Group 2

Bfys Workshop
Hindeloopen, 2012

- What is the status of the V_{CKM} and V_{PMNS}?
  - Marcel: Introduction and origin of V_{CKM} and V_{PMNS}
  - Siim : Experimental status on magnitudes of V_{CKM}
  - Veerle : Experimental status on phases of V_{CKM}

“Limburger” and “Fries”

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Antimatter, CKM, PMNS

- Baryogenesis via quark sector
  - CP Violation in CKM lead to the idea of ***Baryogenesis*** @ GUT scale
    - Baryon number and CP violating processes (and non-equilibrium)
  - However, electroweak phase transition would ***wash-out*** any GUT scale asymmetry in B-L conserving sphaleron transitions
  - EW phase transition is not strong enough to ***generate*** sufficient baryon-asymmetry
    - Higgs mass is too high

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  - CP Violation in PMNS allows for ***Leptogenesis*** via see-saw mechanism with heavy right handed Majorana neutrinos
  - Lepton asymmetry is ***communicated*** to baryon asymmetry via the B-L conserving sphaleron transitions

- LHCb
  - Does ***not really*** study baryogenesis (contrary to what we claim in outreach talks)
  - Precision measurements on CP violation and rare decays in flavour decays to look for ***indirect evidence*** for physics beyond the Standard Model
  - This ***cannot*** be done in a similar way with neutrino physics via neutrino mixing
  - LHCb upgrade allows to study ***Lepton flavor violation*** and searches for specific types of ***Majorana*** neutrinos

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The Origin of CP Violation in the SM in one slide

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  - The Yukawa couplings are free parameters in flavour space (generation 1,2,3).
  - There are four independent 3-by-3 Yukawa matrices.
    - +2/3 quarks, -1/3 quarks, charged leptons and neutrinos

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- After symmetry breaking the fermion fields are re-expressed in terms of mass states (linear combinations).
  - In that case the 3-by-3 Yukawa matrices turn into mass matrices
    - Diagonalized Yukawa couplings are the masses of the fermions
  - The interactions are now no longer diagonal in flavour space
    - Quarks: Relation between the mass states and the interaction states is the CKM matrix in the case of quarks.
      - The CKM matrix has one free complex phase
    - Leptons: Relation between the mass states and the interaction states is the PMNS matrix.

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- The origin of CP violation is directly related to the origin of Mass.
  - Why are the particle masses what they are?
  - Why do we have three generations

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The Amount of CP Violation

Using Standard Parametrization of CKM:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij} \qquad s_{ij} \equiv \sin\theta_{ij}$$

$$J \equiv c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13} \sin\delta = \left(3.0 \pm 0.3\right)\times 10^{-5}$$

(eg.: $J = Im(V_{us} V_{cb} V_{ub}^{*} V_{cs}^{*})$)

(The maximal value J might have = 1/(6√3) ~ 0.1)

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(eg.: $J=Im(V_{us}\, V_{cb}\, V_{ub}^{*}\, V_{cs}^{*})$)

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However, also required is:

$$(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_t^2 - m_u^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_b^2 - m_d^2) \neq 0$$

All requirements for CP violation can be summarized by:

$$\begin{aligned} \Im m\left\{\det\left[M_d M_d^{\dagger}, M_u M_u^{\dagger}\right]\right\} &= -2J\,(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_t^2) \\ &\quad \times (m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_d^2 - m_b^2) \\ &= 6\cdot 10^{-5} \times 4\cdot 10^{10}\ (\mathrm{GeV}^{12}) \neq 0 \rightarrow \text{CP Violation} \end{aligned}$$

Is CP violation maximal? => One has to understand the origin of mass!

A word on Baryogenesis

- Originally, Baryogenesis was proposed at GUT scale:
  - Baryon violation processes "natural"
- Later, Baryogenesis proposed at EW scale
  - Baryon violation provided by sphaleron process
  - CP Violation by Jarlskog determinant
  - Non-equilibrium by phase transition
    - Higgs mass too heavy?
  - Size of CPV at T_c = 100 GeV
    - $J/T_c^{12} \sim 10^{-20}$ but $n_B/n_\gamma = 10^{-10}$
  - With 4 quark generations (heavier quarks) it is a different story!
  - There seem to be other ways out:

Neutrino

A neutrino is feeble, and it is quite small
But it has a brother and he is quite tall

We call him sterile
But he's really "ganz geil"

We observe them in awe
As they make their see-saw

One left, one right
I tell you no "shite"

One heavy, one small

CP in the Standard Model Lagrangian

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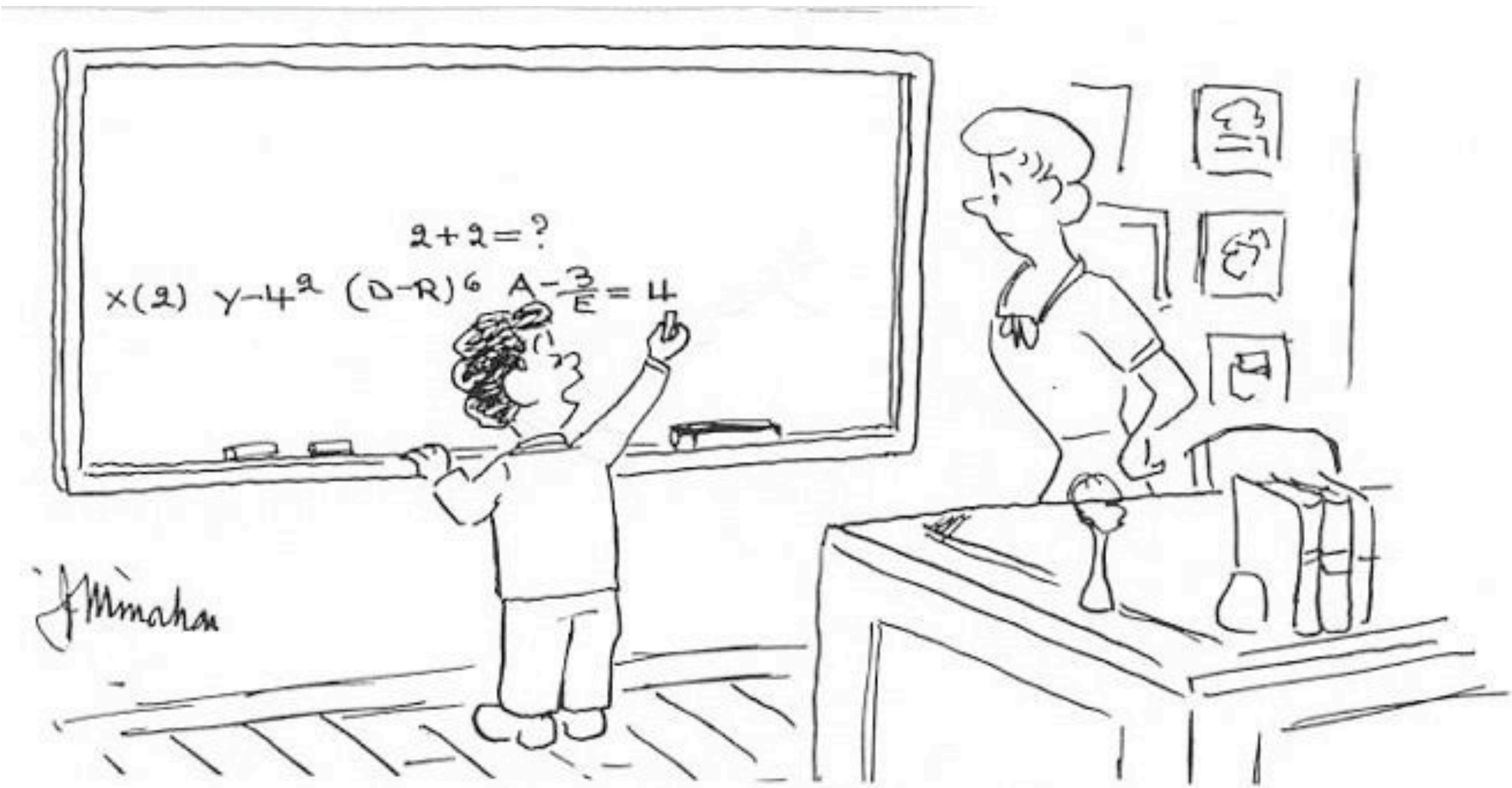

"You do it your way, and I'll do it my way."

CP in the Standard Model Lagrangian

(The origin of the CKM-matrix)

L_{SM} contains:

$L_{Kinetic}$ *: fermion fields*

L_{Higgs} *: the Higgs potential*

L_{Yukawa} *: the Higgs – fermion interactions*

Standard Model gauge symmetry:

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \quad \rightarrow \quad SU(3)_C \times U(1)_{EM}$$

Note Immediately: The weak part is explicitly parity violating

Outline:

- Lorentz structure of the Lagrangian
- Introduce the fermion fields in the SM
- $L_{Kinetic}$: local gauge invariance : fermions ↔ bosons
- L_{Higgs} : spontaneous symmetry breaking
- L_{Yukawa} : the origin of fermion masses
- V_{CKM} : CP violation

The Standard Model Lagrangian

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

- $L_{Kinetic}$: •Introduce the massless fermion fields
  •Require local gauge invariance => gives rise to existence of gauge bosons
  => CP Conserving

- L_{Higgs} : •Introduce Higgs potential with $<\phi> \neq 0$
  •Spontaneous symmetry breaking
  => CP Conserving

  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$

  The W^+, W^-,Z^0 bosons acquire a mass

- L_{Yukawa} : •Ad hoc interactions between Higgs field & fermions
  => CP violating with a single phase

- $L_{Yukawa} \rightarrow L_{mass}$:
  - fermion weak eigenstates:
    -- mass matrix is (3x3) non-diagonal
    => CP-violating
  - fermion mass eigenstates:
    -- mass matrix is (3x3) diagonal
    => CP-conserving!

- $L_{Kinetic}$ in mass eigenstates: CKM – matrix => CP violating with a single phase

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$:The Kinetic Part

$L_{Kinetic}$: Fermions + gauge bosons + interactions

Procedure: Introduce the Fermion fields and demand that the theory is local gauge invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$ transformations.

Start with the Dirac Lagrangian: $L = i\bar{\psi}(\partial^\mu \gamma_\mu)\psi$

Replace: $\partial^\mu \rightarrow D^\mu \equiv \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig' B^\mu Y$

Fields:
- G_a^μ : 8 gluons
- W_b^μ : weak bosons: W_1, W_2, W_3
- B^μ : hypercharge boson

Generators:

L_a	: Gell-Mann matrices:	$\frac{1}{2}\lambda_a$	(3x3)	$SU(3)_C$
T_b	: Pauli Matrices:	$\frac{1}{2}\tau_b$	(2x2)	$SU(2)_L$
Y	: Hypercharge:			$U(1)_Y$

For the remainder we only consider Electroweak: $SU(2)_L \times U(1)_Y$

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Kinetic Part

$$L_{kinetic} : i\overline{\psi}(\partial^{\mu}\gamma_{\mu})\psi \rightarrow i\overline{\psi}(D^{\mu}\gamma_{\mu})\psi$$

$$with \quad \psi = Q^{I}_{Li}, \quad u^{I}_{Ri}, \quad d^{I}_{Ri}, \quad L^{I}_{Li}, \quad l^{I}_{Ri}$$

For example the term with $Q_{Li}{}^{I}$ becomes:

$$L_{kinetic}(Q^{I}_{Li}) = i\overline{Q^{I}_{Li}}\gamma_{\mu}D^{\mu}Q^{I}_{Li}$$

$$= i\overline{Q^{I}_{Li}}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g_{s}G^{\mu}_{a}\lambda_{a} + \frac{i}{2}gW^{\mu}_{b}\tau_{b} + \frac{i}{6}g'B^{\mu}\right)Q^{I}_{Li}$$

and similarly for all other terms ($u_{Ri}{}^{I}, d_{Ri}{}^{I}, L_{Li}{}^{I}, l_{Ri}{}^{I}$).

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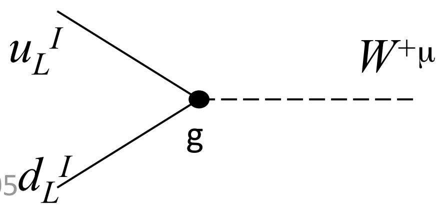
and similarly for all other terms ($u_{Ri}^{\ I}, d_{Ri}^{\ I}, L_{Li}^{\ I}, l_{Ri}^{\ I}$).

Writing out only the weak part for the quarks:

$$L^{Weak}_{kinetic}(u,d)^I_L = i\overline{(u,d)}^I_L\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g\left(W^{\mu}_1\tau_1 + W^{\mu}_2\tau_2 + W^{\mu}_3\tau_3\right)\right)\begin{pmatrix} u \\ d \end{pmatrix}^I_L$$

$$= i\overline{u^I_L}\gamma_{\mu}\partial^{\mu}u^I_L + i\overline{d^I_L}\gamma_{\mu}\partial^{\mu}d^I_L - \frac{g}{\sqrt{2}}\overline{u^I_L}\gamma_{\mu}W^{-\mu}d^I_L - \frac{g}{\sqrt{2}}\overline{d^I_L}\gamma_{\mu}W^{+\mu}u^I_L - \ldots$$



$$L = J_{\mu}W^{\mu}$$

$$W^{+} = (1/\sqrt{2})\,(W_1 + i\,W_2)$$
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and similarly for all other terms ($u_{Ri}{}^I, d_{Ri}{}^I, L_{Li}{}^I, l_{Ri}{}^I$).

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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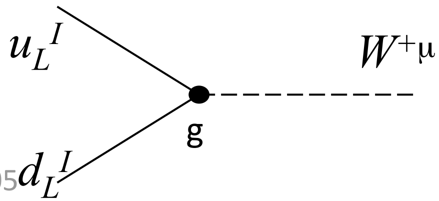

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$$\text{with} \quad \psi = Q^I_{Li},\quad u^I_{Ri},\quad d^I_{Ri},\quad L^I_{Li},\quad l^I_{Ri}$$

Exercise:
Show that this Lagrangian formally violates both P and C
Show that this Lagrangian conserves CP

L_{Kin} = CP conserving

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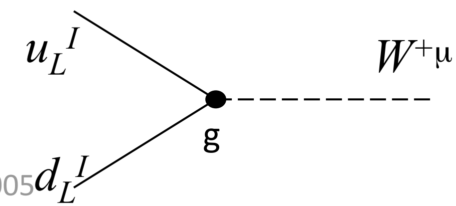

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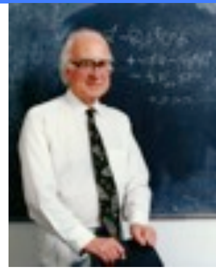
$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Higgs Potential

$$L_{Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_{Higgs}$$

$$V_{Higgs} = \frac{1}{2}\mu^2\left(\phi^\dagger\phi\right) + |\lambda|\left(\phi^\dagger\phi\right)^2$$

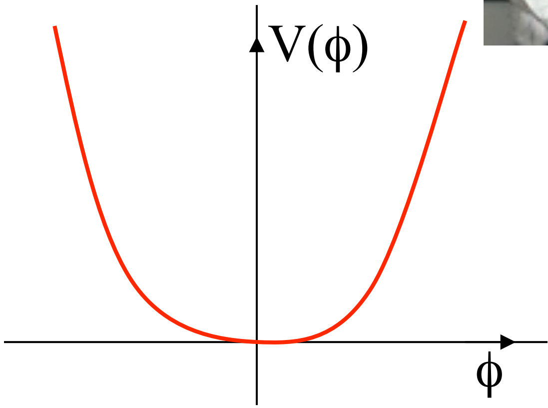


→Note L_{Higgs} = CP conserving

Symmetry

$\mu^2 > 0$:

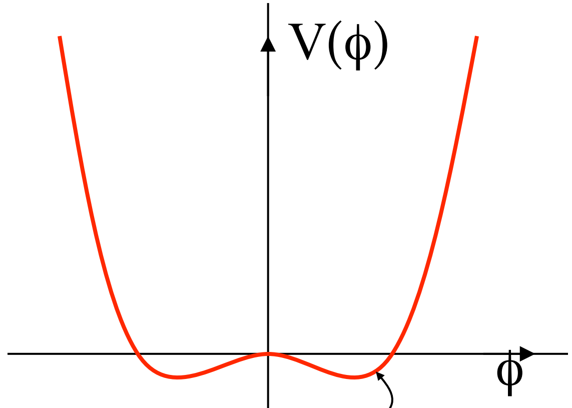
$< \varphi >= 0$



Broken Symmetry

$\mu^2 < 0$:

$$< \varphi >= \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$



$v = \sqrt{-\mu^2/|\lambda|} \sim 246$ GeV

Spontaneous Symmetry Breaking: The Higgs field adopts a non-zero vacuum expectation value

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Higgs Potential

$$L_{Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_{Higgs}$$

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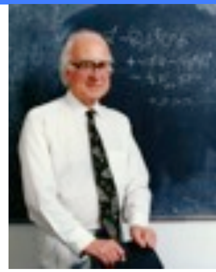

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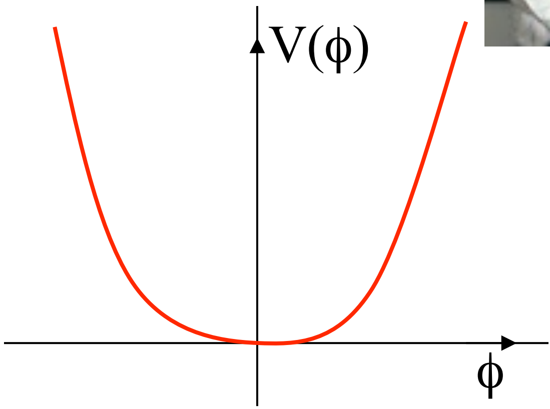

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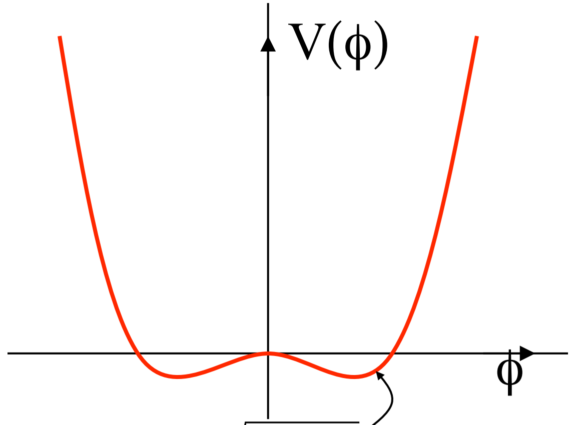

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Spontaneous Symmetry Breaking: The Higgs field adopts a non-zero vacuum expectation value

Procedure:
$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \Re e\varphi^+ + i\Im m\phi^+ \\ \Re e\varphi^0 + i\Im m\phi^0 \end{pmatrix}$$

Substitute:
$$\Re e\varphi^0 = \frac{v + H^0}{\sqrt{2}}$$

And rewrite the Lagrangian (tedious):

(The other 3 Higgs fields are “eaten” by the W, Z bosons)

1. $G_{SM} : \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right) \rightarrow \left(SU(3)_C \times U(1)_{EM}\right)$
2. The W^+, W^-, Z^0 bosons acquire mass
3. The Higgs boson H appears

“The realization of the vacuum breaks the symmetry”

$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$: The Yukawa Part

Since we have a Higgs field we can add (ad-hoc) interactions between ϕ and the fermions in a gauge invariant way.

The result is:

doublets — singlet — $\mathcal{L}$ must be Hermitian (unitary)

$$-\mathcal{L}_{Yukawa} = Y_{ij}\left(\overline{\psi}_{Li}\,\phi\right)\psi_{Rj} + h.c.$$

$$= Y_{ij}^{d}\left(\overline{Q_{Li}^{I}}\,\phi\right)d_{Rj}^{I} + Y_{ij}^{u}\left(\overline{Q_{Li}^{I}}\,\tilde{\phi}\right)u_{Rj}^{I}$$

$$+ Y_{ij}^{l}\left(\overline{L_{Li}^{I}}\,\phi\right)l_{Rj}^{I} + h.c.$$

With: $\tilde{\phi} = i\sigma_2\phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}$

(The C-conjugate of ϕ

To be manifestly invariant under SU(2))

Y_{ij}^{d} , Y_{ij}^{u} , Y_{ij}^{l} are arbitrary complex matrices which operate in family space (3x3)

=> Flavour physics!

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Yukawa Part

Writing the first term explicitly:

$$Y^d_{ij}\,(\overline{u^I_L}, \overline{d^I_L})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d^I_{Rj} \quad =$$

$$\begin{pmatrix} Y^d_{11}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{12}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{13}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \\ Y^d_{21}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{22}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{13}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \\ Y^d_{31}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{32}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{33}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \end{pmatrix} \bullet \begin{pmatrix} d^I_R \\ s^I_R \\ b^I_R \end{pmatrix}$$

$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$: The Yukawa Part

$$-L_{Yukawa} = Y_{ij}\left(\overline{\psi}_{Li}\,\phi\right)\psi_{Rj} + h.c.$$

Formally, CP is violated if:

$$\Im m\left\{\det\left[Y^d Y^{d\dagger}, Y^u Y^{u\dagger}\right]\right\} \neq 0$$

In general $\mathcal{L}_{Yukawa}$ is CP violating

Exercise (intuitive proof)

Show that:

- The hermiticity of the Lagrangian implies that there are terms in pairs of the form:

$$Y_{ij}\overline{\psi}_{Li}\phi\psi_{Rj} + Y_{ij}^{*}\overline{\psi}_{Rj}\phi^{\dagger}\psi_{Li}$$

- However a transformation under CP gives:

$$\overline{\psi}_{Li}\phi\psi_{Rj} \leftrightarrow \overline{\psi}_{Rj}\phi^{\dagger}\psi_{Li}$$

and leaves the coefficients Y_{ij} and Y_{ij}^{*} unchanged

CP is conserved in $\mathcal{L}_{Yukawa}$ only if $Y_{ij} = Y_{ij}^{*}$

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$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Yukawa Part

There are 3 Yukawa matrices (in the case of massless neutrino's):

$$Y_{ij}^{d} \quad , \quad Y_{ij}^{u} \quad , \quad Y_{ij}^{l}$$

Each matrix is 3x3 complex:

- 27 real parameters
- 27 imaginary parameters ("phases")

➢ many of the parameters are equivalent, since the physics described by one set of couplings is the same as another ("rephasing")

➢ It can be shown that the independent parameters are:

- the masses of the fermions
- 3 real mixing angles in the CKM and 3 in PMNS for massive neutrinos
- 1 imaginary phase in CKM
- 1 imaginary Dirac phase in PMNS if neutrinos are Dirac particles
- 2 additional Majorana phases if neutrino's are Majorana particles

➢These phases is the source of CP violation in the Standard Model

$L_{Yukawa} \overset{S.S.B}{\Rightarrow} L_{Mass}$: The Fermion Masses

Start with the Yukawa Lagrangian

$$-L_{Yuk} = Y_{ij}^{d}\,(\overline{u_L^I},\overline{d_L^I})_i \begin{pmatrix}\varphi^{+}\\ \varphi^{0}\end{pmatrix} d_{Rj}^{I} + Y_{ij}^{u}(\ldots) + Y_{ij}^{l}(\ldots)$$

$$S.S.B. \;:\; \Re e\left(\varphi^{0}\right) \rightarrow \frac{v+H}{\sqrt{2}}$$

v is vacuum expectation value of the Higgs potential

After which the following mass term emerges:

$$-L_{Yuk} \rightarrow -L_{Mass} = \overline{d_{Li}^{I}}\, M_{ij}^{d}\, d_{Rj}^{I} + \overline{u_{Li}^{I}}\, M_{ij}^{u}\, u_{Rj}^{I} + \overline{l_{Li}^{I}}\, M_{ij}^{l}\, l_{Rj}^{I} + h.c.$$

with $M_{ij}^{d} \equiv \frac{v}{\sqrt{2}} Y_{ij}^{d}$, $M_{ij}^{u} \equiv \frac{v}{\sqrt{2}} Y_{ij}^{u}$, $M_{ij}^{l} \equiv \frac{v}{\sqrt{2}} Y_{ij}^{l}$

L_{Mass} is CP violating in a similar way as L_{Yuk}

$L_{Yukawa} \overset{S.S.B}{\circledR} L_{Mass}$: The Fermion Masses

Writing in an explicit form:

$$-L_{Mass} = \left(\overline{d^I}, \overline{s^I}, \overline{b^I}\right)_L \cdot \left(M^d\right) \cdot \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R + \left(\overline{u^I}, \overline{c^I}, \overline{t^I}\right)_L \cdot \left(M^u\right) \cdot \begin{pmatrix} u^I \\ c^I \\ t^I \end{pmatrix}_R + \left(\overline{e^I}, \overline{\mu^I}, \overline{\tau^I}\right)_L \cdot \left(M^l\right) \cdot \begin{pmatrix} e^I \\ \mu^I \\ \tau^I \end{pmatrix}_R + h.c.$$

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$L_{Yukawa} \overset{S.S.B}{\circledR} L_{Mass}$: The Fermion Masses

Writing in an explicit form:

$$-L_{Mass} = \left(\overline{d^I}, \overline{s^I}, \overline{b^I}\right)_L \cdot \left(M^d\right) \cdot \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R + \left(\overline{u^I}, \overline{c^I}, \overline{t^I}\right)_L \cdot \left(M^u\right) \cdot \begin{pmatrix} u^I \\ c^I \\ t^I \end{pmatrix}_R + \left(\overline{e^I}, \overline{\mu^I}, \overline{\tau^I}\right)_L \cdot \left(M^l\right) \cdot \begin{pmatrix} e^I \\ \mu^I \\ \tau^I \end{pmatrix}_R + h.c.$$

The matrices M can always be diagonalised by unitary matrices V_L^f and V_R^f such that:

$$V_L^f \; M^f \; V_R^{f\dagger} = M^f_{diagonal}$$

$$\left[\left(\overline{d^I}, \overline{s^I}, \overline{b^I}\right)_L V_L^{f\dagger} \, V_L^f \; M^f \; V_R^{f\dagger} \, V_R^f \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R \right]$$

$L_{Yukawa} \overset{S.S.B}{\circledR} L_{Mass}$: The Fermion Masses

Writing in an explicit form:

$$-L_{Mass} = \left(\overline{d^I},\overline{s^I},\overline{b^I}\right)_L \cdot \left(M^d\right) \cdot \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R + \left(\overline{u^I},\overline{c^I},\overline{t^I}\right)_L \cdot \left(M^u\right) \cdot \begin{pmatrix} u^I \\ c^I \\ t^I \end{pmatrix}_R + \left(\overline{e^I},\overline{\mu^I},\overline{\tau^I}\right)_L \cdot \left(M^l\right) \cdot \begin{pmatrix} e^I \\ \mu^I \\ \tau^I \end{pmatrix}_R + h.c.$$

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$$\left[\left(\overline{d^I},\overline{s^I},\overline{b^I}\right)_L V_L^{f\dagger} V_L^f \; M^f \; V_R^{f\dagger} V_R^f \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R \right]$$

Then the real fermion mass eigenstates are given by:

$$d_{Li} = \left(V_L^d\right)_{ij} \times d^I_{Lj} \qquad d_{Ri} = \left(V_R^d\right)_{ij} \times d^I_{Rj}$$

$$u_{Li} = \left(V_L^u\right)_{ij} \times u^I_{Lj} \qquad u_{Ri} = \left(V_R^u\right)_{ij} \times u^I_{Rj}$$

$$l_{Li} = \left(V_L^l\right)_{ij} \times l^I_{Lj} \qquad l_{Ri} = \left(V_R^l\right)_{ij} \times l^I_{Rj}$$

d_L^I, u_L^I, l_L^I are the weak interaction eigenstates

d_L, u_L, l_L are the mass eigenstates ("physical particles")

$L_{Yukawa} \overset{S.S.B}{\circledR} L_{Mass}$: The Fermion Masses

In terms of the mass eigenstates:

$$-L_{Mass} = \left(\bar{d},\bar{s},\bar{b}\right)_L \cdot \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u},\bar{c},\bar{t}\right)_L \cdot \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \cdot \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

$$+ \left(\bar{e},\bar{\mu},\bar{\tau}\right)_L \cdot \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} \cdot \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R + h.c.$$

$$\begin{aligned} -L_{Mass} = \; & m_u \bar{u}u + m_c \bar{c}c + m_t \bar{t}t \\ + \; & m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b \\ + \; & m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau \end{aligned}$$

= CP Conserving?

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$L_{Yukawa} \overset{S.S.B}{\circledR} L_{Mass}$: The Fermion Masses

In terms of the mass eigenstates:

$$-L_{Mass} = \left(\bar{d},\bar{s},\bar{b}\right)_L \cdot \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u},\bar{c},\bar{t}\right)_L \cdot \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \cdot \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

$$+ \left(\bar{e},\bar{\mu},\bar{\tau}\right)_L \cdot \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} \cdot \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R + h.c.$$

$$-L_{Mass} = m_u \bar{u}u + m_c \bar{c}c + m_t \bar{t}t$$
$$+ m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b$$
$$+ m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau$$

= CP Conserving?

In flavour space one can choose:

Weak basis: The gauge currents are diagonal in flavour space, but the flavour mass matrices are non-diagonal

Mass basis: The fermion masses are diagonal, but some gauge currents (charged weak interactions) are not diagonal in flavour space

In the weak basis: L_{Yukawa} = CP violating

In the mass basis: $L_{Yukawa} \rightarrow L_{Mass}$ = CP conserving

=>What happened to the charged current interactions (in $L_{Kinetic}$) ?

$L_W \text{®} L_{CKM}$: The Charged Current

The charged current interaction for quarks in the ***interaction*** basis is:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \overline{u_{Li}^I} \gamma^\mu d_{Li}^I W_\mu^+$$

The charged current interaction for quarks in the ***mass*** basis is:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \overline{u_{Li}} V_L^u \gamma^\mu V_L^{d\dagger} d_{Li} W_\mu^+$$

The unitary matrix: $V_{CKM} = \left(V_L^u \times V_L^{d\dagger}\right)$ With: $V_{CKM} \times V_{CKM}^\dagger = 1$

is the Cabibbo Kobayashi Maskawa mixing matrix:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \left(\overline{u}, \overline{c}, \overline{t}\right)_L \left(V_{CKM}\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^\mu W_\mu^+$$

$L_W \rightarrow L_{CKM}$: The Charged Current

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$$-L_{W^+} = \frac{g}{\sqrt{2}} \left(\overline{u}, \overline{c}, \overline{t}\right)_L \left(V_{CKM}\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^\mu W_\mu^+$$

Lepton sector: similarly $V_{MNS} = \left(V_L^\nu \cdot V_L^{l\dagger}\right)$

However, for massless neutrino's: V_L^ν = arbitrary. Choose it such that $V_{MNS} = 1$

=> There is no mixing in the lepton sector

Flavour Changing Neutral Currents

To illustrate the SM neutral current take W_3^μ and B^μ term of the Kinetic Lagrangian:

$$-L_{NC}(Q_{Li}^I) = \overline{Q_{Li}^I}\gamma_\mu \left(\frac{g}{2} W_3^\mu \tau_3 + \frac{g'}{6} B^\mu\right) Q_{Li}^I$$

And consider the Z-boson field: $Z^\mu = \cos\theta_W W_3^\mu - \sin\theta_W B^\mu$ and $\tan\theta_W = g'/g$

$$A^\mu = \sin\theta_W W_3^\mu + \cos\theta_W B^\mu$$

Take further $Q_{Li}^I = d_{Li}^I$:

$$-L_Z(d_{Li}^I) = \frac{g}{\cos\theta_W}\left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right)\overline{d_{Li}^I}\gamma_\mu Z^\mu d_{Li}^I$$

$$-L_Z(d_{Li}^I) = \frac{g}{\cos\theta_W}\left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right)\overline{d_{Li}}\gamma_\mu \left(V_L^d\, V_L^{d\dagger}\right)_{ij} d_{Lj}\, Z^\mu$$

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$$-L_Z(d_{Li}^I) = \frac{g}{\cos\theta_W}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)\overline{d_{Li}^I}\gamma_\mu Z^\mu d_{Li}^I$$

$$-L_Z(d_{Li}^I) = \frac{g}{\cos\theta_W}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)\overline{d_{Li}}\gamma_\mu \left(V_L^d\, V_L^{d\dagger}\right)_{ij} d_{Lj}\, Z^\mu$$

Use $(V_L^{u\dagger}V_L^u = V_L^{d\dagger}V_L^d = 1)$ to find in general:

$$-L_Z(Q_{Li}) = \frac{g}{\cos\theta_W}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)\overline{d_{Li}}\gamma_\mu d_{Li}Z^\mu \quad + \ldots(\quad)\overline{u_{Li}}\gamma_\mu u_{Li}Z^\mu$$

In terms of physical fields no non-diagonal contributions occur for the neutral Currents. => GIM mechanism

Standard Model forbids flavour changing neutral currents.

Charged Currents

The charged current term reads:

$$L_{CC} = \frac{g}{\sqrt{2}} \overline{u^I_{Li}} \gamma^\mu W^-_\mu d^I_{Li} \quad + \quad \frac{g}{\sqrt{2}} \overline{d^I_{Li}} \gamma^\mu W^+_\mu u^I_{Li} \quad = \quad J^{\mu -}_{CC} W^-_\mu + J^{\mu +}_{CC} W^+_\mu$$

$$= \frac{g}{\sqrt{2}} \overline{u_i} \left(\frac{1-\gamma^5}{2} \right) \gamma^\mu W^-_\mu V_{ij} \left(\frac{1-\gamma^5}{2} \right) d_j \quad + \quad \frac{g}{\sqrt{2}} \overline{d_j} \left(\frac{1-\gamma^5}{2} \right) \gamma^\mu W^+_\mu V^\dagger_{ji} \left(\frac{1-\gamma^5}{2} \right) u_i$$

$$= \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W^-_\mu V_{ij} \left(1-\gamma^5\right) d_j \quad + \quad \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W^+_\mu V^*_{ij} \left(1-\gamma^5\right) u_i$$

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$$= \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W^-_\mu V_{ij} \left(1-\gamma^5\right) d_j \quad + \quad \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W^+_\mu V^*_{ij} \left(1-\gamma^5\right) u_i$$

Under the CP operator this gives: (Together with (x,t) -> (-x,t))

$$L_{CC} \xrightarrow{CP} \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W^+_\mu V_{ij} \left(1-\gamma^5\right) u_i \quad + \quad \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W^i_\mu V^*_{ij} \left(1-\gamma^5\right) d_j$$

A comparison shows that CP is conserved only if $V_{ij} = V_{ij}^*$

In general the charged current term is CP violating

The Standard Model Lagrangian (recap)

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- $\mathcal{L}_{Kinetic}$: •Introduce the massless fermion fields
  •Require local gauge invariance => gives rise to existence of gauge bosons
  => CP Conserving
- $\mathcal{L}_{Higgs}$: •Introduce Higgs potential with $<\phi> \neq 0$
  •Spontaneous symmetry breaking
  => CP Conserving

  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$

  The W^+, W^-, Z^0 bosons acquire a mass
- $\mathcal{L}_{Yukawa}$: •Ad hoc interactions between Higgs field & fermions
  => CP violating with a single phase
- $\mathcal{L}_{Yukawa} \rightarrow \mathcal{L}_{mass}$:
  - fermion weak eigenstates:
    -- mass matrix is (3x3) non-diagonal
    => CP-violating
  - fermion mass eigenstates:
    -- mass matrix is (3x3) diagonal
    => CP-conserving!
- $\mathcal{L}_{Kinetic}$ in mass eigenstates: CKM – matrix => CP violating with a single phase

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Quark field re-phasing

Under a quark phase transformation:

$$u_{Li} \circledR\ e^{i\phi_{u i}} u_{Li} \qquad\qquad d_{Li} \circledR\ e^{i\phi_{d i}} d_{Li}$$

and a simultaneous rephasing of the CKM matrix:

$$V \circledR \begin{pmatrix} e^{-\phi_u} & & \\ & e^{-\phi_c} & \\ & & e^{-\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-\phi_d} & & \\ & e^{-\phi_s} & \\ & & e^{-\phi_b} \end{pmatrix} \quad \text{or} \quad V_{\alpha j} \rightarrow \exp\left(i\left(\phi_j - \phi_\alpha\right)\right) V_{\alpha j}$$

the charged current $J^{\mu}_{CC} = \overline{u_{Li}} \gamma^{\mu} V_{ij} d_{Lj}$ is left invariant.

Quark field re-phasing

Under a quark phase transformation:

$$u_{Li} \text{ ® } e^{i\phi_{u\,i}} u_{Li} \qquad\qquad d_{Li} \text{ ® } e^{i\phi_{d\,i}} d_{Li}$$

and a simultaneous rephasing of the CKM matrix:

Exercise:
Convince yourself that there are indeed 5 relative quark phases

$$V \text{ ® } \begin{pmatrix} e^{-\phi_u} & & \\ & e^{-\phi_c} & \\ & & e^{-\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-\phi_d} & & \\ & e^{-\phi_s} & \\ & & e^{-\phi_b} \end{pmatrix} \quad \text{or} \quad V_{\alpha j} \rightarrow \exp\left(i\left(\phi_j - \phi_\alpha\right)\right) V_{\alpha j}$$

the charged current $J^{\mu}_{CC} = \overline{u_{Li}} \gamma^{\mu} V_{ij} d_{Lj}$ is left invariant.

Quark field re-phasing

Under a quark phase transformation:

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and a simultaneous rephasing of the CKM matrix:

$$V \text{ ® } \begin{pmatrix} e^{-\phi_u} & & \\ & e^{-\phi_c} & \\ & & e^{-\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-\phi_d} & & \\ & e^{-\phi_s} & \\ & & e^{-\phi_b} \end{pmatrix}$$

or $V_{\alpha j} \rightarrow \exp\left(i\left(\phi_j - \phi_\alpha\right)\right) V_{\alpha j}$

the charged current $J^{\mu}_{CC} = \overline{u_{Li}} \gamma^{\mu} V_{ij} d_{Lj}$ is left invariant.

Exercise:
Convince yourself that there are indeed 5 relative quark phases

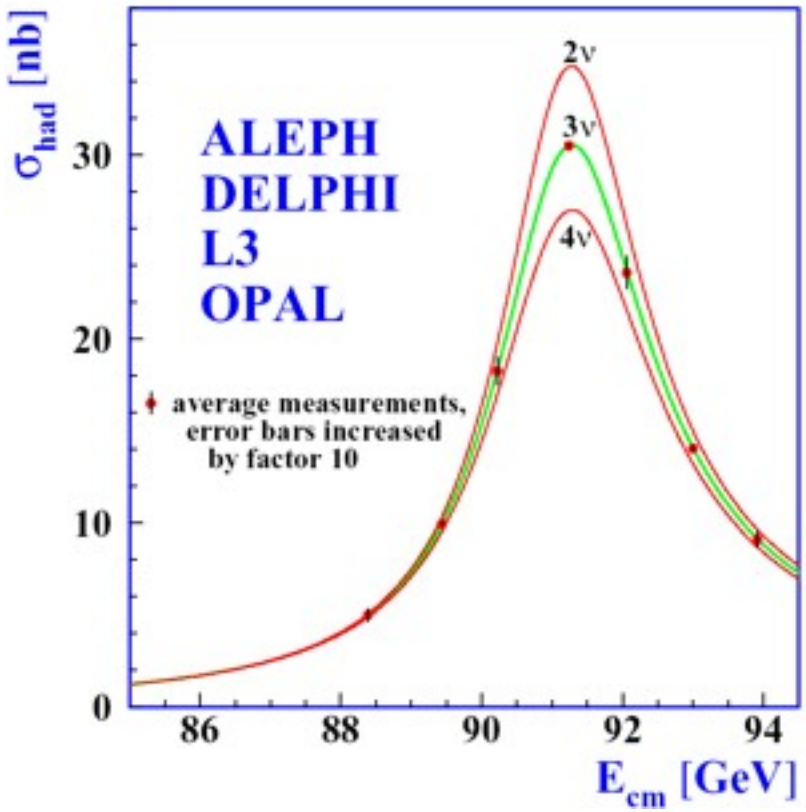
Degrees of freedom in V_{CKM} in	3	N	generations
Number of real parameters:	9	+ N²	
Number of imaginary parameters:	9	+ N²	
Number of constraints ($VV^\dagger = 1$):	-9	- N²	
Number of relative quark phases:	-5	- (2N-1)	
	------------	------------	
Total degrees of freedom:	4	(N-1)²	
Number of Euler angles:	3	N (N-1) / 2	
Number of CP phases:	1	(N-1) (N-2) / 2	

2 generations:

$$V_{CKM} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

No CP violation in SM!
This is the reason Kobayashi and Maskawa first suggested a third family of fermions!

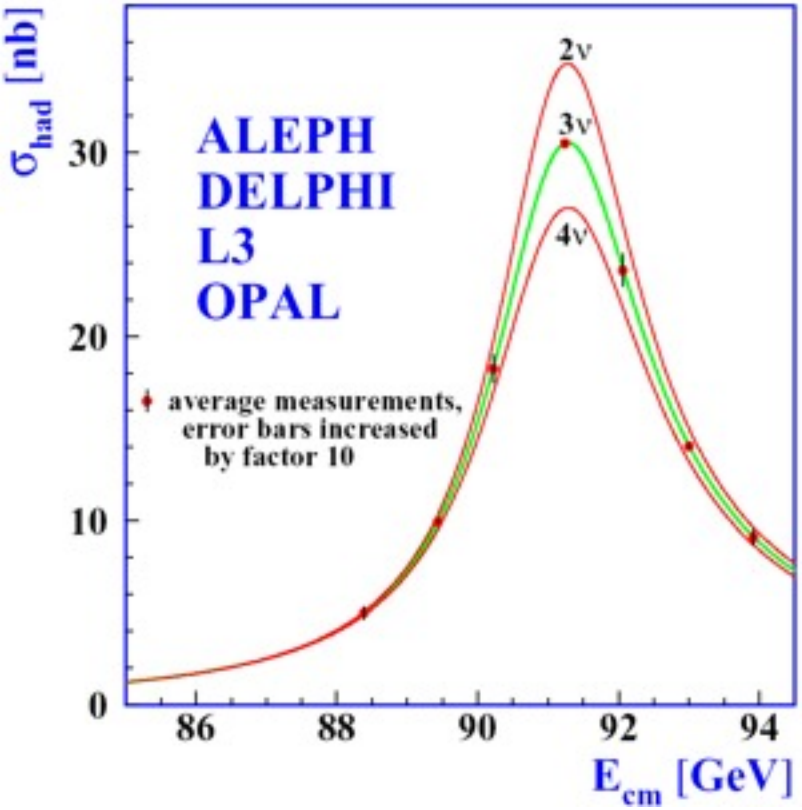
The LEP collider @ CERN



Maybe the most important result of LEP:

"There are 3 generations of neutrino's"

The LEP collider @ CERN



Maybe the most important result of LEP:

"There are 3 generations of neutrino's"

Light, left-handed, "active"

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The lepton sector

- N. Cabibbo: Phys.Rev.Lett. 10, 531 (1963)
  - 2 family flavour mixing in quark sector (GIM mechanism)
- M.Kobayashi and T.Maskawa, Prog. Theor. Phys 49, 652 (1973)
  - 3 family flavour mixing in quark sector
- Z.Maki, M.Nakagawa and S.Sakata, Prog. Theor. Phys. 28, 870 (1962)
  - 2 family flavour mixing in neutrino sector to explain neutrino oscillations

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- In case neutrino masses are of the Dirac type, the situation in the lepton sector is very similar as in the quark sector: $V_{PMNS} \sim V_{CKM}$.
  - There is one CP violating phase in the lepton $PMNS$ matrix
- In case neutrino masses are of the Majorana type (a neutrino is its own anti-particle → no freedom to redefine neutrino phases)
  - There are 3 CP violating phases in the lepton $PMNS$ matrix
    - However, the two extra phases are unobservable in neutrino oscillations
  - There is even a CP violating phase in case $N_{dim} = 2$

CP Violation and quark masses

Note that the massless Lagrangian has a global symmetry for unitary transformations in flavour space.

Let's now assume two quarks with the same charge are degenerate in mass, eg.: $m_s = m_b$

Redefine: $s' = V_{us}\, s + V_{ub}\, b$

$$\bar{u} V_{ud}\, d + \bar{u} V_{us}\, s + \bar{u} V_{ub}\, b$$
$$\rightarrow \bar{u} V_{ud}\, d + \bar{u} V_{us'}\, s' + 0$$

Now the u quark only couples to s' and not to b': i.e. $V_{13}' = 0$

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Using unitarity we can show that the CKM matrix can now be written as:

$$V_{CKM}' = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta\cos\phi & \cos\theta\cos\phi & \sin\phi \\ \sin\theta\sin\phi & -\cos\theta\sin\phi & \cos\phi \end{pmatrix} \longrightarrow$$ CP conserving

Necessary criteria for CP violation:

$$m_u \neq m_c \;, \quad m_c \neq m_t \;, \quad m_t \neq m_u \;,$$
$$m_d \neq m_s \;, \quad m_s \neq m_b \;, \quad m_b \neq m_d$$

The Amount of CP Violation

Using Standard Parametrization of CKM:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij}$$
$$s_{ij} \equiv \sin\theta_{ij}$$

$$J \equiv c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta = (3.0 \pm 0.3)\times 10^{-5}$$

(eg.: $J = Im(V_{us} V_{cb} V_{ub}^* V_{cs}^*)$)

(The maximal value J might have = 1/(6√3) ~ 0.1)

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However, also required is:

$$(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_t^2 - m_u^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_b^2 - m_d^2) \neq 0$$

All requirements for CP violation can be summarized by:

$$\Im m\left\{\det\left[M_d M_d^\dagger, M_u M_u^\dagger\right]\right\} = -2J\,(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_t^2) \times (m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_d^2 - m_b^2)$$

$$= 6\cdot 10^{-5} \times 4\cdot 10^{10}\ (\mathrm{GeV}^{12}) \neq 0 \rightarrow \text{CP Violation}$$

Is CP violation maximal? => One has to understand the origin of mass!

Mass Patterns

Mass spectra ($\mu = M_z$, MS-bar scheme)

m_u ~ 1 - 3 MeV , m_c ~ 0.5 – 0.6 GeV , m_t ~ 180 GeV
m_d ~ 2 - 5 MeV , m_s ~ 35 – 100 MeV , m_b ~ 2.9 GeV

m_e = 0.51 MeV , m_μ = 105 MeV , m_τ = 1777 MeV

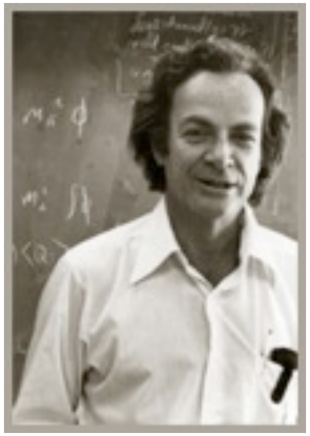
Observe:

$$\frac{m_u}{m_c} \sim \frac{m_c}{m_t} \sim \lambda^4 \quad ,$$

$$\frac{m_d}{m_s} \sim \frac{m_s}{m_b} \sim \lambda^2$$

Why are neutrino's so light? Is it related to the fact that they are the only neutral fermions? See-saw mechanism?

- Do you want to be famous?
- Do you want to be a king?
- Do you want more then the nobel prize?



- Then solve the mass Problem –

R.P. Feynman

Additional Material

Lagrangian Density

Local field theories work with Lagrangian densities:

$$L\left(\vec{x},t\right) = L\left(\phi_j\left(\vec{x},t\right), \partial^\mu \phi_j\left(\vec{x},t\right)\right)$$

with $\phi_j\left(\vec{x},t\right),\ j = 1,2,\ldots,N$ the fields taken at $\vec{x},t$

The fundamental quantity, when discussing symmetries is the Action:

$$A = \int d^4x\, L\left(\vec{x},t\right)$$

If the action is (is not) invariant under a symmetry operation then the symmetry in question is a good (broken) one

=> Unitarity of the interaction requires the Lagrangian to be Hermitian

$$\left(S = \exp iA_{\text{int}}\right)$$

Structure of a Lagrangian

Lorentz structure: interactions can be implemented using combinations of: $\bar{\psi}\, O_i\, \psi$

S: Scalar currents : 1

P: Pseudoscalar currents : γ_5

V: Vector currents : γ_μ

A: Axial vector currents : $\gamma_\mu\gamma_5$

T: Tensor currents : $\sigma_{\mu\nu}$

Dirac field ψ : $(i\gamma^\mu \partial_\mu - m)\psi = 0$

Scalar field ϕ: $(i\partial^\mu \partial_\mu + m^2)\phi = 0$

Example:

Consider a spin-1/2 (Dirac) particle ("nucleon") interacting with a spin-0 (Scalar) object ("meson")

$$L(\vec{x},t) = i\bar{\psi}(\vec{x},t)\gamma^\mu\partial_\mu\psi(\vec{x},t) - m\bar{\psi}(\vec{x},t)\psi(\vec{x},t)$$

$$+ \frac{1}{2}\partial^\mu\phi(\vec{x},t)\partial_\mu\phi(\vec{x},t) - V(\phi(\vec{x},t))^2$$

$$+ \bar{\psi}(\vec{x},t)(a + ib\gamma_5)\psi(\vec{x},t)\phi(\vec{x},t)$$

Nucleon field

Meson potential

Nucleon – meson interaction

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Exercise:

What are the symmetries of this theory under C, P, CP ? Can ***a*** and ***b*** be any complex numbers?

Note: the interaction term contains scalar and pseudoscalar parts

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Violates P, conserves C, violates CP
a and b must be real from Hermeticity

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Transformation Properties

			P	C
	$(\vec{x},t)$	$\rightarrow$	$(-\vec{x},t)$	$(\vec{x},t)$
Scalar Field :	$\phi(\vec{x},t)$	$\rightarrow$	$\phi(-\vec{x},t)$	$\phi^{\dagger}(\vec{x},t)$
Pseudo Field :	$P(\vec{x},t)$	$\rightarrow$	$-P(-\vec{x},t)$	$P^{\dagger}(\vec{x},t)$
Dirac Field :	$\psi(\vec{x},t)$	$\rightarrow$	$\gamma_0\psi(-\vec{x},t)$	$i\gamma^2\gamma^0\bar{\psi}^T(\vec{x},t)$
Vector Field :	$V_\mu(\vec{x},t)$	$\rightarrow$	$V^\mu(-\vec{x},t)$	$-V_\mu^{\dagger}(\vec{x},t)$
Axial Field :	$A_\mu(\vec{x},t)$	$\rightarrow$	$-A^\mu(-\vec{x},t)$	$A_\mu^{\dagger}(\vec{x},t)$

Feynman Metric:

$Q^0 = Q_0 \quad , \quad Q^k = -Q_k$

(Ignoring arbitrary phases)

Transformation properties of Dirac spinor bilinears (interaction terms):

			P	C	CP	T	CPT
$S:$	$\bar{\psi}_1\psi_2$	®	$\bar{\psi}_1\psi_2$	$\bar{\psi}_2\psi_1$	$\bar{\psi}_2\psi_1$	$\bar{\psi}_1\psi_2$	$\bar{\psi}_2\psi_1$
$P:$	$\bar{\psi}_1\gamma_5\psi_2$	$\rightarrow$	$-\bar{\psi}_1\gamma_5\psi_2$	$\bar{\psi}_2\gamma_5\psi_1$	$-\bar{\psi}_2\gamma_5\psi_1$	$-\bar{\psi}_1\gamma_5\psi_2$	$\bar{\psi}_2\gamma_5\psi_1$
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Fields: Notation

$Q = T_3 + Y$

Fermions: $\psi_L = \left(\frac{1-\gamma_5}{2}\right)\psi$; $\psi_R = \left(\frac{1+\gamma_5}{2}\right)\psi$ with $\psi = Q_L, u_R, d_R, L_L, l_R, \nu_R$

Quarks:

Under SU2:
Left handed doublets
Right hander singlets

- $\begin{pmatrix} u^I(3,2,1/6) \\ d^I(3,2,1/6) \end{pmatrix}_{Li} \equiv Q^I_{Li}(3,2,1/6)$

Interaction rep. (I); Left-handed (L); generation index (i); $SU(3)_C$; $SU(2)_L$; Y

- $u^I_{Ri}(3,1,2/3)$
- $d^I_{Ri}(3,1,-1/3)$

Leptons:

- $\begin{pmatrix} \nu^I(1,2,-1/2) \\ l^I(1,2,-1/2) \end{pmatrix}_{Li} \equiv L^I_{Li}(1,2,-1/2)$
- $l^I_{Ri}(1,1,-1)$
- ~~ν^I_{Ri}~~

Scalar field:

- $\phi(1,2,1/2) = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$

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Note:
Interaction representation: standard model interaction is independent of generation number

maandag 11 juni 2012

Fields: Notation

$Q = T_3 + Y$

<u>Explicitly:</u>

- The left handed quark doublet :

$$Q_{Li}^I(3,2,1/6) = \begin{pmatrix} u_r^I, u_g^I, u_b^I \\ d_r^I, d_g^I, d_b^I \end{pmatrix}_L, \begin{pmatrix} c_r^I, c_g^I, c_b^I \\ s_r^I, s_g^I, s_b^I \end{pmatrix}_L, \begin{pmatrix} t_r^I, t_g^I, t_b^I \\ b_r^I, b_g^I, b_b^I \end{pmatrix}_L \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = 1/6)$$

- Similarly for the quark singlets:

$$u_{Ri}^I(3,1,\ 2/3) = \left(u_r^I, u_r^I, u_r^I\right)_R, \left(c_r^I, c_r^I, c_r^I\right)_R, \left(t_r^I, t_r^I, t_r^I\right)_R \quad \left(Y = 2/3\right)$$

$$d_{Ri}^I(3,1,-1/3) = \left(d_r^I, d_r^I, d_r^I\right)_R, \left(s_r^I, s_r^I, s_r^I\right)_R, \left(b_r^I, b_r^I, b_r^I\right)_R \quad \left(Y = -1/3\right)$$

- The left handed leptons:

$$L_{Li}^I(1,2,-1/2) = \begin{pmatrix} \nu_e^I \\ e^I \end{pmatrix}_L, \begin{pmatrix} \nu_\mu^I \\ \mu^I \end{pmatrix}_L, \begin{pmatrix} \nu_\tau^I \\ \tau^I \end{pmatrix}_L \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad \left(Y = -1/2\right)$$

- And similarly the (charged) singlets: $l_{Ri}^I(1,1,-1) = e_R^I, \mu_R^I, \tau_R^I \quad \left(Y = -1\right)$

Intermezzo: Local Gauge Invariance in a single transparancy

Basic principle: The Lagrangian must be invariant under local gauge transformations

Example: massless Dirac Spinors in QED: $\mathcal{L} = i\bar{\psi}\left(\gamma^{\mu}\partial_{\mu}\right)\psi$

"global" U(1) gauge transformation: $\psi(x) \rightarrow \psi'(x) = e^{i\alpha}\psi(x)$

"local" U(1) gauge transformation: $\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$

Is the Lagrangian invariant?

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x) \quad ; \quad \bar{\psi}(x) \rightarrow e^{-i\alpha(x)}\bar{\psi}(x)$$

$$\partial_{\mu}\psi(x) \rightarrow e^{i\alpha(x)}\partial_{\mu}\psi(x) + ie^{i\alpha(x)}\psi(x)\partial_{\mu}\alpha(x)$$

Then: $i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \rightarrow i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \bar{\psi}\gamma^{\mu}\psi\partial_{\mu}\alpha(x)$

Not invariant!

=> Introduce the covariant derivative: $D_{\mu} \equiv \partial_{\mu} - ieA_{\mu}$

and <u>demand</u> that A_{μ} transforms as: $A_{\mu} \rightarrow A_{\mu}' = A_{\mu} + \frac{1}{e}\partial_{\mu}\alpha(x)$

Then it turns out that:

$$\mathcal{L} \rightarrow \mathcal{L}' = \mathcal{L}$$

is invariant!

Conclusion:

- Introduce charged fermion field (electron)
- Demand invariance under local gauge transformations (U(1))
- The price to pay is that a gauge field A_{μ} must be introduced at the same time (the photon)

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Yukawa Part

There are 3 Yukawa matrices (in the case of massless neutrino's):

$$Y_{ij}^{d} \quad , \quad Y_{ij}^{u} \quad , \quad Y_{ij}^{l}$$

Each matrix is 3x3 complex:
- 27 real parameters
- 27 imaginary parameters ("phases")

➢ many of the parameters are equivalent, since the physics described by one set of couplings is the same as another

➢ It can be shown (see ref. [Nir]) that the independent parameters are:
  - 12 real parameters
  - 1 imaginary phase

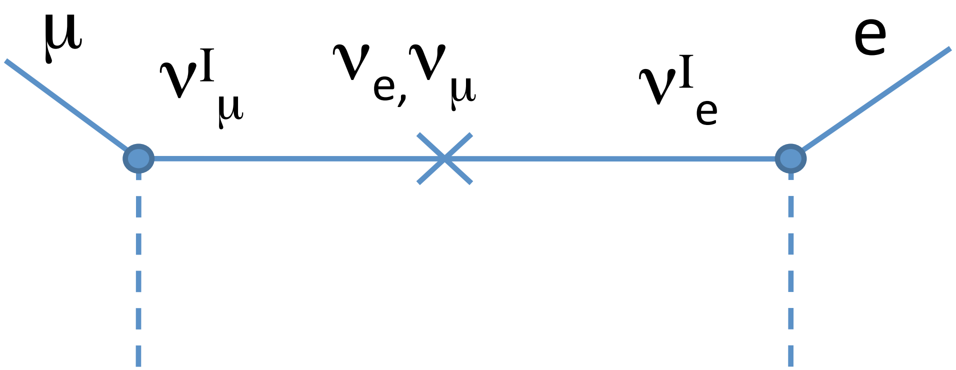
➢ This single phase is the source of all CP violation in the Standard Model

……Revisit later

Lepton mixing and neutrino oscillations

Question:

- In the CKM we write by convention the mixing for the down type quarks; in the lepton sector we write it for the (up-type) neutrinos. Is it relevant?
  - If yes: why?
  - If not, why don't we measure charged lepton oscillations rather then neutrino oscillations?



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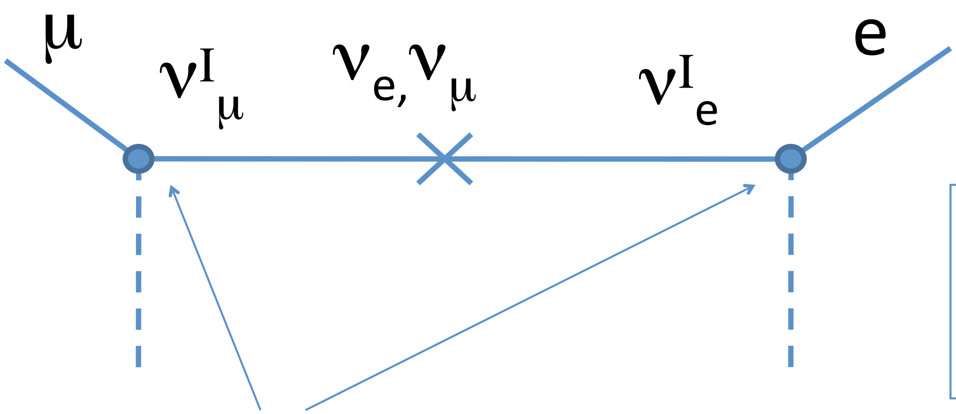

However, observation of neutrino oscillations is possible due to small neutrino mass differences.

$$J^\mu_{CC} = \frac{g}{\sqrt{2}} \bar{l}_i \gamma^\mu \left(V_{ij} \nu_j \right) = \frac{g}{\sqrt{2}} \left(\bar{l}_i V_{ij} \right) \gamma^\mu \nu_j$$

Rephasing Invariants

The standard representation of the CKM matrix is:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{ij} = \cos\theta_{ij}$$
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- $V^{\dagger}V=1$: Unitarity triangle: $V_{ud}\, V_{cd}{}^{*} + V_{us}\, V_{cs}{}^{*} + V_{ub}\, V_{cb}{}^{*} = 0$
  - Multiply the equation by $V_{us}{}^{*}\, V_{cs}$ and take the imaginary part:
  - $Im\,(V_{us}{}^{*}\, V_{cs}\, V_{ud}\, V_{cd}{}^{*}) = -\,Im\,(V_{us}{}^{*}\, V_{cs}\, V_{ub}\, V_{cb}{}^{*})$
  - $J = Im\, Q_{udcs} = -\,Im\, Q_{ubcs}$
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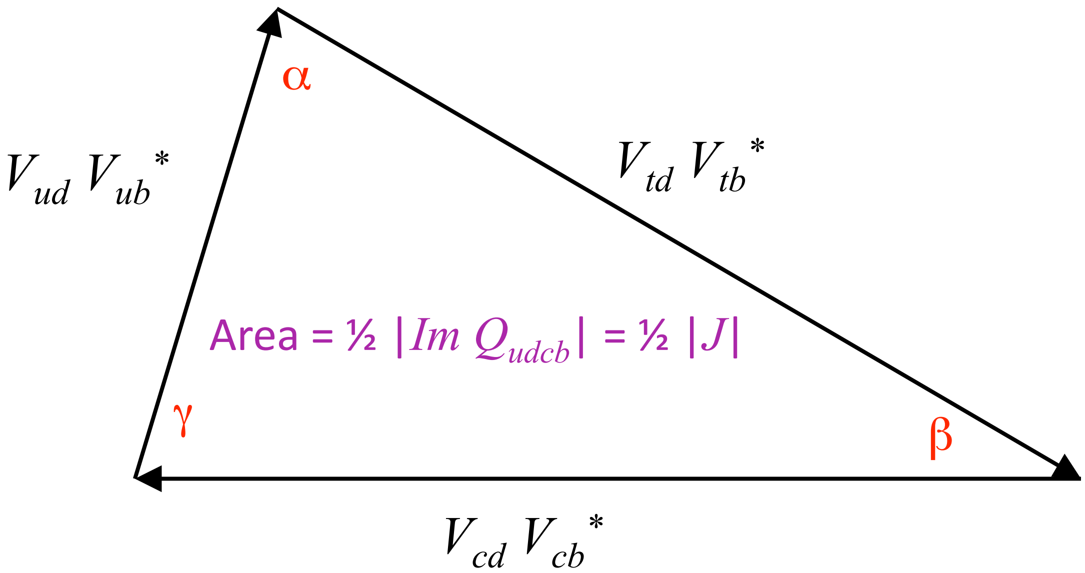
Amount of CP violation is proportional to *J*

The Unitarity Triangle

The "*db*" triangle: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

unitarity: $V_{CKM}^\dagger V_{CKM} = 1$



$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = \arg\left(-Q_{ubtd}\right)$$

$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = \arg\left(-Q_{tbcd}\right)$$

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \arg\left(-Q_{cbud}\right)$$

Under re-phasing: $V_{\alpha j} \to \exp\left(i\left(\phi_j - \phi_\alpha\right)\right) V_{\alpha j}$ the unitary angles are invariant

(In fact, rephasing implies a rotation of the whole triangle)

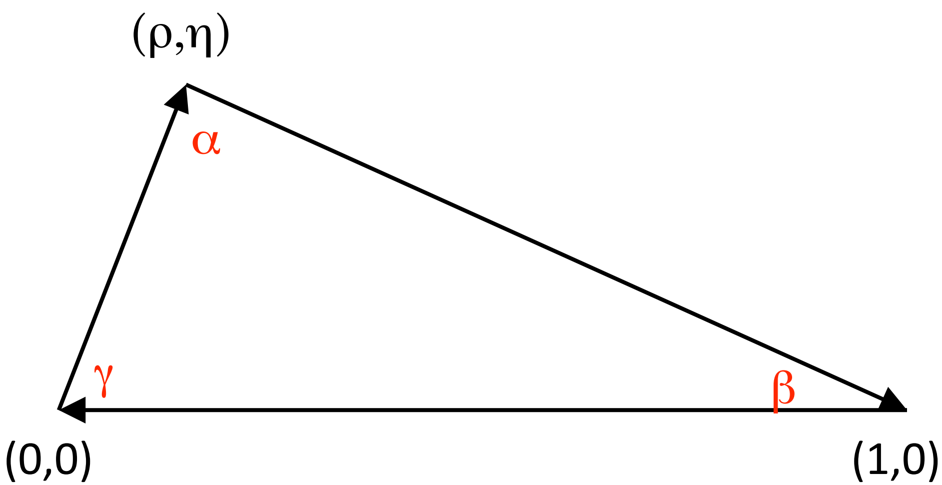
Wolfenstein Parametrization

Wolfenstein realised that the non-diagonal CKM elements are relatively small compared to the diagonal elements, and parametrized as follows:

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$$\approx \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & |V_{ts}| & |V_{tb}| \end{pmatrix}$$

Normalised CKM triangle:



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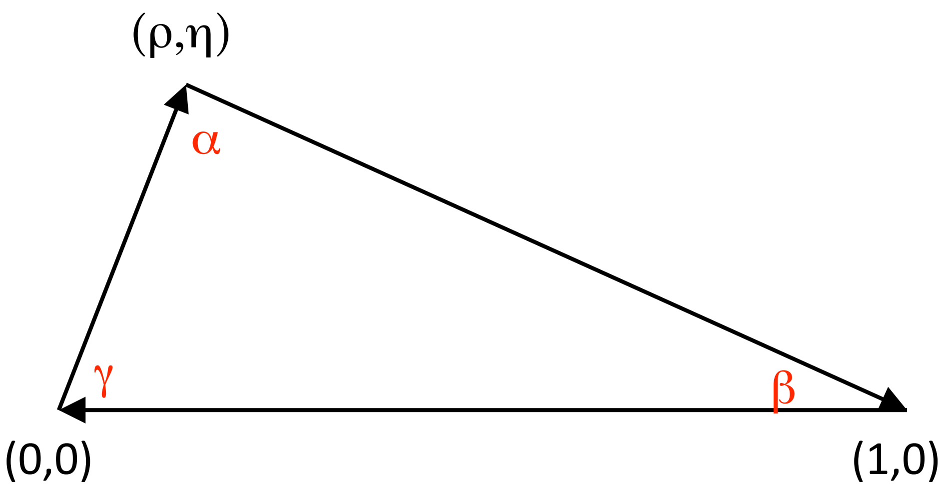

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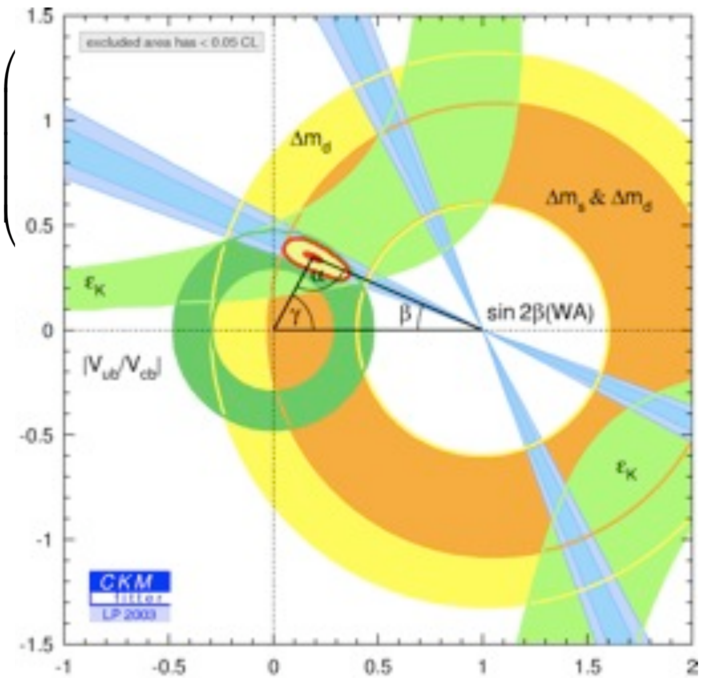

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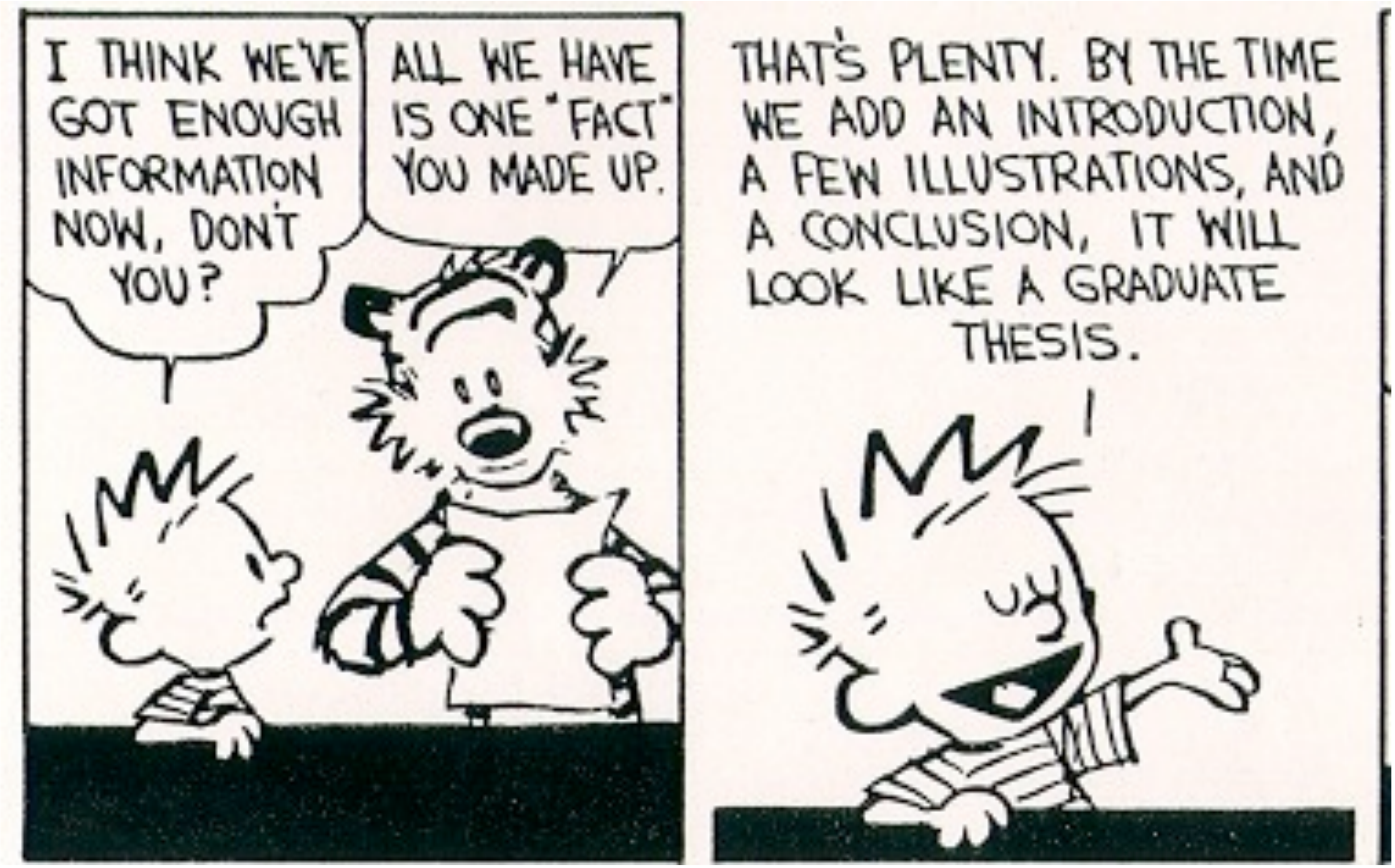
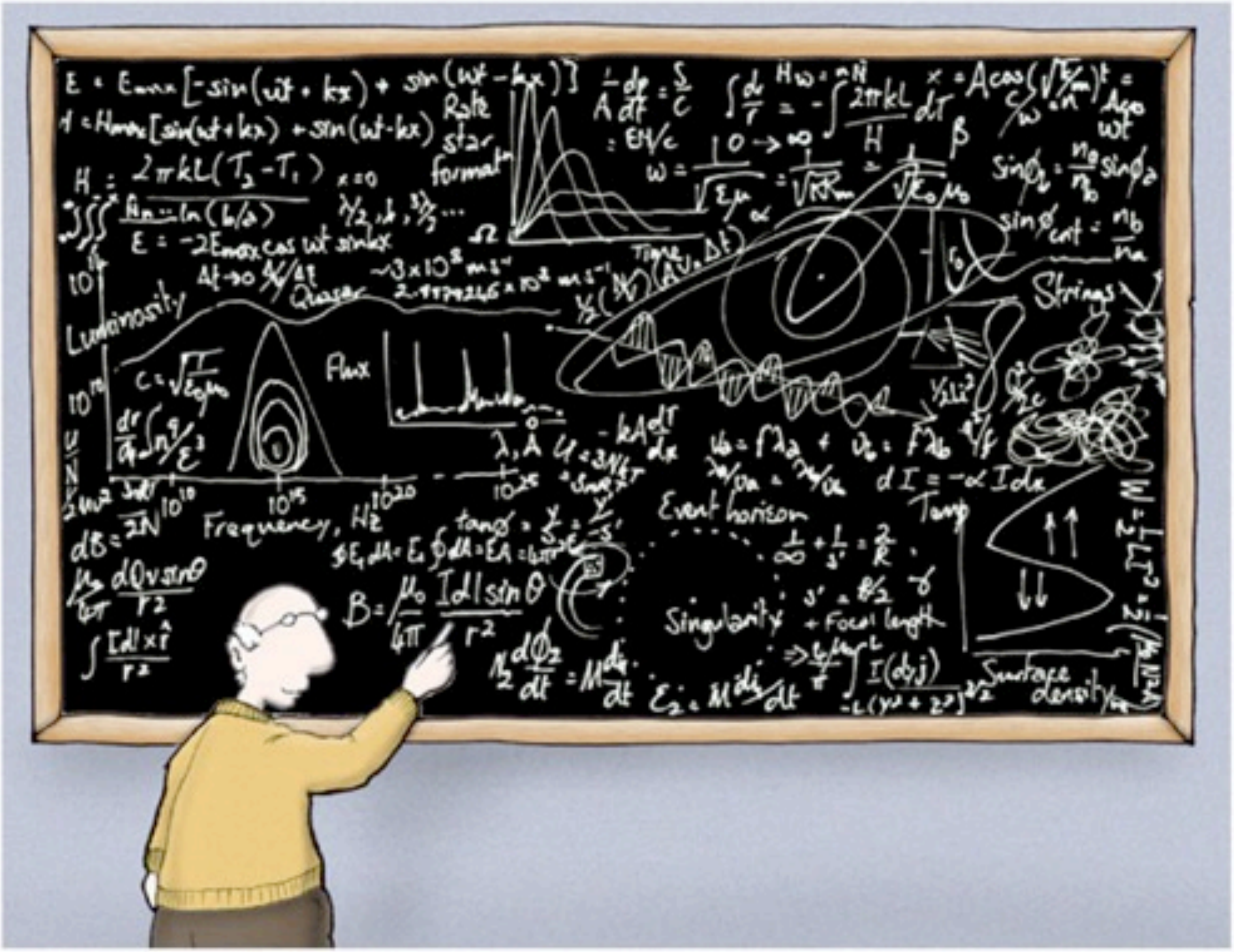

I THINK WE'VE GOT ENOUGH INFORMATION NOW, DON'T YOU?
ALL WE HAVE IS ONE "FACT" YOU MADE UP.
THAT'S PLENTY. BY THE TIME WE ADD AN INTRODUCTION, A FEW ILLUSTRATIONS, AND A CONCLUSION, IT WILL LOOK LIKE A GRADUATE THESIS.



Astrophysics made simple