

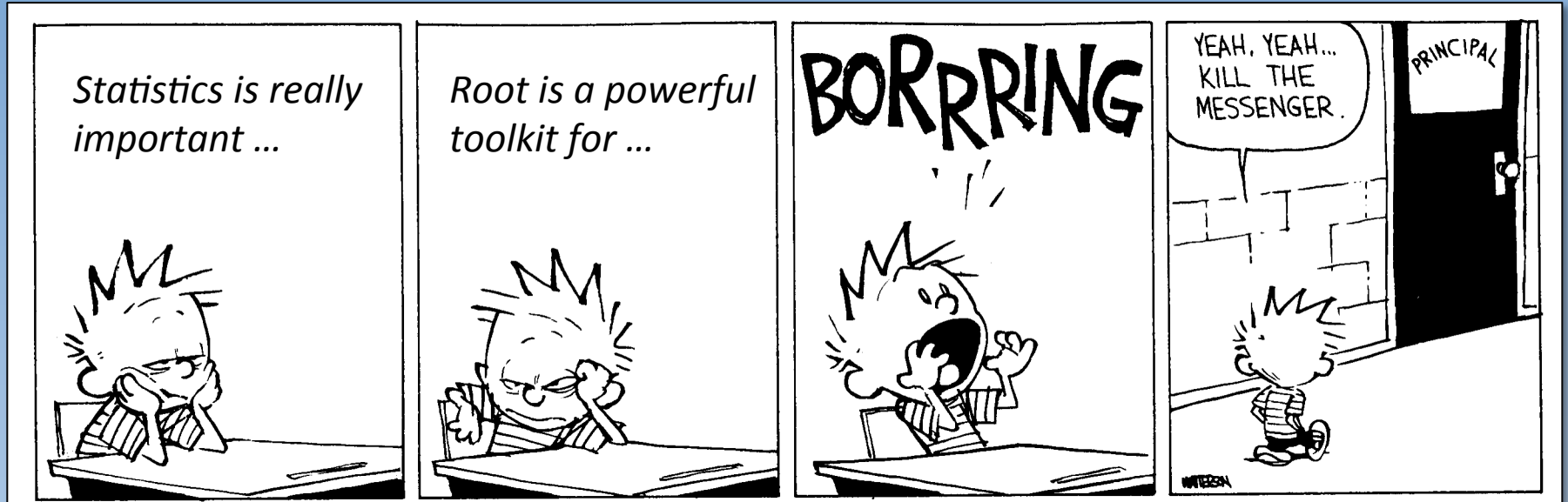
## Statistics exercises

Tuesday 3.5 hours (14:00-15:30 & 16:00-18:00)

Wednesday 3.5 hours (11:00-12:30 & 14:00-16:00)

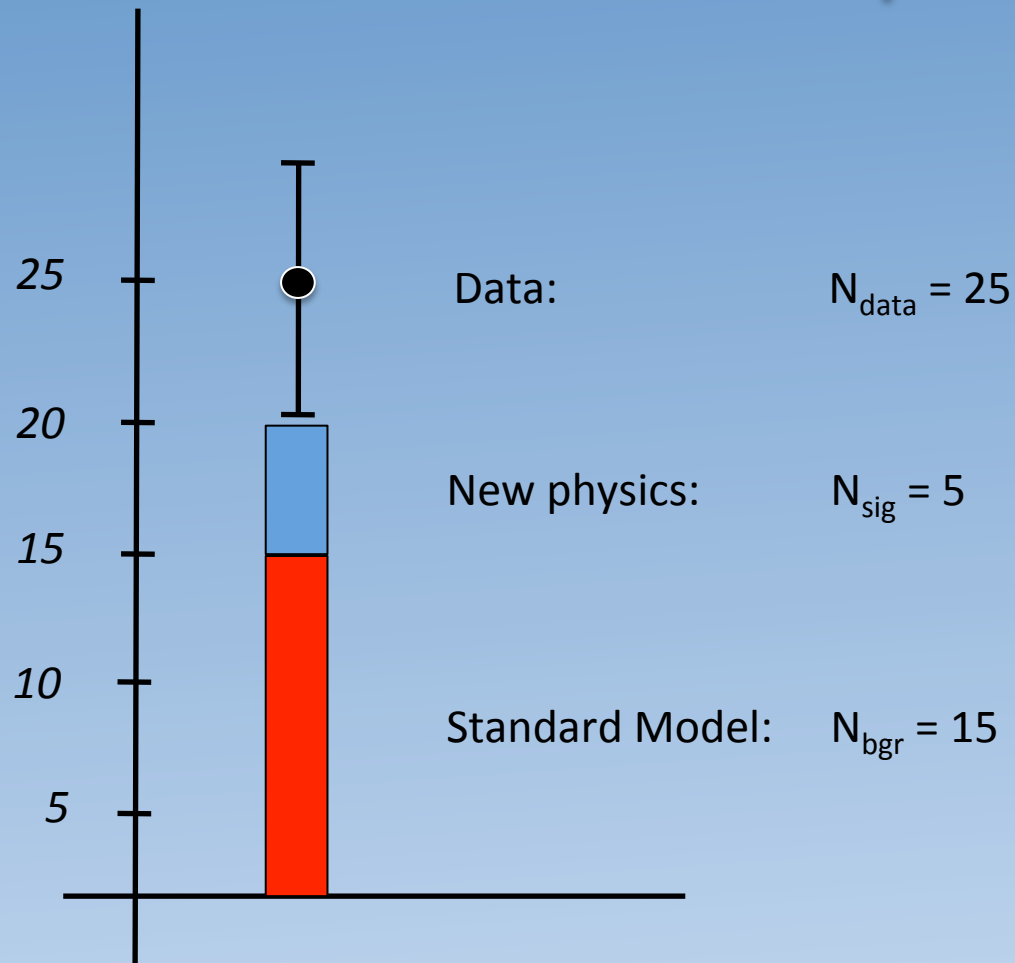
Ivo van Vulpen (UvA/Nikhef)

# A short lecture on Root and statistics



Note: will be 95% of your work. Master this and you can focus on physics

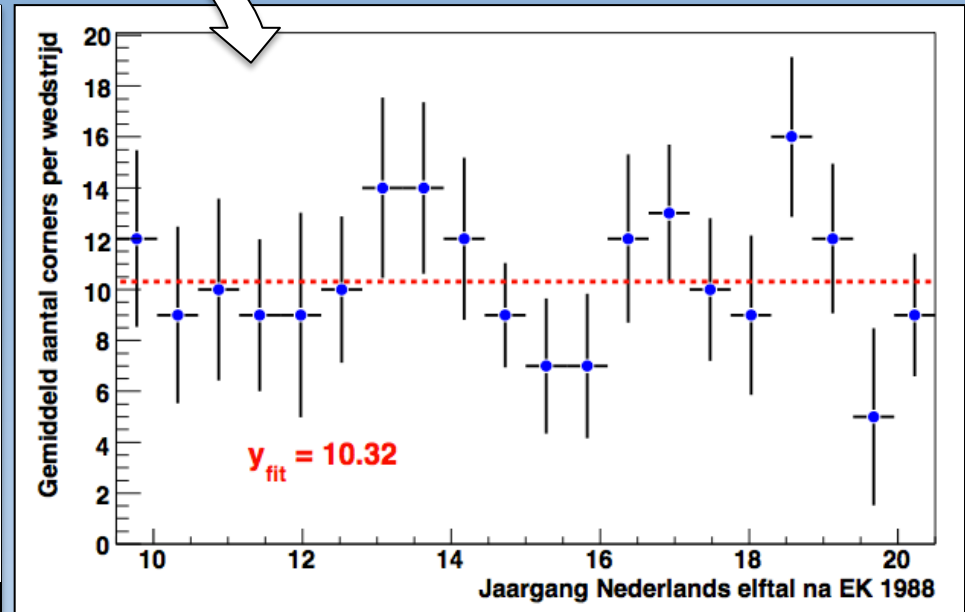
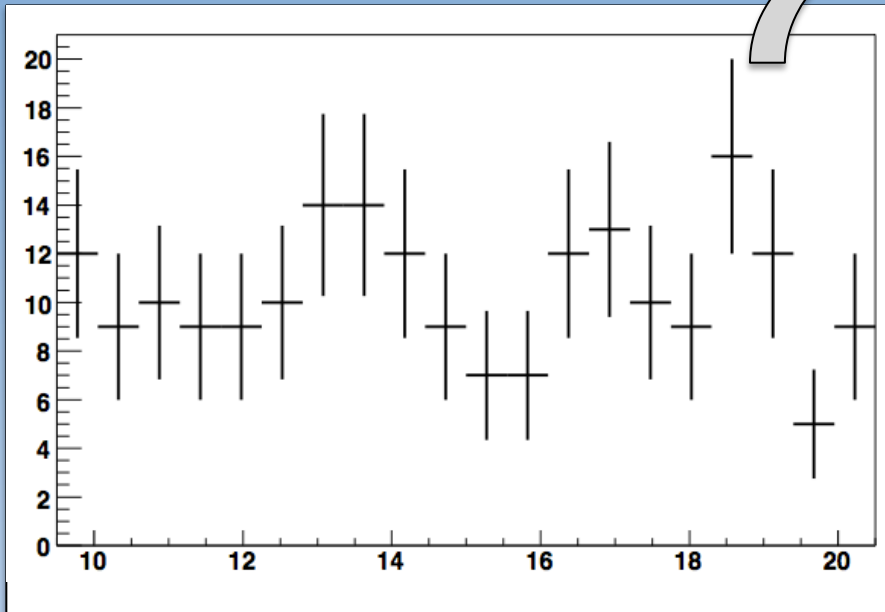
# Example: significance



*What is the significance of the excess ?*

# Example: poisson statistics & likelihood fit

Can everybody do this ?





# Statistics is everywhere in science and industry

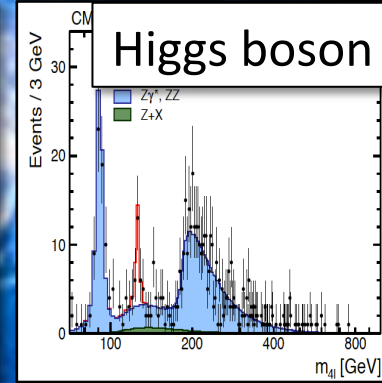
Risk analyses



Banking/consultancy



Higgs boson



- Many mysteries, folklore, buzz-words, bluffing etc. but you **need** to master it to quantify the results of any analysis. Do **not** just follow ‘what everybody else does’ or your supervisor tells you.
- RooFit, Roostats, BAT, TMVA, BDT’s are excellent and very powerful tools. Make sure you understand the basics so you know what you ask it to do.

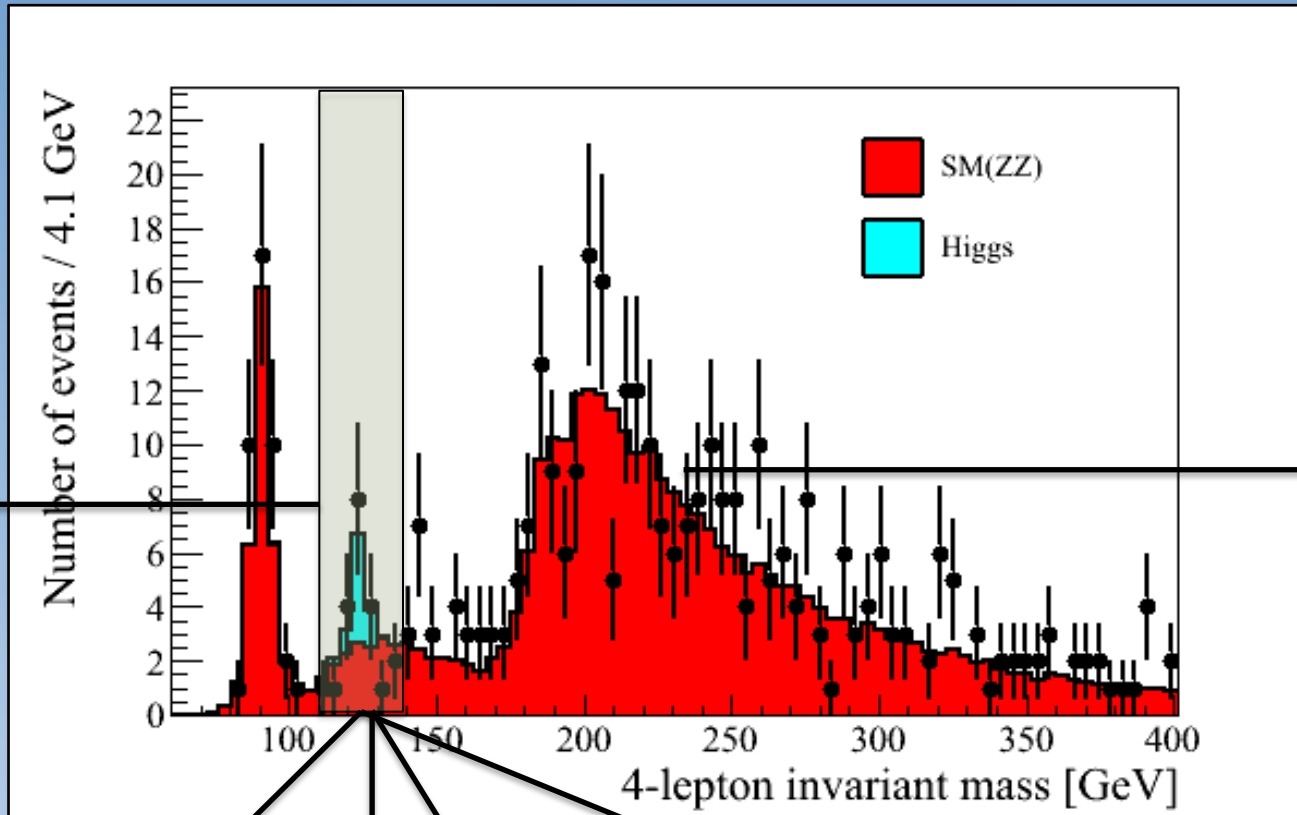
## Goals of the hands-on exercises:

- manipulate histograms in root file
- understand and compute significances
- likelihood fit and parameter estimation (with error)

Extra: Convey message that you can and should do analysis steps yourself first to understand the various concepts

Only then use power of RooFit and RooStats.

# Data-set for the exercises: 4 lepton mass



Cross-section measurement

Exclusions

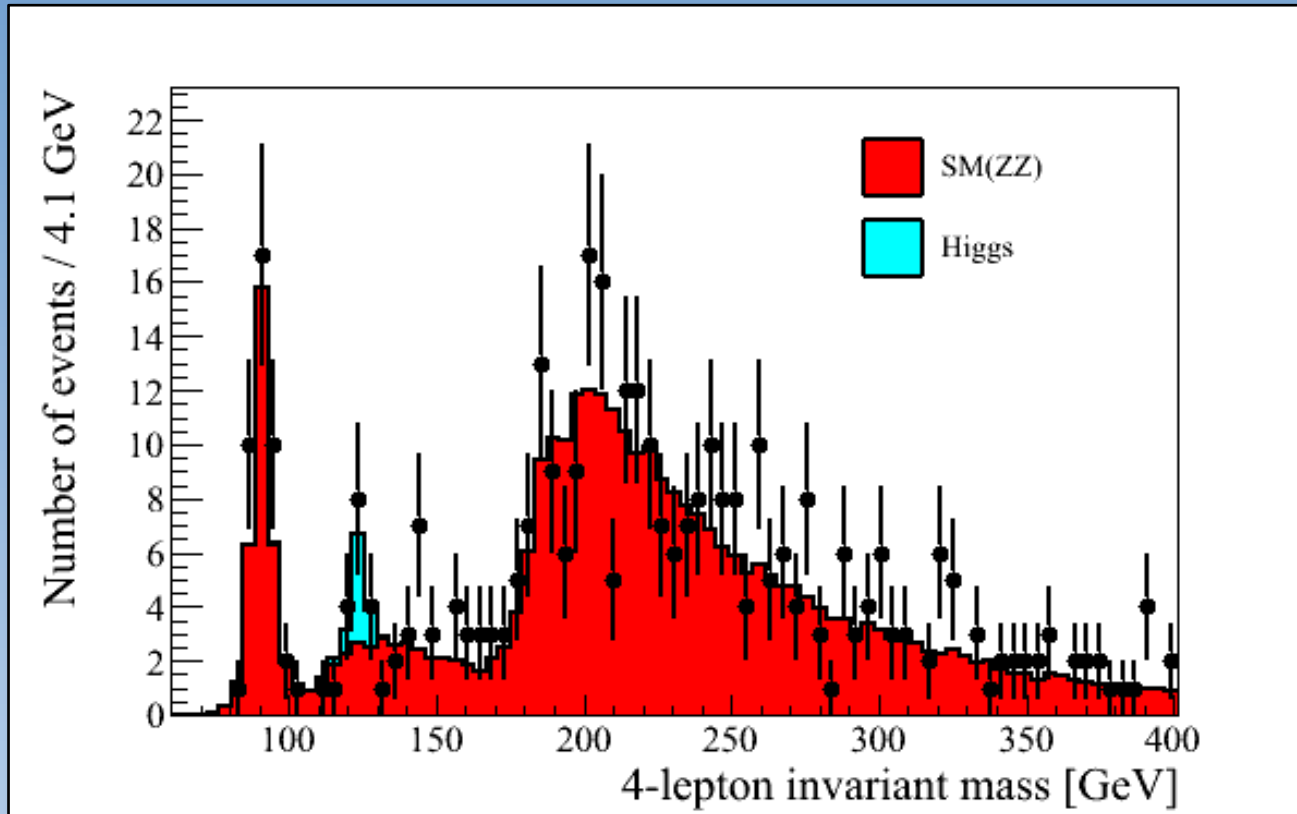
Significance optimization

Test statistic (Toy-MC)

Mass measurement

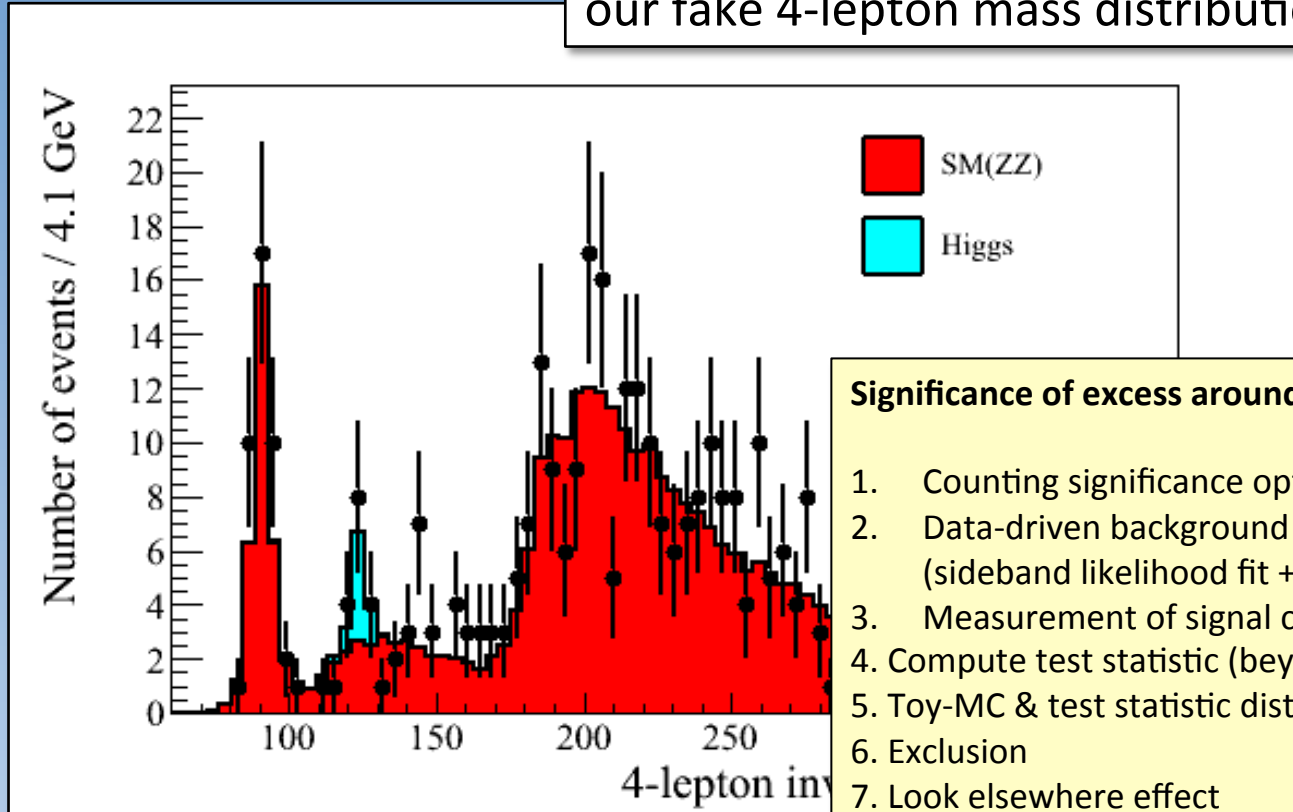
Data-driven background estimate  
(likelihood fit using side bands)

# Data-set for the exercises: 4 lepton mass



Note: - Original histograms have 200 MeV bins  
- This is fake data

our fake 4-lepton mass distribution



**Significance of excess around 125 GeV**

1. Counting significance optimization
2. Data-driven background estimate (sideband likelihood fit + toy MC Poisson)
3. Measurement of signal cross-section
4. Compute test statistic (beyond counting)
5. Toy-MC & test statistic distribution
6. Exclusion
7. Look elsewhere effect
8. Complex/correlated measurements

Full set of exercises (this talk):

[http://www.nikhef.nl/~ivov/Talks/2015\\_04\\_21\\_DESY\\_HandsOnExercises.pdf](http://www.nikhef.nl/~ivov/Talks/2015_04_21_DESY_HandsOnExercises.pdf)

# Basic material for the exercises:

- 1) Download tarball: **DesyExercises.tgz**
- 2) Unpack everything: **tar -vzxf DesyExercises.tgz**

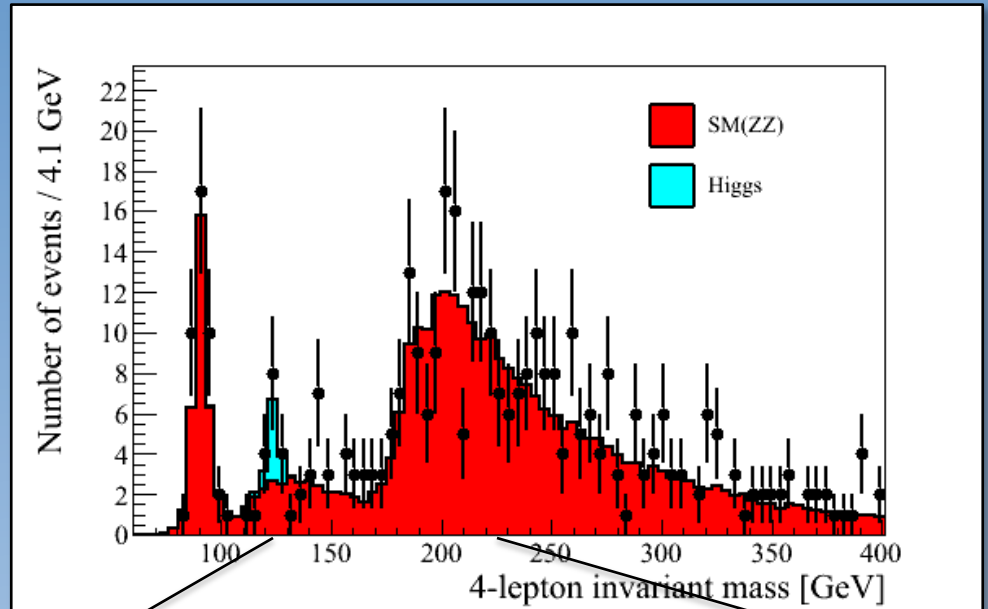
- a) **Histograms\_fake.root**  
4 histograms with the 4 lepton invariant mass (H125, H200, ZZ, data)
- b) **DESY\_skeleton.C**  
Some skeleton code (different levels, as minimal as possible)
- c) **Rootlogon.C**  
Some standard Root blabla

## Create the 4-lepton mass plot

```
root> .L DESY_skeleton.C++  
root> MassPlot(20)
```

↓  
Rebin-factor

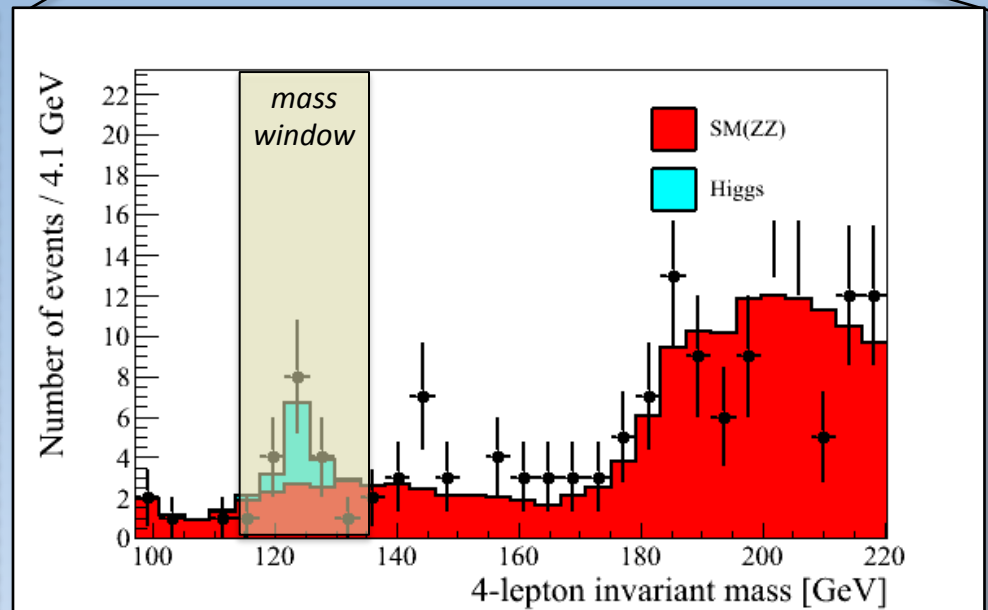
*hist: h\_bgr, h\_sig, h\_data*



## Summary in signal mass region (using 200 MeV bin and 10 GeV window)

Ndata = 16  
Nbgr = 6.42  
Nsig = 5.96

Exercises: significance



# DESY\_skeleton.C

```
//=====
void MassPlot(int Irebin){
//=====
//-----
// Goal: produce SM+Higgs+data plot
// Note: rebinning is only for plotting
//-----

//-----
//-- Standard stuff and prepare canvas
//-----
gROOT->Clear();
gROOT->Delete();

//-- Prepare canvas and plot histograms
TCanvas * canvas1 = new TCanvas( "canvas1","Standard Canvas",600,400);
canvas1->SetLeftMargin(0.125);
canvas1->SetBottomMargin(0.125);
canvas1->cd();

//-----
//-- [1] Prepare histograms
//--   o Get histograms from the files (signal, background and data)
//--   o Make cumulative histograms (for signal and background)
//-----

//-- Get histograms from the files (higgs, zz and data)
TH1D *h_sig, *h_bgr, *h_data;
h_sig = GetMassDistribution(125);
h_bgr = GetMassDistribution(1);
h_data = GetMassDistribution(2);

//-----
//-- [2] Plot histograms and make gif
//--   o rebin histograms
//--   o prepare cumulative histogram
//--   o make plot + opsmuk + gif
//-----

//-- Rebin histograms (only for plotting)
h_sig->Rebin(Irebin);
h_bgr->Rebin(Irebin);
h_data->Rebin(Irebin);

//-- Prepare cumulative histogram for signal + background
TH1D *h_sig_plus_bgr = (TH1D*) h_bgr->Clone("h_sig_plus_bgr");
h_sig_plus_bgr->Reset();
for (int i_bin = 1; i_bin < h_bgr->GetNbinsX(); i_bin++){
    h_sig_plus_bgr->SetBinContent( i_bin, h_sig->GetBinContent(i_bin) + h_bgr->GetBinContent(i_bin));
    printf(" REBINNED HISTOGRAM: bin %d, Ndata = %d\n",i_bin,(int)h_data->GetBinContent(i_bin));
}

//-- prepare histograms and plot them on canvas
double Data_max = h_data->GetBinContent(h_data->GetMaximumBin());
double Ymax_plot = 1.10* (Data_max + TMath::Sqrt(Data_max));
h_sig_plus_bgr->SetFillColor(7);
h_sig_plus_bgr->SetAxisRange(0.,Ymax_plot,"Y");
h_sig_plus_bgr->SetAxisRange(0.,400.,"X");
h_bgr->SetFillColor(2);
h_sig_plus_bgr->Draw("hist");
h_bgr->Draw("same");
h_data->Draw("axis same");
h_data->Draw("e same");

//-- some nice axes and add legend
AddText( 0.900, 0.035, "4-lepton invariant mass [GeV]",0.060, 0.,"right");
AddText( 0.040, 0.900, Form("Number of events / %3.1F GeV",h_bgr->GetBinWidth(1)), 0.060,90.,"right"); // X-axis
TLegend *leg1 = new TLegend(0.65,0.65,0.90,0.85);
leg1->SetBorderSize(0); leg1->SetFillColor(0);
TLegendEntry *leg1a = leg1->AddEntry(h_bgr, " SM(ZZ)", "F"); leg1a->SetTextSize(0.04);
TLegendEntry *leg1b = leg1->AddEntry(h_sig_plus_bgr, " Higgs", "F"); leg1b->SetTextSize(0.04);
leg1->Draw();

//-- prepare gif
canvas1->Print(Form("./MassPlot_rebin%d.gif",Irebin));

return;

//=====
} // end MassPlot()
//=====
```

Define canvas

Get histograms from root-file

Rebin histograms

Print bin content

Make cumulative histogram

Histogram characteristics & plot

Add text

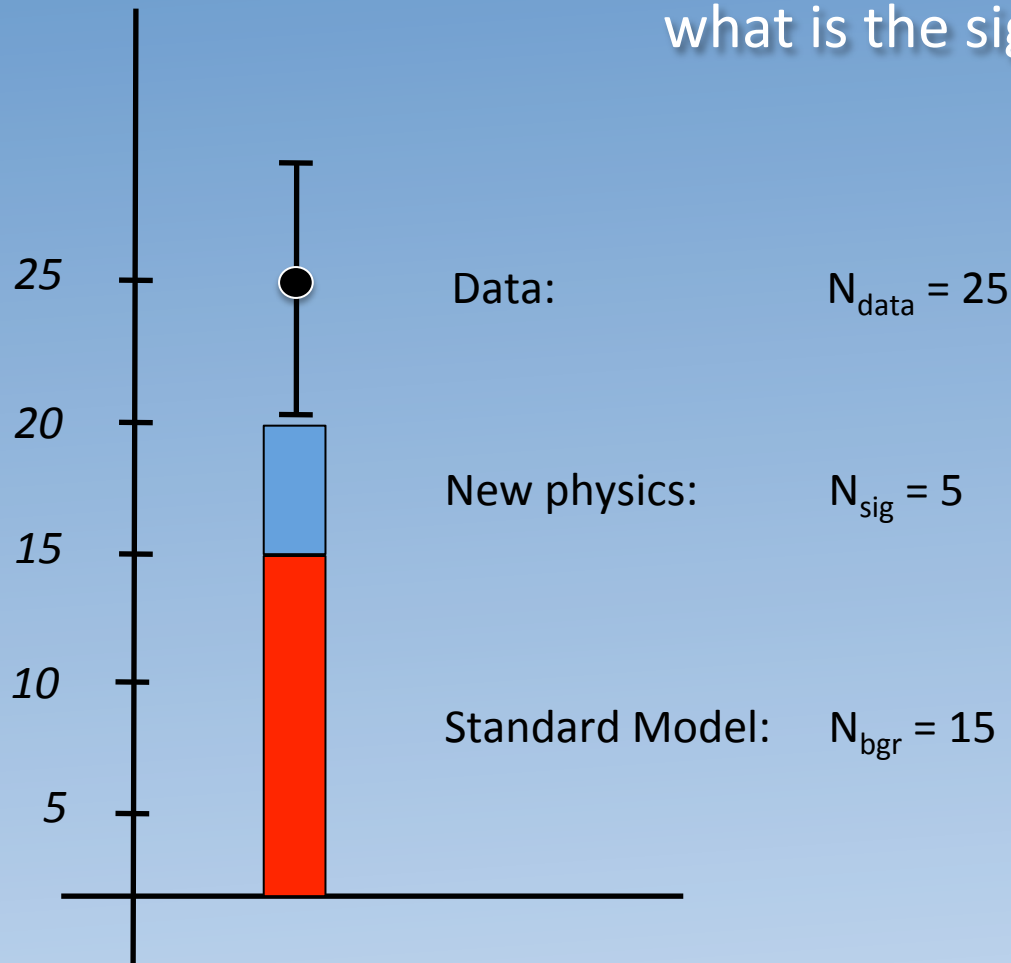
Save plot as gif in your directory



**10-slide mini lecture on significance:**

**- discovery and exclusion -**

General remark :  
what is the significance ?



*Significance for  $N$  events: probability to observe  $N$  events (or even more) under the background-only hypothesis*

## Observed significance:

$$\int_{25}^{\infty} \text{Poisson}(N | 15) dN = 0.0112 \quad \leftarrow p\text{-value}$$
$$= 2.28 \text{ sigma} \quad \leftarrow \text{significance}$$

## Expected significance:

$$\int_{20}^{\infty} \text{Poisson}(N | 15) dN = 0.1248$$
$$= 1.15 \text{ sigma}$$

Discovery if  $p\text{-value} < 2.87 \times 10^{-7}$

→ 39 events

# Poisson distribution

# The Poisson distribution

Binomial with  $n \rightarrow \infty$ ,  $p \rightarrow 0$  and  $np = \lambda$

$$P(n | \lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$$

*Poisson distribution*

Probability to observe  $n$  events  
when  $\lambda$  are expected

$$P(0 | 4.0) = 0.01832$$

$$P(2 | 4.0) = 0.14653 \quad !$$

$$P(3 | 4.0) = 0.19537$$

$$P(4 | 4.0) = 0.19537$$

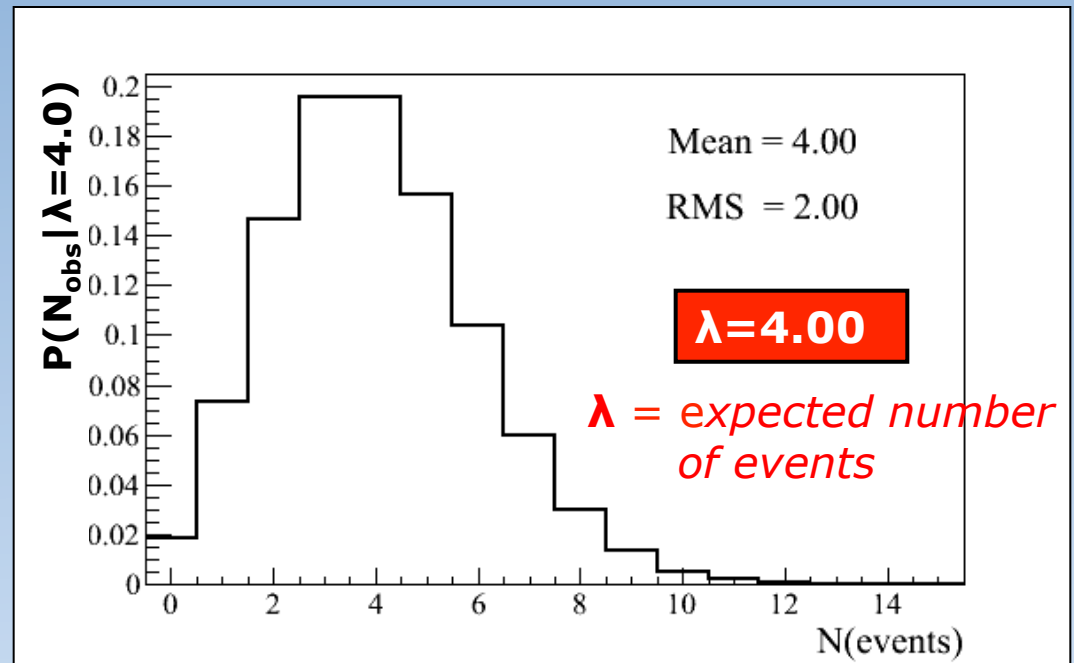
$$P(6 | 4.0) = 0.10420 \quad !$$

#observed

$\lambda$  hypothesis

varying

fixed



# Known $\lambda$ (Poisson)

Binomial with  $n \rightarrow \infty$ ,  $p \rightarrow 0$   $np = \lambda$

$$P(n | \lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$$

*Poisson distribution*

Probability to observe  $n$  events  
when  $\lambda$  are expected

$$P(0 | 4.9) = 0.00745$$

$$P(2 | 4.9) = 0.08940$$

$$P(3 | 4.9) = 0.14601$$

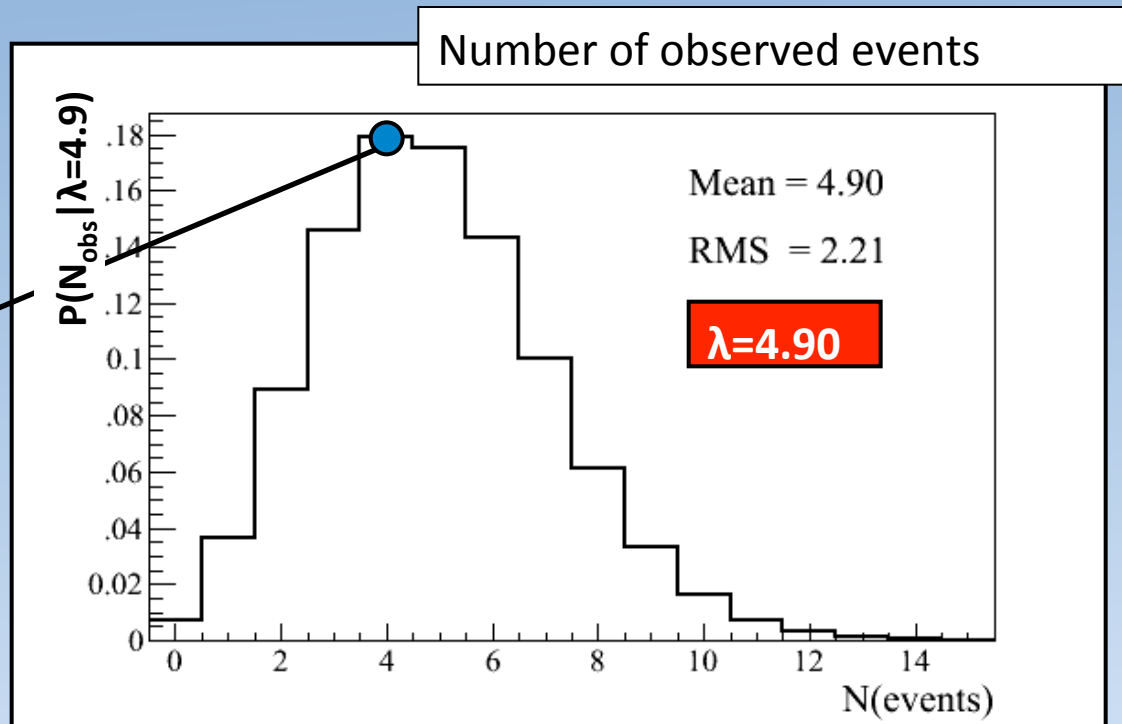
$$P(4 | 4.9) = 0.17887$$

#observed

varying

$\lambda$  hypothesis

fixed

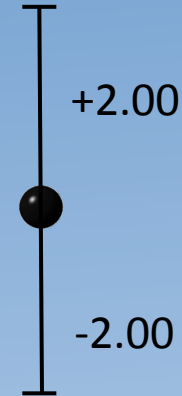


the famous  $\sqrt{N}$

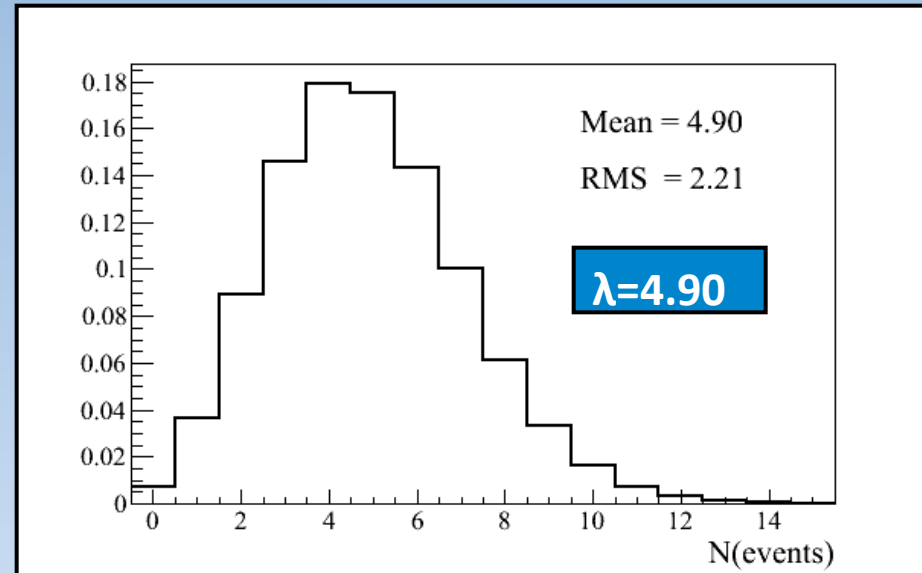
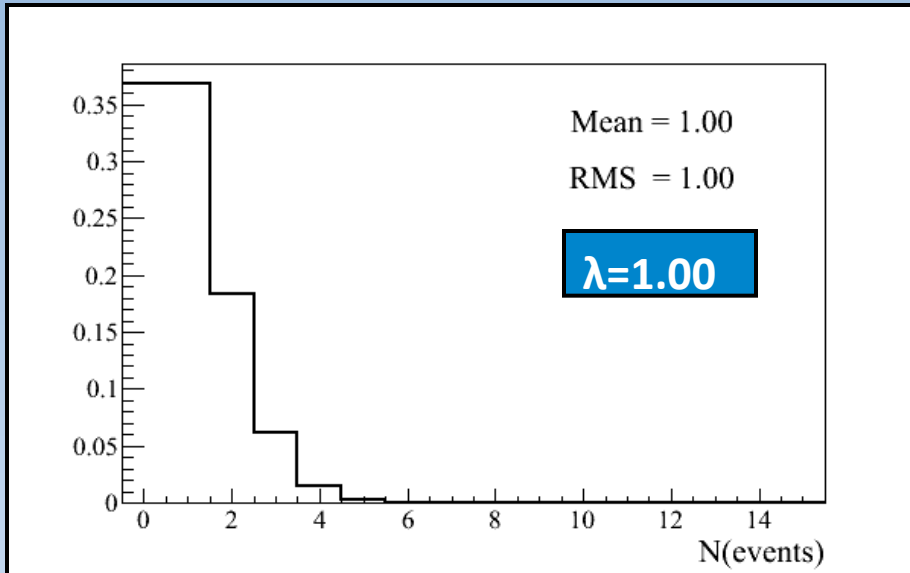
### Properties Poisson distribution

- (1) Mean:  $\langle n \rangle = \lambda$
- (2) Variance:  $\langle (n - \langle n \rangle)^2 \rangle = \lambda$
- (3) Most likely: first integer  $\leq \lambda$

Usual way to represent the error on a data-point



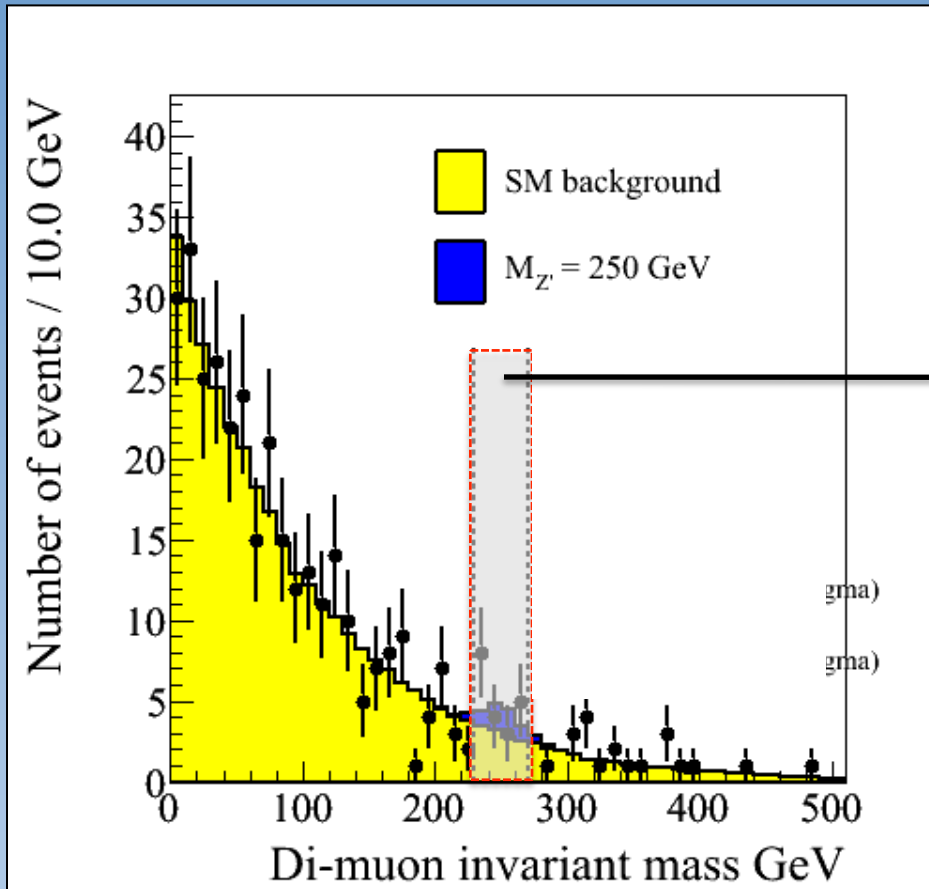
Not default in Root



Significance example





# Counting events in a mass window



Standard Model

SM	10
Higgs	5
Data	12

Ok, now what ?

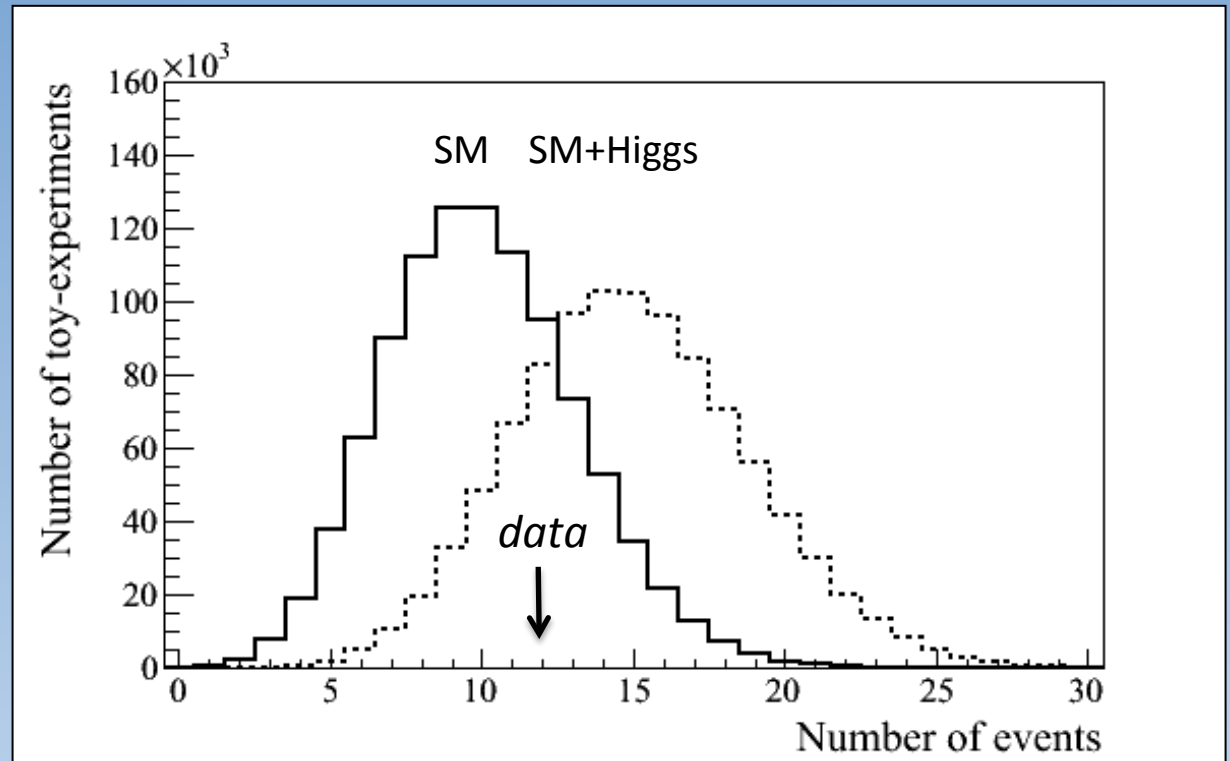
Next slides:  *discovery*  
 *exclusion*

Standard Model

SM	10
Higgs	5
Data	12

Ok, now what ?

*Poisson distribution*



*Significance for N events: probability to observe N events (or even more) under the background-only hypothesis*

# Interpretation

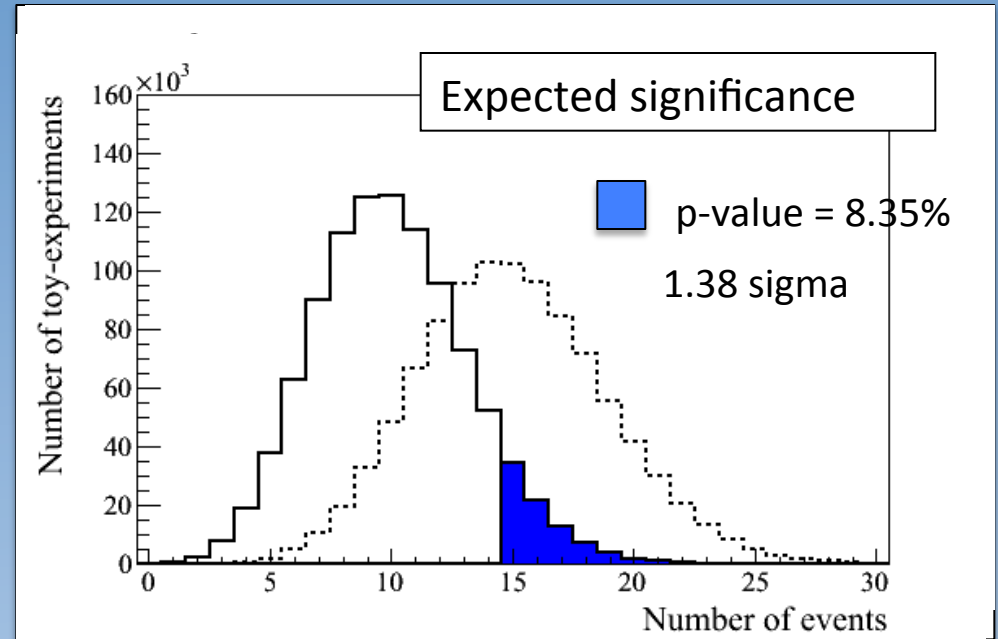
optimistic: discovery

*Incompatibility with SM-hypothesis*

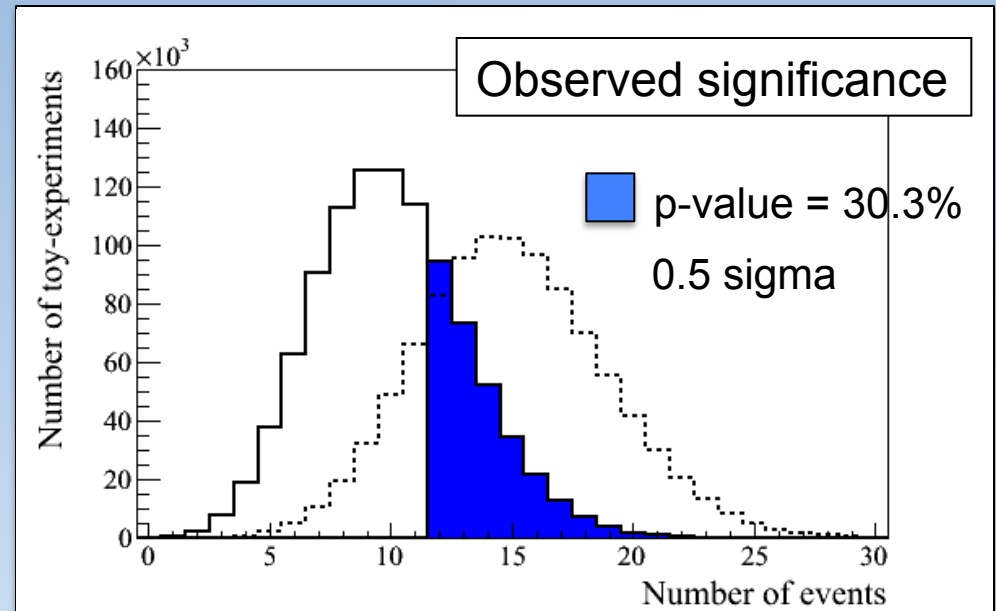
# p-value: incompatibility with SM-only hypothesis

SM	10
Higgs	5
Data	12

1) What is the **expected** significance ?



2) What is the **observed** significance ?



# p-value: incompatibility with SM-only hypothesis

SM	10
Higgs	5

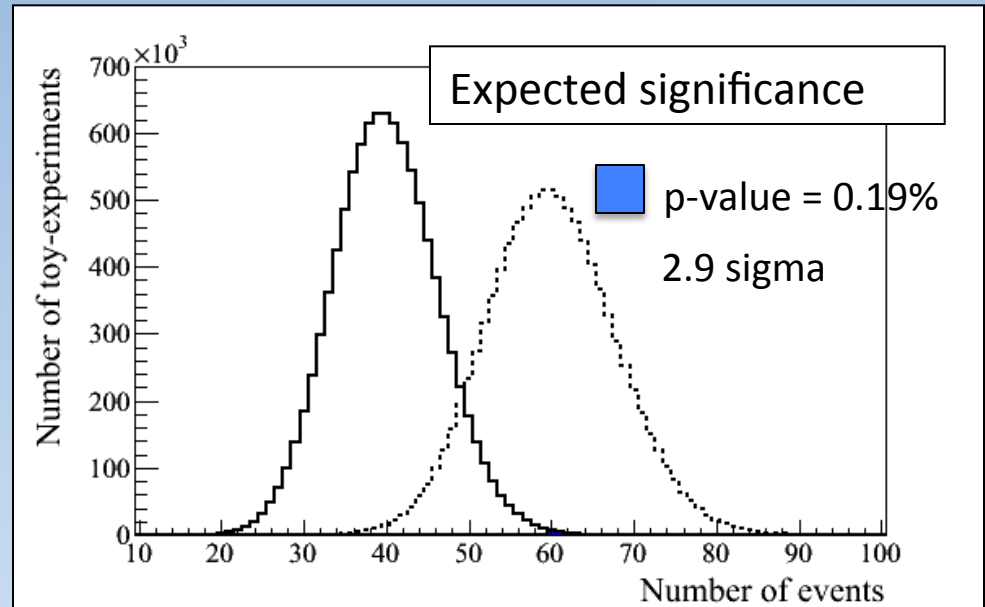
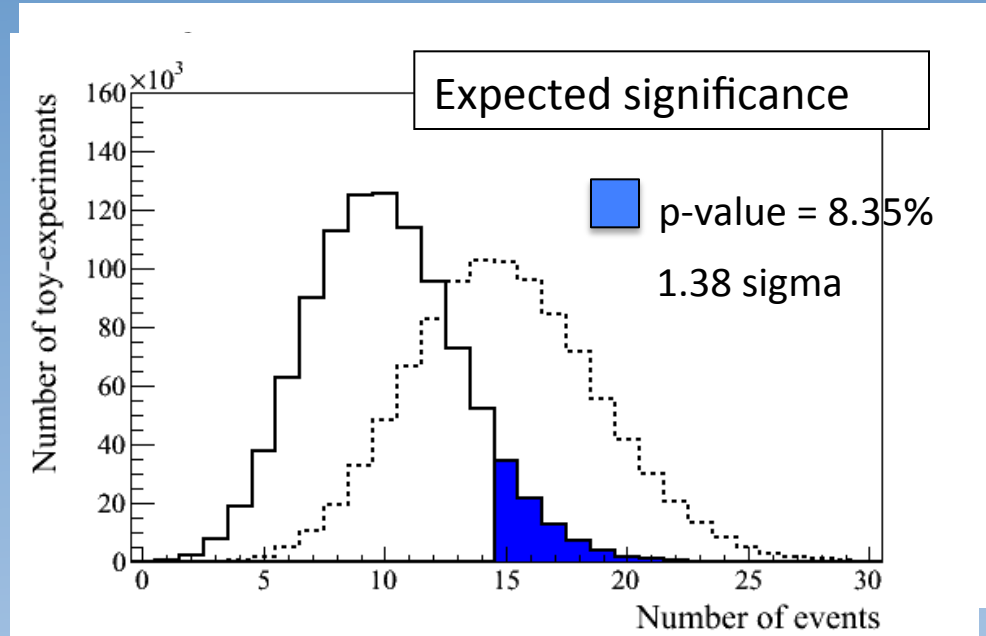
3) At what Lumi do you expect to be able to claim a discovery?

3 TIMES MORE LUMINOSITY



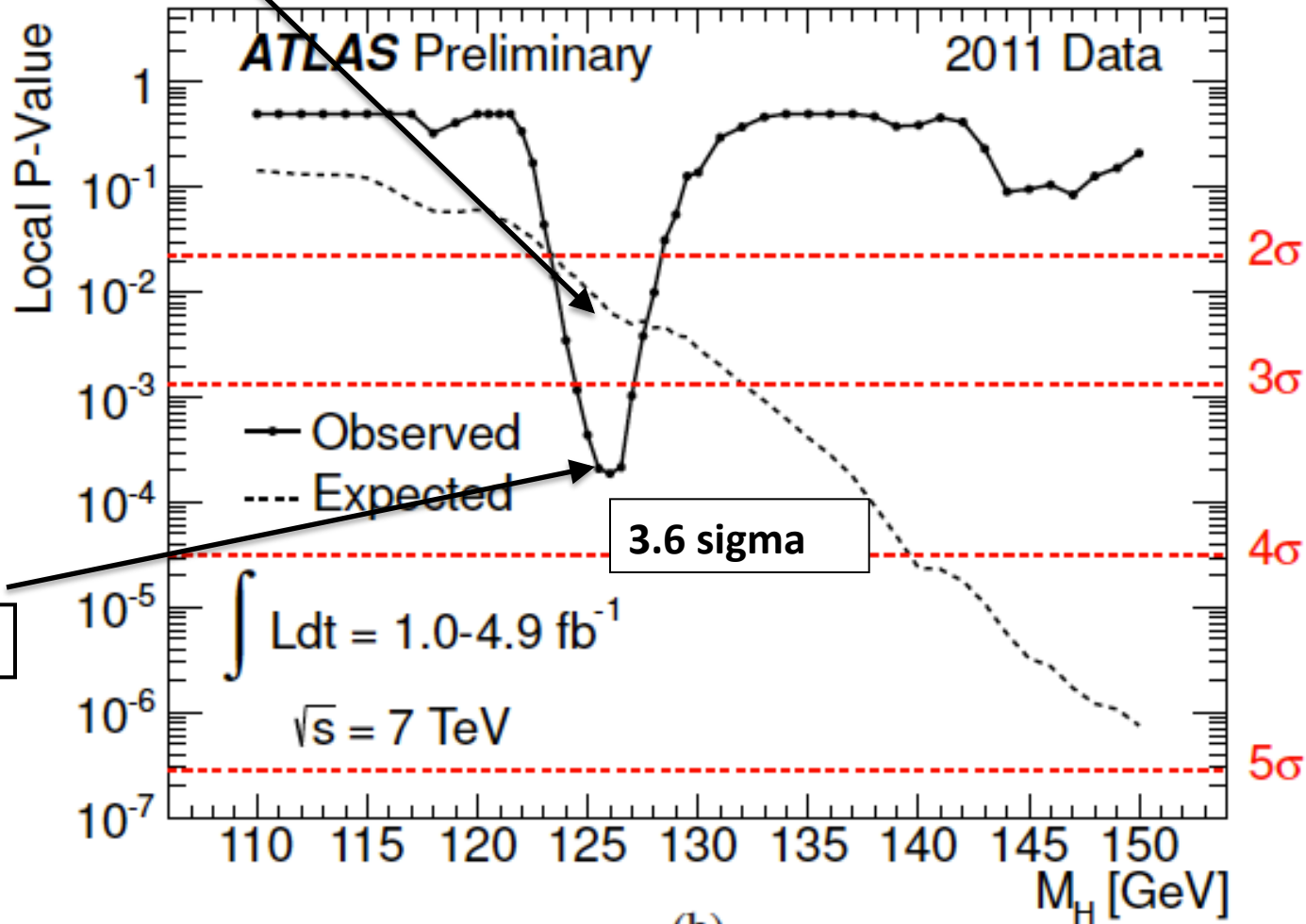
SM	30
Higgs	15

Discovery if p-value  $< 2.87 \times 10^{-7}$



# Standard HEP p-value plot

exected p-value



observed p-value

# Interpretation

pessimistic: exclusion

*Incompatibility with New Physics-hypothesis*

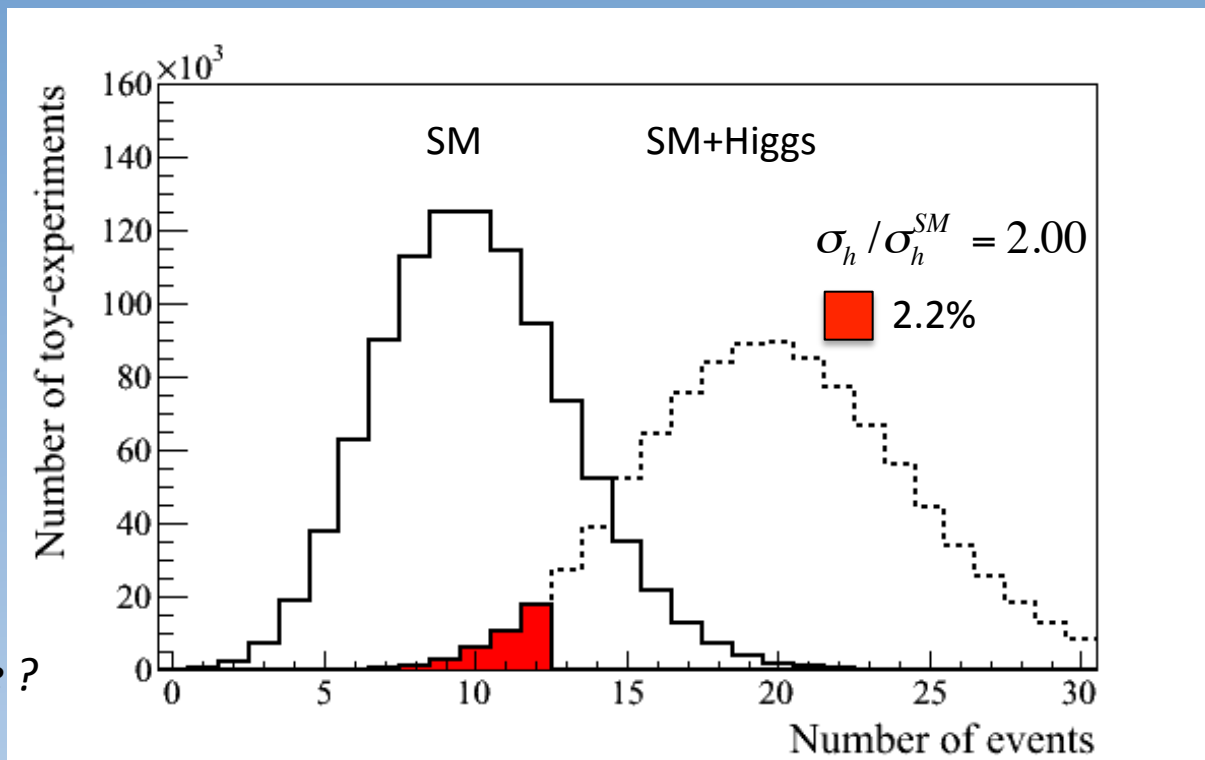
# Excluding a signal: Incompatibility with s+b hypothesis

Standard Model

SM	10
Higgs	5
Data	12

Can we exclude the SM+Higgs hypothesis?

What  $\sigma_h/\sigma_h^{SM}$  can we exclude?



Exclusion: probability to observe  $N$  events (or even less) under the signal + background hypothesis



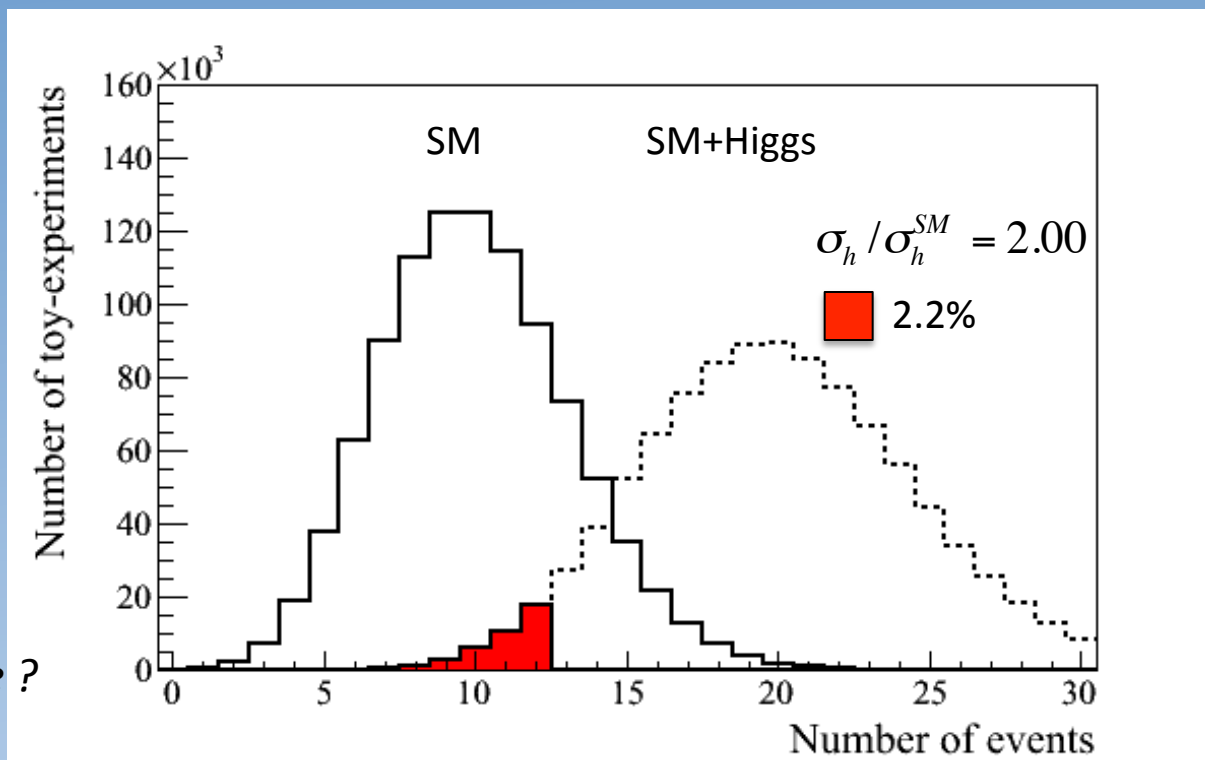
# Excluding a signal: Incompatibility with s+b hypothesis

Standard Model

SM	10
Higgs	5
Data	12

Can we exclude the SM+Higgs hypothesis?

What  $\sigma_h/\sigma_h^{SM}$  can we exclude?



$\sigma/\sigma_{SM}$	SM	# data	SM+Higgs	
1.0	10	12	15.0	18.5 %
1.5	10	12	17.5	6.8%
2.0	10	12	20.0	2.2%

*excluded*

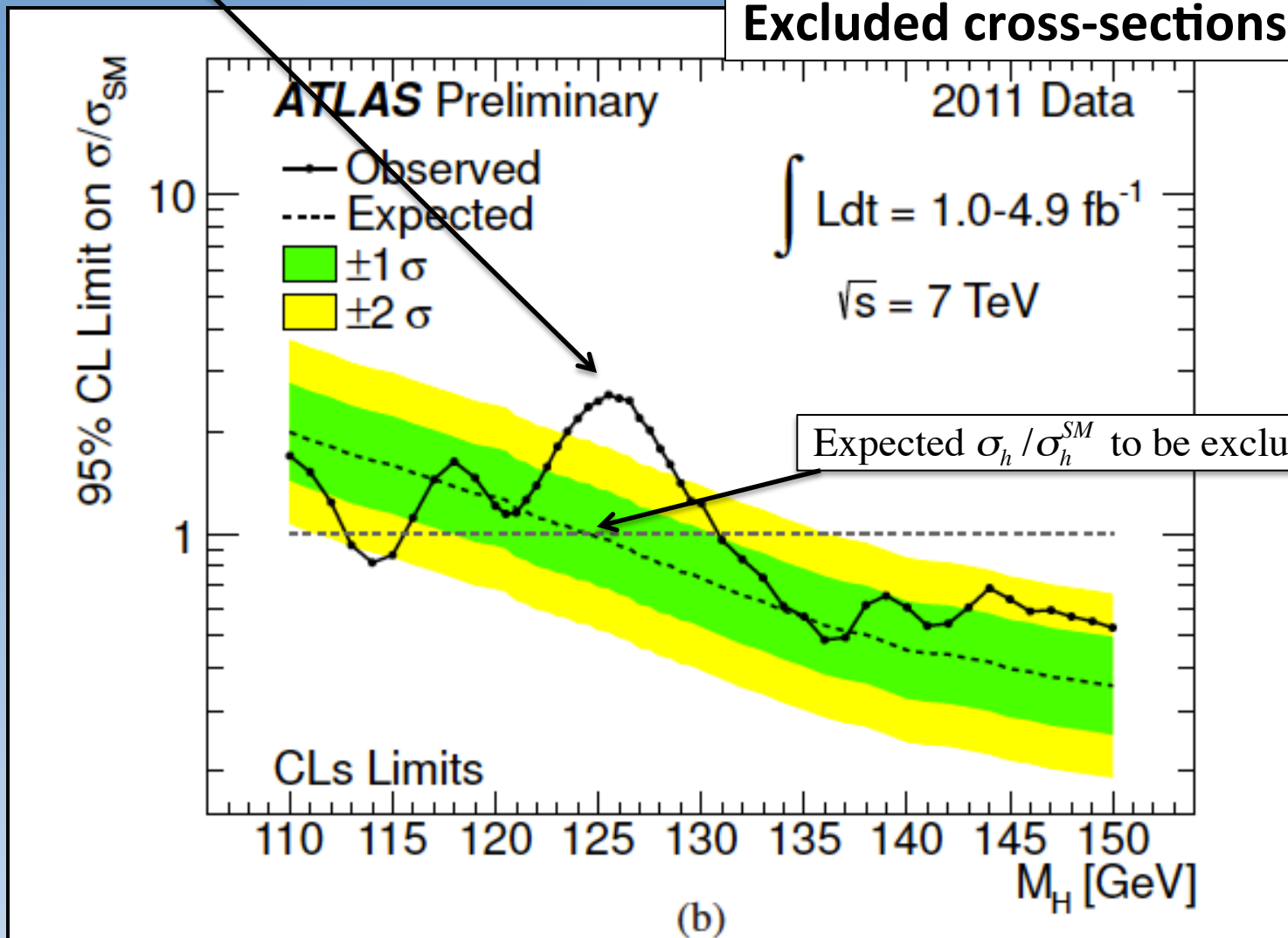
Expected exclusion? Use mean SM instead of Ndata

**Observed** excluded cross-section,  $\sigma_h/\sigma_h^{SM}$ , = 1.64

# Standard HEP exclusion plot

Observed  $\sigma_h / \sigma_h^{SM}$  to be excluded

Excluded cross-sections





Exercises

PART 1

# **1) significance optimization**

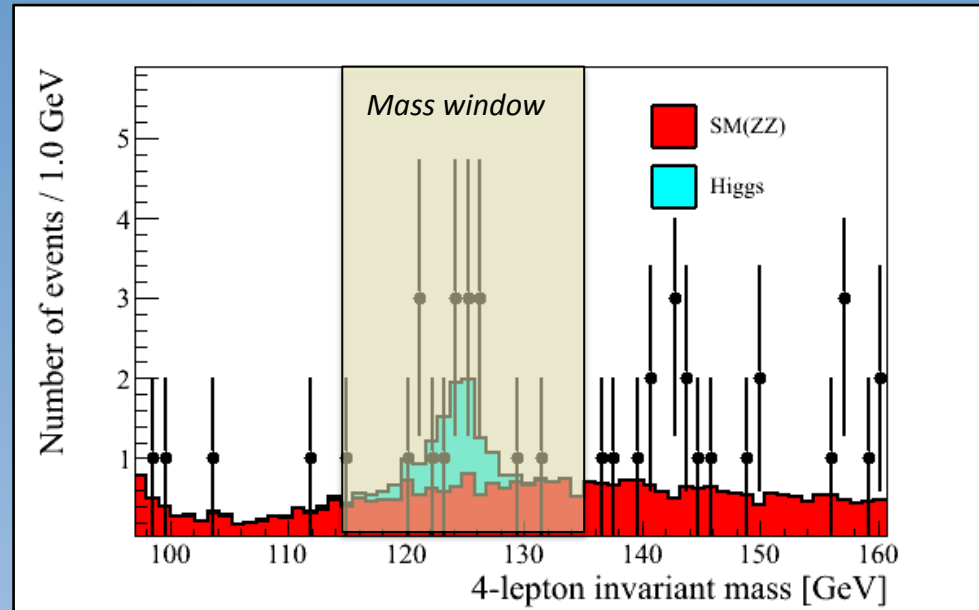
# Exercise 1:

## Optimizing the counting experiment

Code you could use:

```
IntegratePoissonFromRight()
```

```
Significance_Optimization()
```



### Exercise 1: significance optimization of search window (Poisson counting)

- 1.1** Find the window that optimizes the expected significance
- 1.2** Find the window that optimizes the observed significance (and never do it again)
- 1.3** Find the window that optimizes the expected significance for 5x higher luminosity
- 1.4** At what luminosity do you expect to be able to make a discovery ?



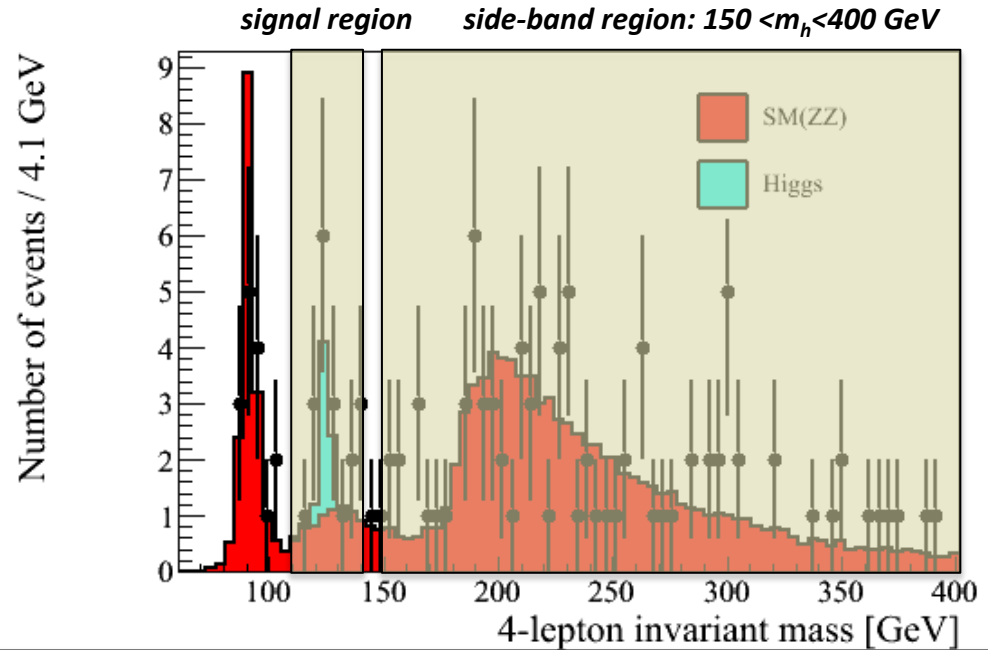
**2) background estimate  
from side-band fit**

# Exercise 2:

Data driven bkg estimate in 10 GeV ,mass window or optimal one from Exercise 1

Code you could use:

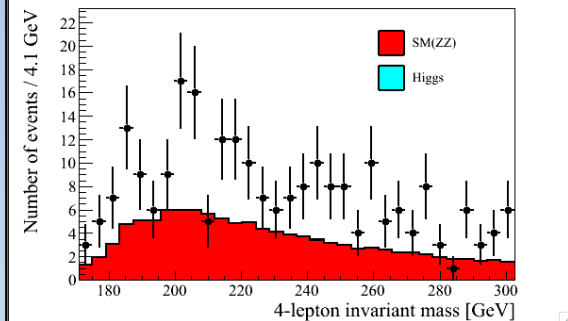
```
SideBandFit()
```



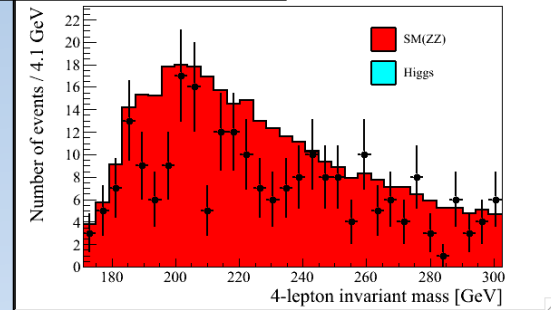
## Exercise 2: background estimation from side-band fit

- 2.1 What is the optimal scale-factor for the background ( $\alpha$ ) ?  
Do a likelihood fit to the side-band region  $150 \leq m_h \leq 400$  GeV

$\alpha = 0.50$  (too small)



$\alpha = 1.50$  (too large)



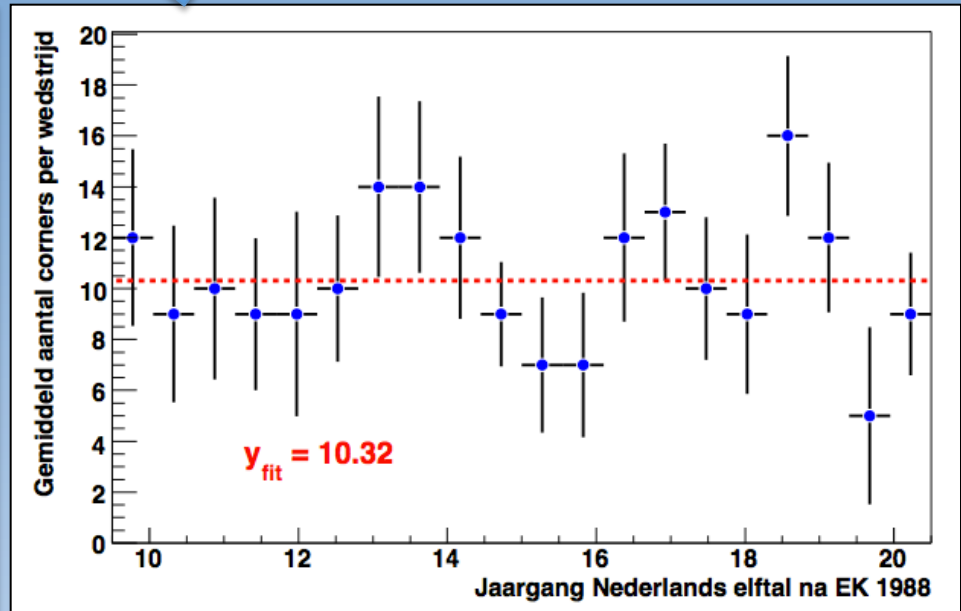
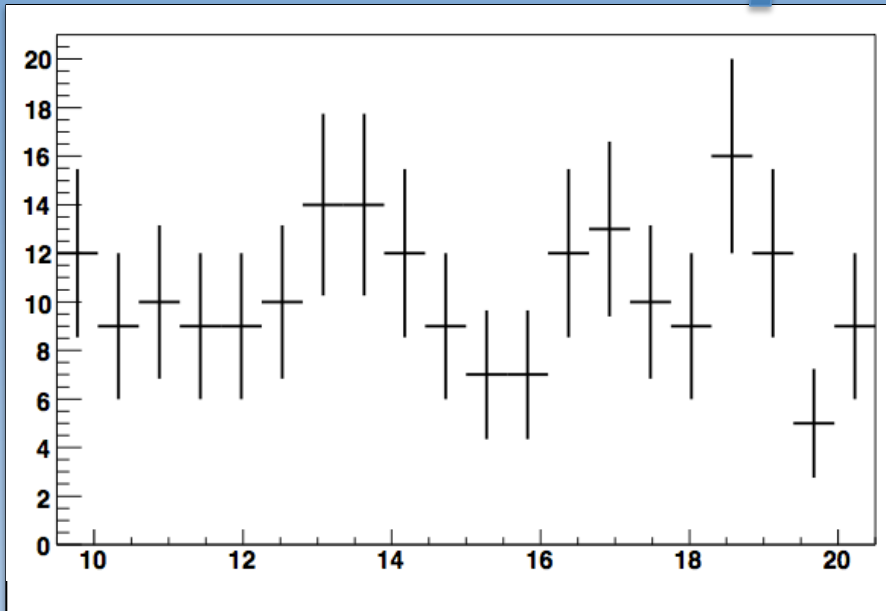
# **10-slide mini lecture on fitting**

**- Likelihood fits and uncertainties -**



# Simple likelihood fit

Can everybody do this ?

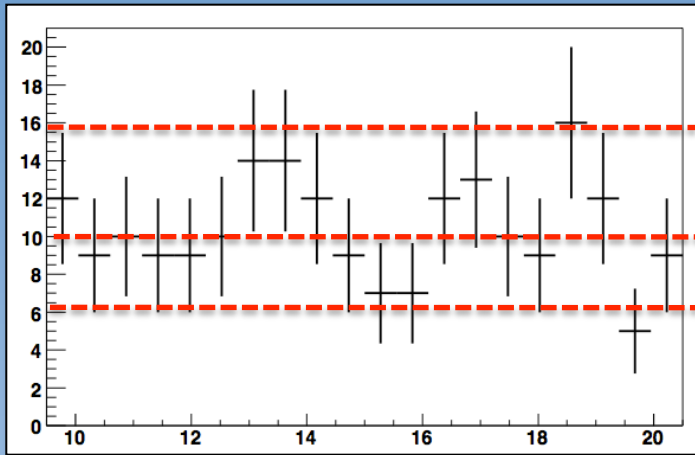


If you want to reproduce this plot, but cannot please let me know

`TMmath::Poisson( Nevt_bin, alpha )`

<http://www.nikhef.nl/~ivov/SimpleFit/>

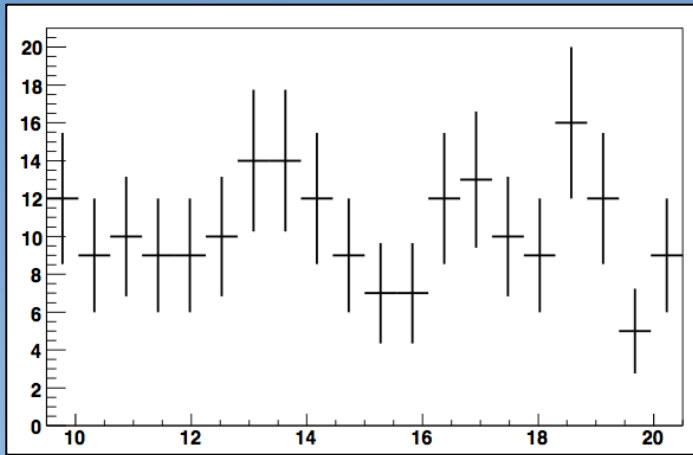
# Fitting in 1 slide



You model:  $f(x) = \lambda$

Try different values of  $\lambda$   
and for each one compute:

# Fitting in 1 slide



You model:  $f(x) = \lambda$

Try different values of  $\lambda$  and for each one compute the **compatibility** of the model with the data

## $\chi^2$ -fit

**Compatibility number :**

$$\chi^2 = \sum_{bins} \frac{(N_{bin}^{data} - \lambda_{bin}^{expected})^2}{N_{bin}^{data}}$$

**Best value:**

Value of  $\lambda$  that minimizes  $\chi^2$  ( $\chi_{min}^2$ )

**Errors:**

Values of  $\lambda$  for which  $\chi^2 = \chi_{min}^2 + 1$

## Likelihood-fit

**Compatibility number :**

$$-2\log(L) = -2 \cdot \sum_{bins} \log(\text{Poisson}(N_{bin}^{data} | \lambda))$$

`TMath::Poisson( Nevt_bin,  $\lambda$  )`

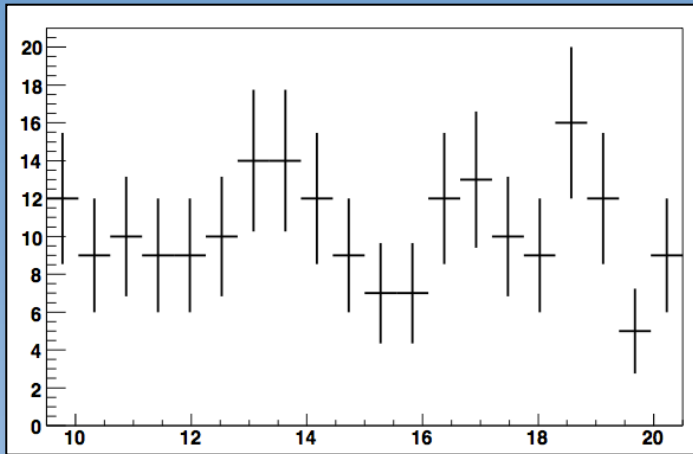
**Best value:**

Value of  $\lambda$  that minimizes  $-2\log(L)$  ( $-2\log(L)_{min}$ )

**Errors:**

Values of  $\lambda$  for which  $2\log(L) = (-2\log(L)_{min}) + 1$

# Fitting in 1 slide



You model:  $f(x) = \lambda$

Try different values of  $\lambda$  and for each one compute the **compatibility** of the model with the data

## Recipe for each value of $\lambda$ :

- Set LogLik = 0
- Loop over all bins:
  - o For each bin: compute prob. to observe  $N_i$  evts when you expect  $\lambda$ . **Poisson distribution**
  - o take  $-2 \cdot \text{Log}$  of bin-probability
  - o Add to existing LogLik
- Output LogLik (1 number)

## Likelihood-fit

### Compatibility number :

$$-2\log(L) = -2 \cdot \sum_{bins} \log(\text{Poisson}(N_{bin}^{data} | \lambda))$$

`TMath::Poisson( Nevt_bin,  $\lambda$  )`

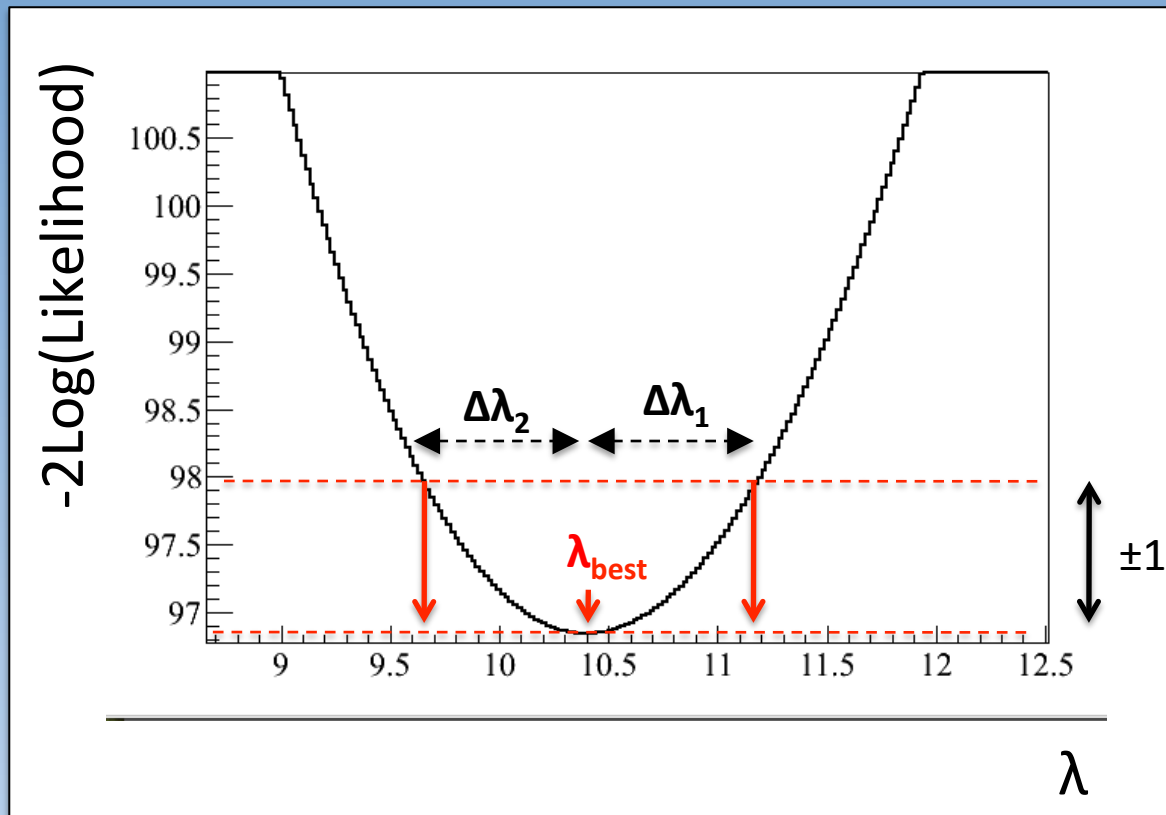
### Best value:

Value of  $\lambda$  that minimizes  $-2\text{Log}(L)$  ( $-2\log(L)_{\min}$ )

### Errors:

Values of  $\lambda$  for which  $2\text{Log}(L) = (-2\log(L)_{\min}) + 1$

# Result from the fit



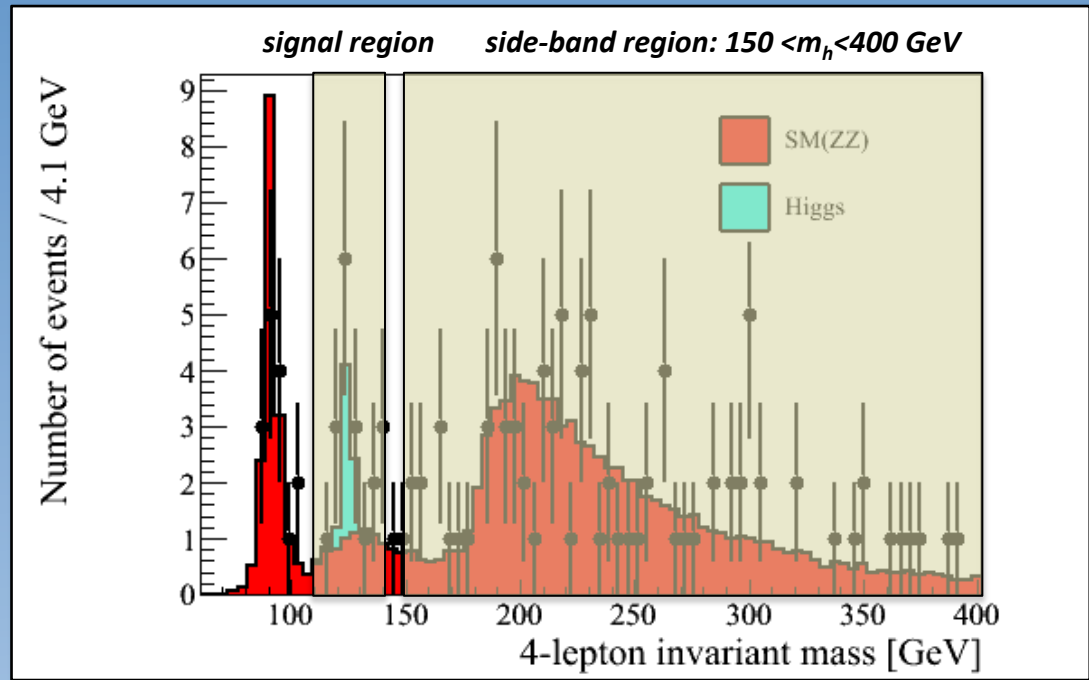
result :  $\lambda = \lambda_{\text{best}}^{+\Delta\lambda_1}_{-\Delta\lambda_2}$

# Exercise 2:

continued

Code you could use:

```
SideBandFit()
```



**Exercise 2: significance optimization of mass/search window** (use Poisson counting)

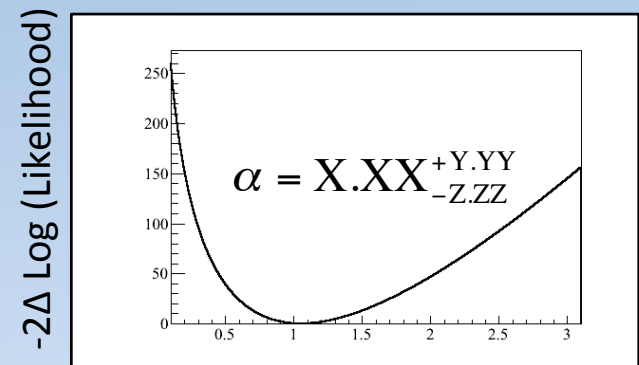
**2.1** What is the optimal scale-factor for the background ( $\alpha$ ) ?

Do a likelihood fit to the side-band region  $150 \leq m_h \leq 400$  GeV

**Computing the likelihood:**

For each 'guess' of  $\alpha$ :

$$-2\log(L) = -2 \cdot \sum_{bins} \log(\text{Poisson}(N_{bin}^{data} \mid \alpha \cdot f_{bin}^{SM}))$$



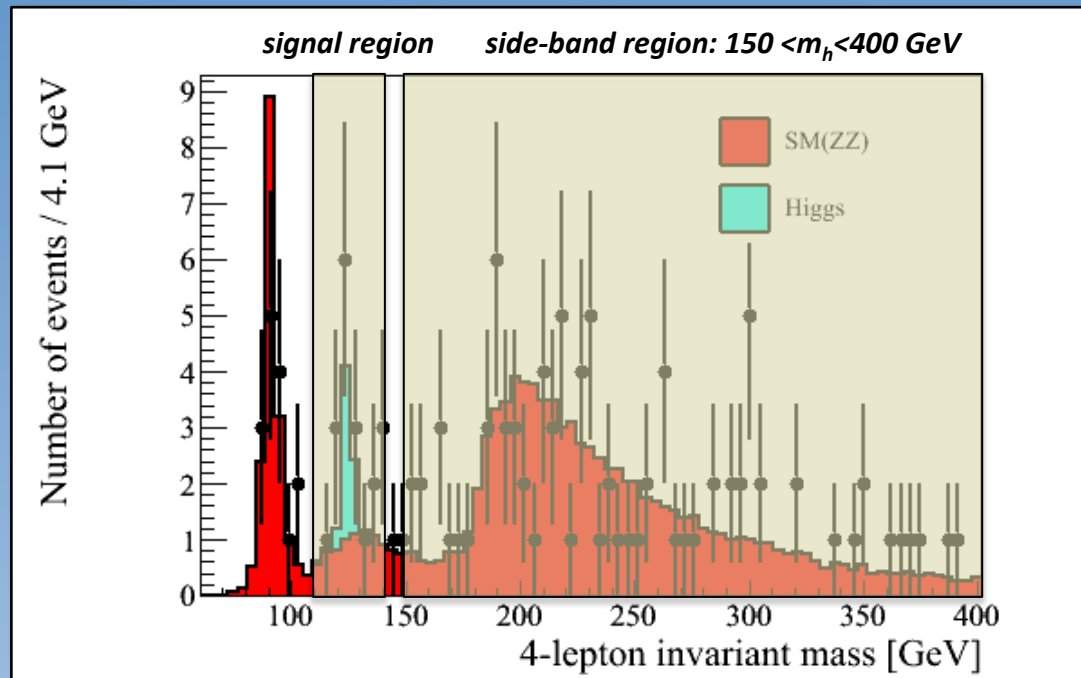
Background scale factor ( $\alpha$ )

# Exercise 2:

continued

Code to use:

none



**2.2** Estimate background and its uncertainty  $b \pm \Delta b$  in the mass window around 125 GeV (your optimal one from Exercise 1 or a simply a 10 GeV window)

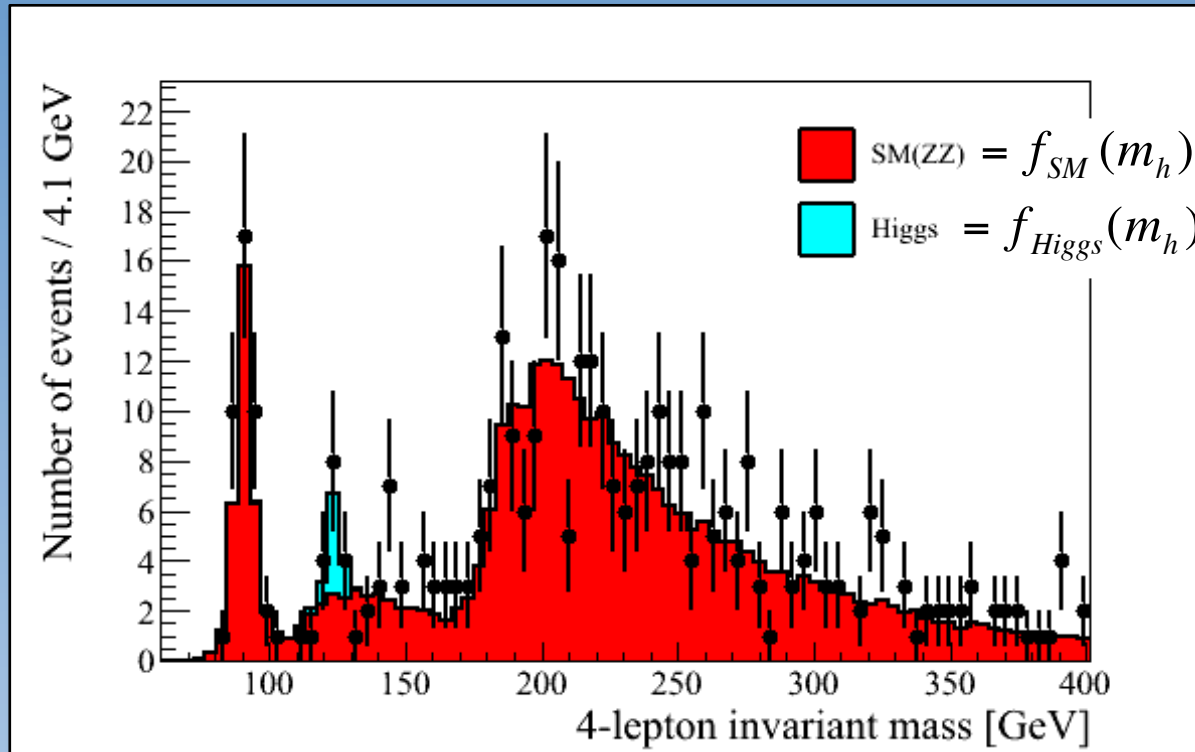
**2.3** Compute the expected and observed significance using Toy-MC

Note: Draw random # events in the mass window (for b-only and s+b)  
For each toy-experiment, not just draw a Poisson number,  
but also take a new central value using the (Gauss)  $\Delta b$  from 3.2

Compare it to the significance in exercise 1

## **3) signal cross-section**





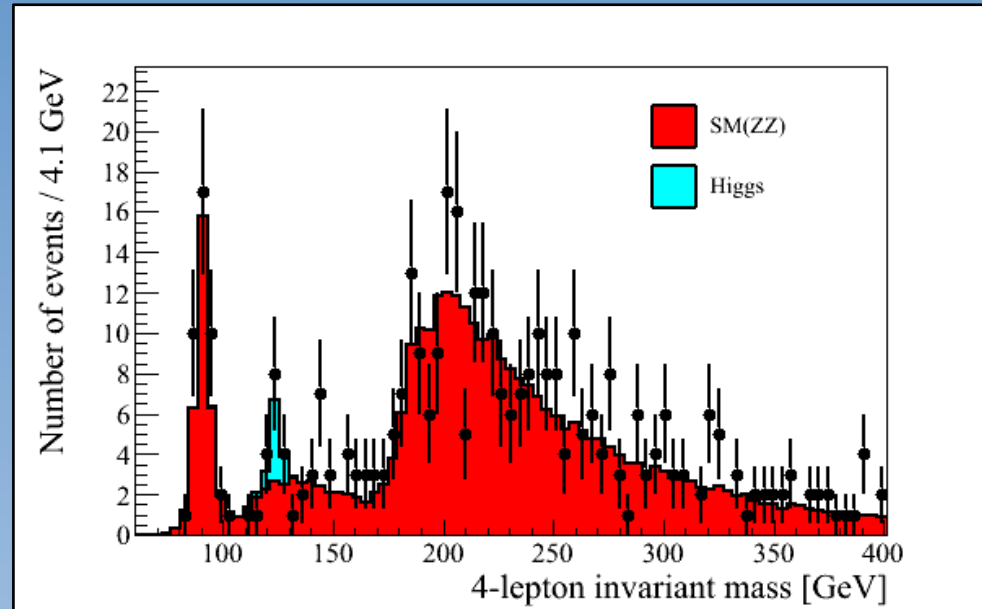
$$f(m_h) = \mu \times f_{Higgs}(m_h) + \alpha \times f_{SM}(m_h)$$

Scale factor for the Higgs

Scale factor for the SM background

# Exercise 3:

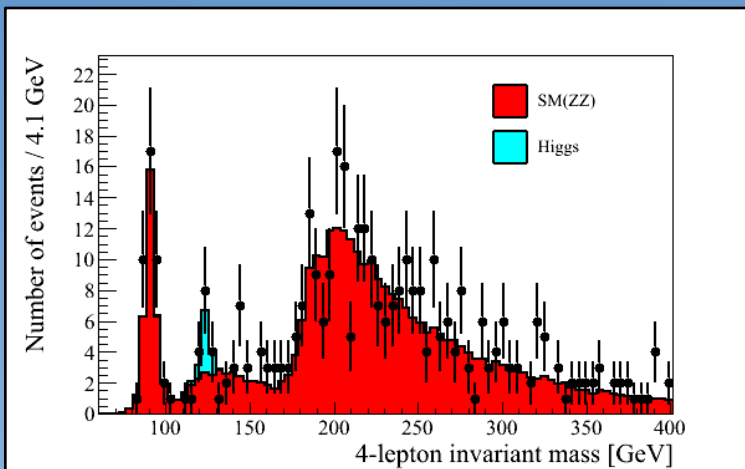
## Estimate of Higgs cross-section



$$-2 \cdot \log(\text{Likelihood}) = -2 \cdot \sum_{\text{bins}} \log(\text{Poisson}(N_{\text{bin}}^{\text{data}} \mid \mu \cdot f_{\text{bin}}^{\text{Higgs}} + \alpha \cdot f_{\text{bin}}^{\text{SM}}))$$

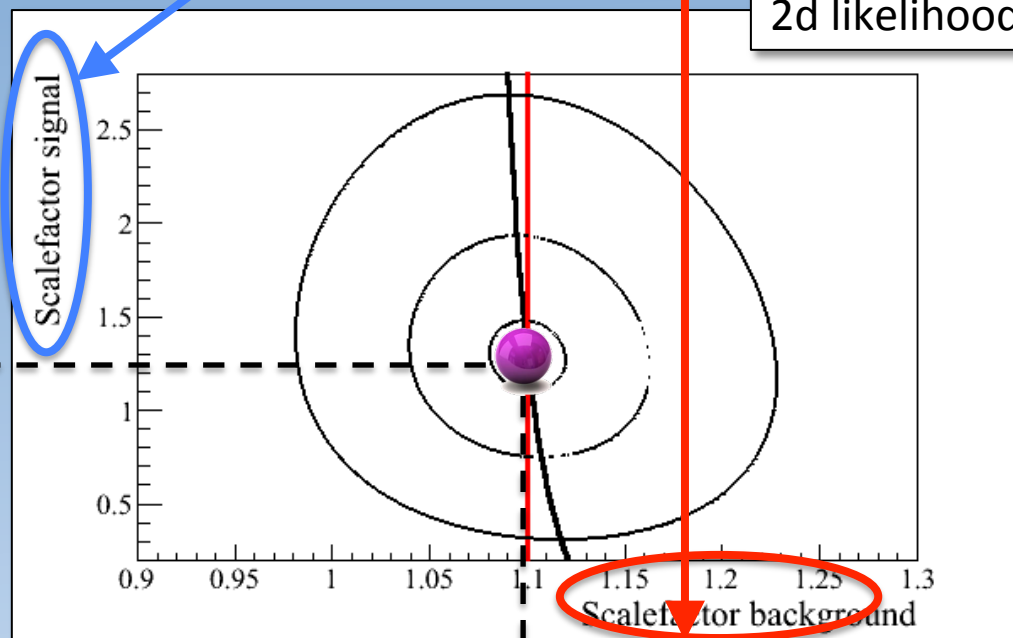
### Exercise 3: Measurement of the signal cross-section

- 3.1 Do a fit where you fix background (to level from exercise 2) and leave the signal cross-section ( $\mu$ ) free. What is the best value for  $\mu$  and what is its uncertainty?
- 3.2 Do a fit where you leave both  $\alpha$  and  $\mu$  free. What are the optimal values? How would you estimate the uncertainty on each of the parameters?



$$f(m_h) = \mu \times f_{\text{Higgs}}(m_h) + \alpha \times f_{\text{SM}}(m_h)$$

2d likelihood



$$\mu_{\text{bgr}}^{\text{best}} = 1.29$$

$$\alpha_{\text{bgr}}^{\text{best}} = 1.10$$

Good luck





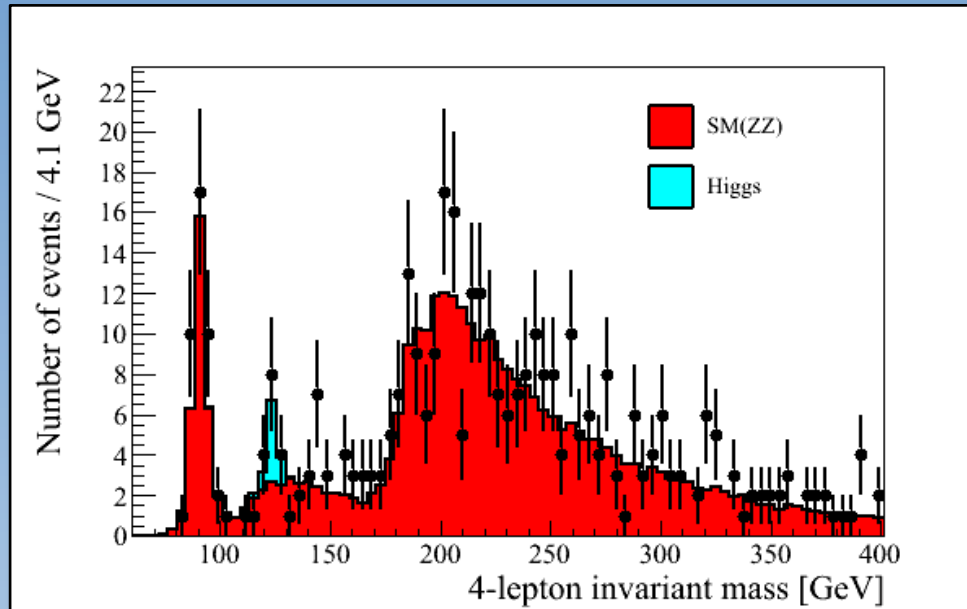


Exercises

PART 2

## **4) More complex test statistics**

# Beyond simple counting: profile likelihood ratio test-statistic



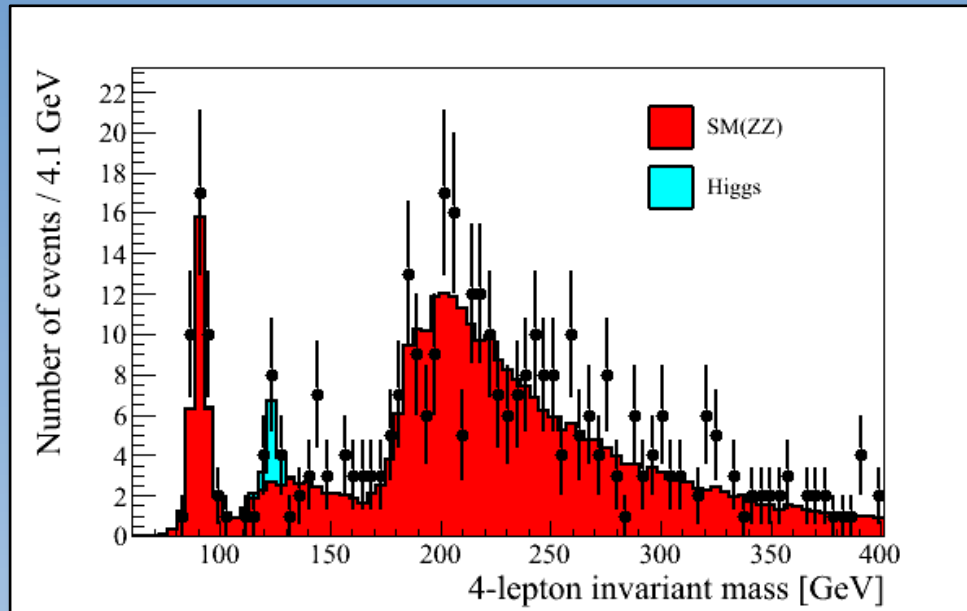
Condense data in  
**one** number:  $X$

LHC experiments:

$$X(\mu) = -2\ln(Q(\mu)), \quad \text{with} \quad Q(\mu) = \frac{L(\mu, \hat{\theta}(\mu))}{L(\hat{\mu}, \hat{\theta})}$$

We'll use something a bit simpler, but same idea

# Beyond simple counting: likelihood ratio test-statistic



$$-2 \cdot \log(\text{Likelihood}) = -2 \cdot \sum_{\text{bins}} \log(\text{Poisson}(N_{\text{bin}}^{\text{data}} \mid \mu \cdot f_{\text{bin}}^{\text{Higgs}} + \alpha \cdot f_{\text{bin}}^{\text{SM}}))$$

$$X = -2 \ln(Q), \text{ with } Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)}$$

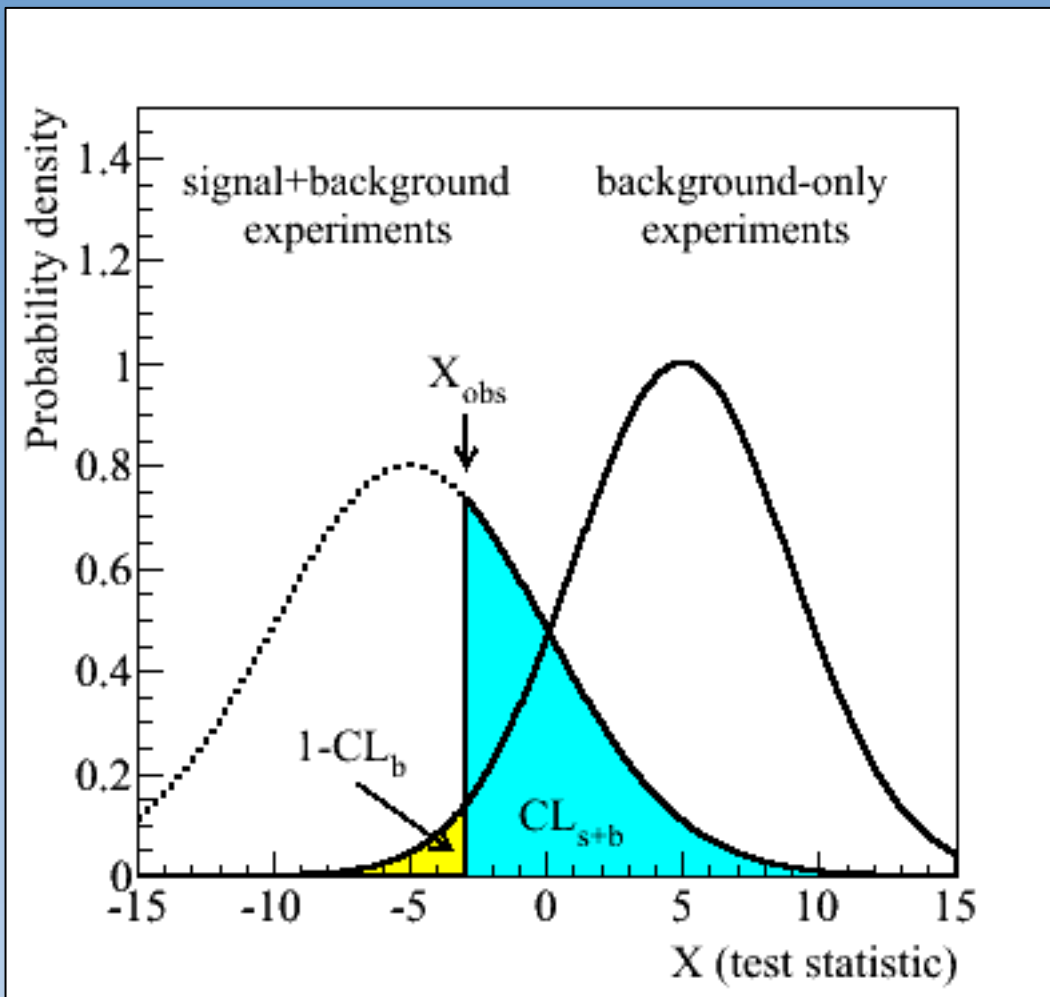
Likelihood assuming  $\mu_s=1$  (signal+background)

**Hypothesis 1**

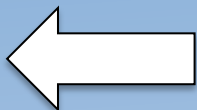
**Hypothesis 0**

Likelihood assuming  $\mu_s=0$  (only background)





signal like



background like

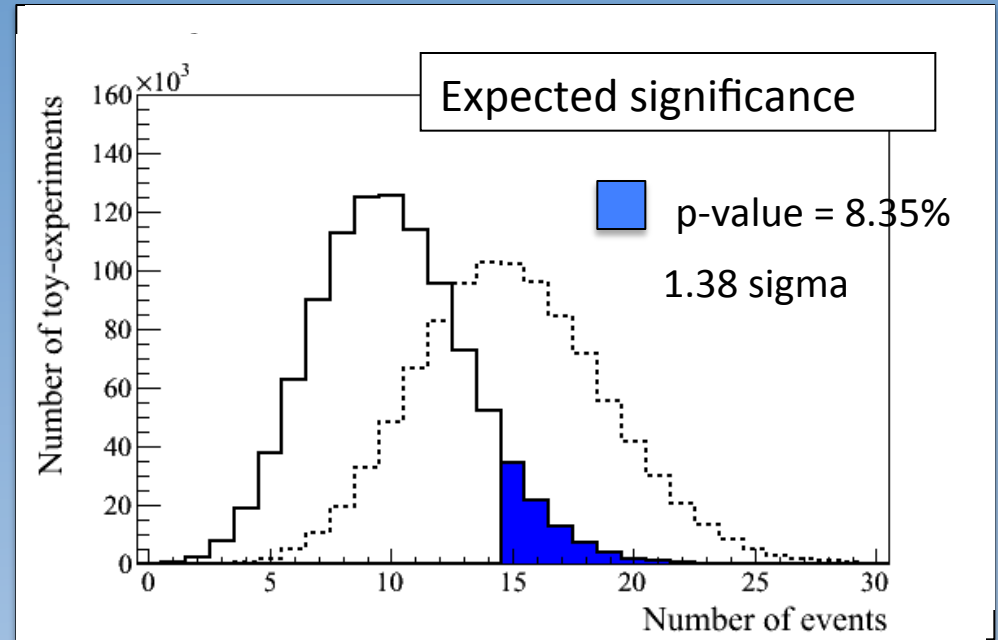


# Discovery-aimed: p-value and significance

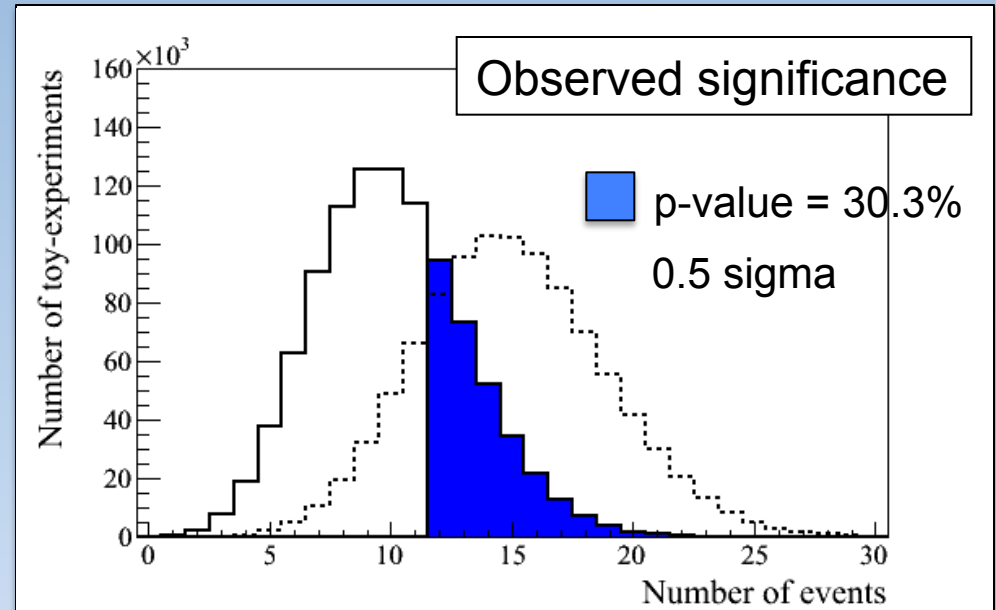
incompatibility with SM-only hypothesis

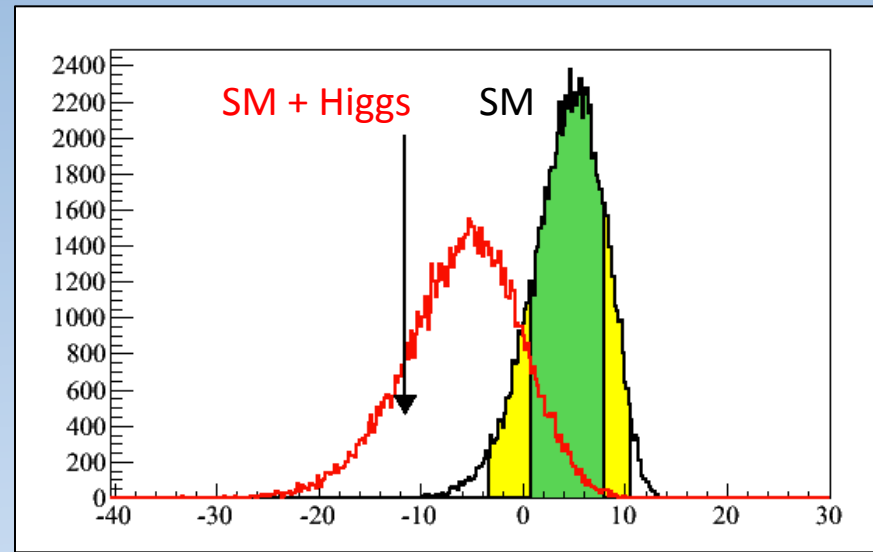
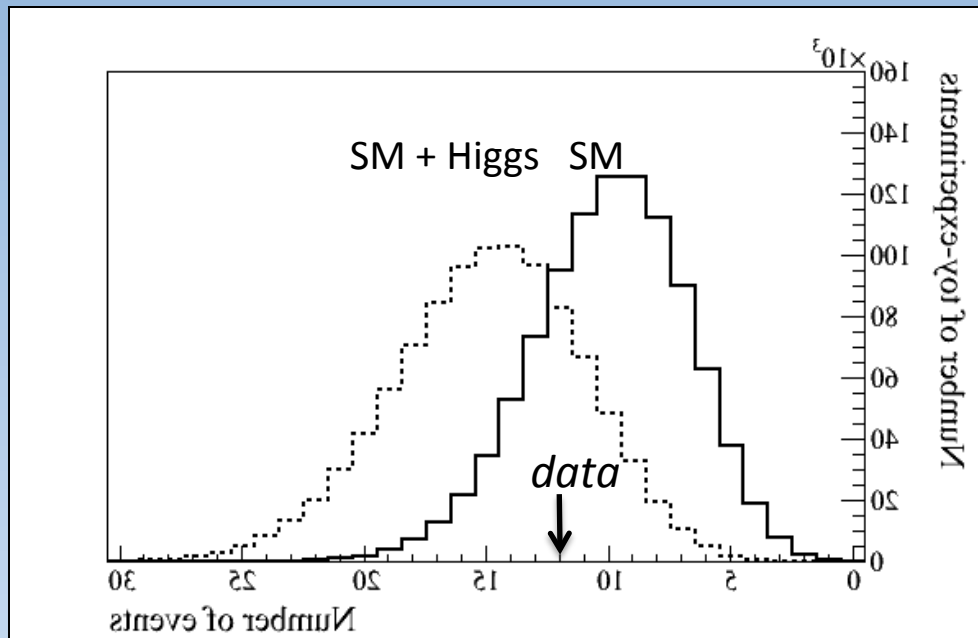
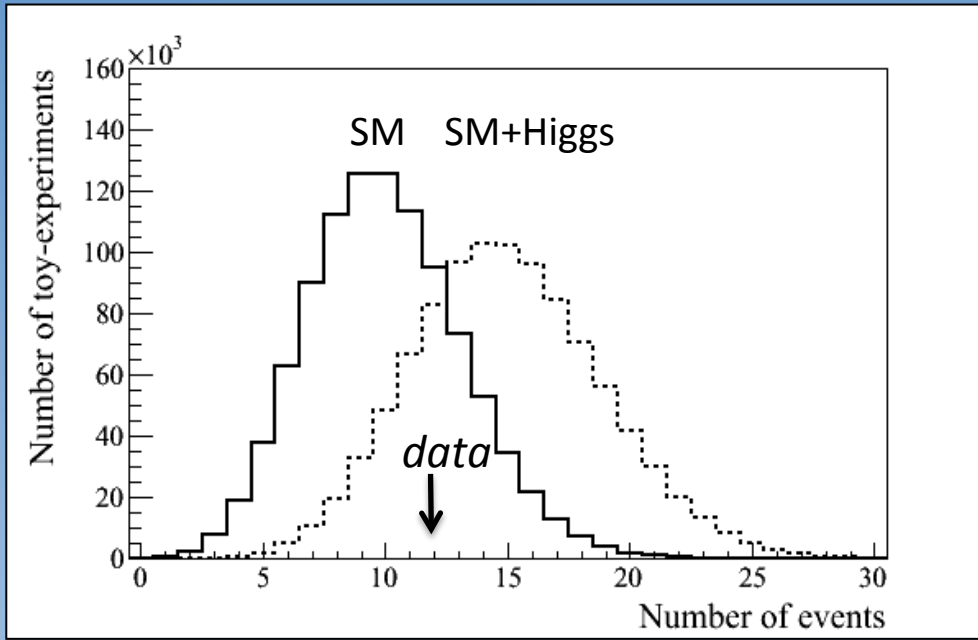
SM	10
Higgs	5
Data	12

1) What is the **expected** significance ?



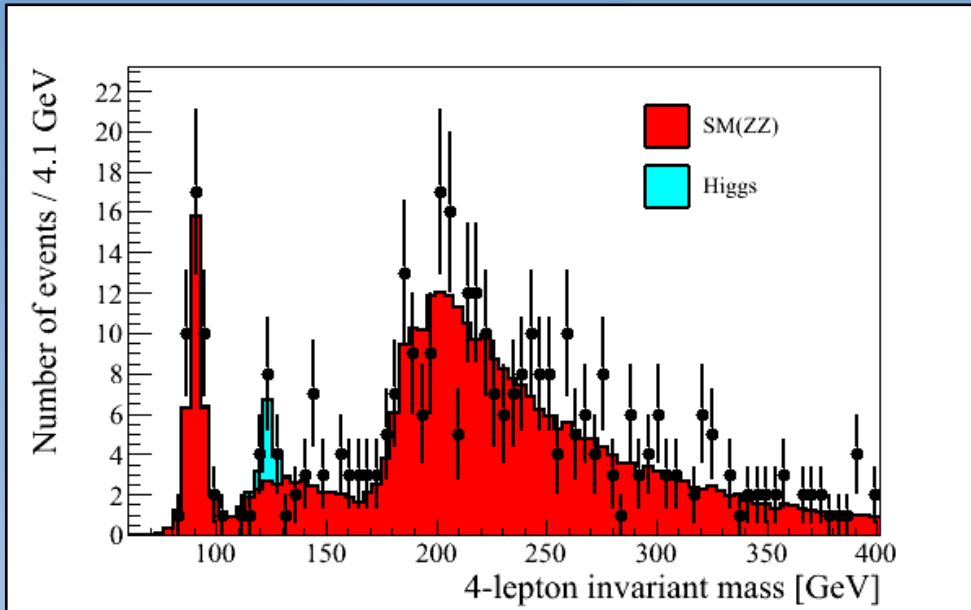
2) What is the **observed** significance ?





# Question: does the window not matter ?

$$X = -2\ln(Q), \text{ with } Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)}$$



$$X = \log(a/b) = \log(A) - \log(B)$$

What happens if you add a bin at 300 GeV ?  
Will it not dilute the channel like in counting ?

In that bin  $\text{Lik}_{\text{bin}} = \text{Constant} = C$

$$\begin{aligned} X = \log(a/b) &= [\log(A) + \log(C)] - [\log(B) + \log(C)] \\ &= \log(A) - \log(B) \end{aligned}$$

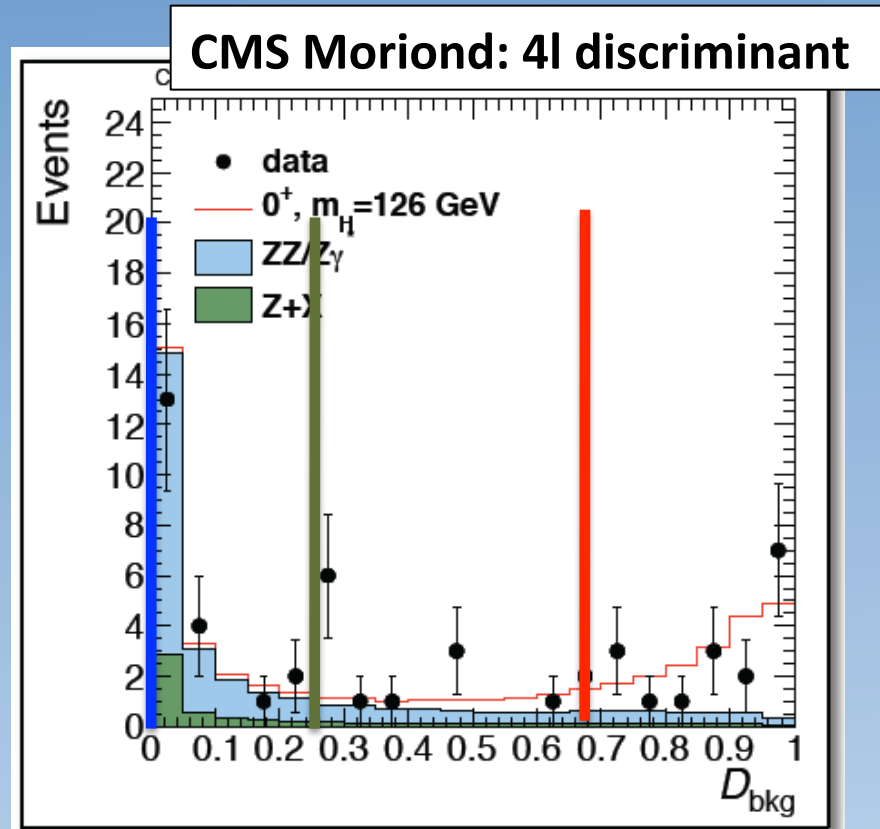
**ANY discrimination  
info is good !**

# Question: what about more info than mass alone ?

1) Optimal for counting

2) Optimal for LR test stat.

3) Normal procedure



Why: because the ‘information’ you add below  $D < 0.25$  is maybe difficult to verify in terms of correctness: needs signal description in very background-like region: systematics. Need to find optimum.

Note: they still evaluate, like you:  $X = -2\ln(Q)$ , with  $Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)}$

# We will use a very simple form for the test statistic

Our exercise ( $\alpha=1$  or from Ex.3):

$$X = -2\ln(Q), \text{ with } Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)} = \frac{\text{red}}{\text{blue}}$$

Tevatron-style:

$$X = -2\ln(Q), \text{ with } Q = \frac{L(\mu_s = 1, \hat{\theta}_{(\mu_s=1)})}{L(\mu_s = 0, \hat{\theta}_{(\mu_s=0)})}$$

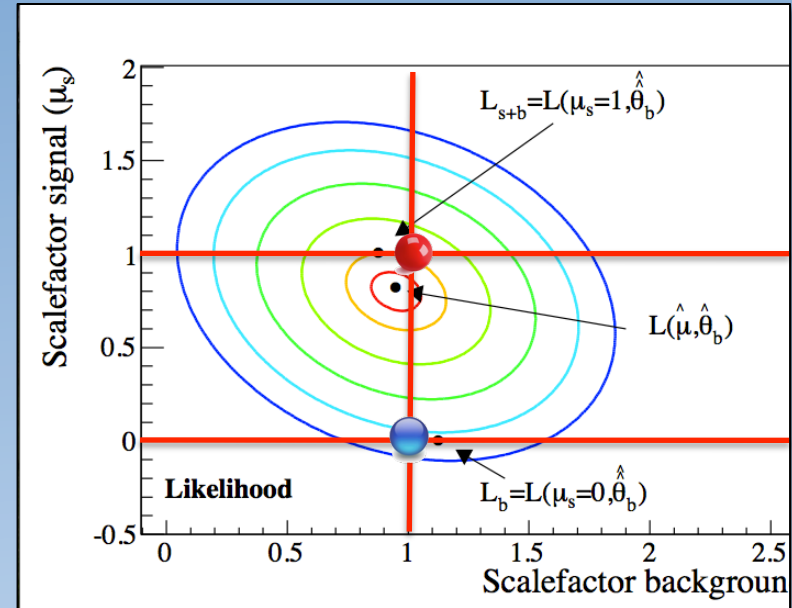
LHC experiments:

$$X(\mu) = -2\ln(Q(\mu)), \text{ with } Q(\mu) = \frac{L(\mu, \hat{\theta}(\mu))}{L(\hat{\mu}, \hat{\theta})}$$

**Note:**

$\alpha_{\text{bgr}}$  is just one of the nuisance parameters  $\theta$  in a 'real' analysis

2-dimensional fit ( $\alpha$  and  $\mu$  free)



# Exercise 4:

## Likelihood ratio test statistic (X)

$$X = -2\ln(Q), \text{ with } Q = \frac{L(\mu_s = 1)}{L(\mu_s = 0)} \begin{array}{l} \longrightarrow \text{Likelihood assuming } \mu_s=1 \text{ (signal+background)} \\ \longrightarrow \text{Likelihood assuming } \mu_s=0 \text{ (only background)} \end{array}$$

### Exercise 4: create the likelihood ratio test statistic – beyond simple counting

**4.1** Write a routine that computes the likelihood ratio test-statistic for a given data-set

`double Get_TestStatistic(TH1D *h_mass_dataset, TH1D *h_template_bgr, TH1D *h_template_sig)`

$$-2\text{Log}(\text{Likelihood}_{(\mu, \alpha = 1)}) = -2 \cdot \sum_{bins} \log(\text{Poisson}(N_{bin}^{data} \mid \mu \cdot f_{bin}^{Higgs} + \alpha \cdot f_{bin}^{SM}))$$

**Note:**  $\log(a/b) = \log(a) - \log(b)$

**4.2** Compute the likelihood ratio test-statistic for the ‘real’ data

**bonus:** Implement the conditional profile likelihood ratio, i.e. find for each of the two hypotheses ( $\mu_s=1$  and  $\mu_s=0$ ) the best value for the background scaling ( $\alpha_{bgr}$ )

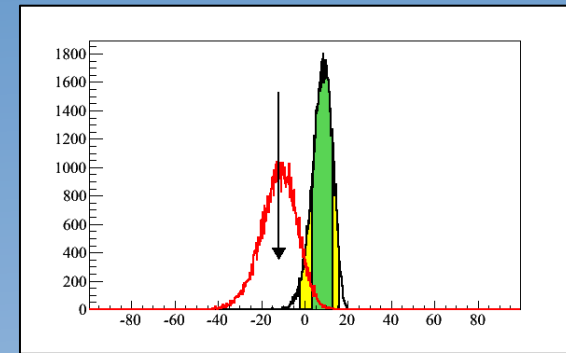
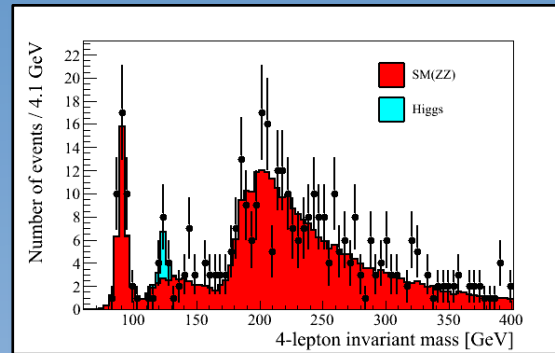
$$X = -2\ln(Q), \text{ with } Q = \frac{L(\mu_s = 1, \hat{\theta}_{(\mu_s=1)})}{L(\mu_s = 0, \hat{\theta}_{(\mu_s=0)})}$$

- 5) - Toy Monte Carlo**
  - distribution of test statistic for different hypotheses**



# Exercise 5:

- Generate toy data-sets
- Test statistic distribution



## Exercise 5: create toy data-sets

5.1 Write a routine that generates a toy data-set from a MC template (b or s+b)

`TH1D * GenerateToyDataSet(TH1D *h_mass_template)`

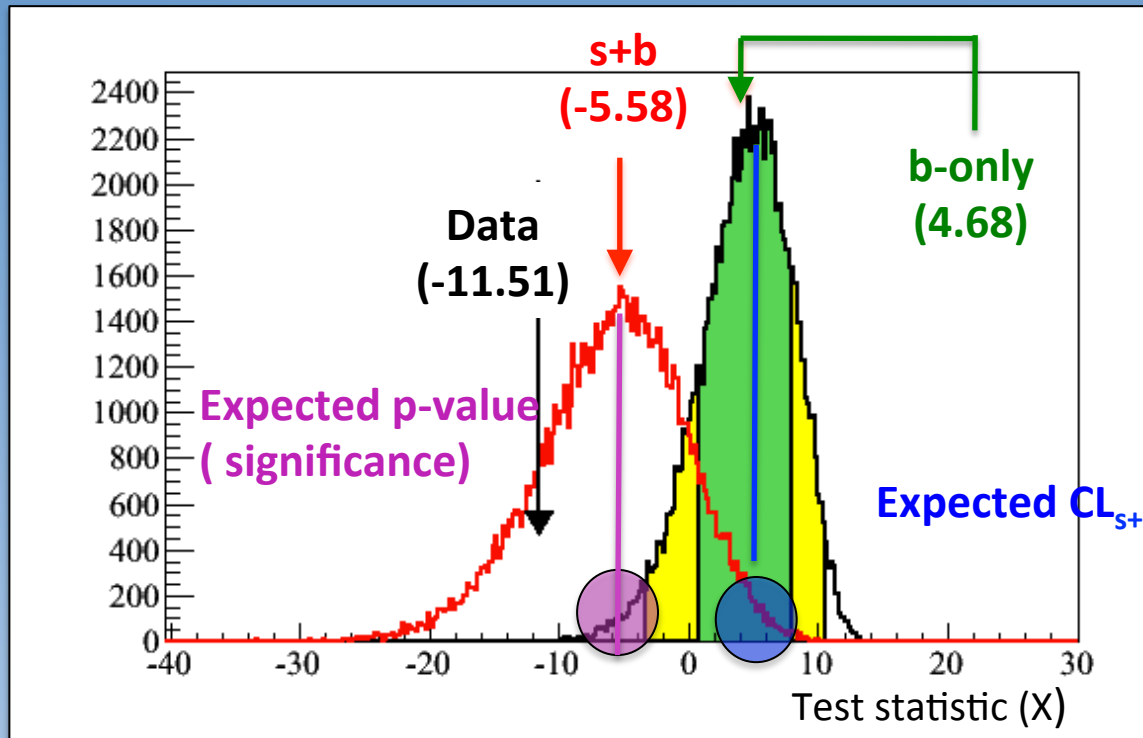
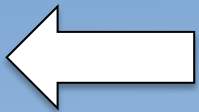
How: Take the histogram `h_mass_template` and draw a Poisson random number in each bin using the bin content in `h_mass_template` as the central value. Return the new fake data-set.

5.2 Generate 1000 toy data-sets for *background-only* & get test statistic distribution  
Generate 1000 toy data-sets for *signal+background* & get test statistic distribution

→ plot both in one plot

5.3 Add the test-statistic from the data(exercise 4.2) to the plot

signal like

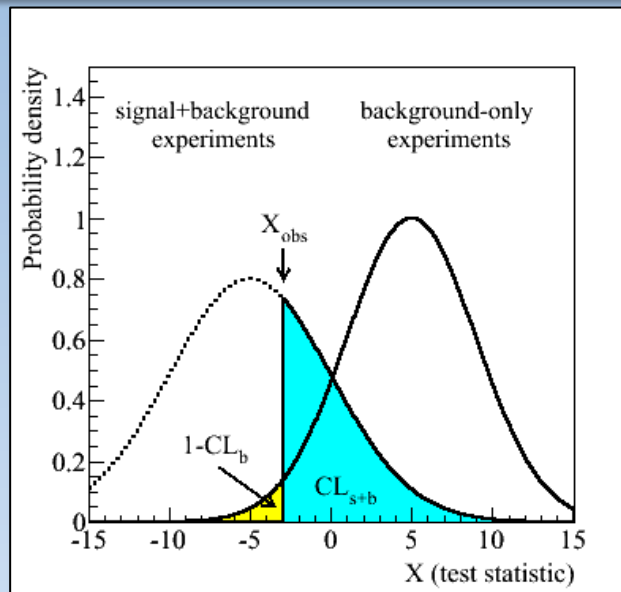
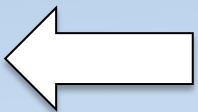


background like



Discovery:  $1-CL_b < 2.87 \times 10^{-7}$   
Incompatibility with b-only hypothesis

signal like



Exclusion:  $CL_{s+b} < 0.05$   
Incompatibility with s+b hypothesis

background like



## **6) Discovery potential**

# Exercise 6

Summarize separation power: conclusion

## Exercise 5: compute p-value

- 6.1** Compute the p-value or  $1-Cl_b$  (under the background-only hypothesis):
- For the average(median) b-only experiment
  - For the average(median) s+b-only experiment [expected significance]
  - For the data [observed significance]
- 6.2** Draw conclusions:
- Can you claim a discovery ?
  - Did you expect to make a discovery ?
  - At what luminosity did/do you expect to be able to make a discovery ?

## **7) Excluding hypotheses**

# Exercise 6 continued

Exclude a cross-section for a given Higgs boson mass

Some shortcomings, but  
we'll use it anyway



$$\sigma_h(m_h) = \xi \cdot \sigma_h^{SM}(m_h)$$



Scale factor wrt SM prediction

**Exercise 6: compute  $CL_{s+b}$  and exclude Higgs masses or cross-sections**

**6.3** Compute the  $CL_{s+b}$ :

- For the average(median) s+b experiment
- For the average(median) b-only experiment
- For the data

**6.4** Draw conclusions:

- Can you exclude the  $m_h=200$  GeV hypothesis ? What  $\xi$  can you exclude ?
- Did you expect to be able to exclude the  $m_h=200$  GeV hypothesis ?  
What  $\xi$  did you expect to be able to exclude ?

BACKUP

# Exercise 8

## Pulls

**Exercise 8: Is your procedure unbiased and has correct uncertainty ?**

**8.1** Generate 100 fake data-sets (at  $\mu=1$ ) and extract  $\mu$  and its error  $\rightarrow$  extract pull

... more later



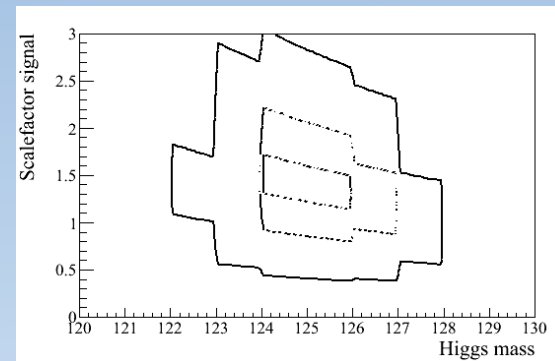
# Exercise 9

## Mass versus $\mu$

### Exercise 9: Imass versus pull

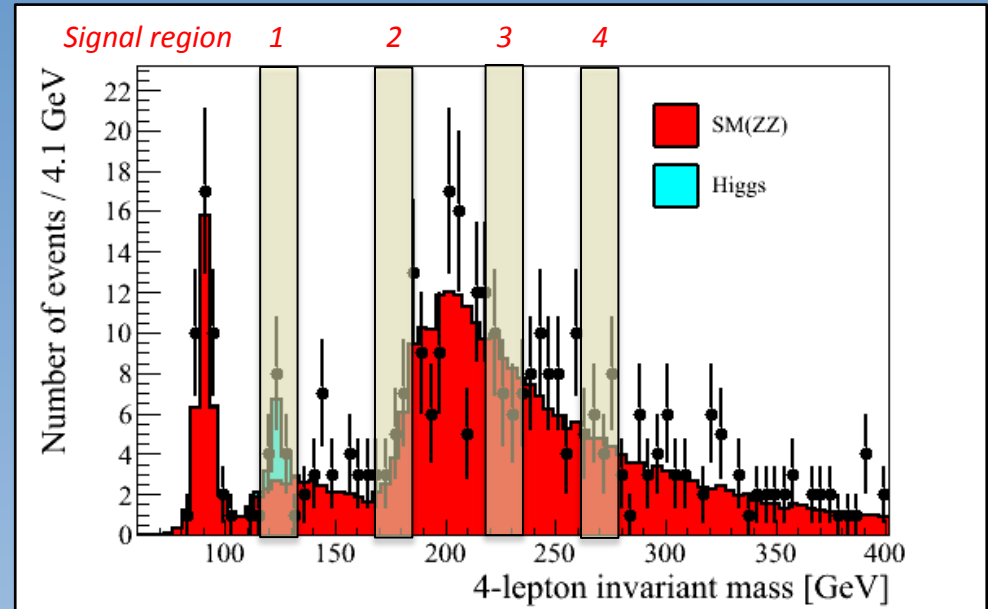
#### 9.1 Do a 2d fit on $m_h$ and $\mu$

... more later



# Exercise 3

Look-Elsewhere effect (Trial factor)



## Exercise 3: trial factors – global versus local p-values (Look Elsewhere effect)

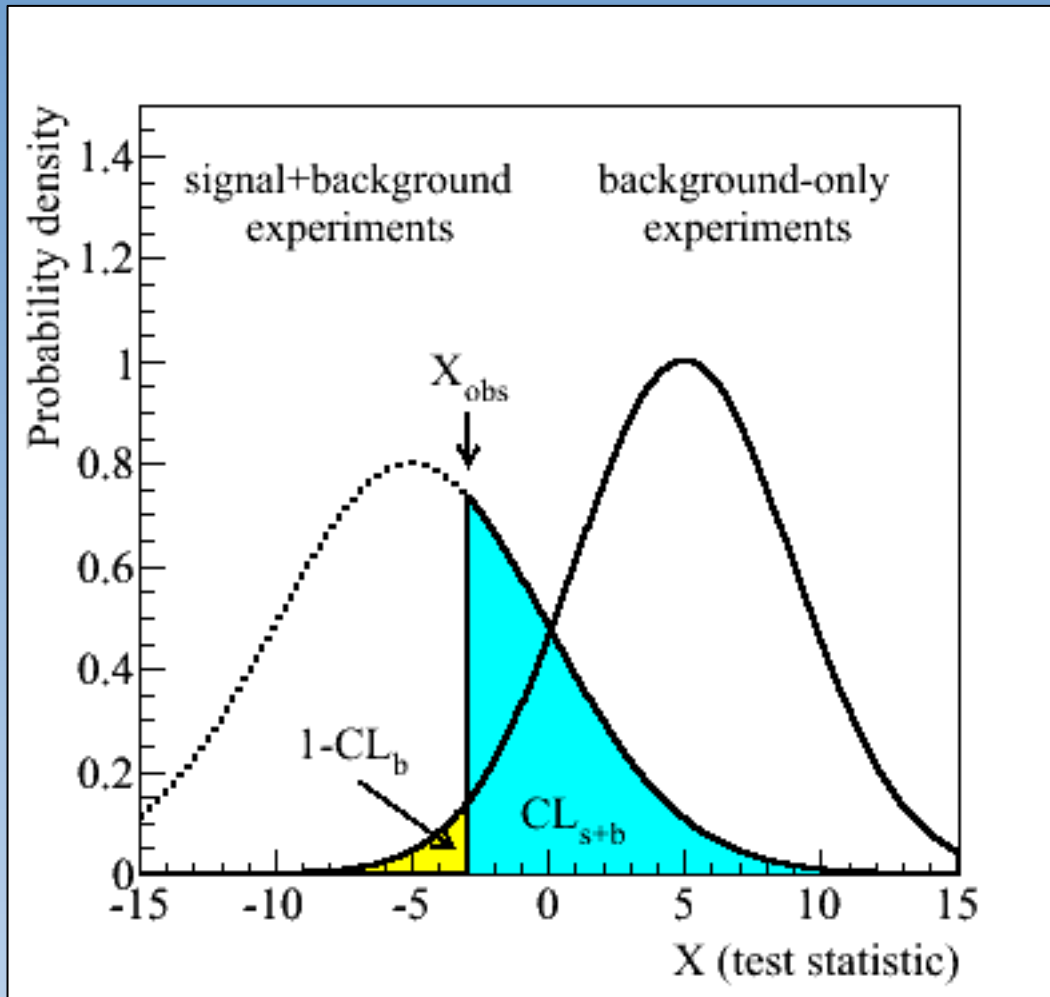
**3.1** Simulate different event yields in 4 possible mass regions. What fraction of LHC experiments is expected to have an excess  $\geq 2$  sigma: at 125, 175, 225 & 275 GeV ?  
Use 4 Poisson Random numbers in a 10 GeV mass window per toy-experiment

**3.2** What is the fraction of toy-experiments that have a maximum excess (at least one out of the 4) above 2 sigma?



To what global significance does a 2 sigma local significance correspond ?

# $X_{\text{obs}}$ : rules for discovery and exclusion



Discovery:  $1-CL_b < 2.87 \times 10^{-7}$   
Incompatibility with b-only hypothesis

Exclusion:  $CL_{s+b} < 0.05$   
Incompatibility with s+b hypothesis