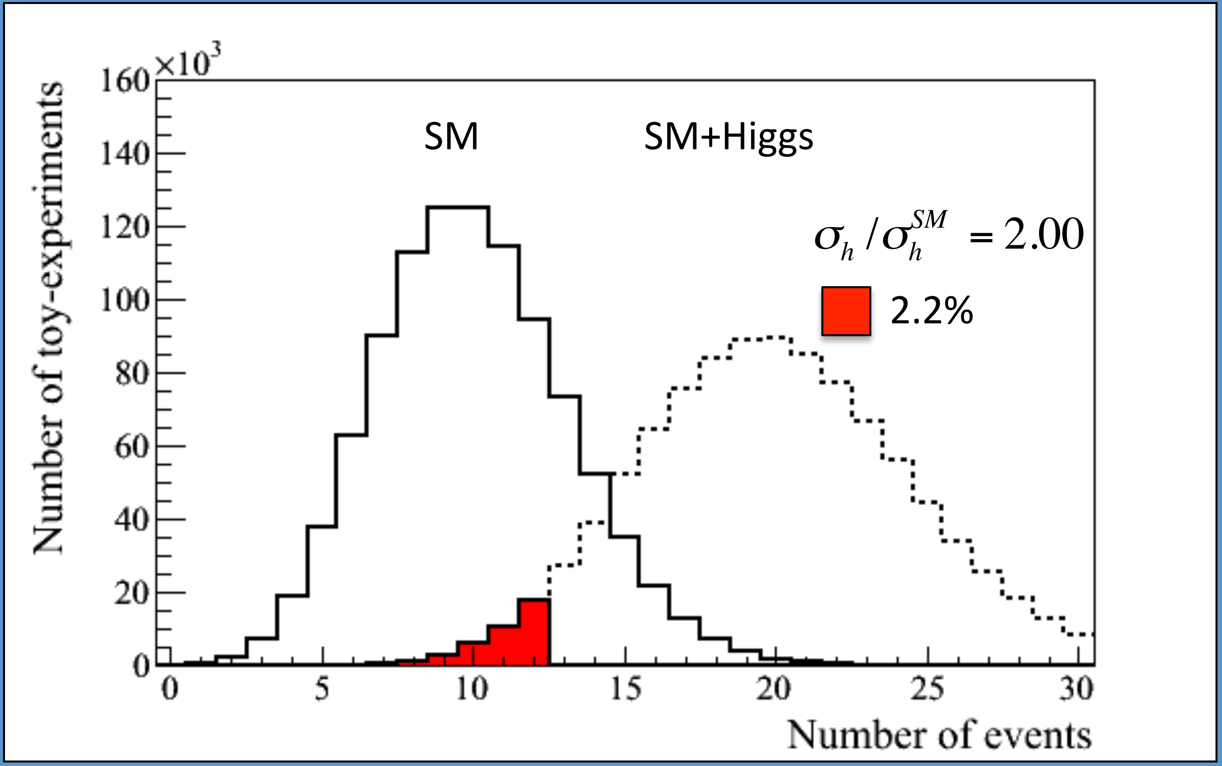
Exclusion: probability to observe N events (or even less) under the signal + background hypothesis

Excluding a signal: Incompatibility with s+b hypothesis

SM	10
Higgs	5
Data	12

Can we exclude the SM+Higgs hypothesis ?

What σ_h/σ_h^{SM} can we exclude ?



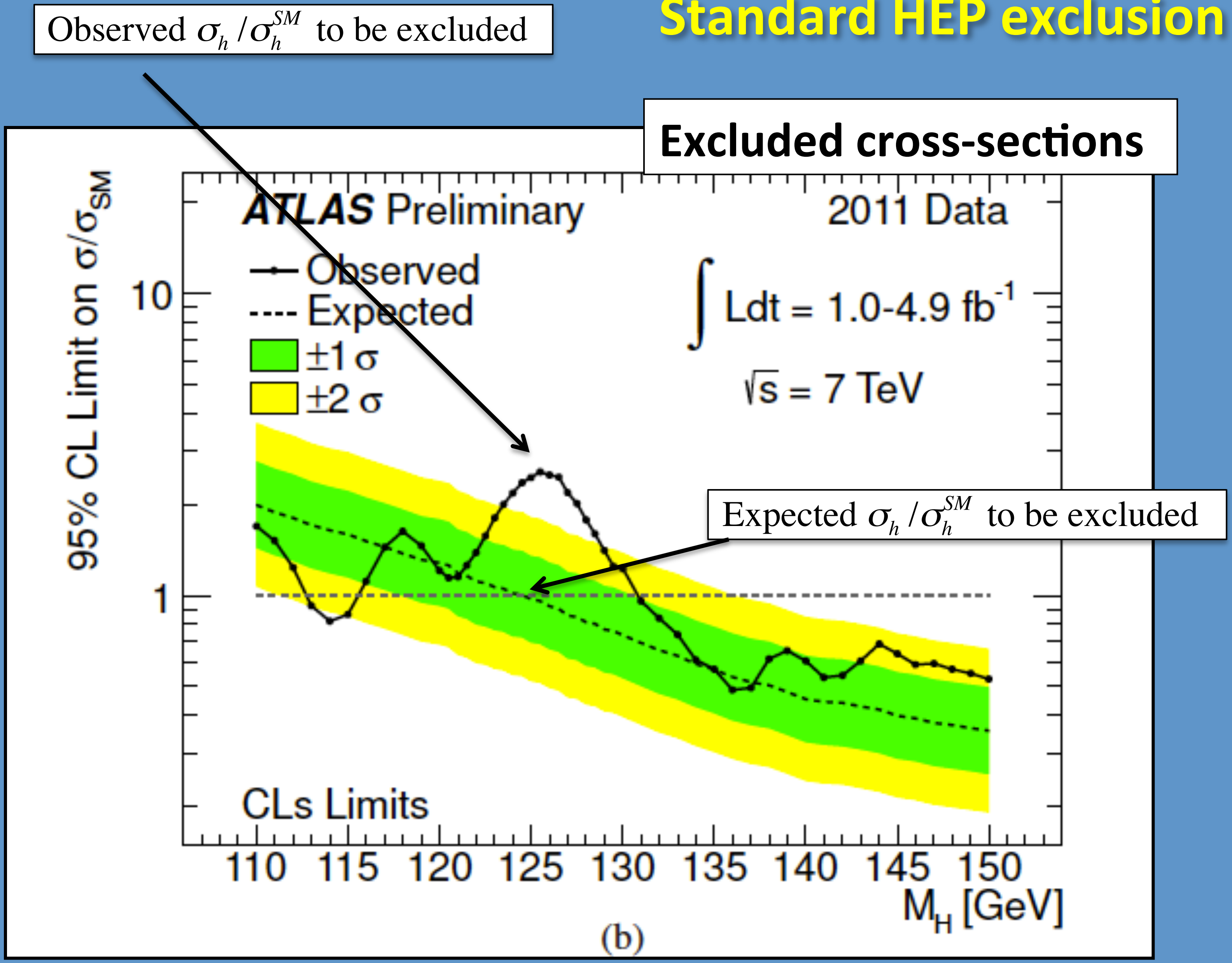
σ/σ_{SM}	SM	# data	SM+Higgs	
1.0	10	12	15.0	18.5 %
1.5	10	12	17.5	6.8%
2.0	10	12	20.0	2.2%

excluded

Expected exclusion ? Use mean SM instead of Ndata

Observed excluded cross-section, σ_h/σ_h^{SM} , = 1.64

Standard HEP exclusion plot



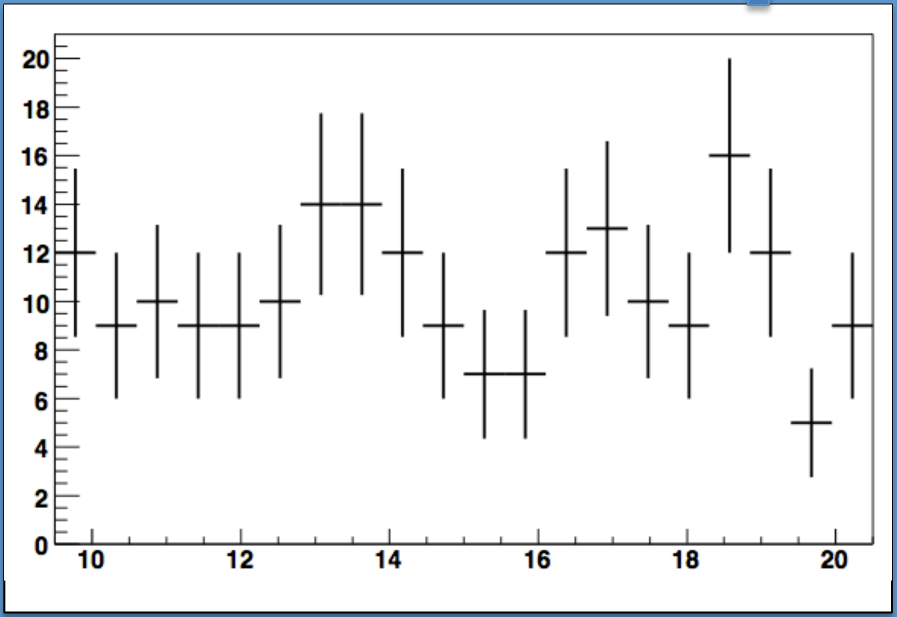
(b)

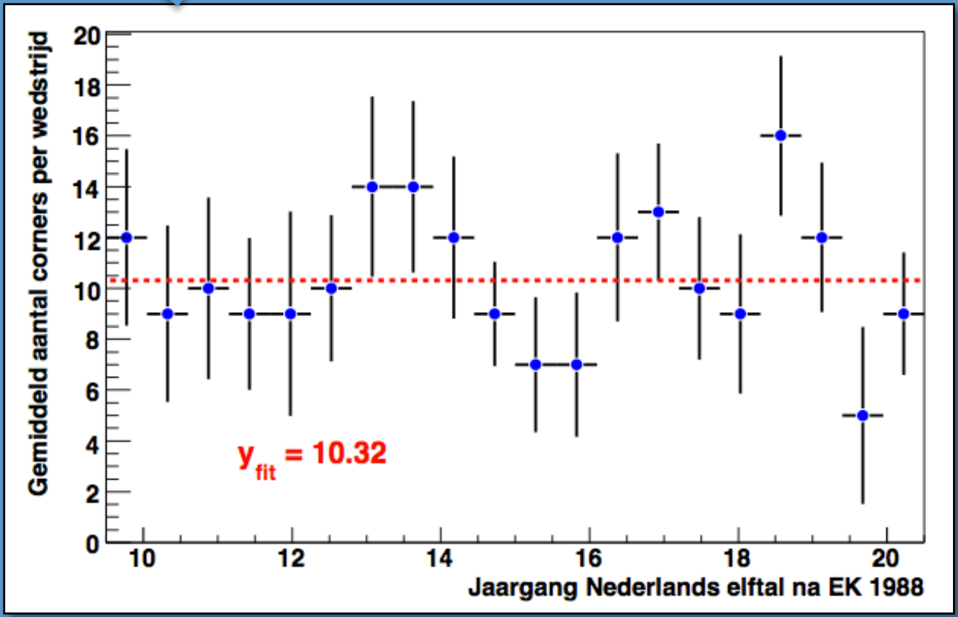
10-slide mini lecture on fitting

- Likelihood fits and uncertainties -

Simple likelihood fit

Can everybody do this ?



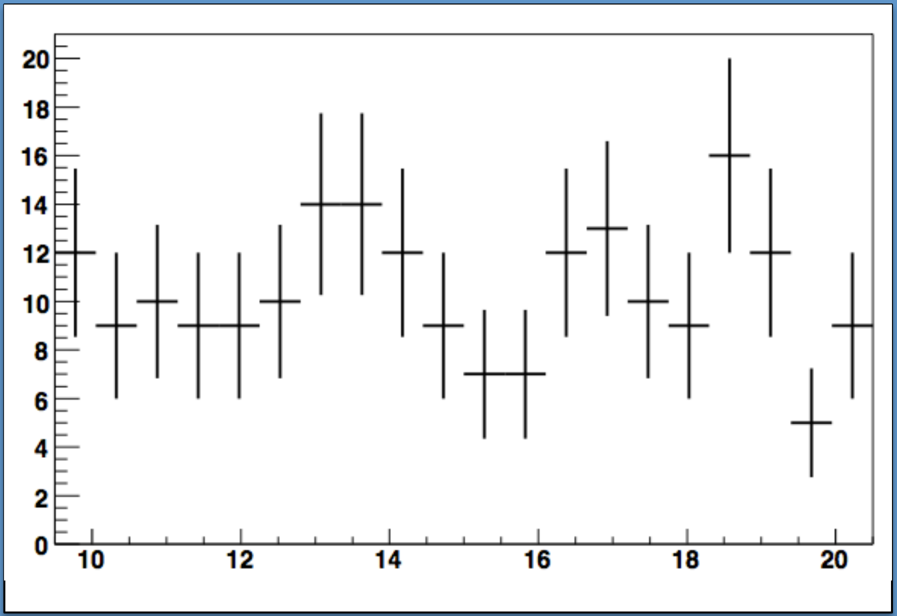


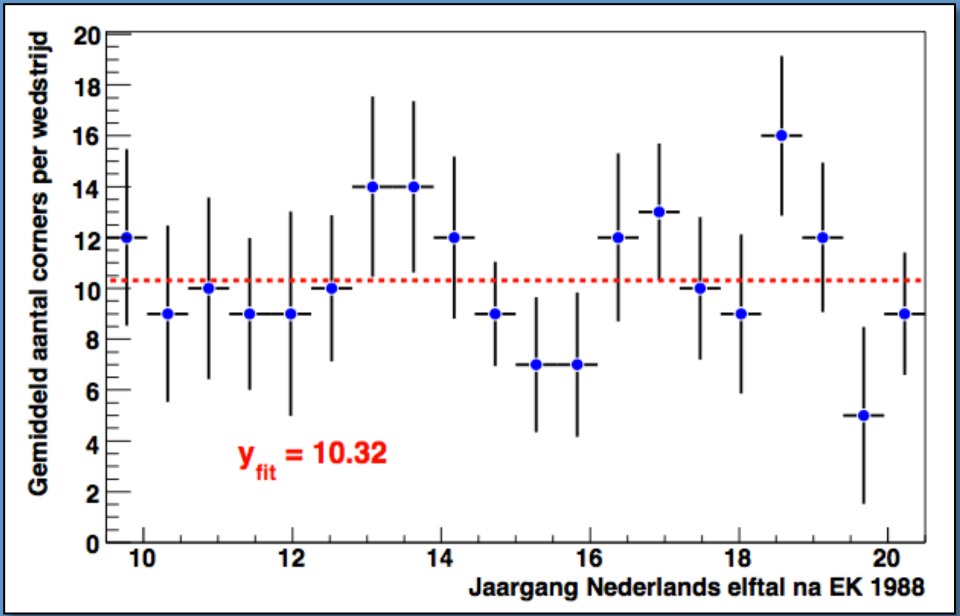
If you want to reproduce this plot, but cannot please let me know

TMath::Poisson(Nevt_bin, alpha)

http://www.nikhef.nl/~ivov/SimpleFit/

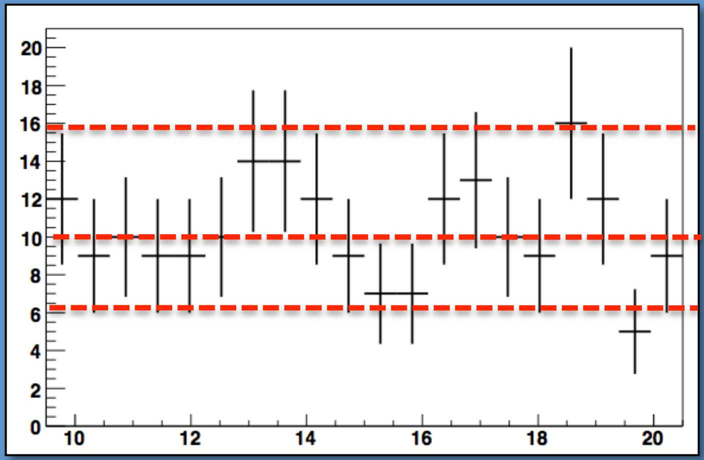
http://www.nikhef.nl/~ivov/SimpleFit/





TMath::Poisson(Nevt_bin, alpha)

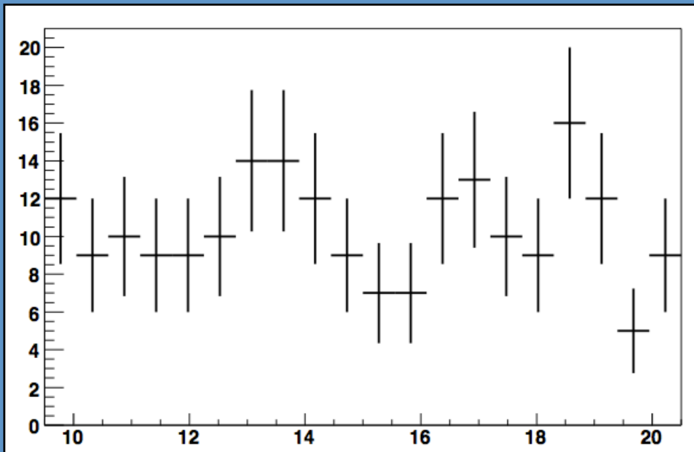
Fitting in 1 slide



You model: $f(x) = \lambda$

Try different values of λ and for each one compute ***compatibility*** of the model with the data

Fitting in 1 slide



You model: $f(x) = \lambda$

Try different values of λ and for each one compute ***compatibility*** of the model with the data

χ^2-fit

Metric:

$$\chi^2 = \sum_{bins} \frac{\left(N_{bin}^{data} - \lambda_{bin}^{expected}\right)^2}{N_{bin}^{data}}$$

Best value:
Value of λ that minimizes χ^2 (χ_{min}^2)

Errors:
Values of λ for which $\chi^2 = \chi_{min}^2 + 1$

Likelihood-fit

Metric:

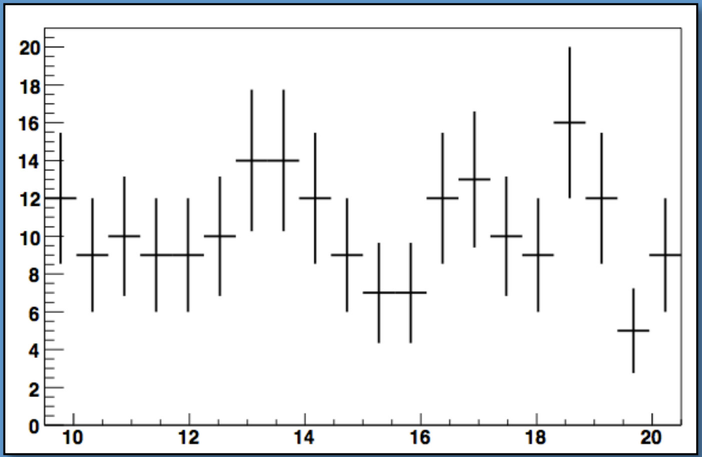
$$-2\log(L) = -2 \cdot \sum_{bins} \log(Poisson(N_{bin}^{data} \mid \lambda))$$

TMath::Poisson(Nevt_bin, λ)

Best value:
Value of λ that minimizes -2Log(L) ($-2\log(L)_{min}$)

Errors:
Values of λ for which 2Log(L) = ($-2\log(L)_{min}$) +1

Fitting in 1 slide



You model: $f(x) = \lambda$

Try different values of λ and for each one compute ***compatibility*** of the model with the data

Recipe for each value of λ:

- Set LogLik = 0
- Loop over all bins:
  - o For each bin: compute prob. to observe N_i evts when you expect λ. **Poisson distribution**
  - o take -2*Log of bin-probability
  - o Add to existing LogLik

→ Output LogLik (1 number)

Likelihood-fit

Compatibility number :

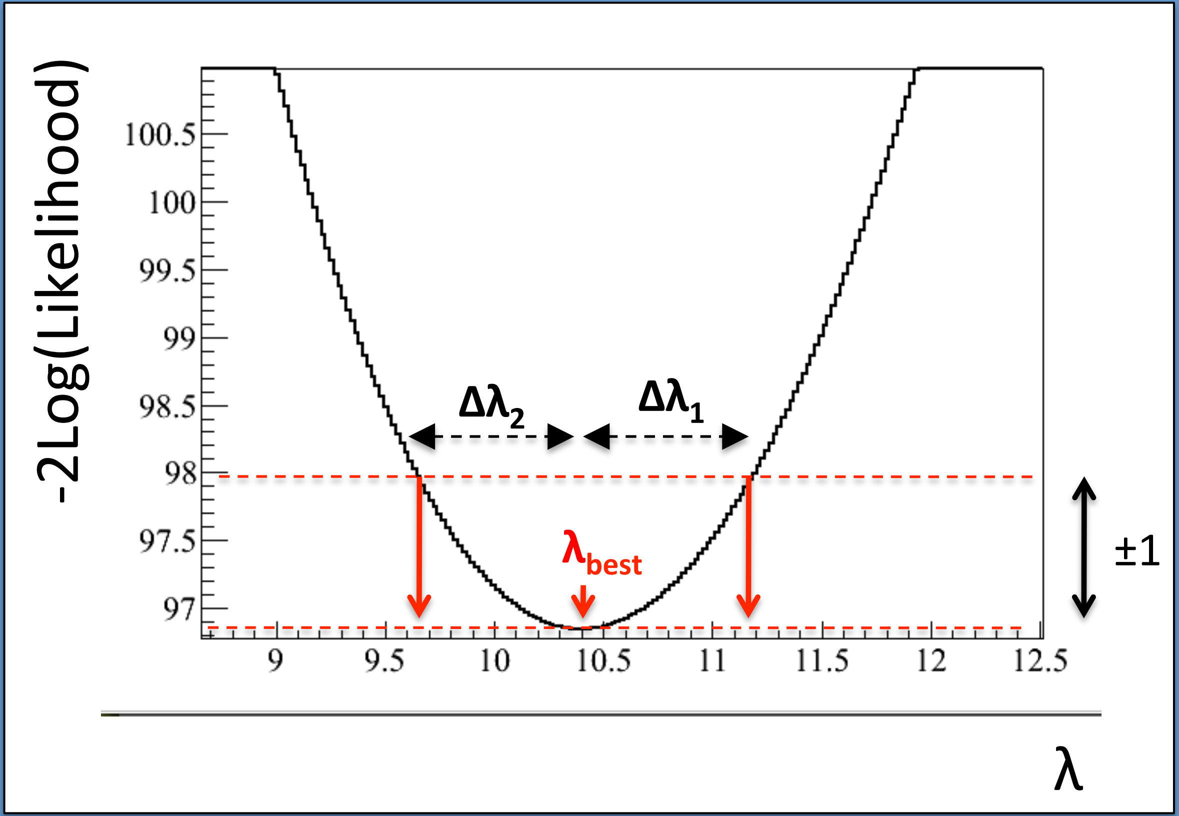
$$-2\log(L) = -2\cdot\sum_{bins}\log(Poisson(N_{bin}^{data} \mid \lambda))$$

TMath::Poisson(Nevt_bin, λ)

Best value:
Value of λ that minimizes -2Log(*L*) (-2log(*L*)$_{min}$)

Errors:
Values of λ for which 2Log(*L*) = (-2log(*L*)$_{min}$) +1

Result from the fit



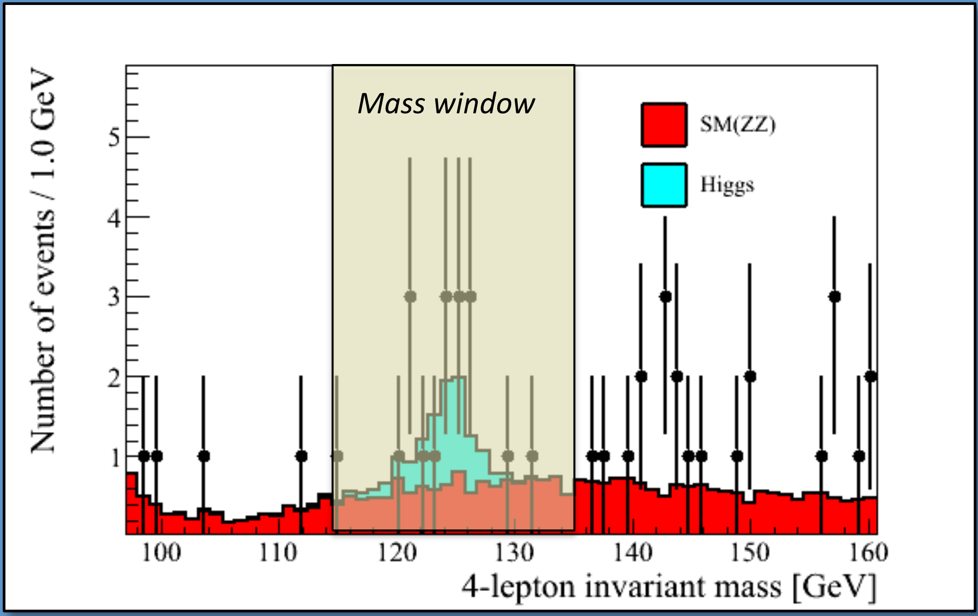
$$\text{result}: \quad \lambda = \lambda_{\text{best}}{}^{+\Delta\lambda_1}_{-\Delta\lambda_2}$$


Exercises
PART 1

Exercise 1:
significance optimization

Exercise 1:

Optimizing the counting experiment



Code you could use:

```
IntegratePoissonFromRight()

Significance_Optimization()
```

Exercise 1: significance optimization of search window (Poisson counting)

1.1 Find the window that optimizes the expected significance

1.2 Find the window that optimizes the observed significance (and never do it again)

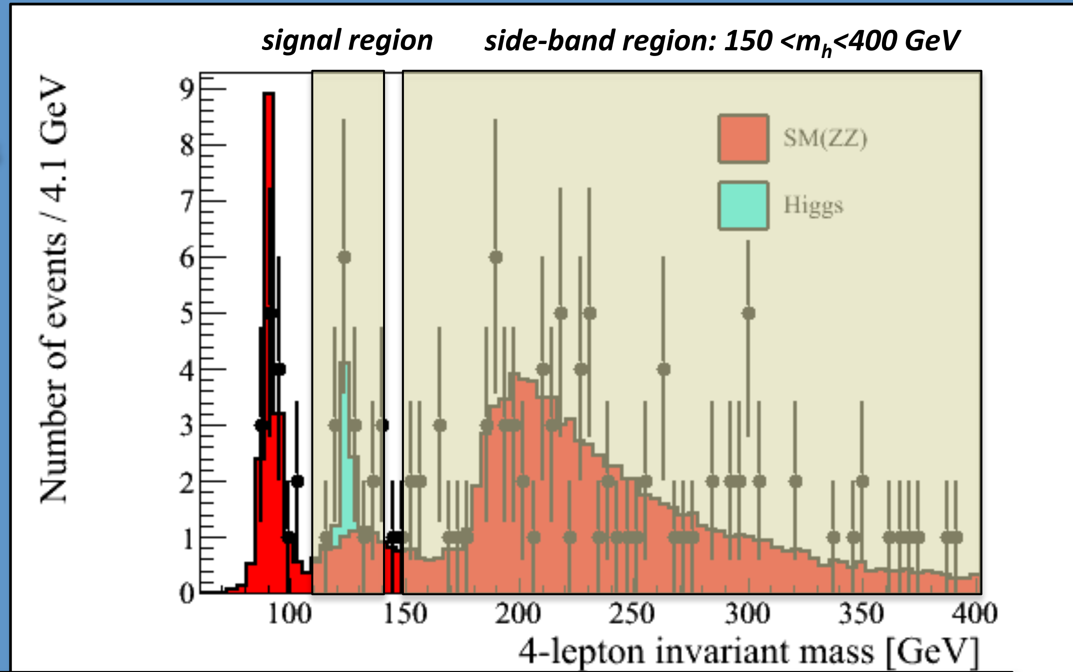
1.3 Find the window that optimizes the expected significance for 5x higher luminosity

1.4 At what luminosity do you expect to be able to make a discovery ?

Exercise 2:
background estimate, side-band fit

Exercise 2:

Data driven bkg estimate in 10 GeV ,mass window or optimal one from Exercise 1



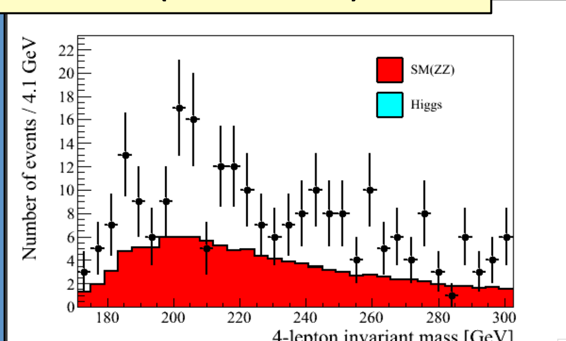
Code you could use:

```
SideBandFit()
```

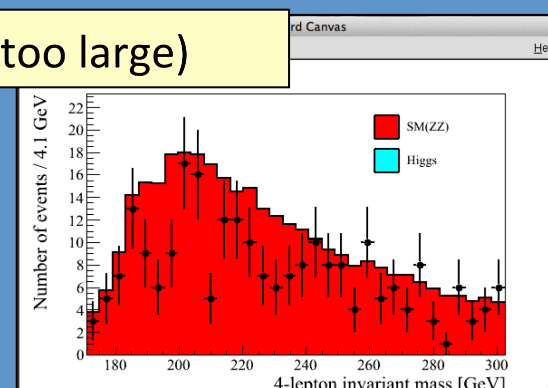
Exercise 2: background estimation from side-band fit

2.1 What is the optimal scale-factor for the background (α) ?
Do a likelihood fit to the side-band region $150 \leq m_h \leq 400$ GeV

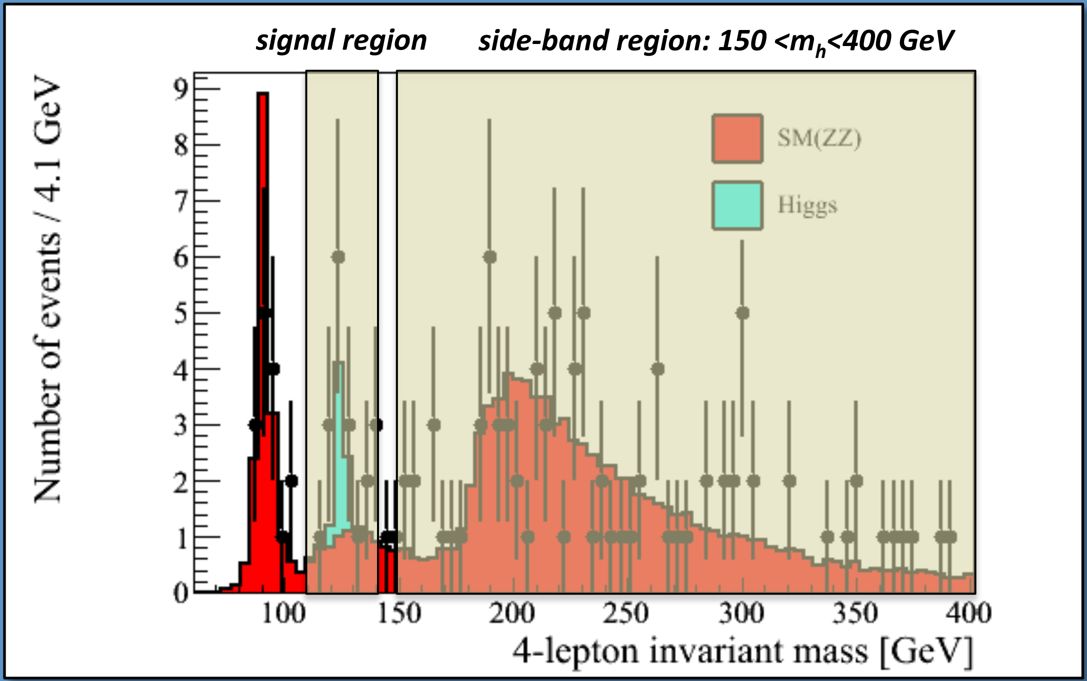
α = 0.50 (too small)



α = 1.50 (too large)



Exercise 2:
continued



Code you could use:

```
SideBandFit()
```

Exercise 2: significance optimization of mass/search window (use Poisson counting)

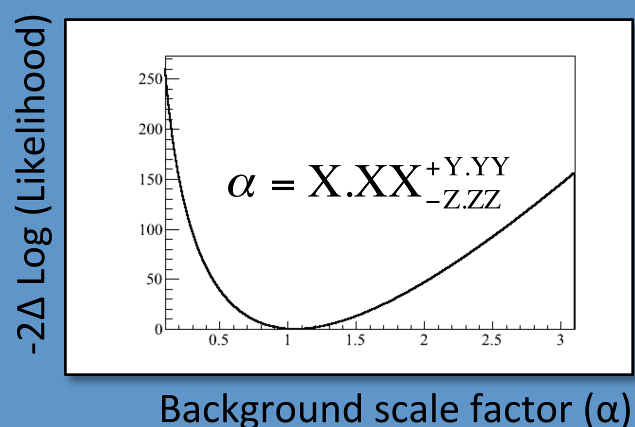
2.1 What is the optimal scale-factor for the background (α) ?

Do a likelihood fit to the side-band region $150 \leq m_h \leq 400$ GeV

Computing the likelihood:

For each 'guess' of α:

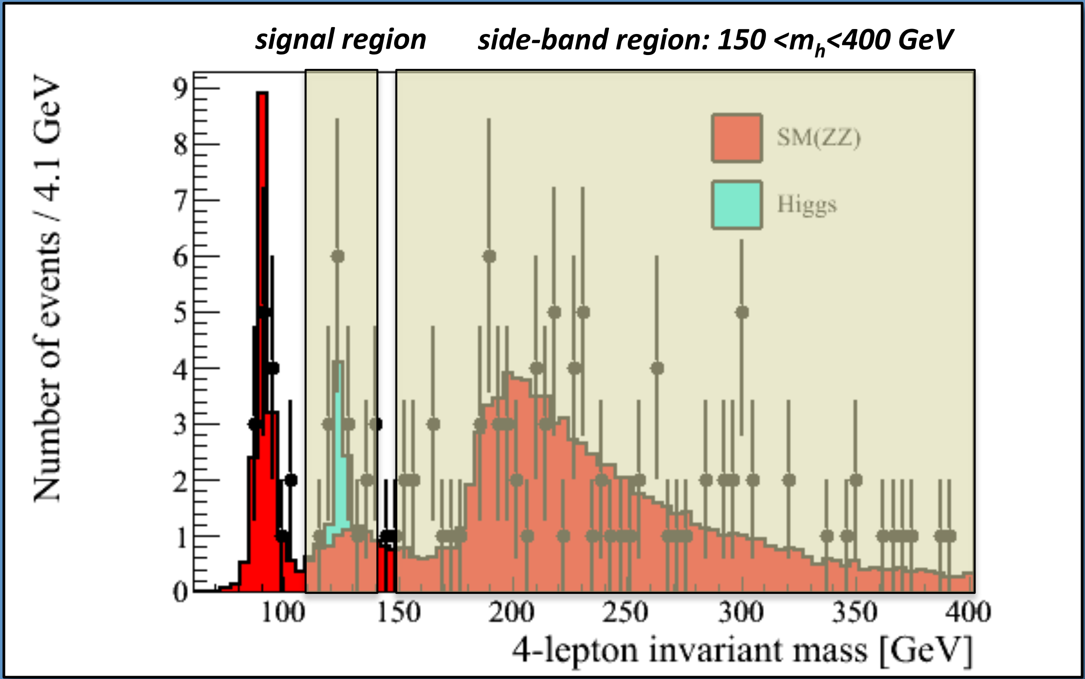
$$-2\log(L) = -2\cdot \sum_{bins} \log(Poisson(N_{bin}^{data} \mid \alpha \cdot f_{bin}^{SM}))$$



Exercise 2:
continued

Code to use:

None



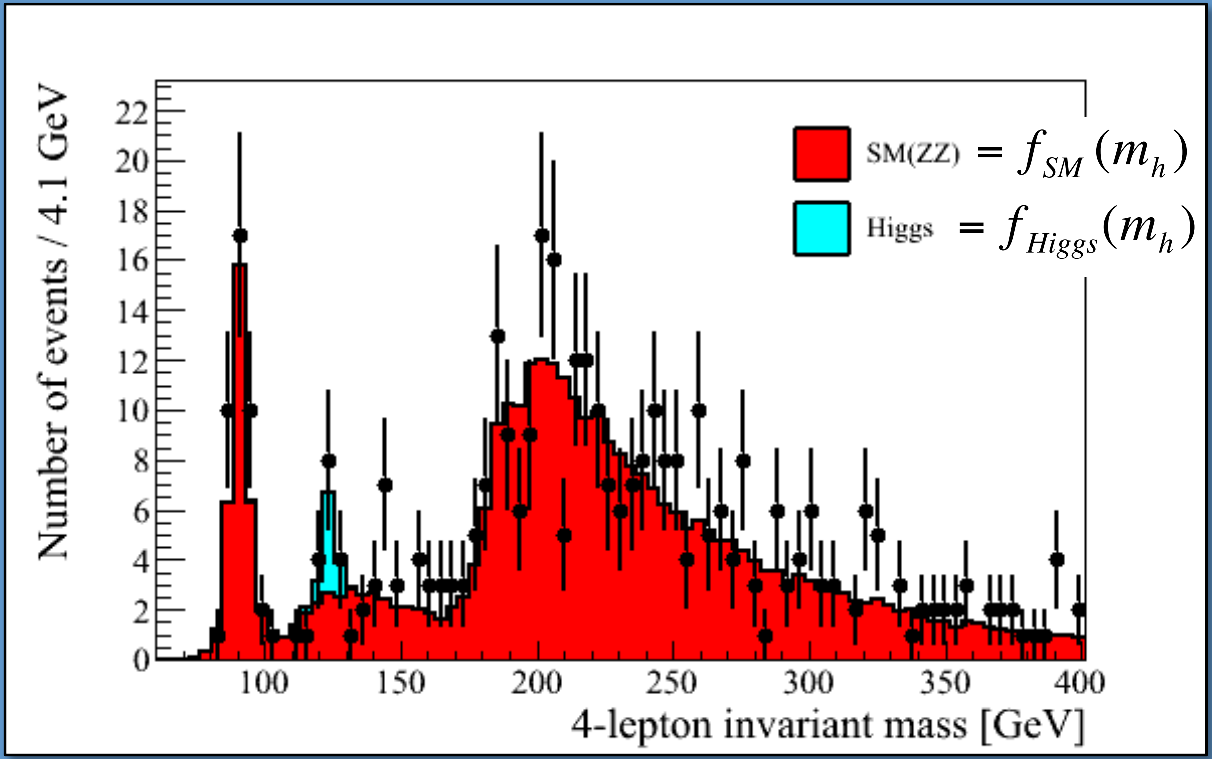
2.2 Estimate background and its uncertainty $b \pm \Delta b$ in the mass window around 125 GeV (your optimal one from Exercise 1 or a simply a 10 GeV window)

2.3 Compute the expected and observed significance using Toy-MC

Note: Draw random # events in the mass window (for b-only and s+b)
For each toy-experiment, not just draw a Poisson number,
but also take a new central value using the (Gauss) Δb from 2.2

Compare it to the significance in exercise 1

Exercise 3:
signal cross-section



$$f(m_h) = \mu \times f_{Higgs}(m_h) + \alpha \times f_{SM}(m_h)$$

Scale factor Higgs

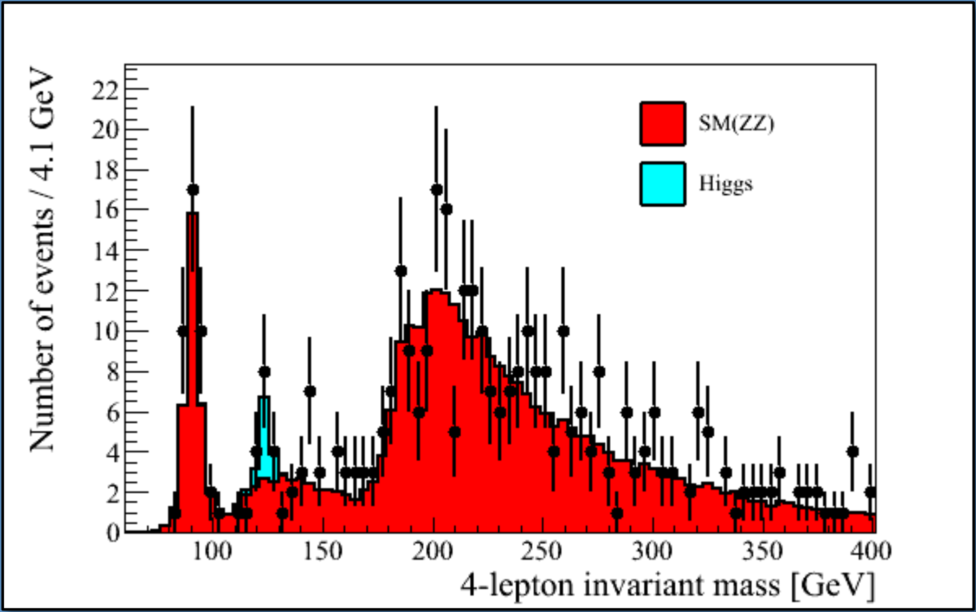
Scale factor SM background

Exercise 3:

Estimate of Higgs cross-section

Code to use:

None (use Exercise 2)

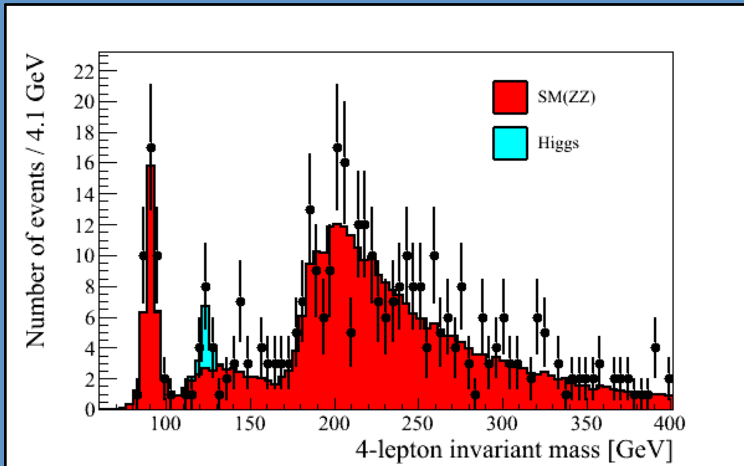


$$-2 \cdot \log\left(Likelihood\right) = -2 \cdot \sum_{bins} \log\left(Poisson(N_{bin}^{data} \mid \mu \cdot f_{bin}^{Higgs} + \alpha \cdot f_{bin}^{SM})\right)$$

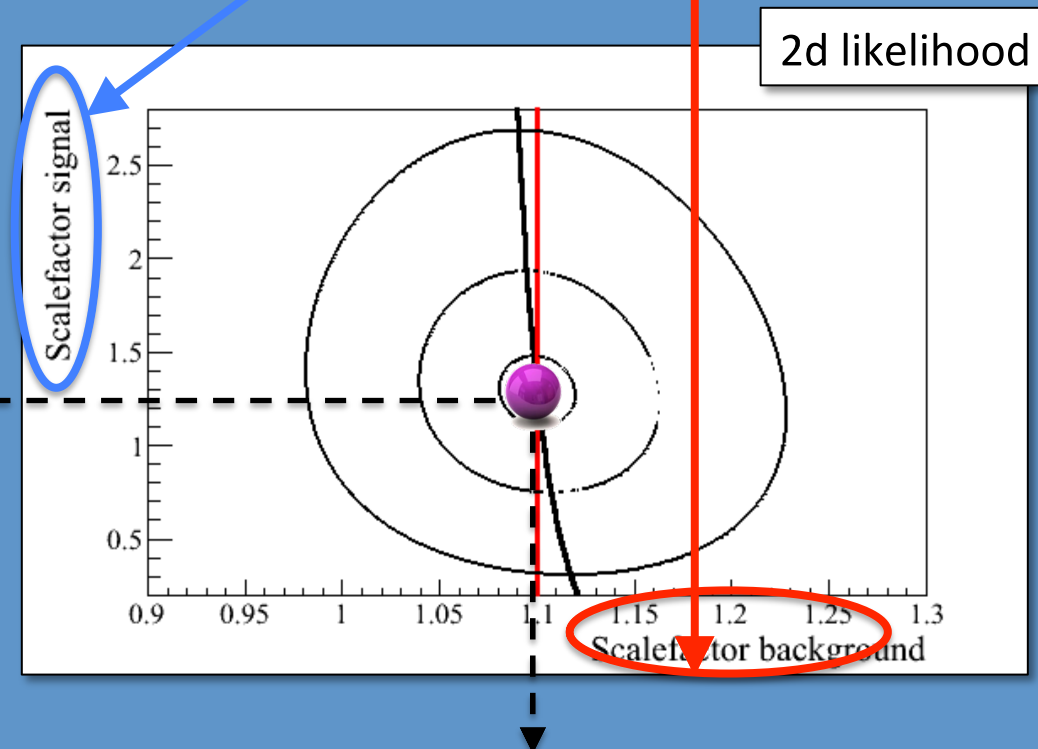
Exercise 3: Measurement of the signal cross-section

3.1 Do a fit where you fix background (to level from exercise 2) and leave the signal cross-section (μ) free. What is the best value for μ and what is its uncertainty ?

3.2 Do a fit where you leave both α and μ free. What are the optimal values ? How would you estimate the uncertainty on each of the parameters ?



$$f(m_h) = \mu \times f_{Higgs}(m_h) + \alpha \times f_{SM}(m_h)$$



$$\mu_{bgr}^{best} = 1.29$$

$$\alpha_{bgr}^{best} = 1.10$$

Tomorrow:

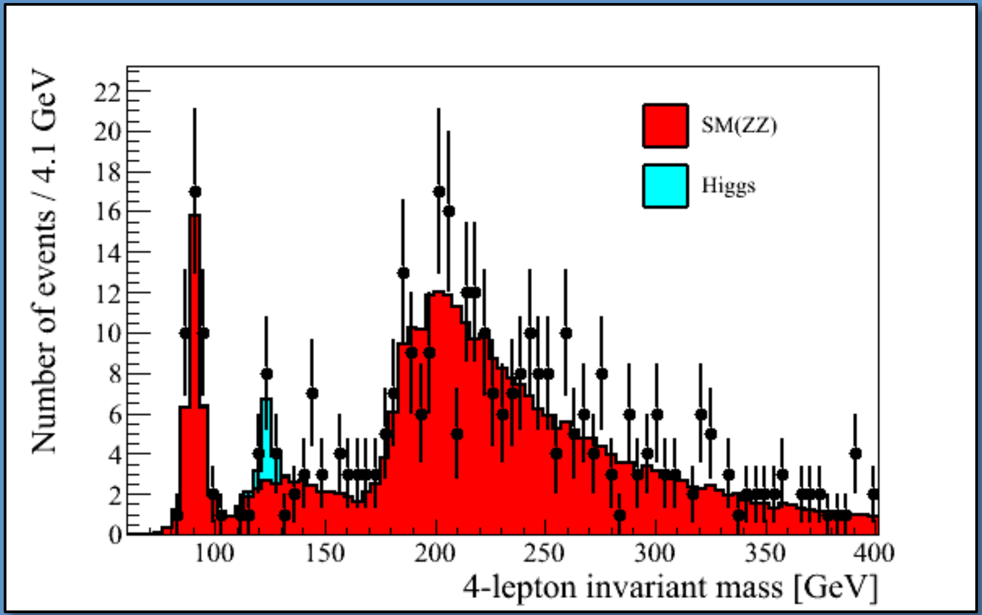
- Test statistics
- Toy Monte Carlo data-sets
- Sensitivity: discovery / exclusion

Good luck!


Exercises
PART 2

Exercise 4:
More complex test statistics

Beyond simple counting: likelihood ratio test-statistic



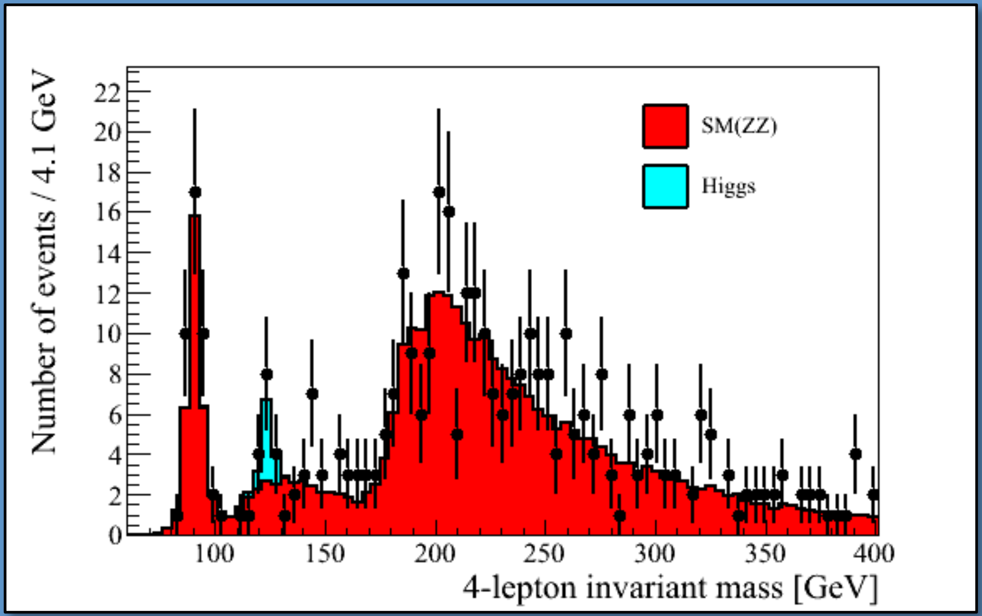
Condense data in **one** number: X

LHC experiments:

$$X(\mu) = -2\ln(Q(\mu)), \text{ with } Q(\mu) = \frac{L(\mu,\hat{\hat{\theta}}(\mu))}{L(\hat{\mu},\hat{\theta})}$$

We'll use something a bit simpler, but same idea

Beyond simple counting: likelihood ratio test-statistic



$$-2\cdot \log\left(Likelihood\right) = -2\cdot \sum_{bins} \log\left(Poisson(N_{bin}^{data} \mid \mu\cdot f_{bin}^{Higgs} + \alpha\cdot f_{bin}^{SM})\right)$$

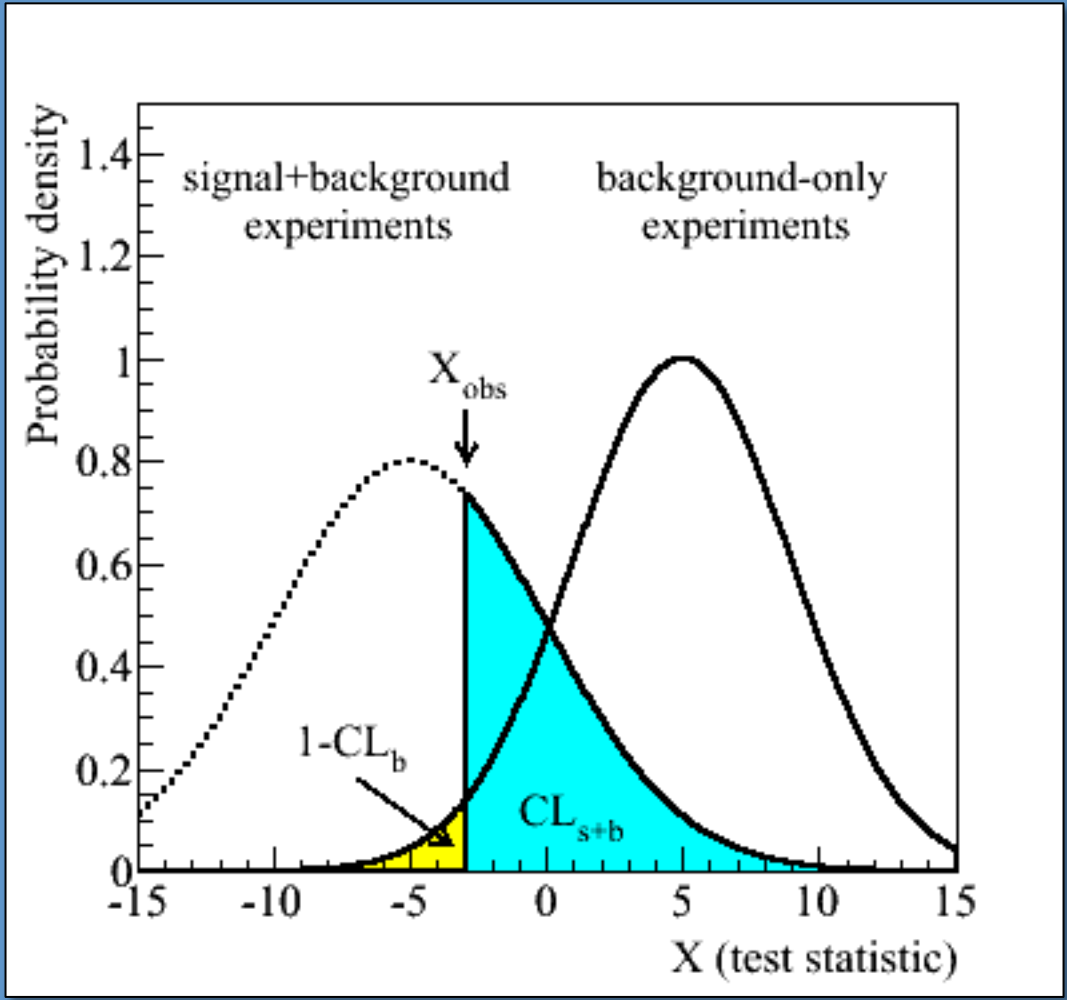
Likelihood assuming μ_s=1 (signal+background)

Hypothesis 1

$$X = -2\ln(Q), \quad \text{with} \quad Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)}$$

Hypothesis 0

Likelihood assuming μ_s=0 (only background) signal like

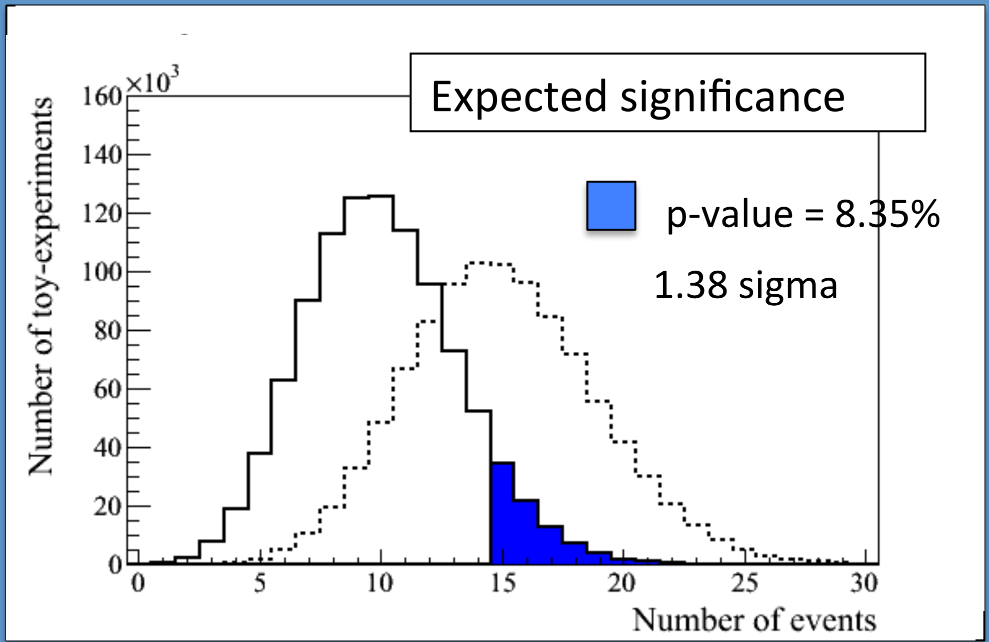


background like

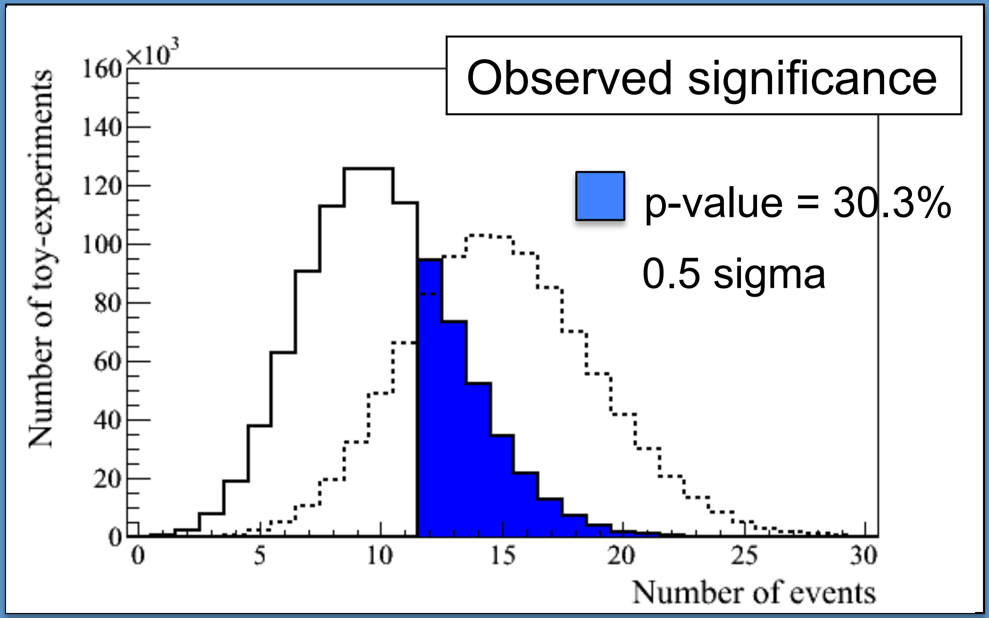
Discovery-aimed: p-value and significance

incompatibiliy with SM-only hypothesis

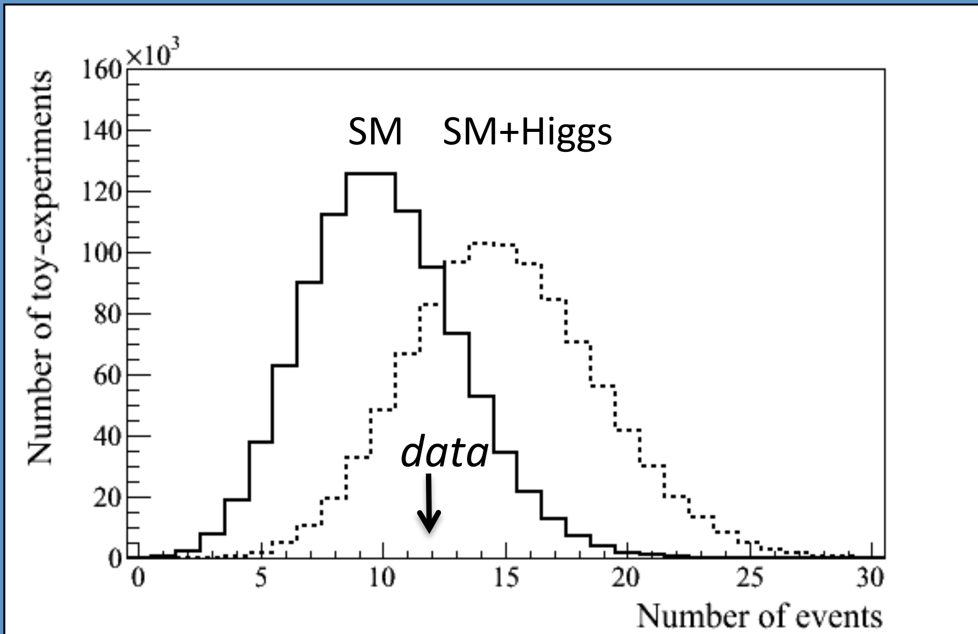
SM	10
Higgs	5
Data	12

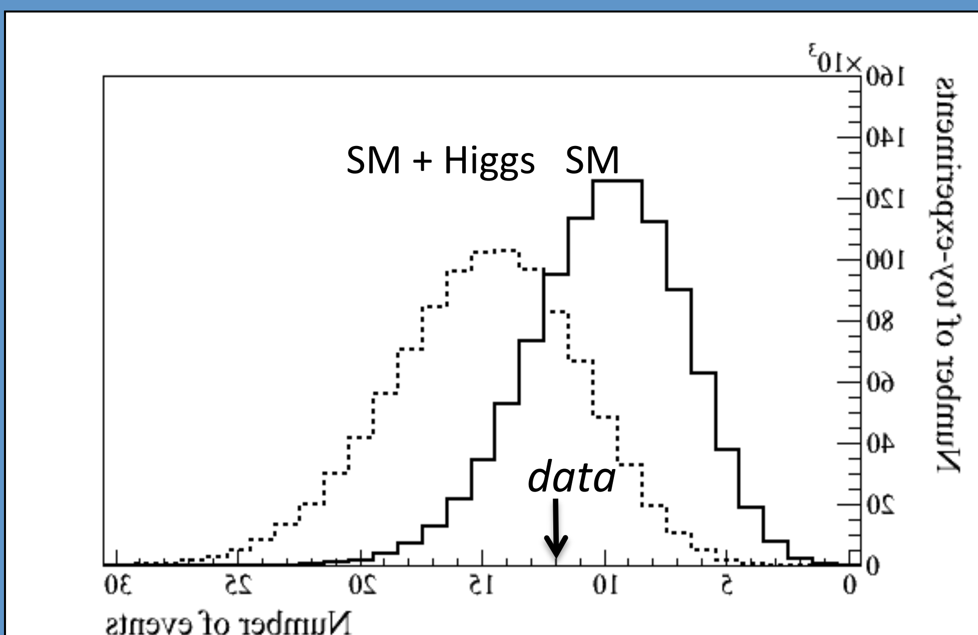


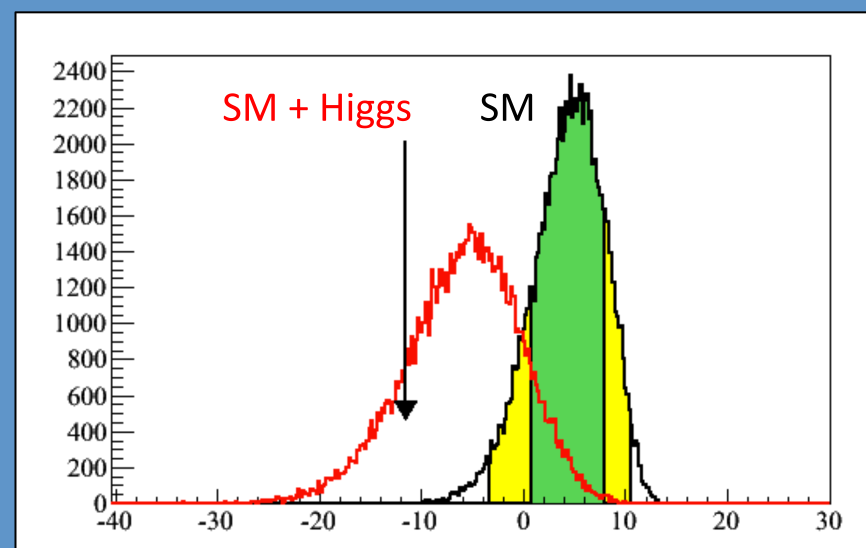
*1) What is the **expected** significance ?*



*2) What is the **observed** significance ?*

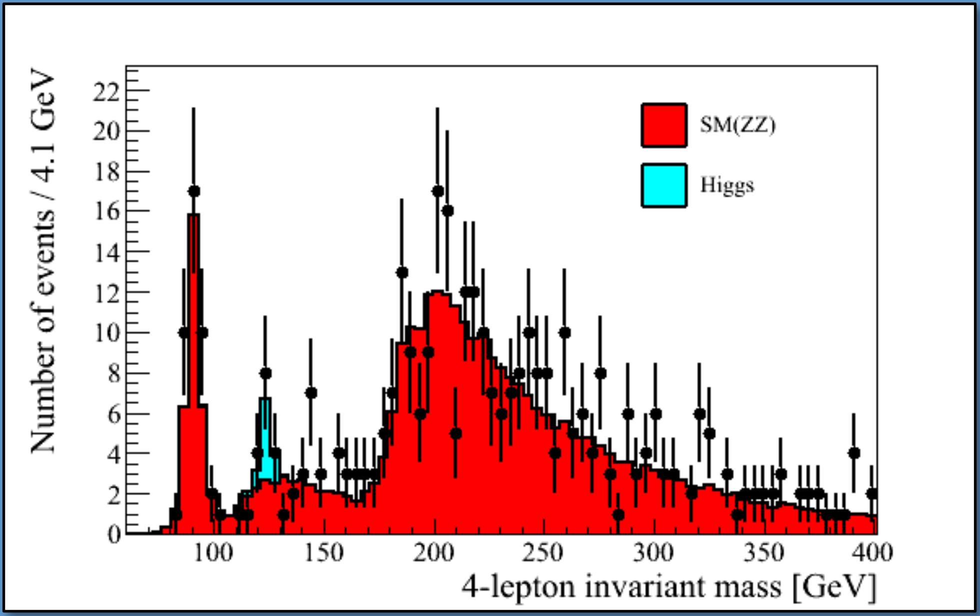

SM
SM+Higgs
data
Number of toy-experiments
Number of events
×10^3


SM + Higgs
SM
data


SM + Higgs
SM

Question: does the window not matter ?

$$X = -2\ln(Q), \quad \text{with} \quad Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)}$$



X = log(a/b) = log(A) – log(B)

What happens if you add a bin at 300 GeV ?
Will it not dilute the channel like in counting ?

In that bin Lik_{bin} = Constant = C

X = log(a/b) = [log(A) + log(C)] – [log(B) + log (C)]
= log(A) – log(B)

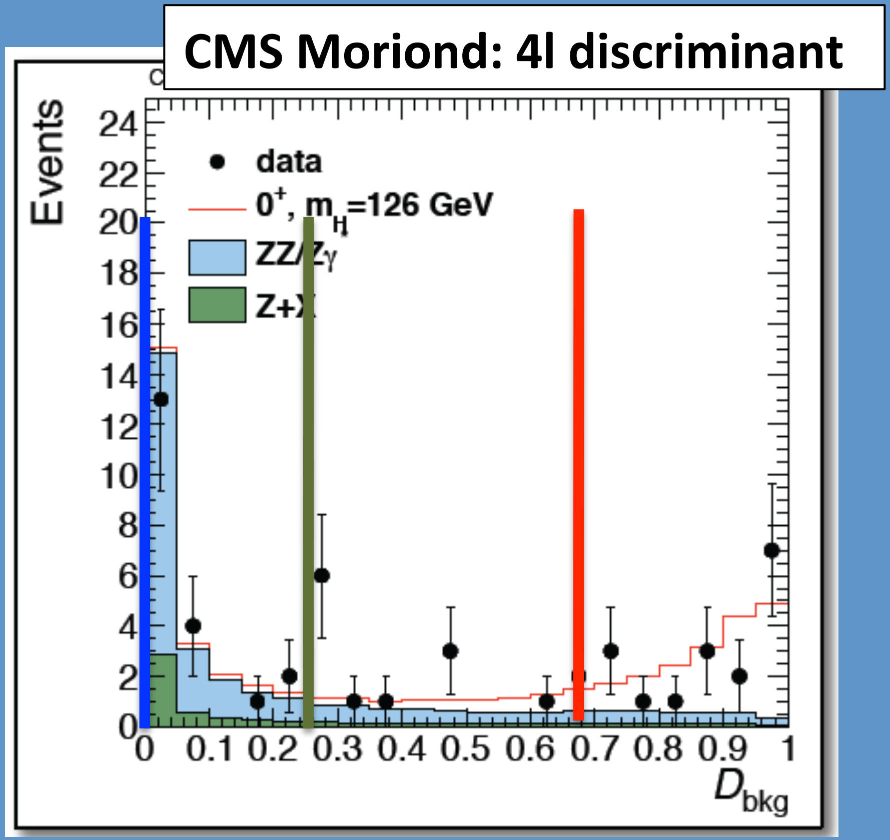
ANY discrimination info is good !

Question: what about more info than mass alone ?

1) Optimal for counting

2) Optimal for LR test stat.

3) Normal procedure



Why: because the 'information' you add below D< 0.25 is maybe difficult to verify in terms of correctness: needs signal descripption in very background-like region: systematics. Need to find optimum.

Note: they still evaluate, like you: $X = -2\ln(Q)$, with $Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)}$

We will use a very simple form for the test statistic

Our exercise (α=1 or from Ex.3):

$$X = -2\ln(Q), \quad \text{with} \quad Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)}$$

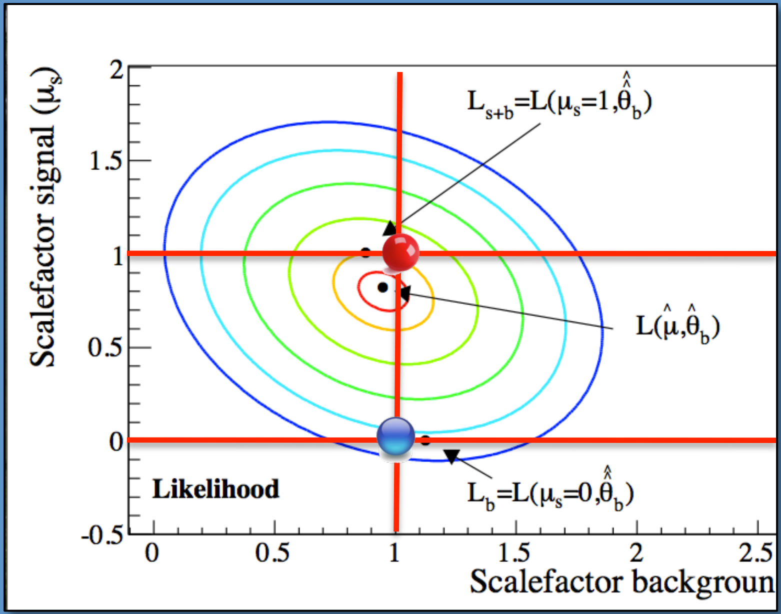
Tevatron-style:

$$X = -2\ln(Q), \quad \text{with} \quad Q = \frac{L(\mu_s = 1, \hat{\hat{\theta}}_{(\mu_s=1)})}{L(\mu_s = 0, \hat{\hat{\theta}}_{(\mu_s=0)})}$$

LHC experiments:

$$X(\mu) = -2\ln(Q(\mu)), \quad \text{with} \quad Q(\mu) = \frac{L(\mu, \hat{\hat{\theta}}(\mu))}{L(\hat{\mu}, \hat{\theta})}$$

2-dimensional fit (α and μ free)



Note: α_{bgr} is just one of the nuissance parameters θ in a ‘real’ analysis

Exercise 4:

Likelihood ratio test statistic (X)

$$X = -2\ln(Q), \quad \text{with} \quad Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)}$$

→ Likelihood assuming μ_s=1 (signal+background)

→ Likelihood assuming μ_s=0 (only background)

Exercise 4: create the likelihood ration test statistic – beyond simple counting

4.1 Write a routine that computes the likelihood ratio test-statistic for a given data-set

```
double Get_TestStatistic(TH1D *h_mass_dataset, TH1D *h_template_bgr, TH1D *h_template_sig)
```

$$-2Log\left(Likelihood_{(\mu,\alpha\ =\ 1)}\right) = -2\cdot\sum_{bins}\log\left(Poisson(N_{bin}^{data} \mid \mu\cdot f_{bin}^{Higgs} + \alpha\cdot f_{bin}^{SM})\right)$$

Note: log(a/b) = log(a) – log(b)

4.2 Compute the likelihood ratio test-statistic for the 'real' data

bonus: Implement the conditional profile likelihood ratio, i.e. find for each of the two hypotheses (μ_s=1 and μ_s=0) the best value for the background scaling (α_{bgr})

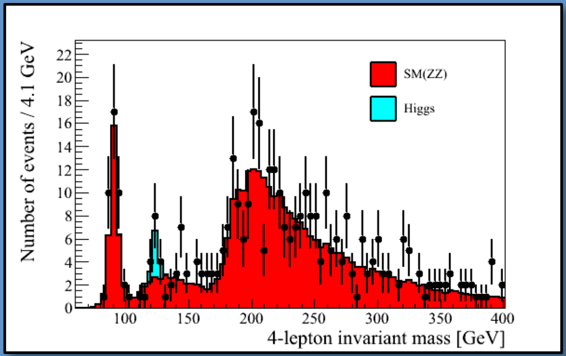
$$X = -2\ln(Q), \quad \text{with} \quad Q = \frac{L(\mu_s = 1, \hat{\hat{\theta}}_{(\mu_s = 1)})}{L(\mu_s = 0, \hat{\hat{\theta}}_{(\mu_s = 0)})}$$

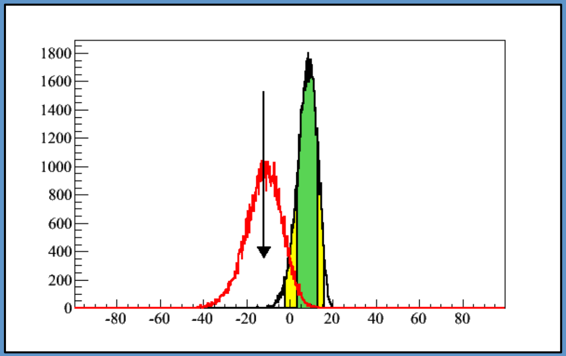
Exercise 5:

- Toy Monte Carlo
- distribution of test statistic for different hypotheses

Exercise 5:

- Generate toy data-sets
- Test statistic distribution





Exercise 5: create toy data-sets

5.1 Write a routine that generates a toy data-set from a MC template (b or s+b)

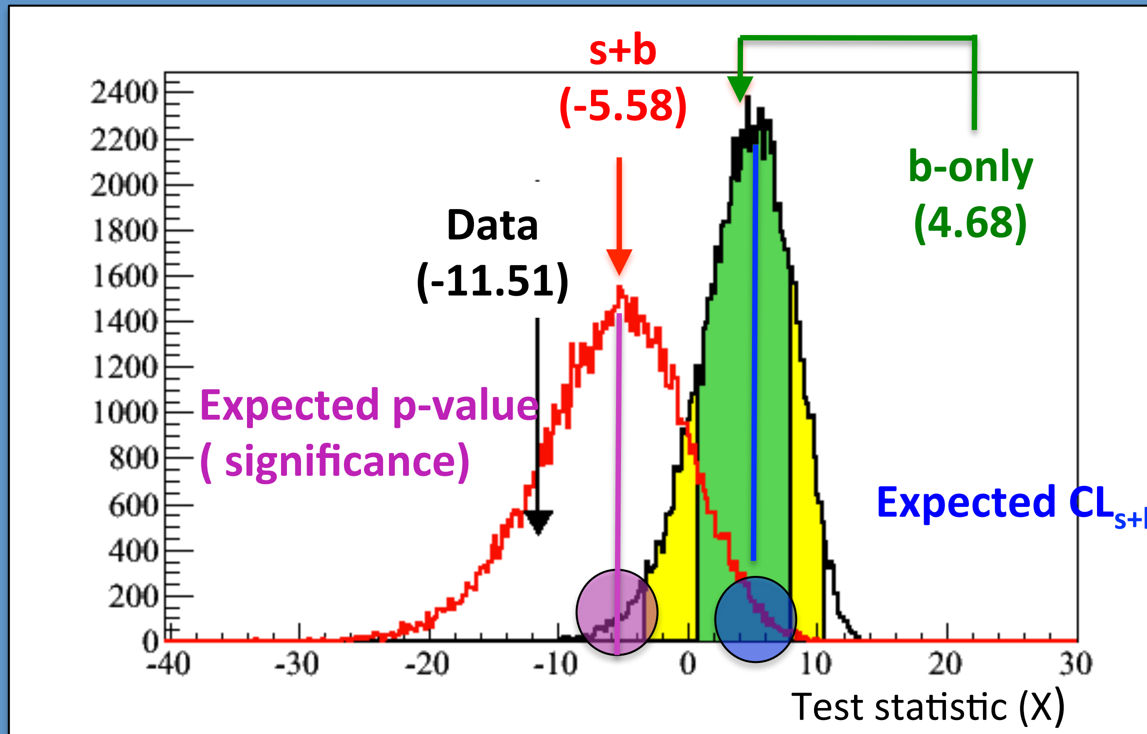
TH1D * GenerateToyDataSet(TH1D *h_mass_template)

How: Take the histogram h_mass_template and draw a Poisson random number in each bin using the bin content in h_mass_template as the central value. Return the new fake data-set.

5.2 Generate 1000 toy data-sets for *background-only* & get test statistic distribution
Generate 1000 toy data-sets for *signal+background* & get test statistic distribution

→ plot both in one plot

5.3 Add the test-statistic from the data(exercise 4.2) to the plot signal like

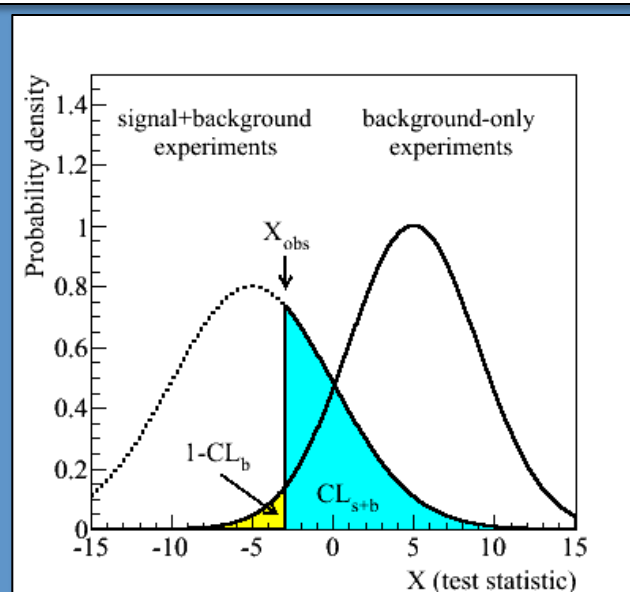


background like

Discovery: $1\text{-}Cl_b < 2.87 \times 10^{-7}$

Incompatibility with b-only hypothesis

signal like



Exclusion: $Cl_{s+b} < 0.05$

Incompatibility with s+b hypothesis

background like

Exercise 6:
Discovery potential

Exercise 6

Summarize separation power: conclusion

Exercise 5: compute p-value

6.1 Compute the p-value or 1-Cl_b (under the background-only hypothesis):

- For the average(median) b-only experiment
- For the average(median) s+b-only experiment [expected significance]
- For the data [observed significance]

6.2 Draw conclusions:

- Can you claim a discovery ?
- Did you expect to make a discovery ?
- At what luminosity did/do you expect to be able to make a discovery ?

Exercise 7:
Excluding hypotheses

Exercise 6 continued

Exclude a cross-section for a given Higgs boson mass

Some shortcomings, but we'll use it anyway

$$\sigma_h(m_h) = \zeta \cdot \sigma_h^{SM}(m_h)$$

Scale factor wrt SM prediction

Exercise 6: compute CL_{s+b} and exclude Higgs masses or cross-sections

6.3 Compute the CL_{s+b}:

- For the average(median) s+b experiment
- For the average(median) b-only experiment
- For the data

6.4 Draw conclusions:

- Can you exclude the m_h=200 GeV hypothesis ? What ς can you exclude ?

- Did you expect to be able to exclude the m_h=200 GeV hypothesis ?
  What ς did you expect to be able to exclude ?

BACKUP

From p-value to sigma

ATLAS-PHYS-PUB-2011-11
CMS Note-2011/005

Procedure for the LHC Higgs boson search combination in Summer 2011

The ATLAS Collaboration
The CMS Collaboration
The LHC Higgs Combination Group

https://cds.cern.ch/record/1379837/files/NOTE2011_005.pdf

To convert the p-value into a significance Z, we adopt the convention of a "one-sided Gaussian tail":

$$p = \int_Z^\infty \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)\, dx \;=\; \frac{1}{2} P_{\chi_1^2}(Z^2), \tag{11}$$