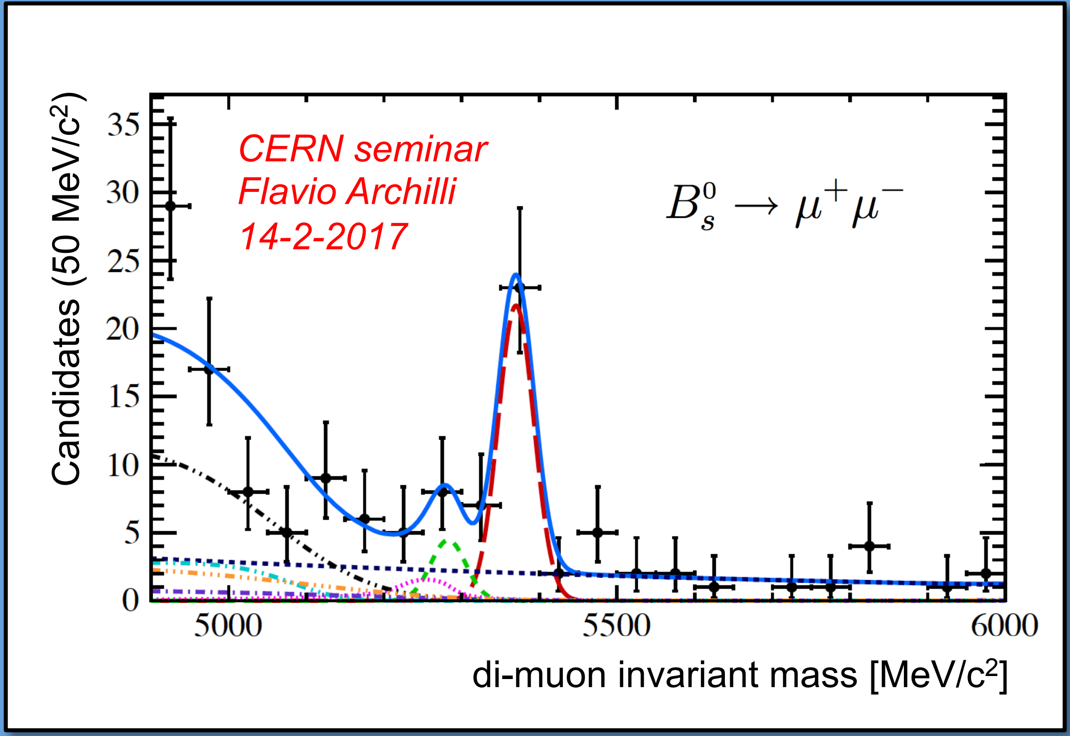
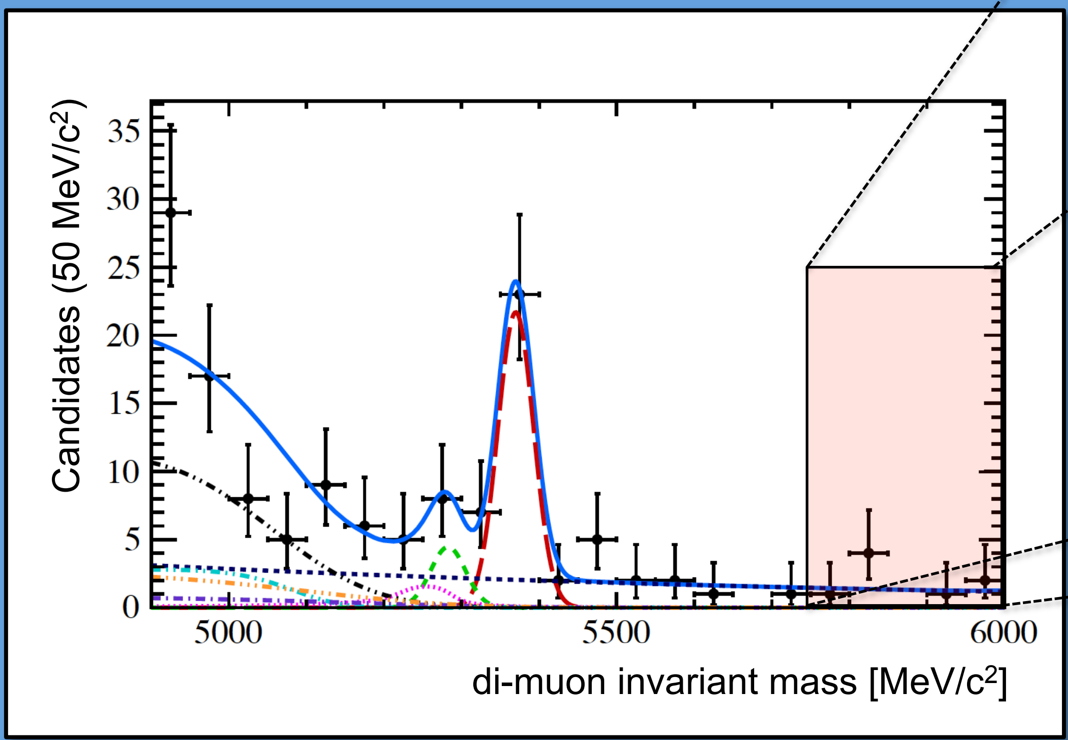
The asymmetric uncertainties on data points in Roofit

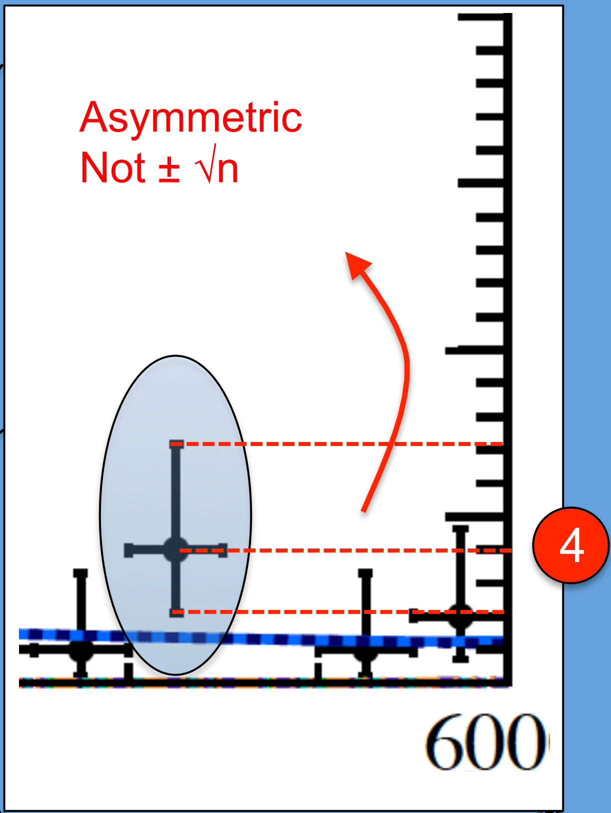
Few slides on 'easy/trivial' topic that will hopefully leave you confused



Ivo van Vulpen
(Nikhef/UvA, ATLAS)

How are our asymmetric uncertainties on data points defined ?






statistics

Solving statistics problems in general

Discussions (in large experiments):

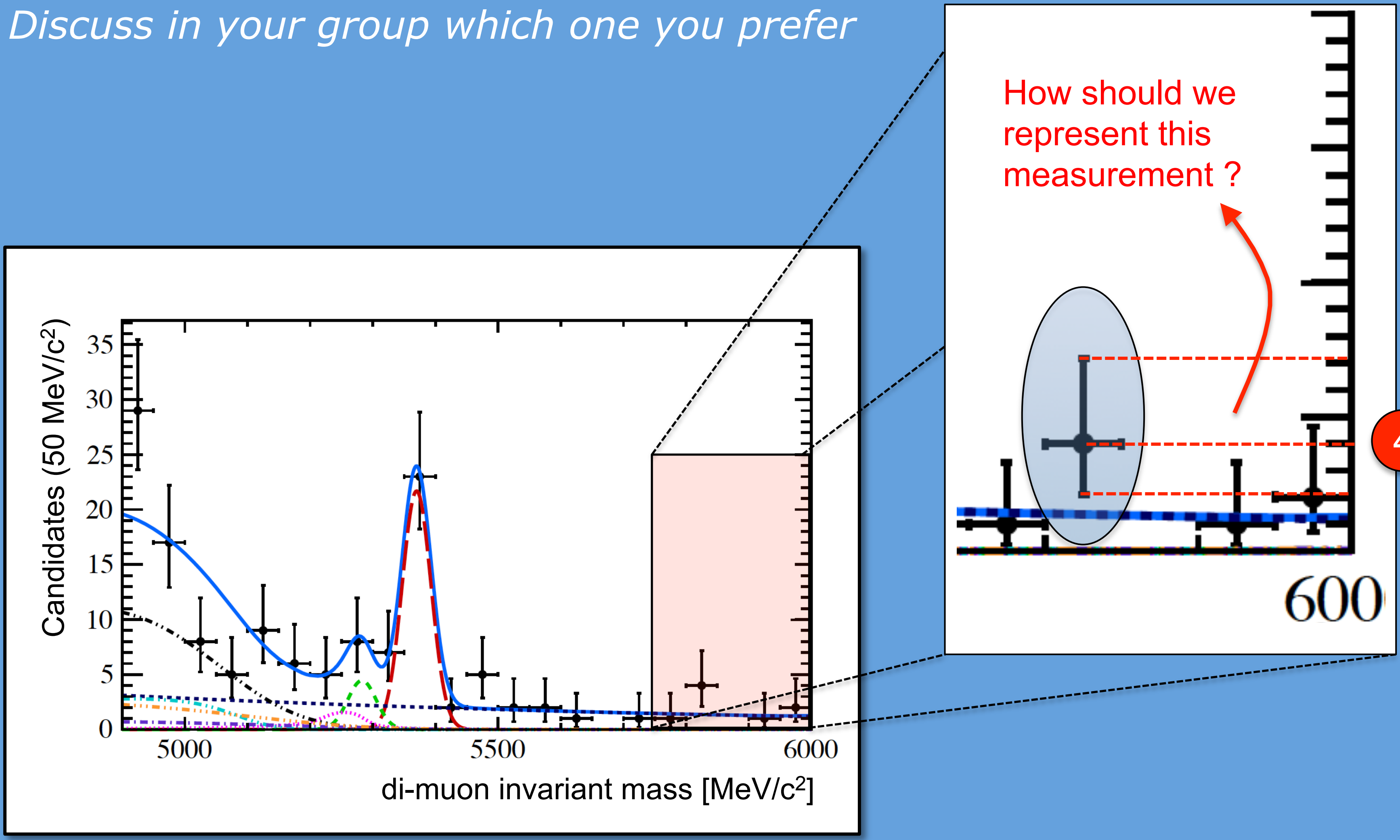
- strong opinions, very outspoken 'experts'
- use **RooSomethingFancy**: made by experts & debugged
- let's do what we did last time, let's be conservative
- …

You are responsible for how you summarize **your** measurement

The tools you use have assumptions, biases, pitfalls, … , so you know best how others (and you) should interpret your measurement

Six reasonable options

Discuss in your group which one you prefer



Basis for this talk: http://www.nikhef.nl/~ivov/Statistics/PoissonError/BobCousins_Poisson.pdf

Option 1: assign NO uncertainty

± 0.00

The number of observed events is what it is: 4

The uncertainty is in the interpretation step, i.e. on the model-parameters that you extract from it

Comfortable territory: Poisson distribution

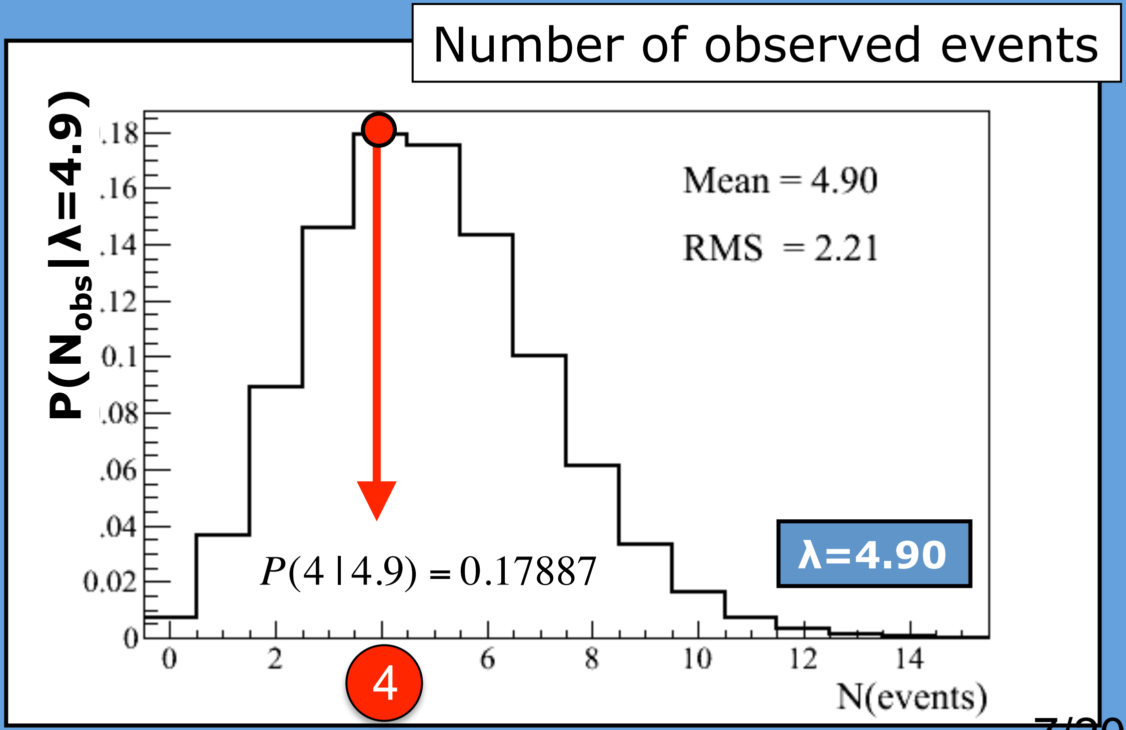
$$P(n\,|\,\lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$$

Binomial with n→∞, p→0 en np=λ

#observed

λ hypothesis

Probability to observe *n* events when λ are expected



Properties of the Poisson distribution

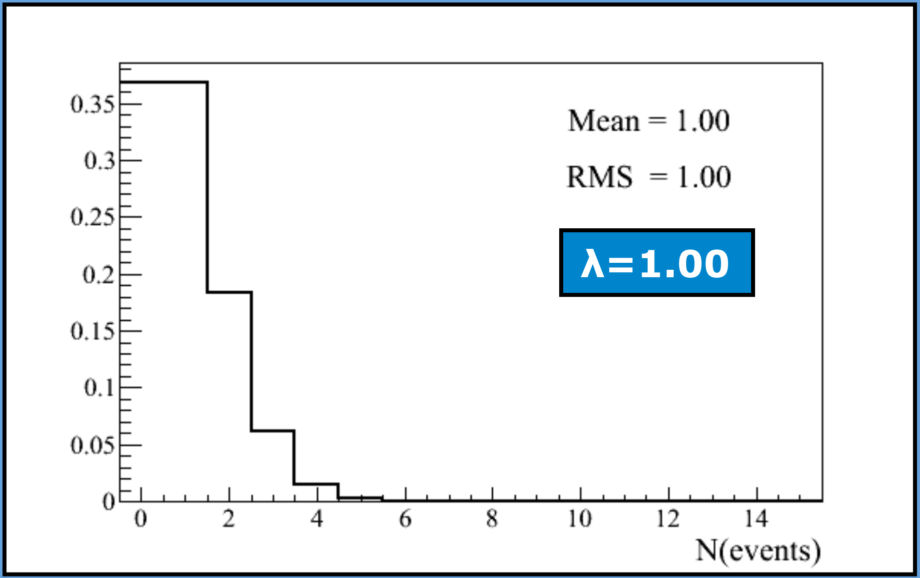
the famous $\sqrt{n}$

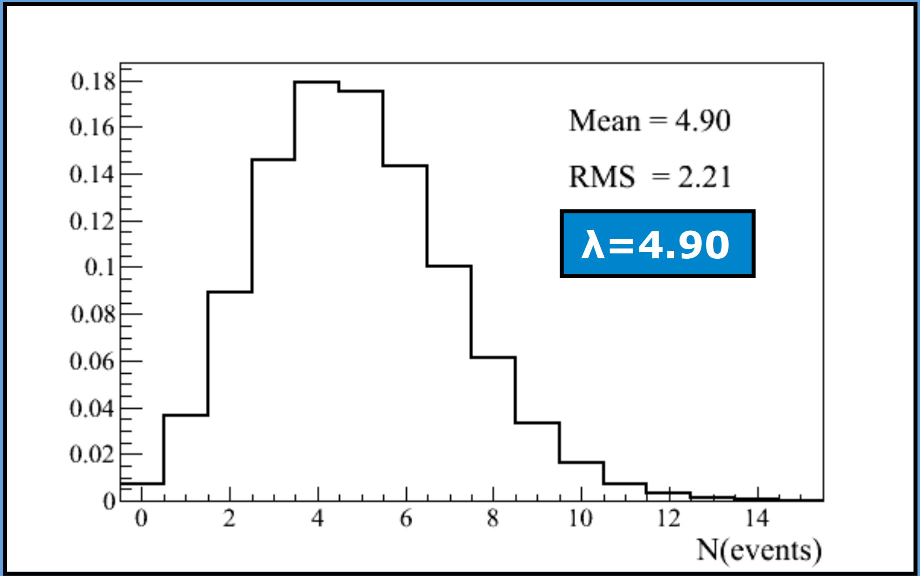
properties

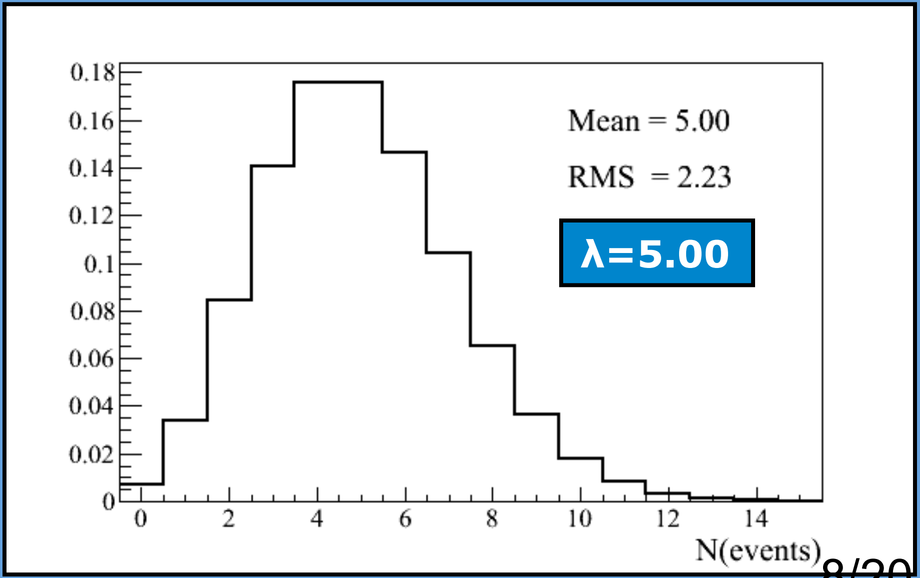
(1) Mean: $\langle n \rangle = \lambda$

(2) Variance: $\langle (n - \langle n \rangle)^2 \rangle = \lambda$

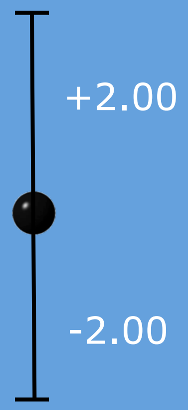
(3) Most likely: first integer $\leq \lambda$







Option 2: the famous sqrt(n)



Poisson variance for λ equal to measured number of events

… but Poisson distribution is asymetric: $\begin{cases} P(4 \mid 2) = 0.09022 \\ P(4 \mid 6) = 0.13385 \end{cases}$

Just treat it like a normal measurement

What you have:

$$P(N_{obs} \mid \lambda)$$

1) construct the Likelihood
   λ as free parameter

↓

2) Find value of λ that
   maximizes the Likelihood

↓

3) Determine error interval:
   Δ(-2Log(Lik.)) = +1

Likelihood

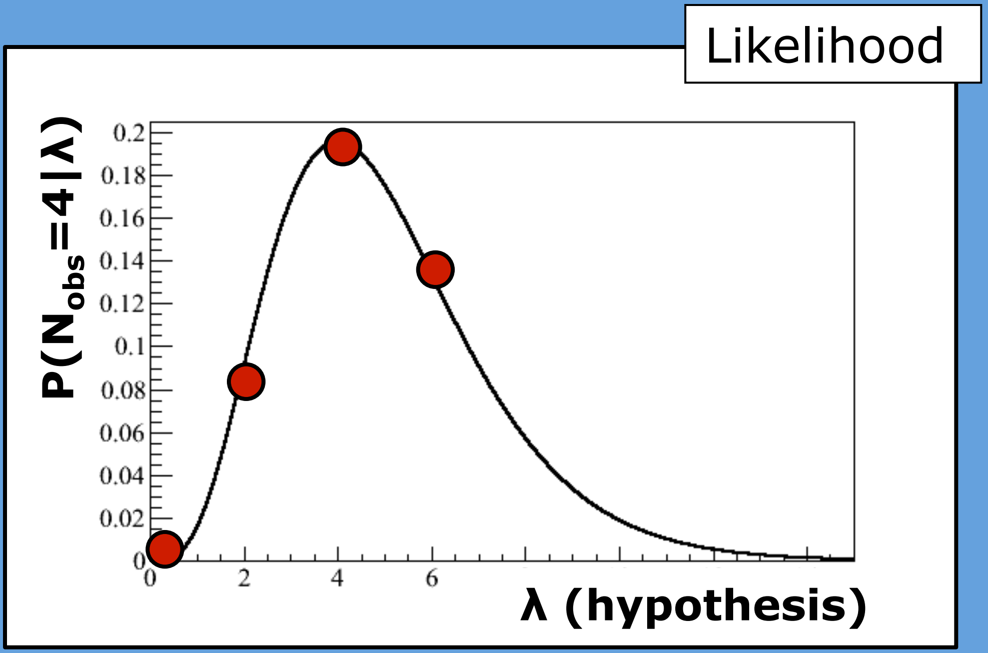
$$L(n \mid \lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$$

Likelihood
Poisson: probability to observe *n* events when λ are expected

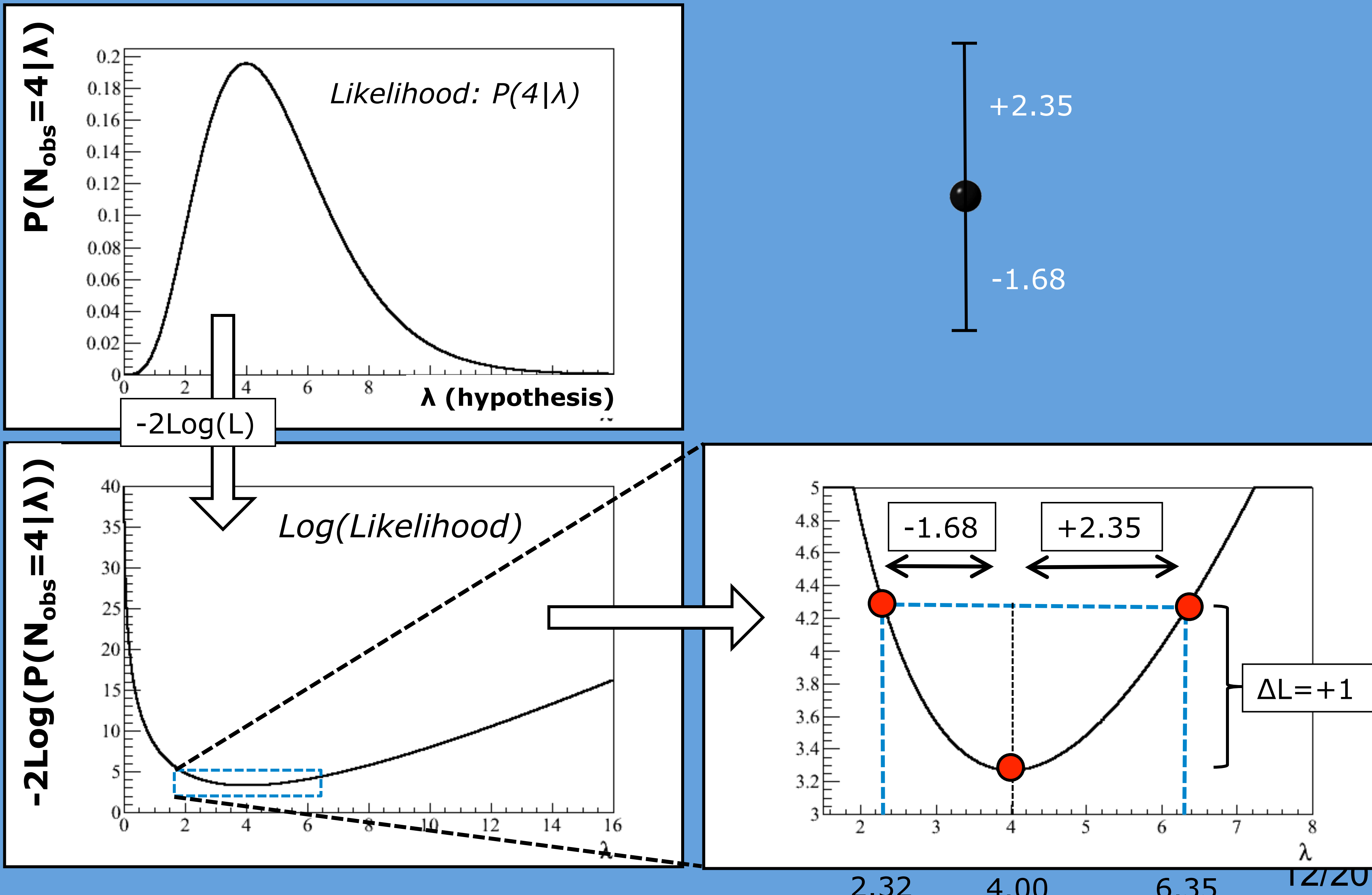
$$P(4 \mid 0) = 0.00000$$
$$P(4 \mid 2) = 0.09022$$
$$P(4 \mid 4) = 0.19537$$
$$P(4 \mid 6) = 0.13385$$

#observed — **fixed**

λ hypothesis — **scan**



Option 3: Likelihood



Bayesian: statement on value of λ

What you have:

What you want:

Likelihood

Probability to observe N_{obs} events
… given a specific hypothesis (λ)

pdf for λ

What can we say about theory (λ)
… given # of observed events (4)

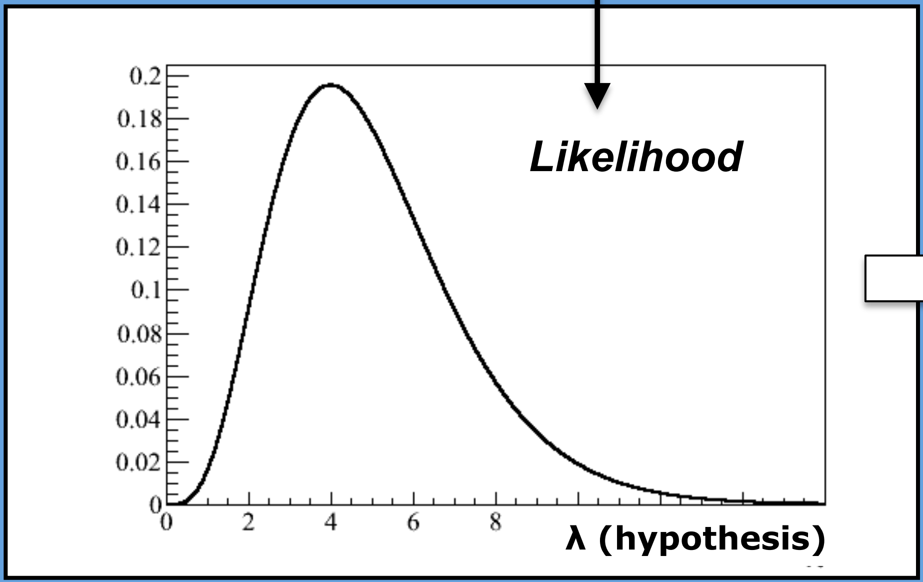
$$P(\lambda \mid N_{obs}) = P(N_{obs} \mid \lambda) P(\lambda)$$

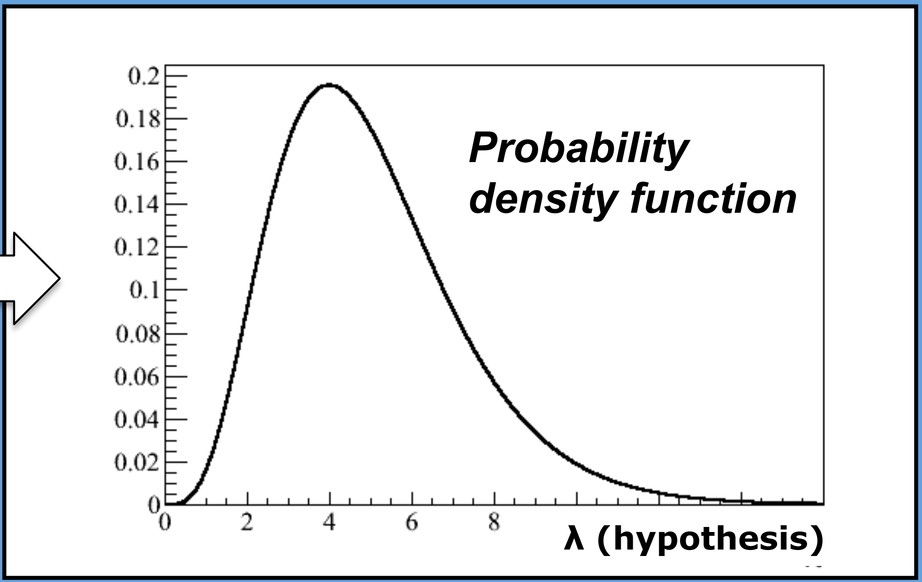
Bayesian: statement on value of λ

$$P(\lambda \mid N_{obs}) = P(N_{obs} \mid \lambda) P(\lambda)$$

Choice of prior P(λ):
(basis for religious wars)

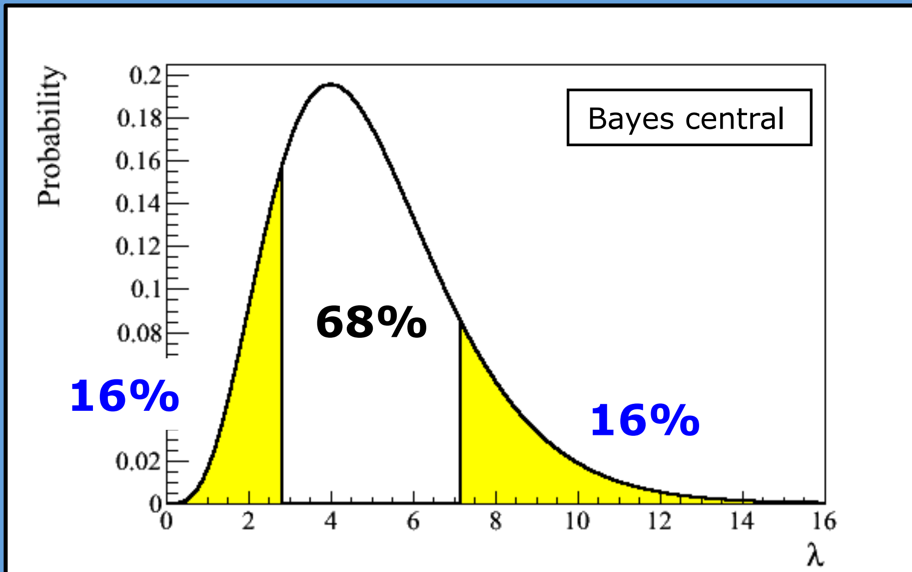
Here: assume all values for λ equally likely

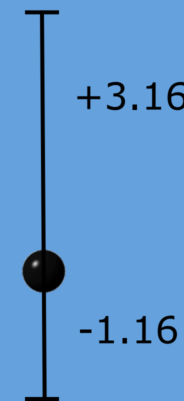


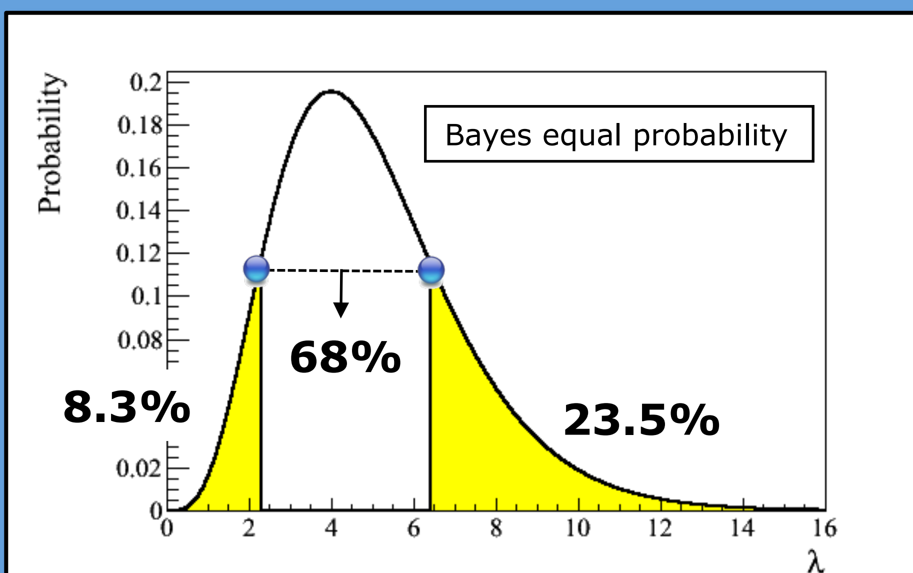


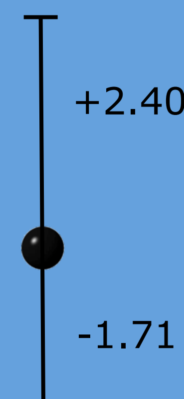
Posterior PDF for λ
→ Integrate to get confidence interval

Option 4 & 5: representing the Bayesian posterior





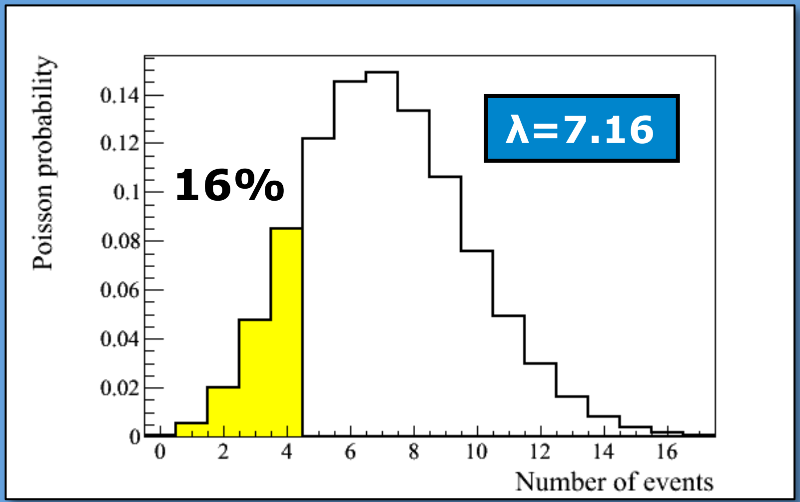




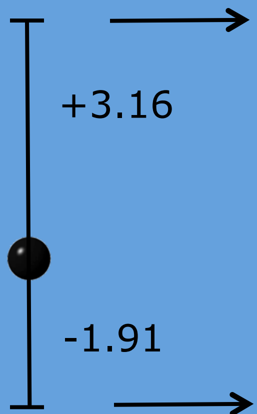
Option 6: Frequentist approach

Find values of λ that are on border of being compatible with observed #events

If $\lambda > 7.16$ then probability to observe 4 events **(or less)** <16%

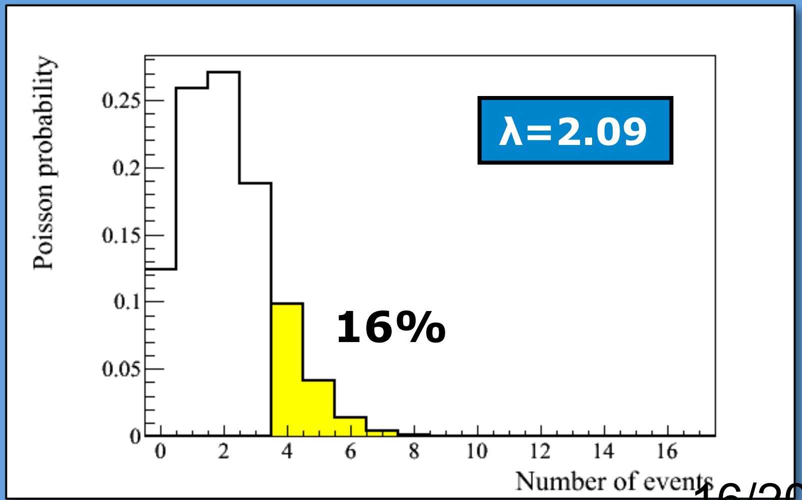


Note: also uses 'data you didn't observe',
i.e. a bit like definition of significance

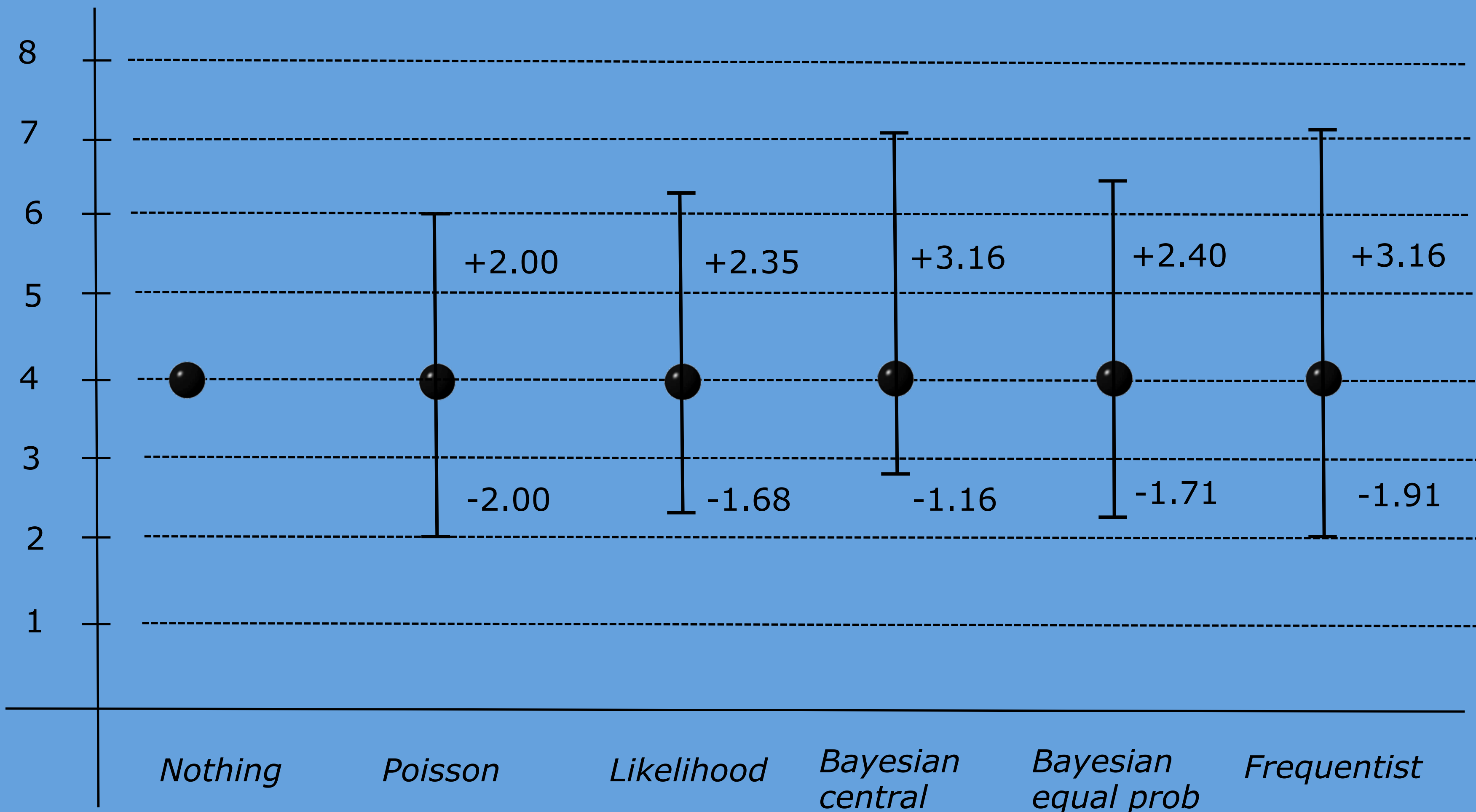


smallest λ (>n) for which $P(n \leq n_{obs} | \lambda) \leq 0.159$

largest λ (<n) for which $P(n \geq n_{obs} | \lambda) \leq 0.159$

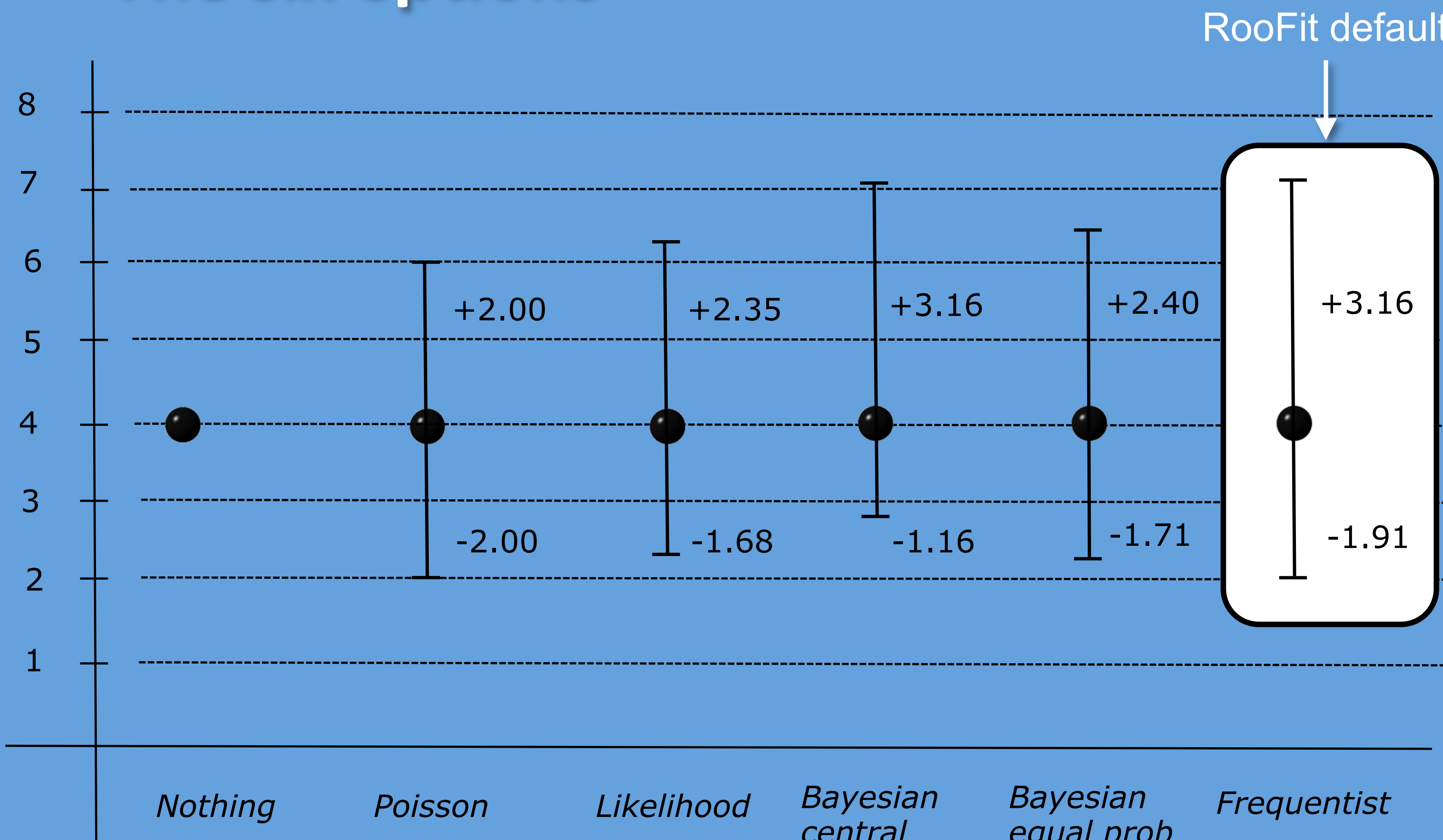


The six options



Δλ: 0.00 4.00 4.03 4.32 4.11 5.07

The six options



$\Delta\lambda$:	0.00	4.00	4.03	4.32	4.11	5.07

Discussions in other experiment:

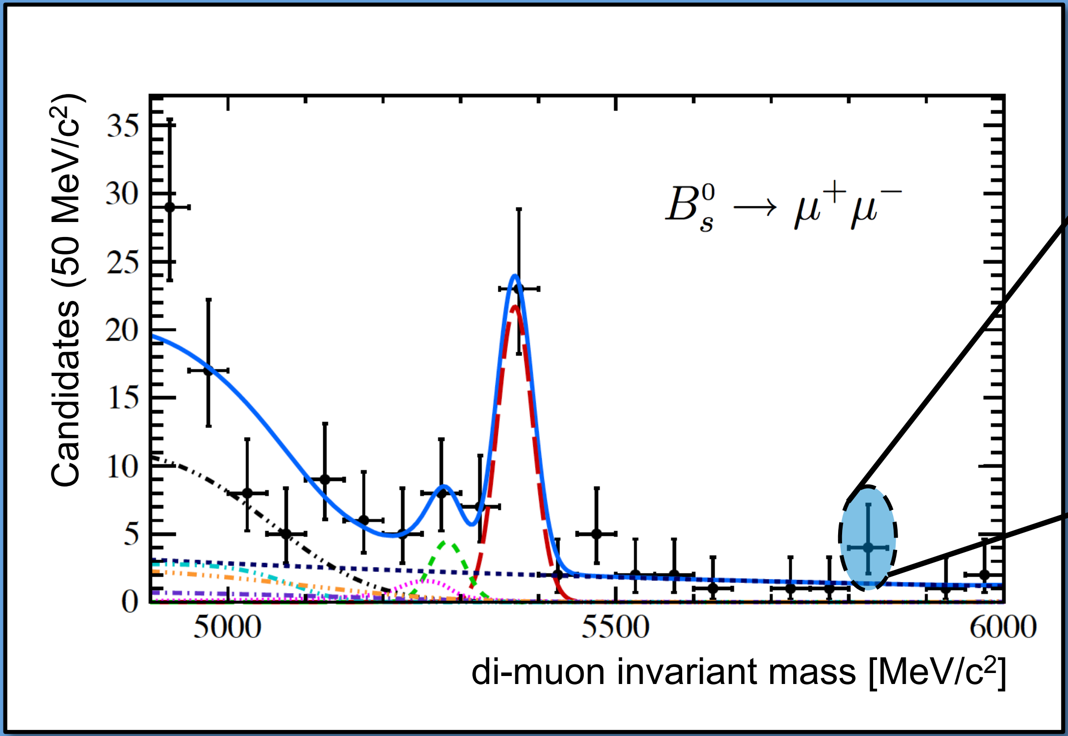
Example → discussion in CDF:
https://www-cdf.fnal.gov/physics/statistics/notes/pois_eb.txt

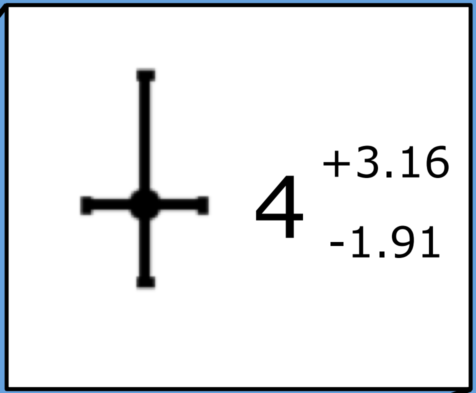
We feel it is important to have a relatively simple rule that is readily understood by readers. A reader does not want to have to work hard simply to understand what an error bar on a plot represents.

<…>

Since the use of +-sqrt(n) is so widespread, the argument in favour of an alternative should be convincing in order for it to be adopted.

Summary





- You now know how Roofit 'Poisson errors' are defined
  Note: choice has no impact on likelihood fits

- Do you agree with RooFit default? What about empty bins then?

- Perfect topic to confuse and irritate people over coffee → do it!