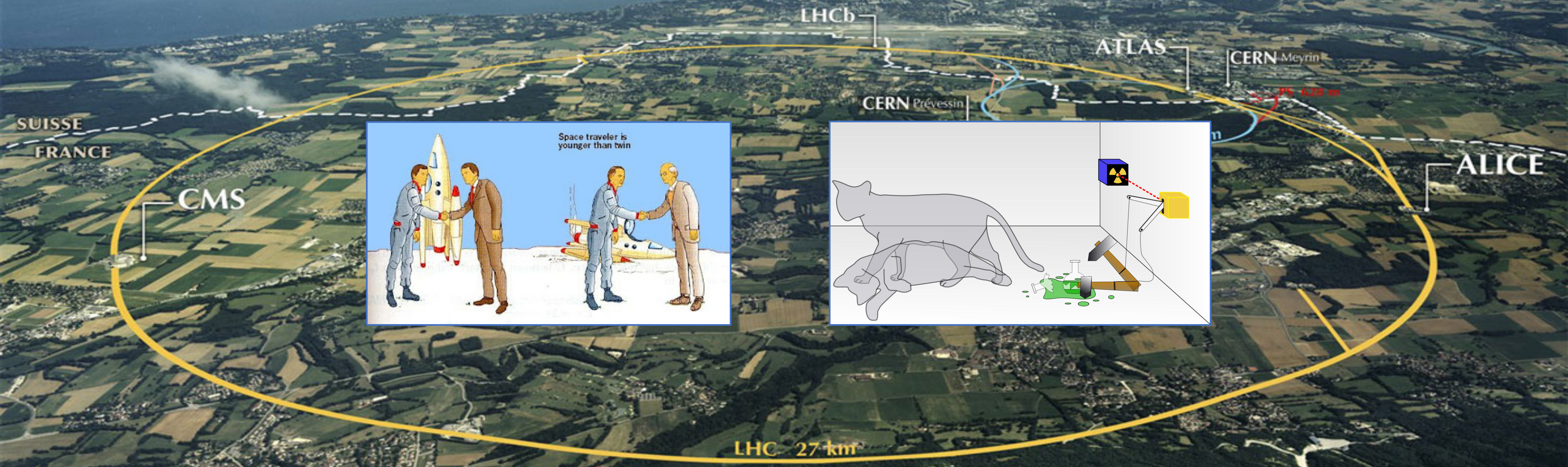
The Relativistic Quantum World

A lecture series on Relativity Theory and Quantum Mechanics

Marcel Merk



University of Maastricht, Sept 16 – Oct 14, 2020

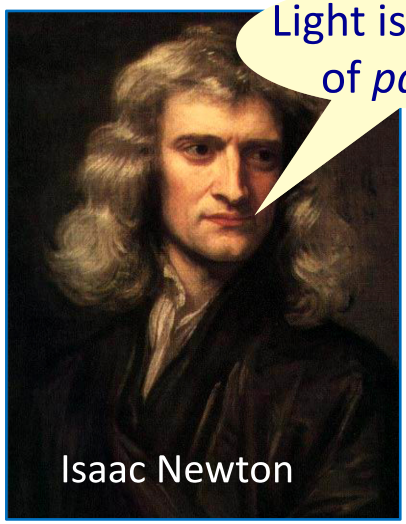
The Relativistic Quantum World

Relativity	Sept. 16:	Lecture 1: The Principle of Relativity and the Speed of Light Lecture 2: Time Dilation and Lorentz Contraction
	Sept. 23:	Lecture 3: The Lorentz Transformation and Paradoxes Lecture 4: General Relativity and Gravitational Waves
Quantum Mechanics	Sept. 30:	Lecture 5: The Early Quantum Theory Lecture 6: Feynman's Double Slit Experiment
	Oct. 7:	Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat Lecture 8: Quantum Reality and the EPR Paradox
Standard Model	Oct. 14:	Lecture 9: The Standard Model and Antimatter Lecture 10: The Large Hadron Collider

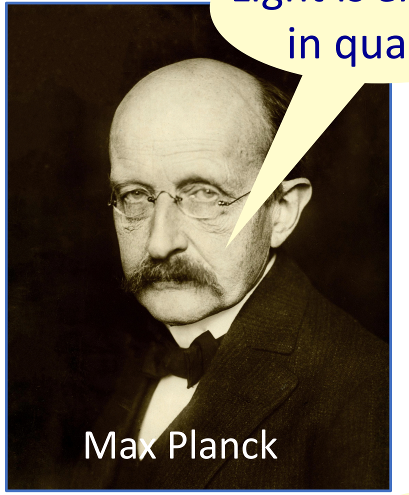
Lecture notes, written for this course, are available: www.nikhef.nl/~i93/Teaching/
Prerequisite for the course: High school level physics & mathematics.

Quantum Mechanics

Light is a stream of *particles*

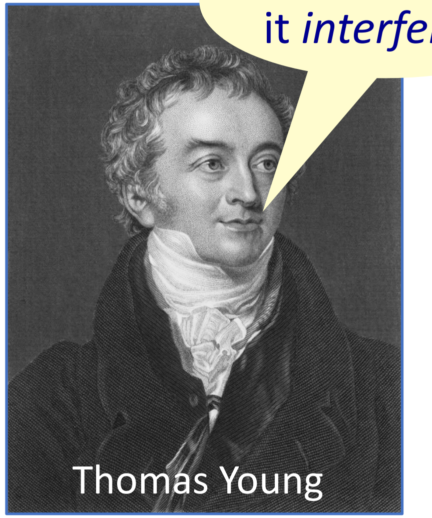


Light is *emitted* in quanta

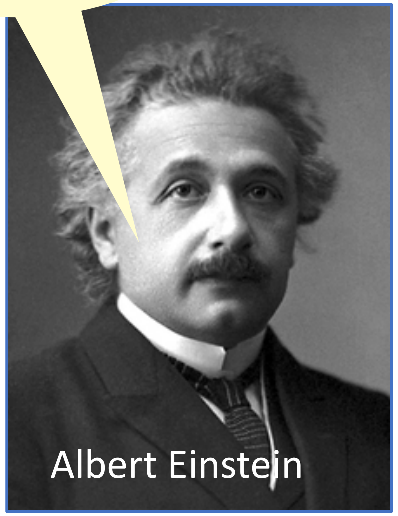


No, similar to sound light consists of *waves*

Yes, because it *interferes*

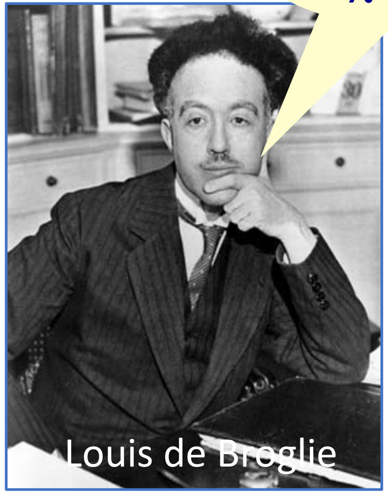


The *nature* of light is quanta

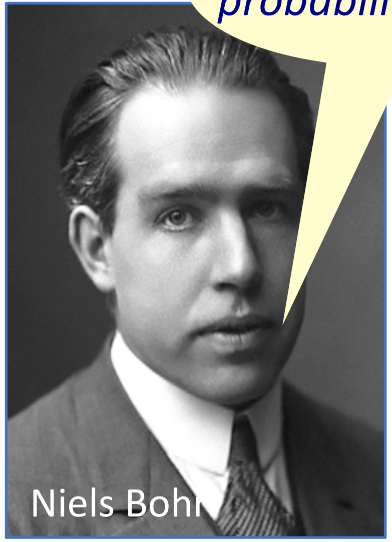


Yes, because photons collide!

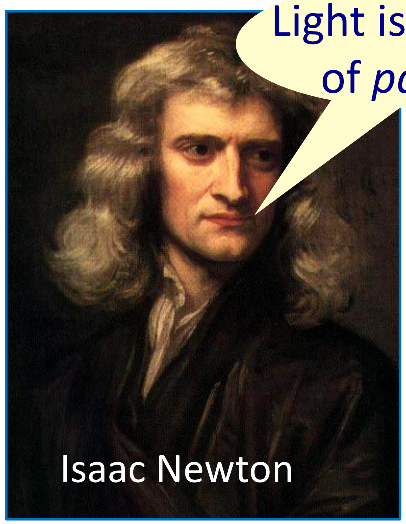
Particles have a *wave nature*: $\lambda = h/p$



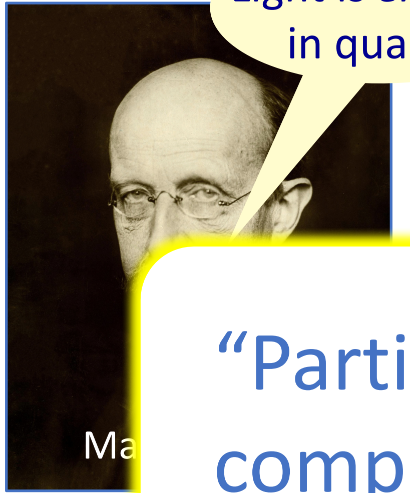
Particles are *probability waves*



Quantum Mechanics



Light is a stream of *particles*



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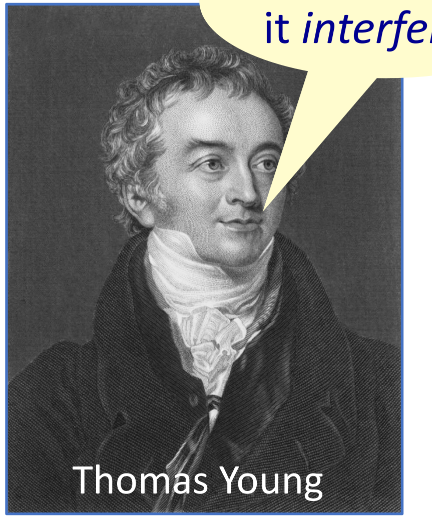
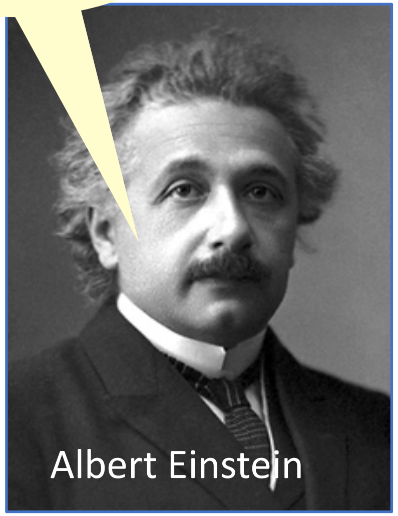


"Particle" and "Wave" are complementary aspects.

The *nature* of light is quanta

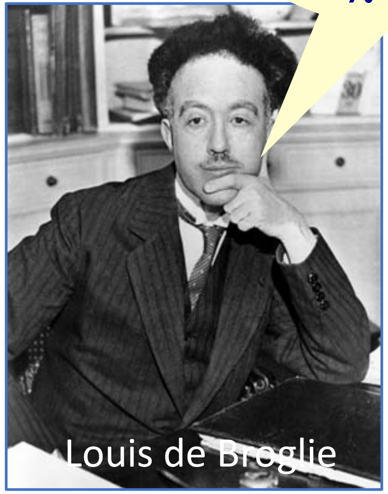


photons collide!

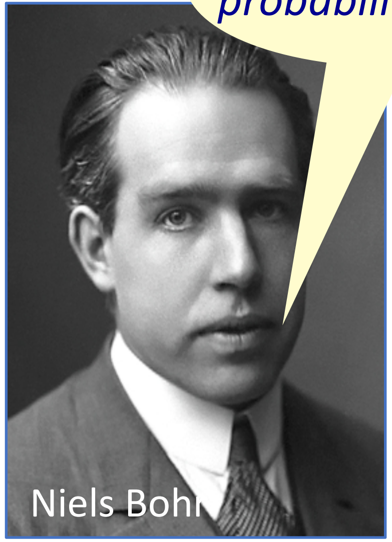
e a

:

$\lambda = h/p$



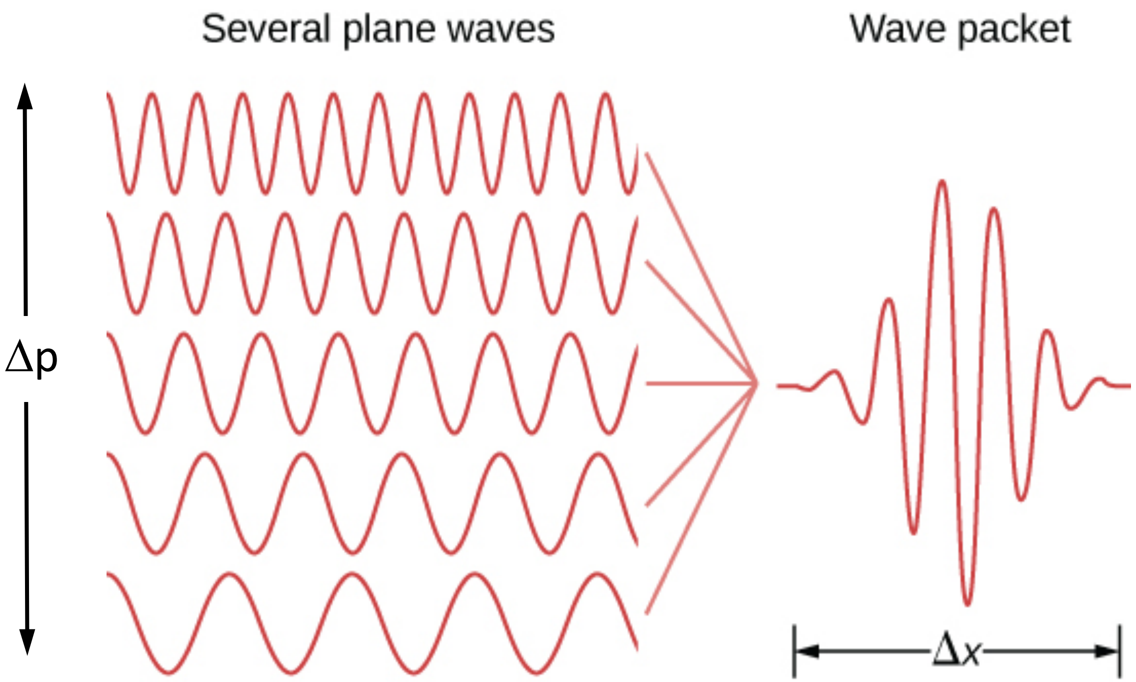
Particles are *probability waves*



Uncertainty Relation

It is ***not*** possible to determine ***position*** and ***momentum*** at the same time:

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

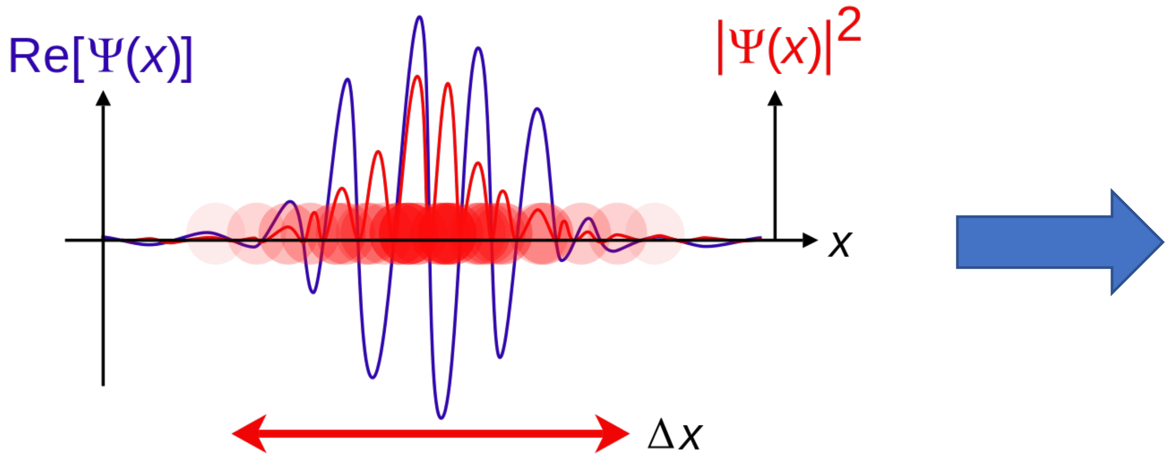


$$p = \frac{h}{\lambda} = \frac{hf}{c}$$

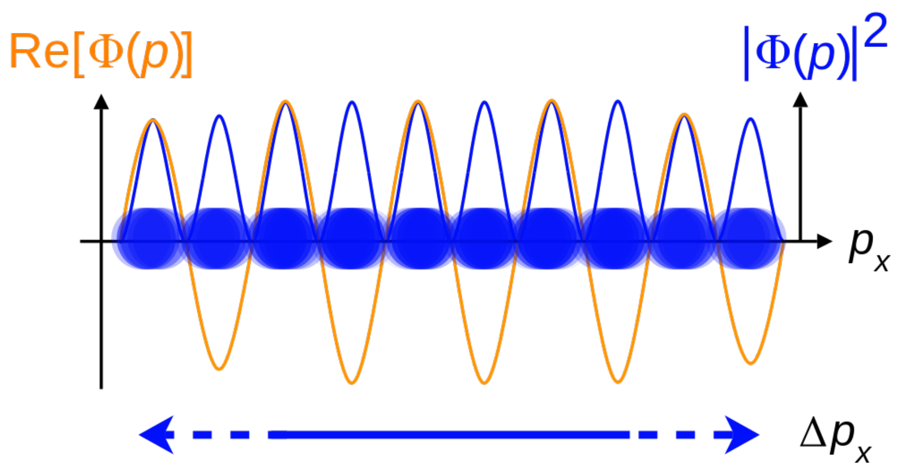
A particle ***does not have*** well defined position and momentum at the same time.

The wave function ψ

Position fairly known



Momentum badly known



The wave function ψ

Position fairly known

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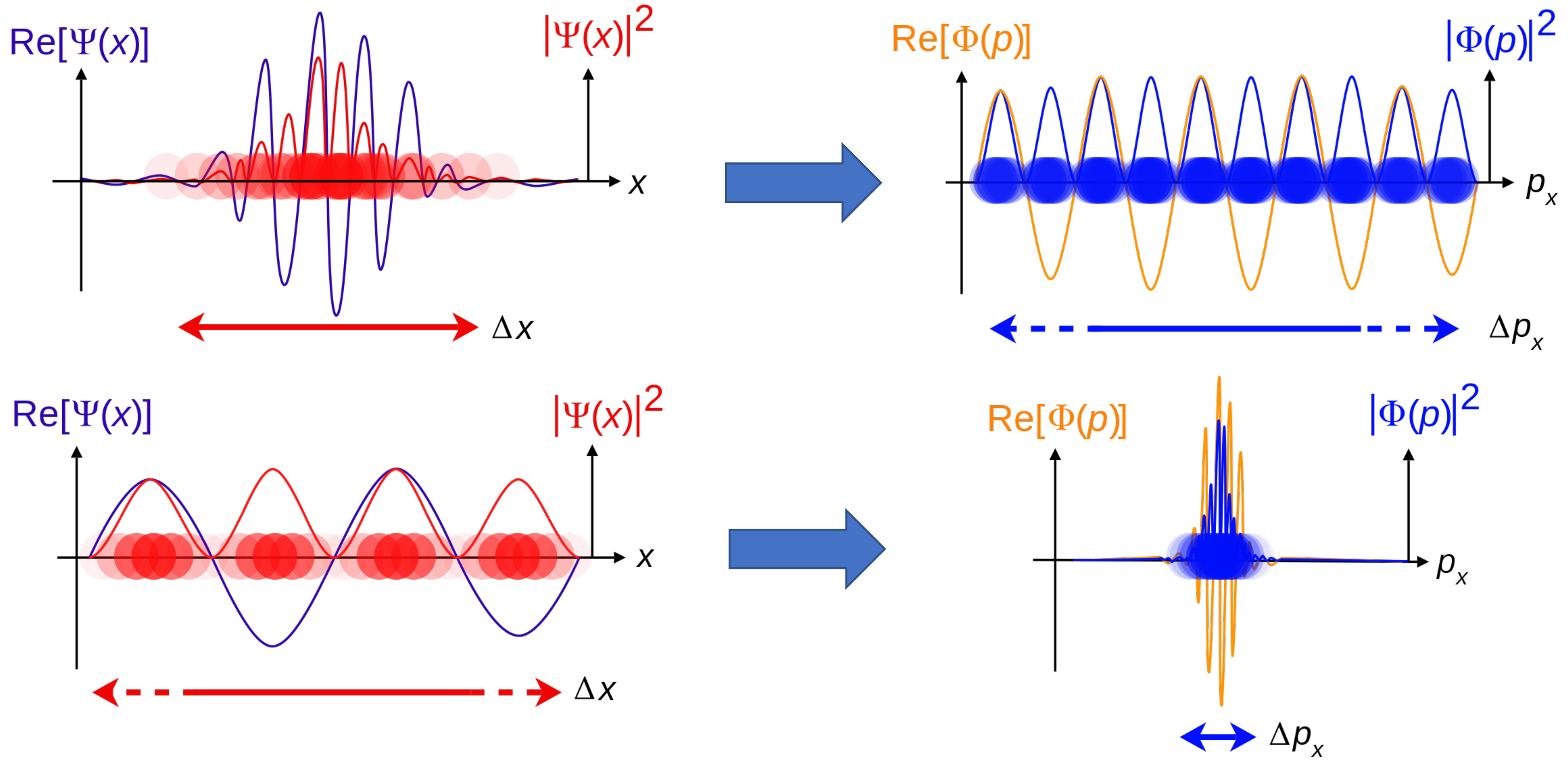


Position badly known

Momentum fairly known

Lecture 6

Feynman’s Double Slit Experiment

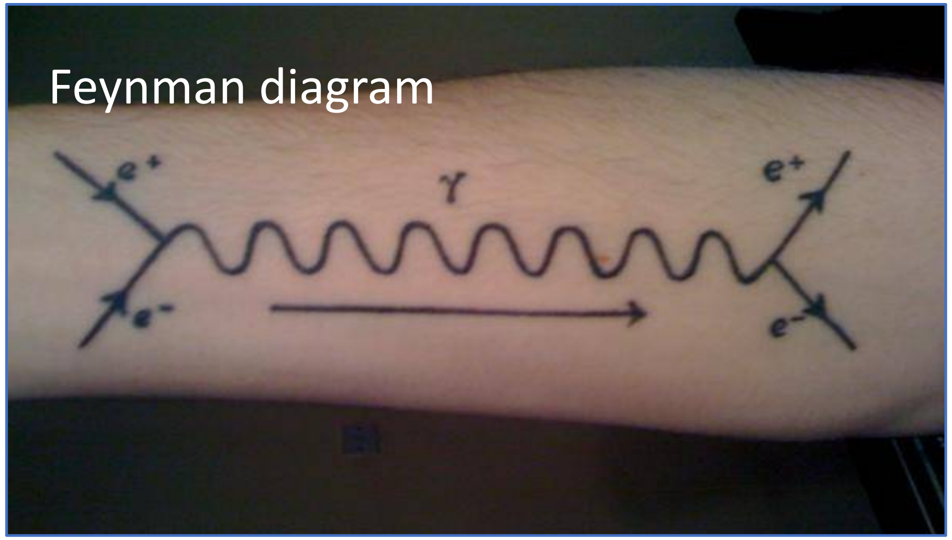
“It doesn’t matter how beautiful your theory is, it doesn’t matter how smart you are. If it doesn’t agree with experiment it’s wrong.”
- Richard Feynman

Richard Feynman (1918 – 1988)

Nobelprize 1965: Quantum Electrodynamics
(Path Integral formulation of quantum mechanics)

Mostly known from:
- Feynman diagrams
- Challenger investigation
- Popular books

Challenger disaster



Feynman diagram

Richard Feynman (1918 – 1988)

Nobelprize 1965: Quantum Electrodynamics
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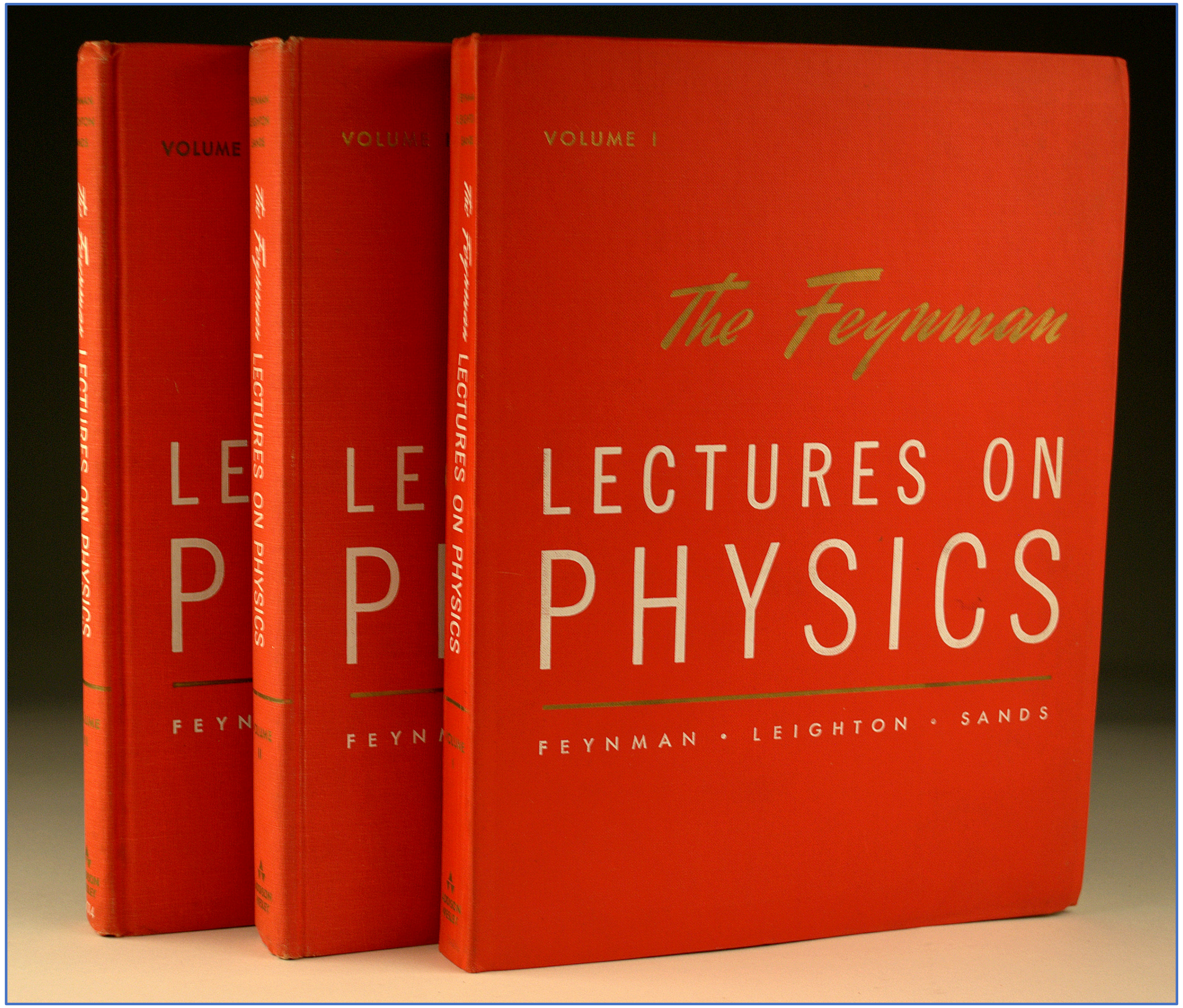
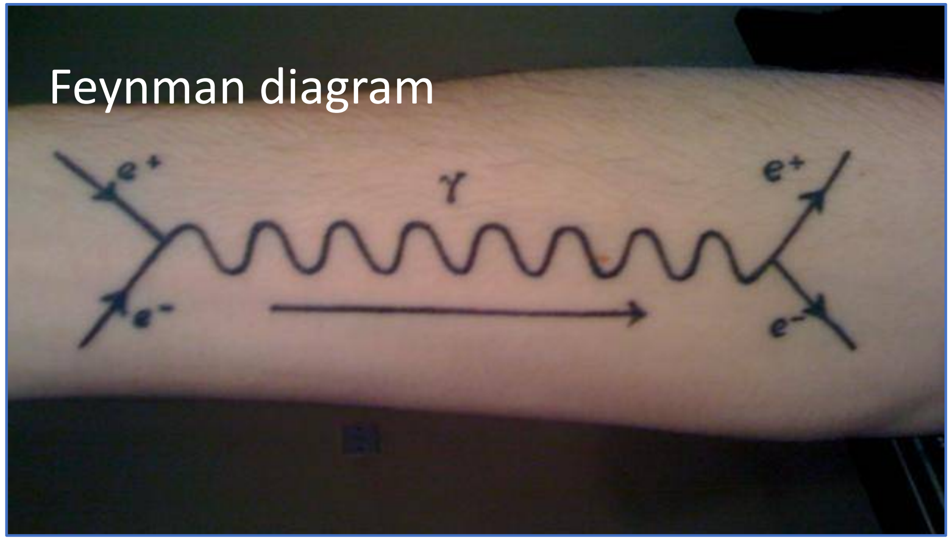




Richard Feynman and the double slit experiment

The double slit experiment demonstrates the fundamental aspect of the quantum world.

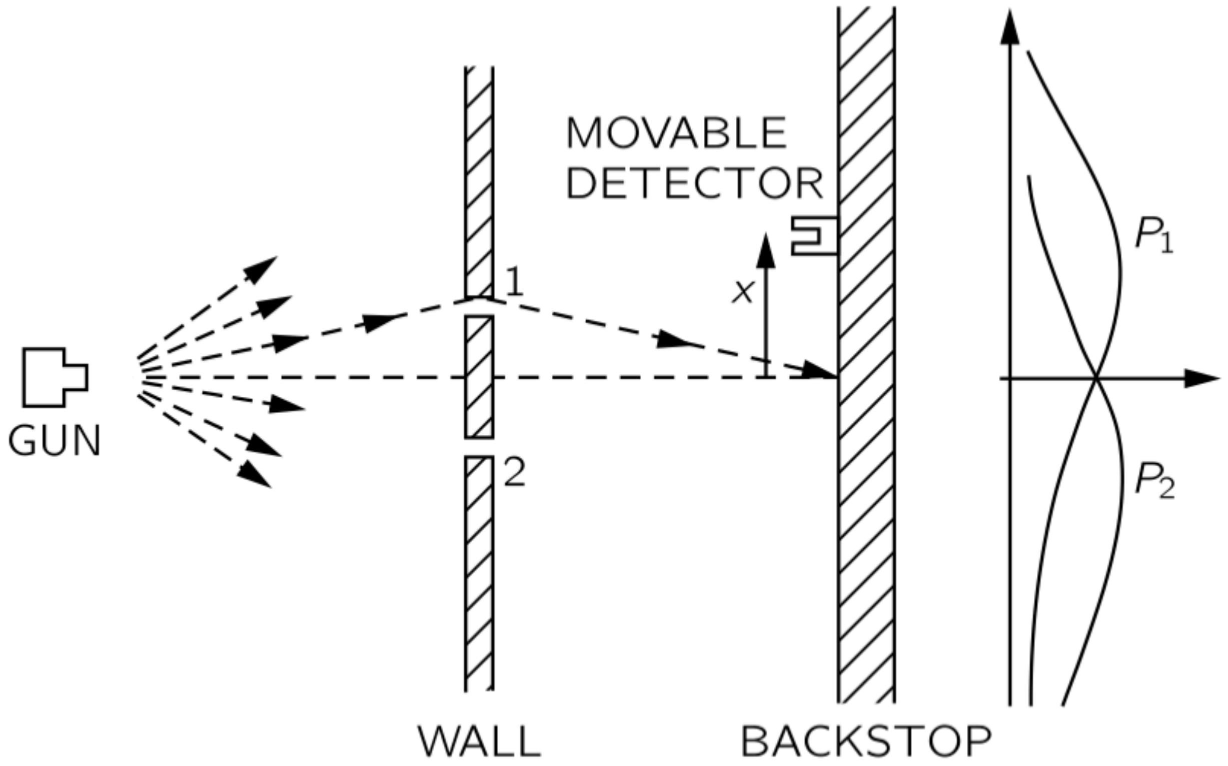
The Double Slit Experiment

Case 1:

An Experiment with Bullets

Case 1: Experiment with Bullets

A gun fires bullets in random direction. Slits 1 and 2 are openings through which bullets can pass. A moveable detector "collects" bullets and counts them.



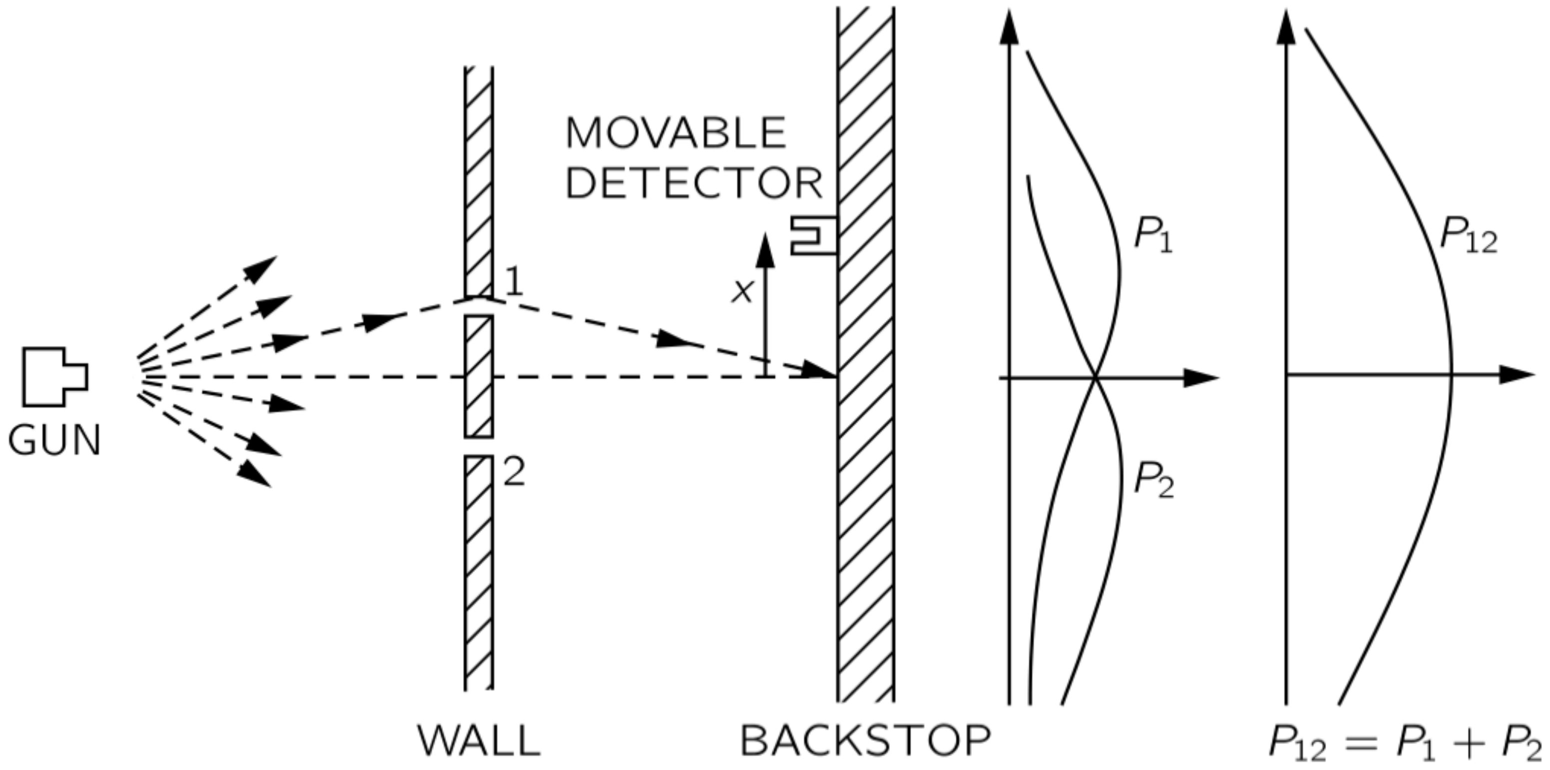
Observation:
Bullets come in "lumps".

P_1 is the probability curve when only slit 1 is open
P_2 is the probability curve when only slit 2 is open

What is the probability curve when both slit 1 and slit 2 are open?

Case 1: Experiment with Bullets

A gun fires bullets in random direction. Slits 1 and 2 are openings through which bullets can pass. A moveable detector "collects" bullets and counts them.



P_1 is the probability curve when only slit 1 is open
P_2 is the probability curve when only slit 2 is open

When both slits are open: $P_{12} = P_1 + P_2$

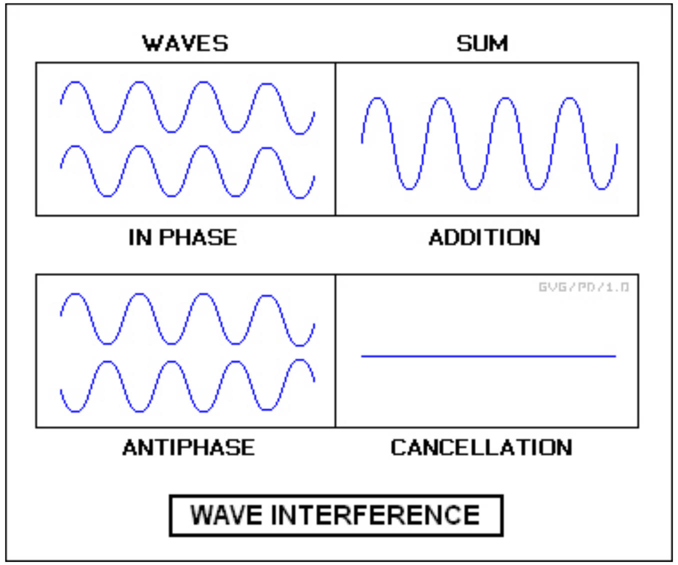
We can just add up the probabilities.

The Double Slit Experiment

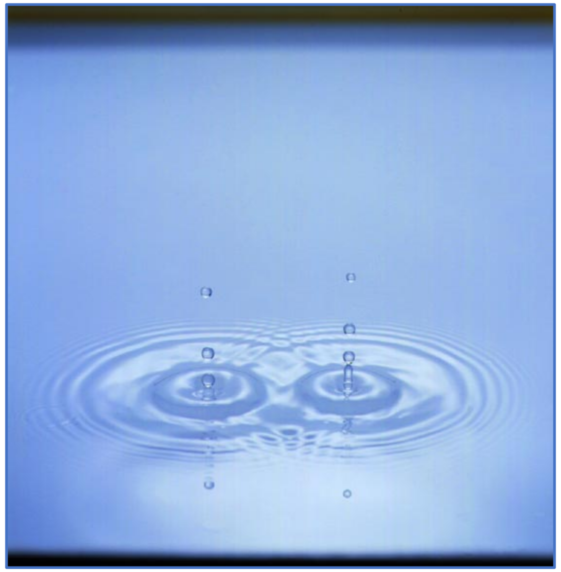
Case 2:
An Experiment with Waves

Waves & Interference : water, sound, light

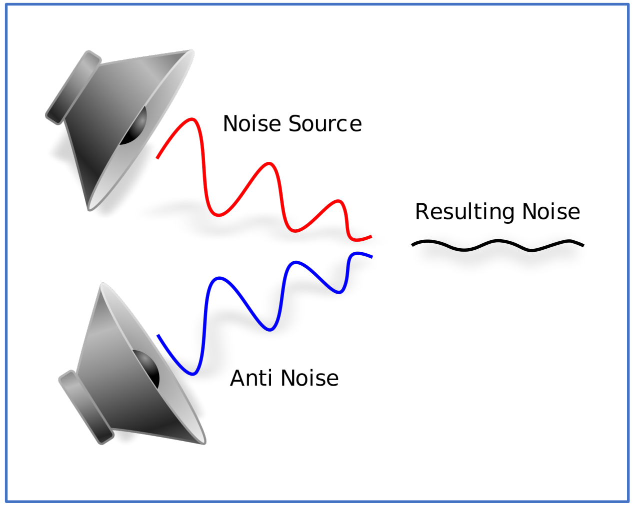
Waves: Interference principle:



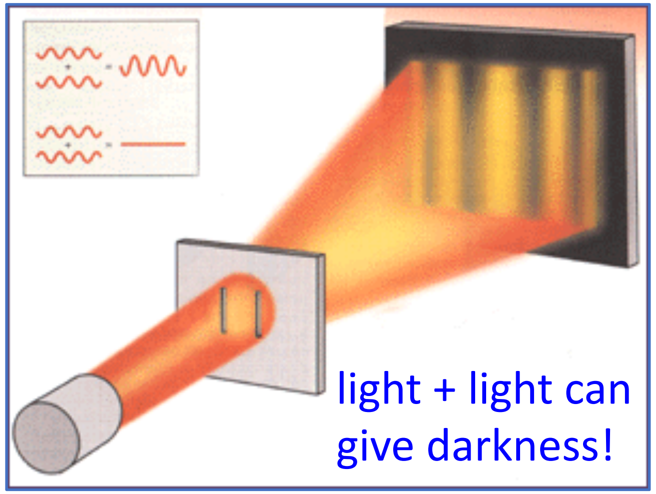
Water: Interference pattern:



Sound: Active noise cancellation:

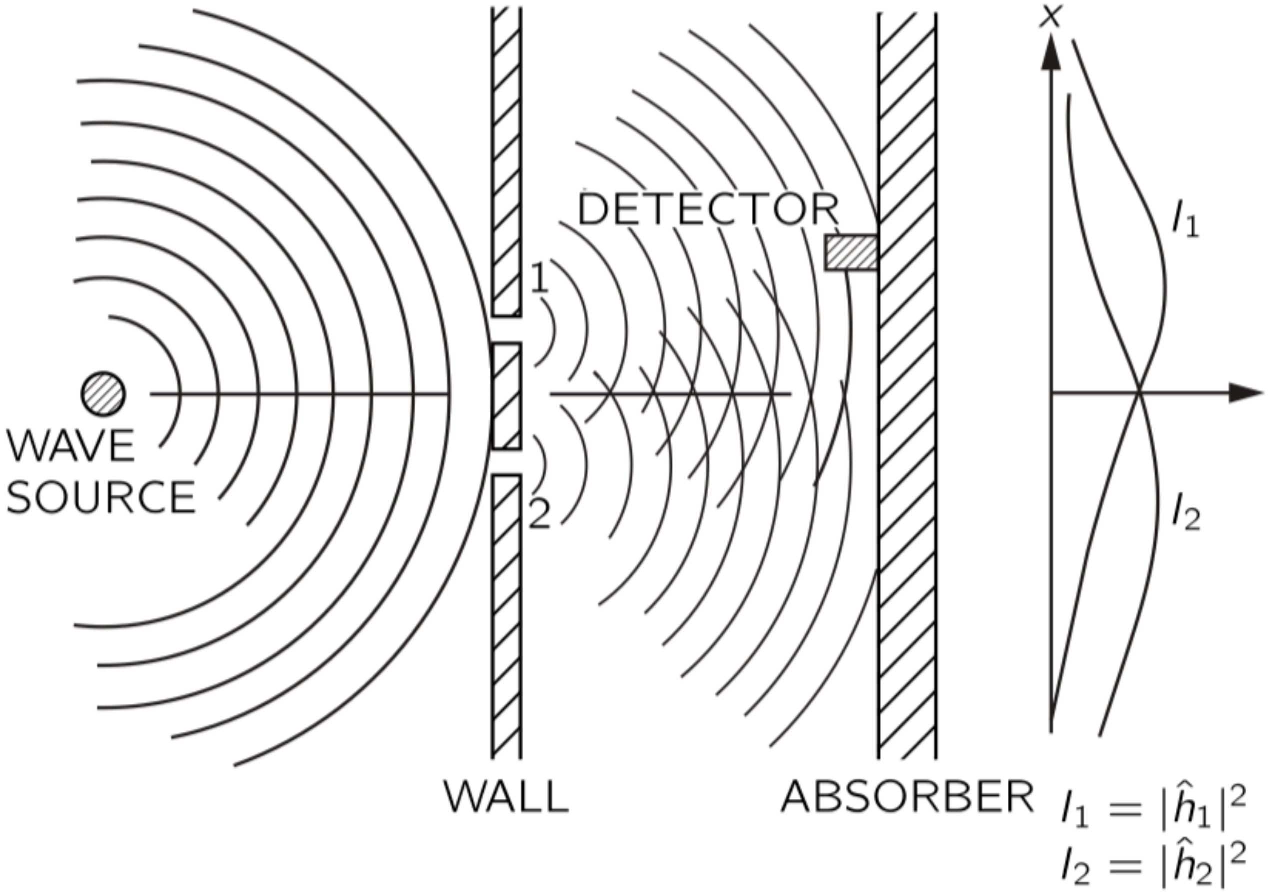


Light: Thomas Young experiment:



Case 2: Experiment with Waves

We replace the gun by a wave generator: think of water waves. Slits 1 and 2 act as new wave sources. The detector measures now the intensity (energy) in the wave.



Observation:
Waves do not
Come in "lumps".

$I_1 = |h_1|^2$

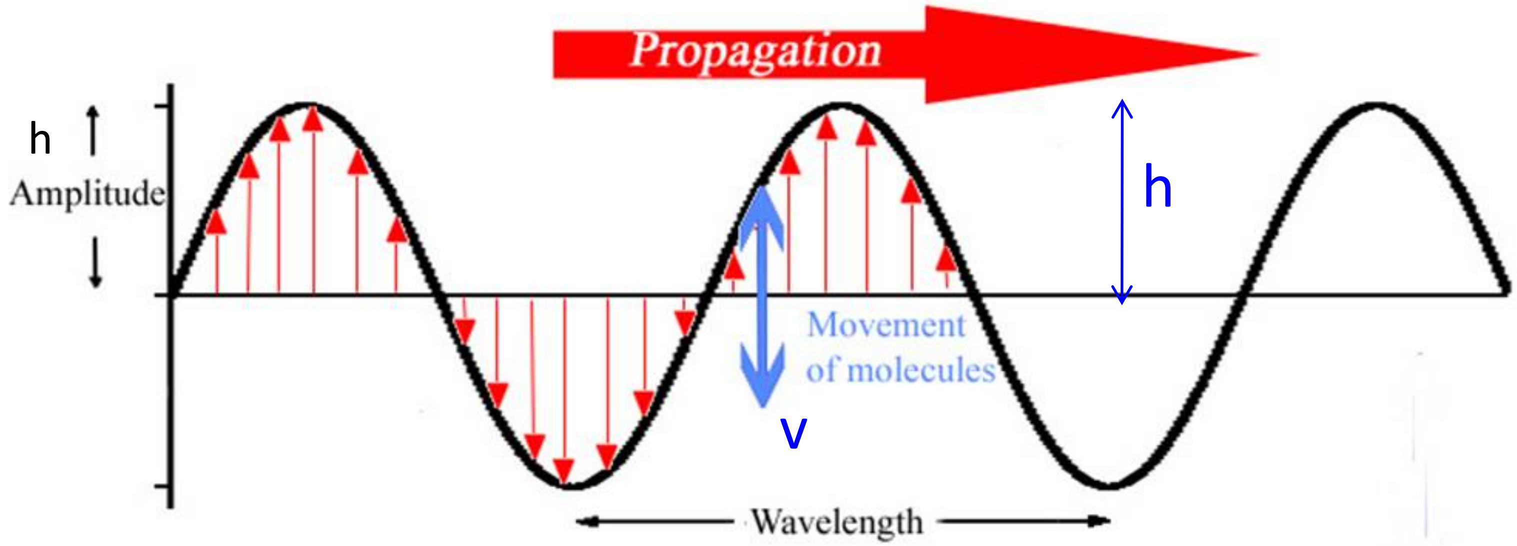
$I_2 = |h_2|^2$

$I_{12} = ??$

The intensity of a wave is the square of the amplitude…

Intermezzo: Wave Oscillation & Intensity

Energy in the oscillation (up-down) movement of the molecules:
$E_{kin} = {}^1\!/_2\, mv^2$ and v is proportional to the amplitude or height: $v \approx h$
So that the intensity of the wave is: $I \approx h^2$



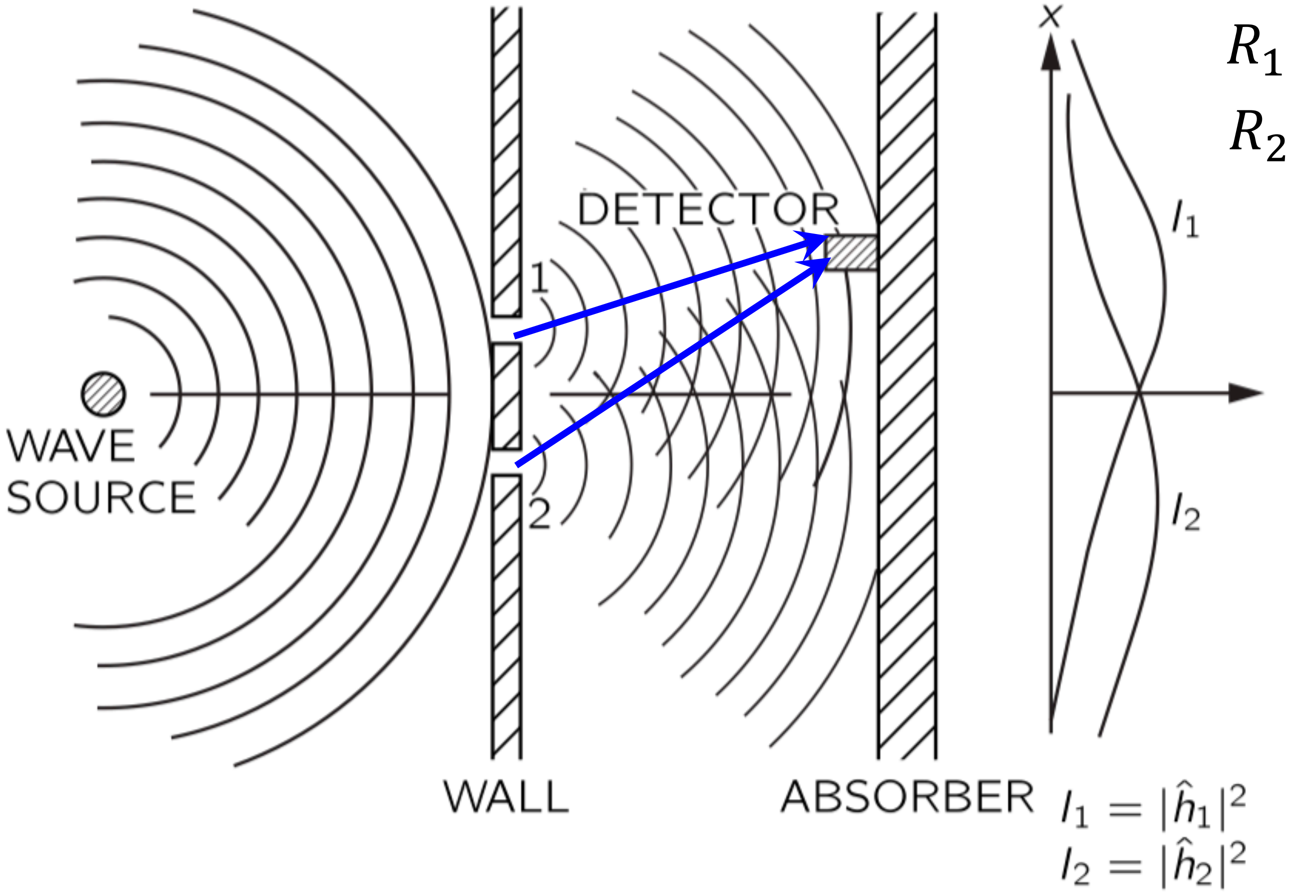
Formula for the resulting oscillation of a water molecule somewhere in the wave:

$R(t) = h\cos(2\pi f t + \phi)$
and the Intensity: $I = h^2$

f = frequency
ϕ = phase

Case 2: Experiment with Waves

When both slits are open there are two contributions to the wave the oscillation at the detector: $R(t) = R_1(t) + R_2(t)$



$$R_1(t) = h_1 \cos(2\pi f\ t + \phi_1)$$
$$R_2(t) = h_2 \cos(2\pi f\ t + \phi_2)$$

ϕ_1 and ϕ_2 depend on distance to 1 and 2

$$I_1 = |h_1|^2$$
$$I_2 = |h_2|^2$$
$$I_{12} = ??$$

First combine: $R(t) = R_1(t) + R_2(t)$
Afterwards look at the amplitude and intensity of the resulting wave!

Mathematics for the die-hards

$$\boldsymbol{R_{12}(t) = h_1 \cos(2\pi f t + \phi_1) + h_2 \cos(2\pi f\, t + \phi_2)}$$

Assume equal size waves: $\boldsymbol{h_1 = h_2 = h}$

First find amplitude of sum wave $\boldsymbol{R_{12}(t)}$.

From math textbook:

$$\cos(A) + \cos(B) = 2\cos\left(\frac{1}{2}(A - B)\right)\cos\left(\frac{1}{2}(A + B)\right)$$

Use this to find:

$$\boxed{\begin{array}{c} \boldsymbol{R_{12}(t) = h' \;\cos\left(2\pi f t + \frac{1}{2}(\phi_1 + \phi_2)\right)} \\ \text{With} \quad \boldsymbol{h' = 2h \;\cos\left(\frac{1}{2}(\phi_1 - \phi_2)\right)} \end{array}}$$

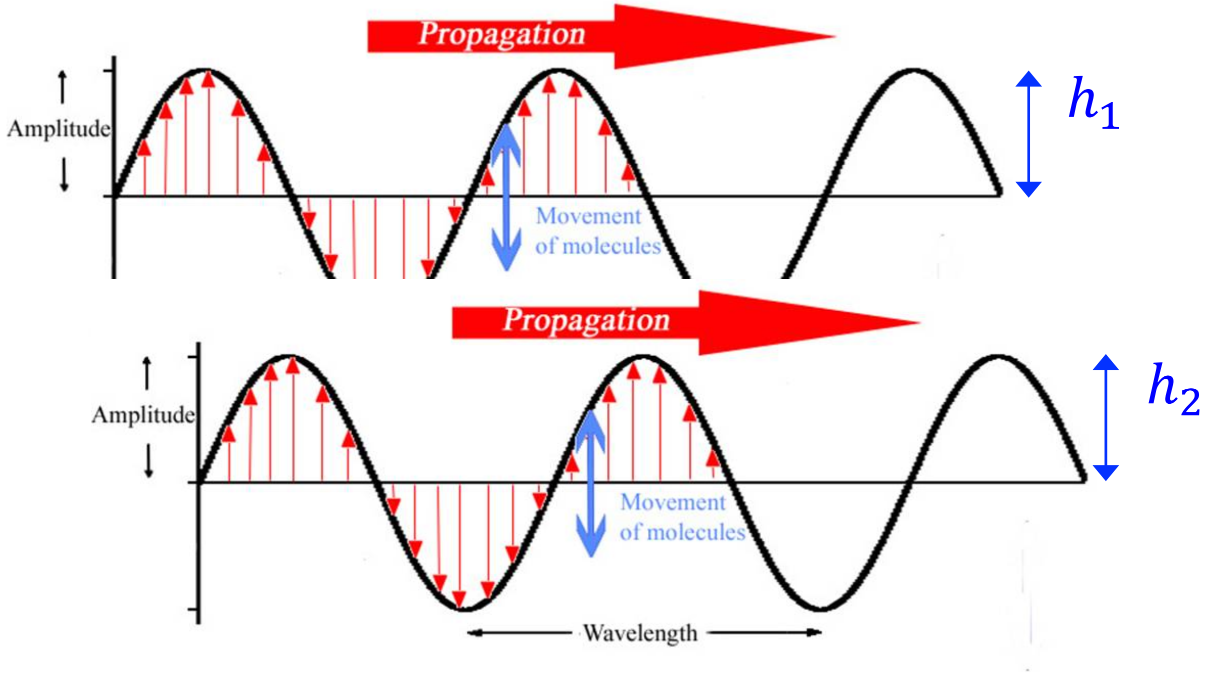
Resulting wave has the intensity: $\boldsymbol{I_{12} = h'^2 = 4h^2 \cos^2\left(\frac{1}{2}(\phi_1 - \phi_2)\right)}$

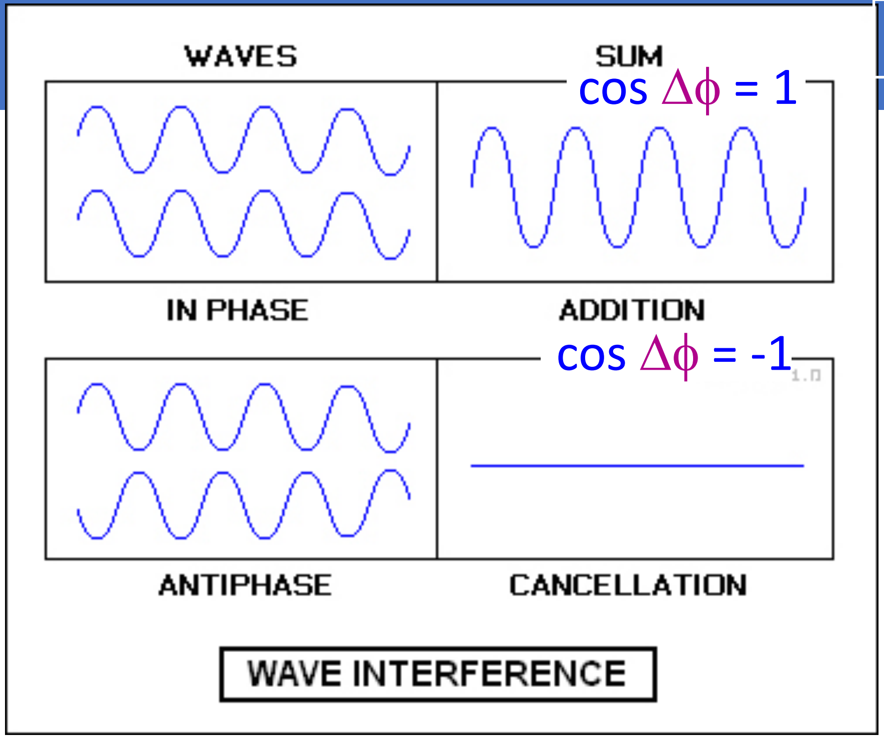
Use math textbook: $\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A,$ so:

$$\boxed{\boldsymbol{I_{12} = 2h^2 + 2h^2 \cos(\phi_1 - \phi_2)}}$$

Interference!

Interference of Waves





Interfering waves:

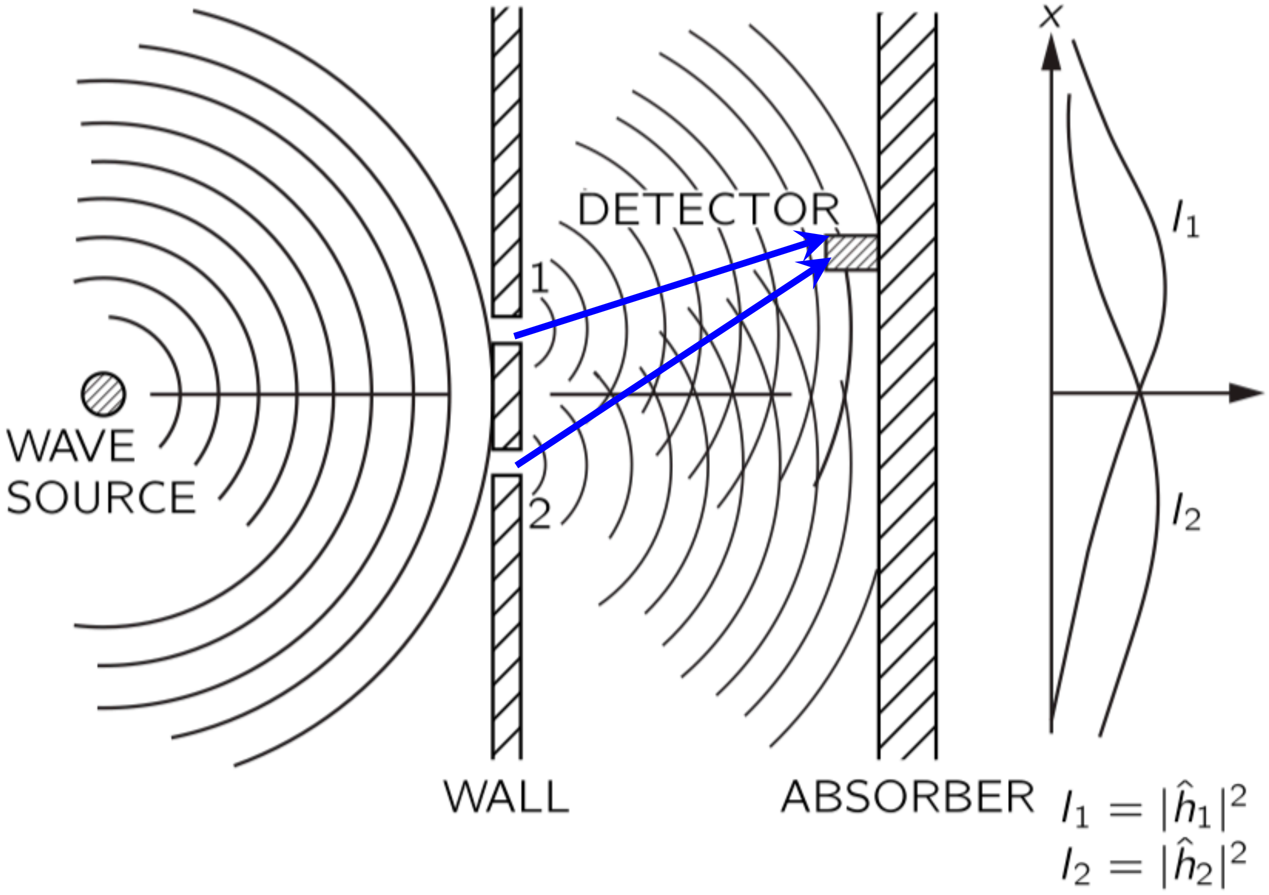
$$I_{12} = |R_1 + R_2|^2 = h_1^2 + h_2^2 + 2h_1 h_2 \cos(\Delta\phi)$$

Regions of *constructive* interference: $I_{12} = 2\times(I_1 + I_2)$

Regions of *destructive* interference: $I_{12} = 0$

Case 2: Experiment with Waves

When both slits are open there are two contributions to the wave the oscillation at the detector: $R(t) = R_1(t) + R_2(t)$



First combine: $R(t) = R_1(t) + R_2(t)$
Afterwards look at the amplitude and intensity of the resulting wave!

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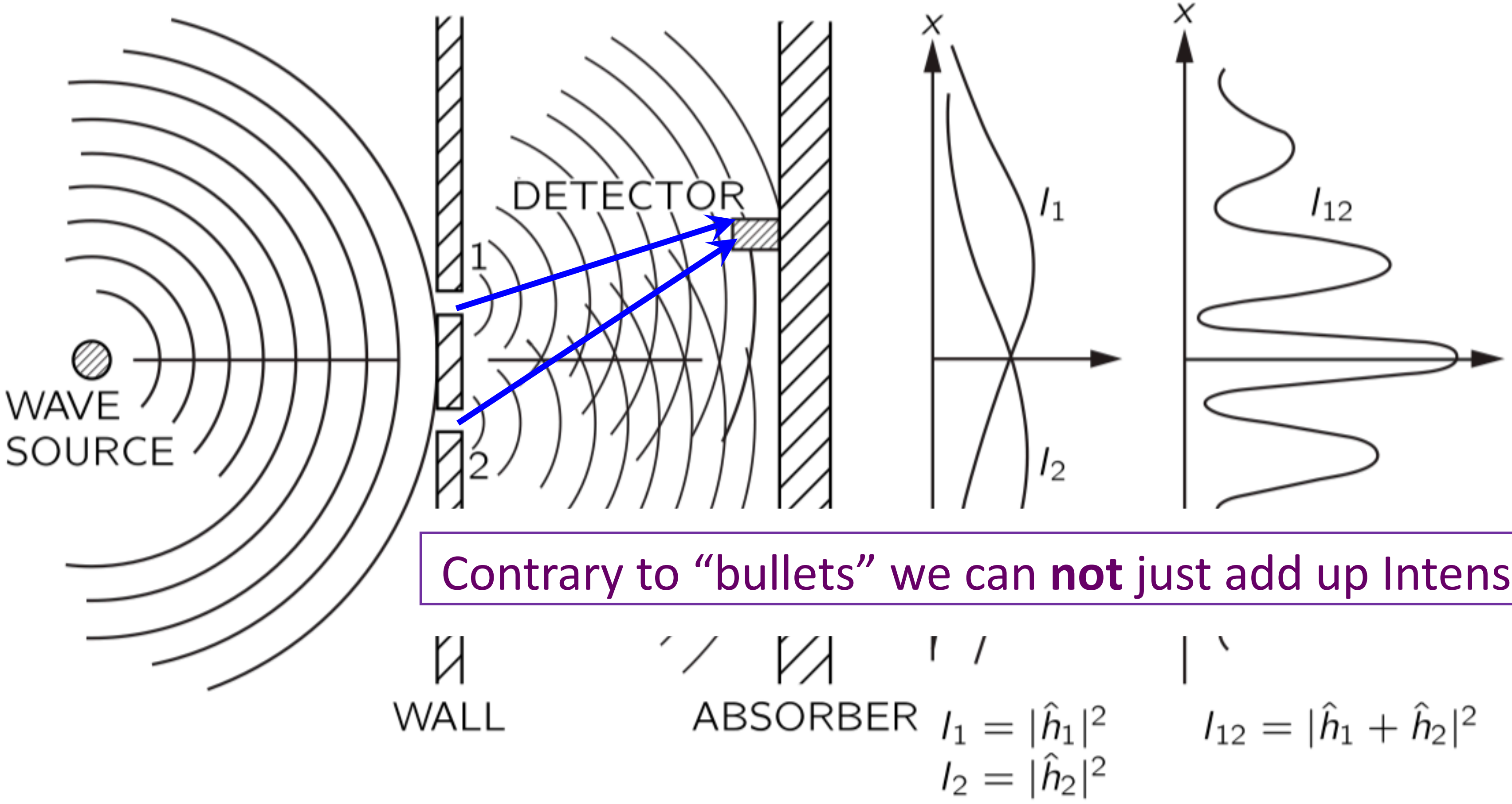
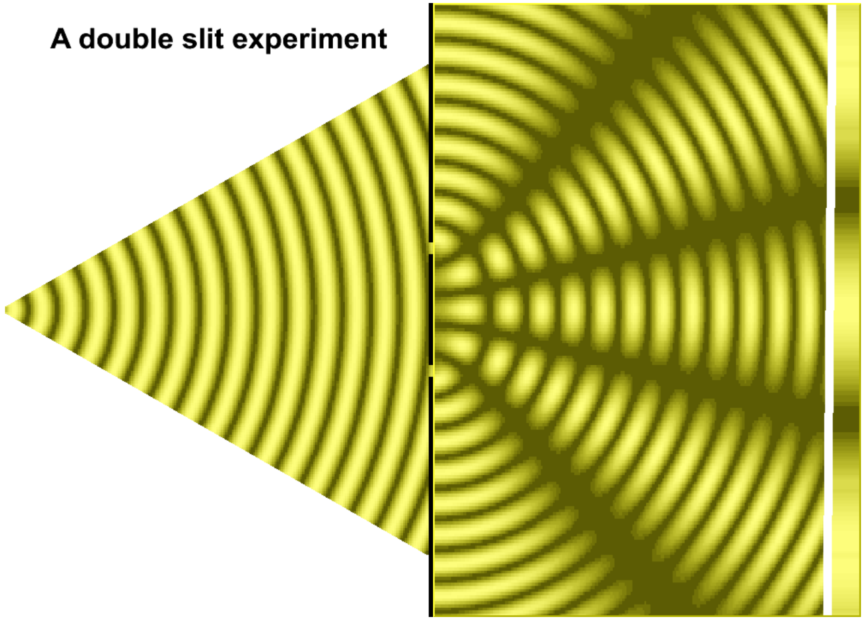
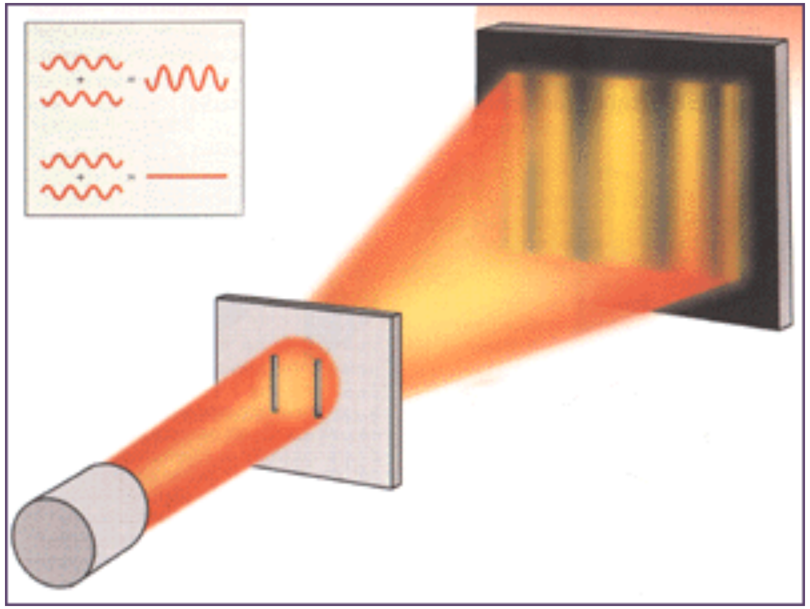


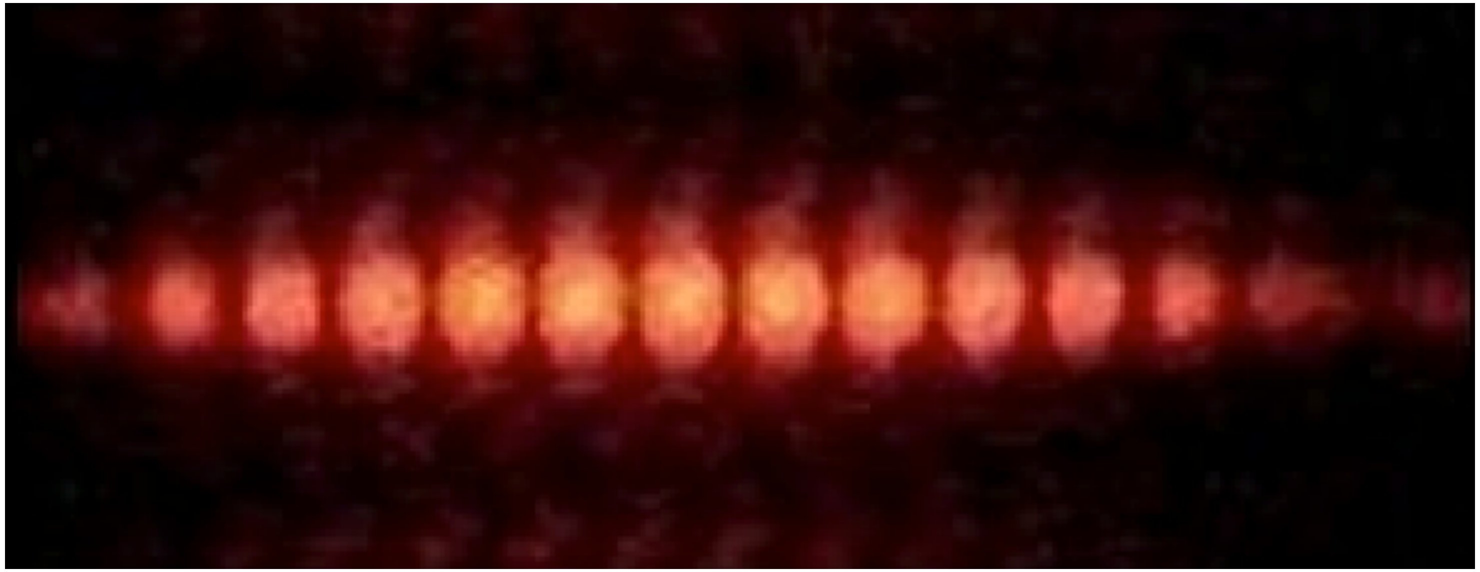
Contrary to "bullets" we can **not** just add up Intensities.

Interference pattern: $I_{12} = |R_1 + R_2|^2 = h_1^2 + h_2^2 + 2h_1h_2\cos(\Delta\phi)$
Regions where waves are *amplified* and regions where waves are *cancelled*.

Double Slit Experiment with Light (Young)





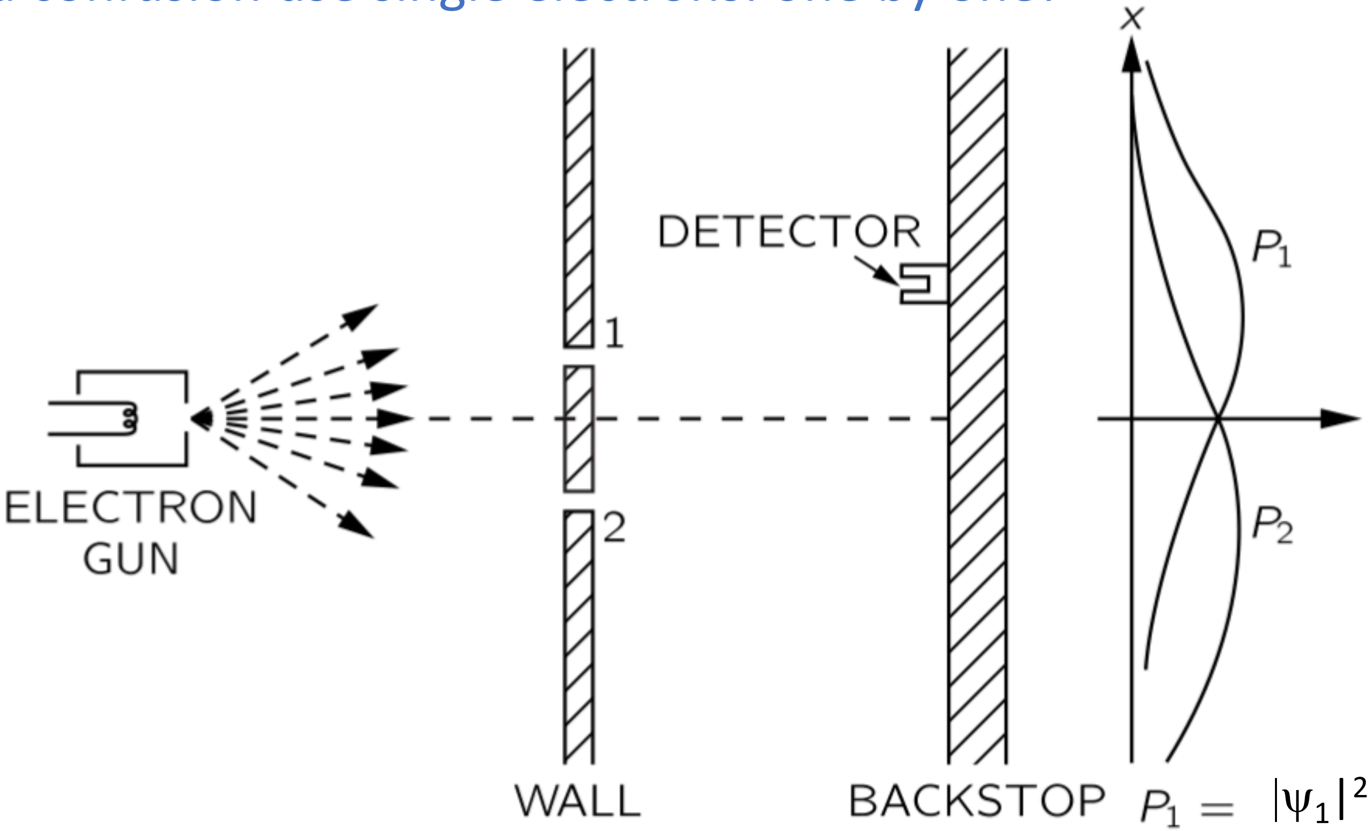


Case 3:
An Experiment with Electrons

Case 3: Experiment with Electrons

From the detector counts deduce again the probabilities P_1 and P_2

To avoid confusion use single electrons: one by one!



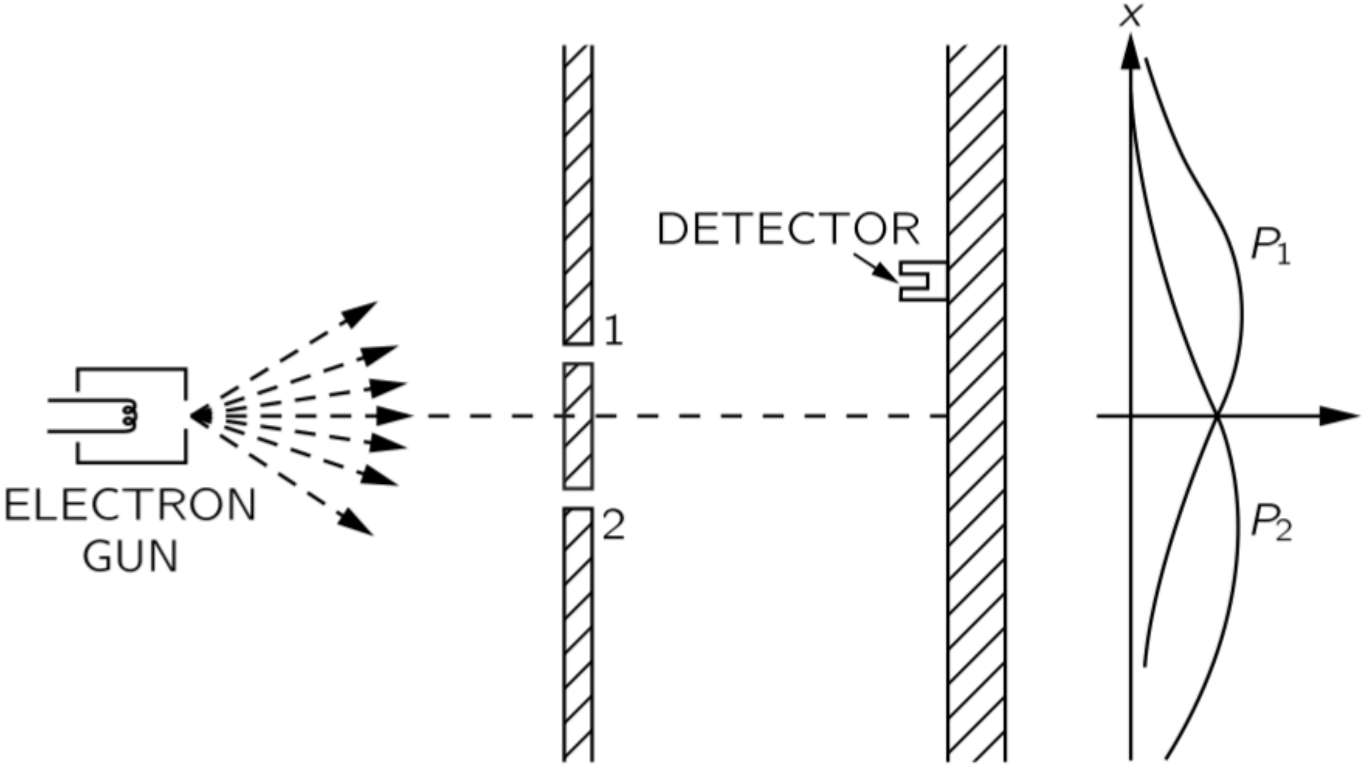
$P_1 = |\psi_1|^2$

$P_2 = |\psi_2|^2$

<u>Observation:</u>
Electrons come in "lumps", like bullets

What do we expect when both slits are open?

Case 3: Experiment with Electrons



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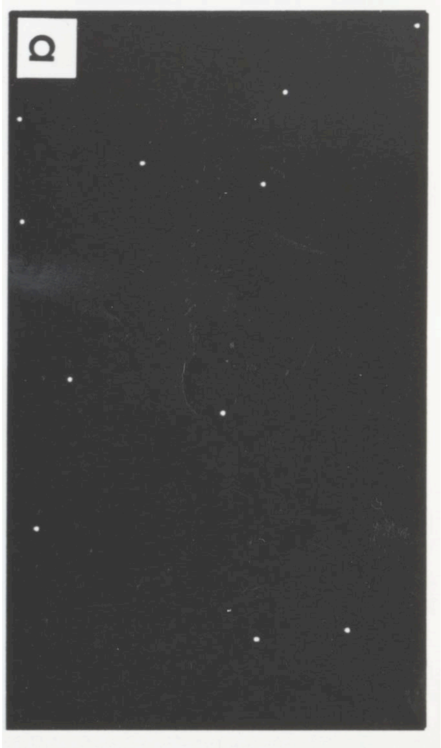

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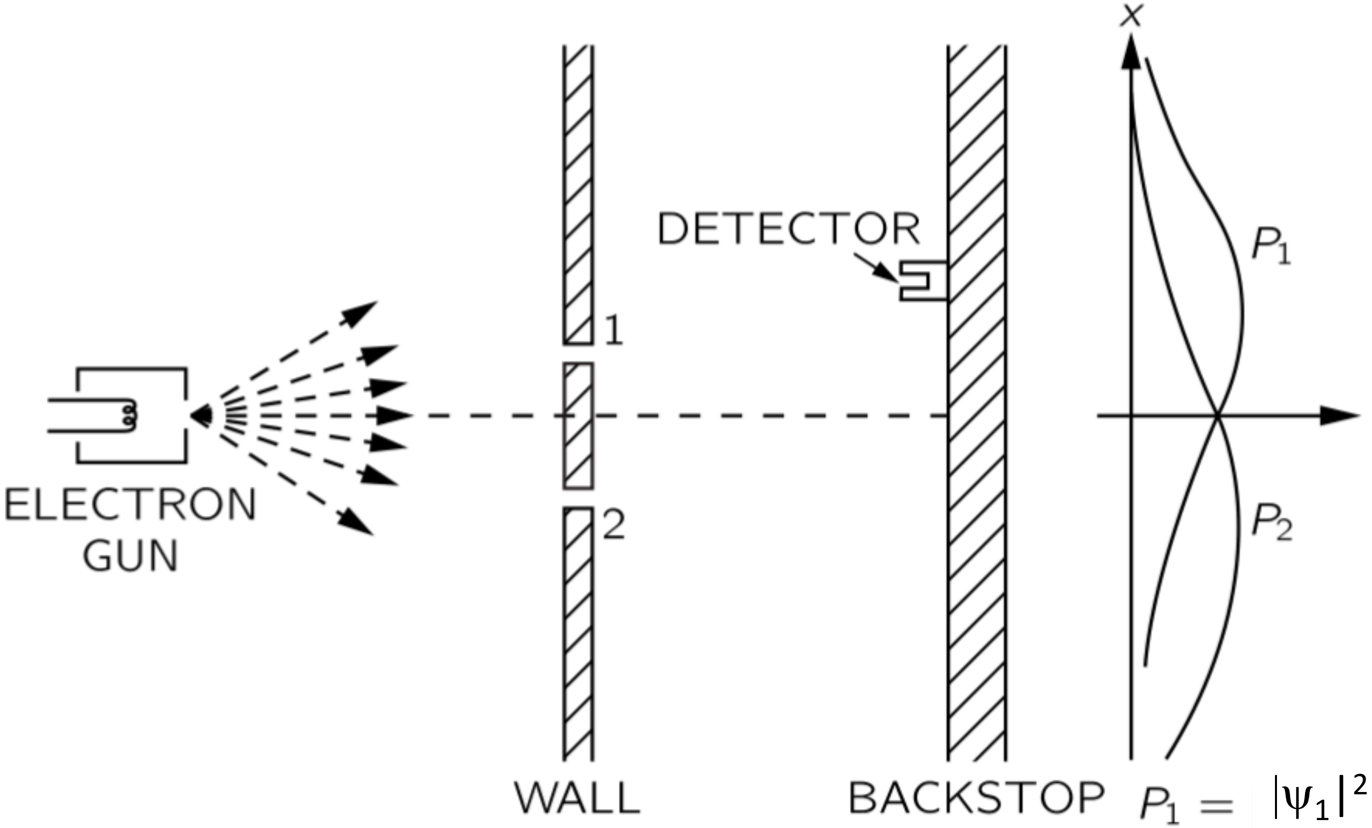

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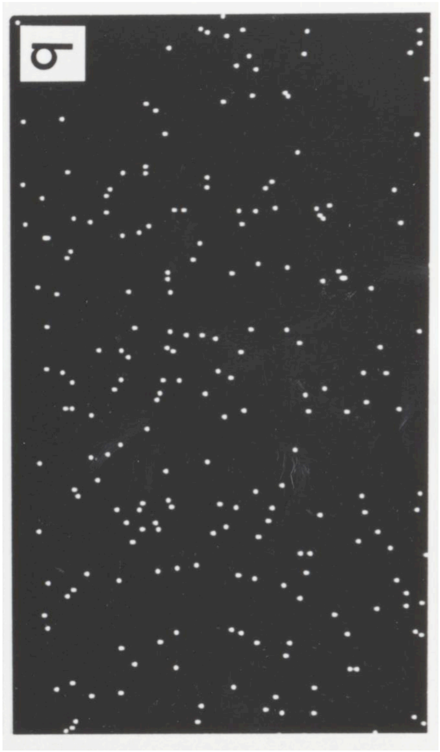
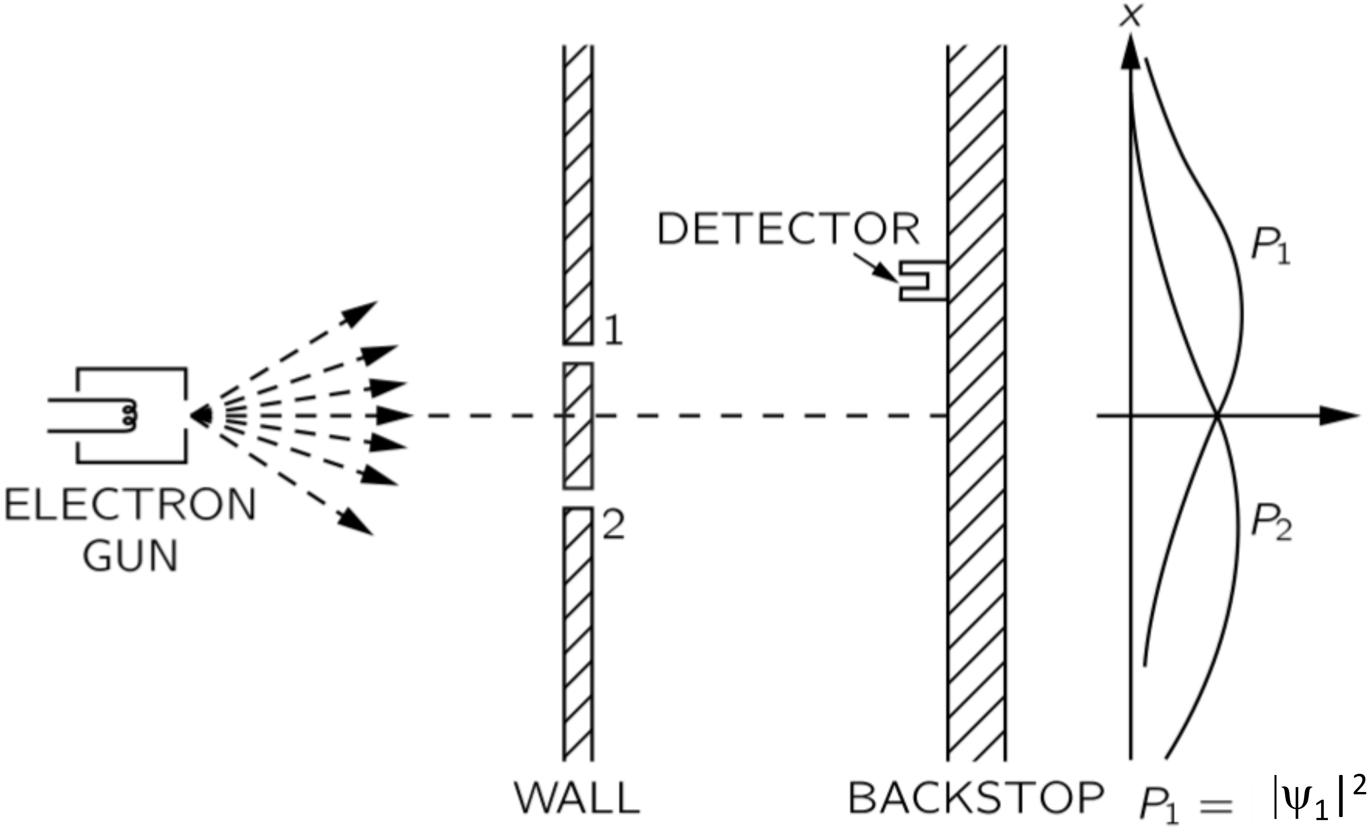


Case 3: Experiment with Electrons



$$P_1 = |\psi_1|^2$$
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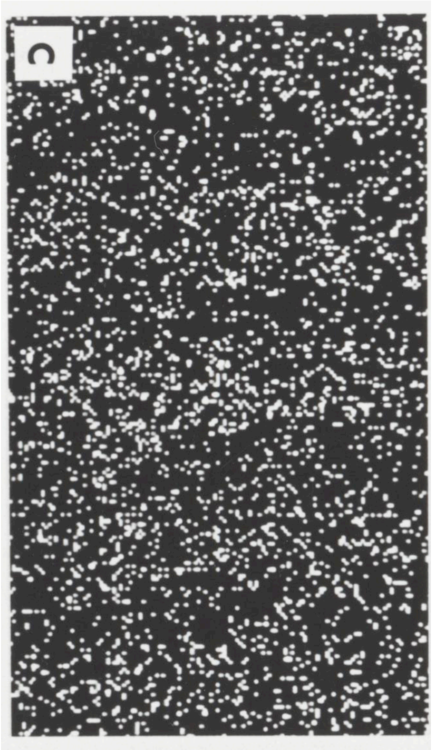

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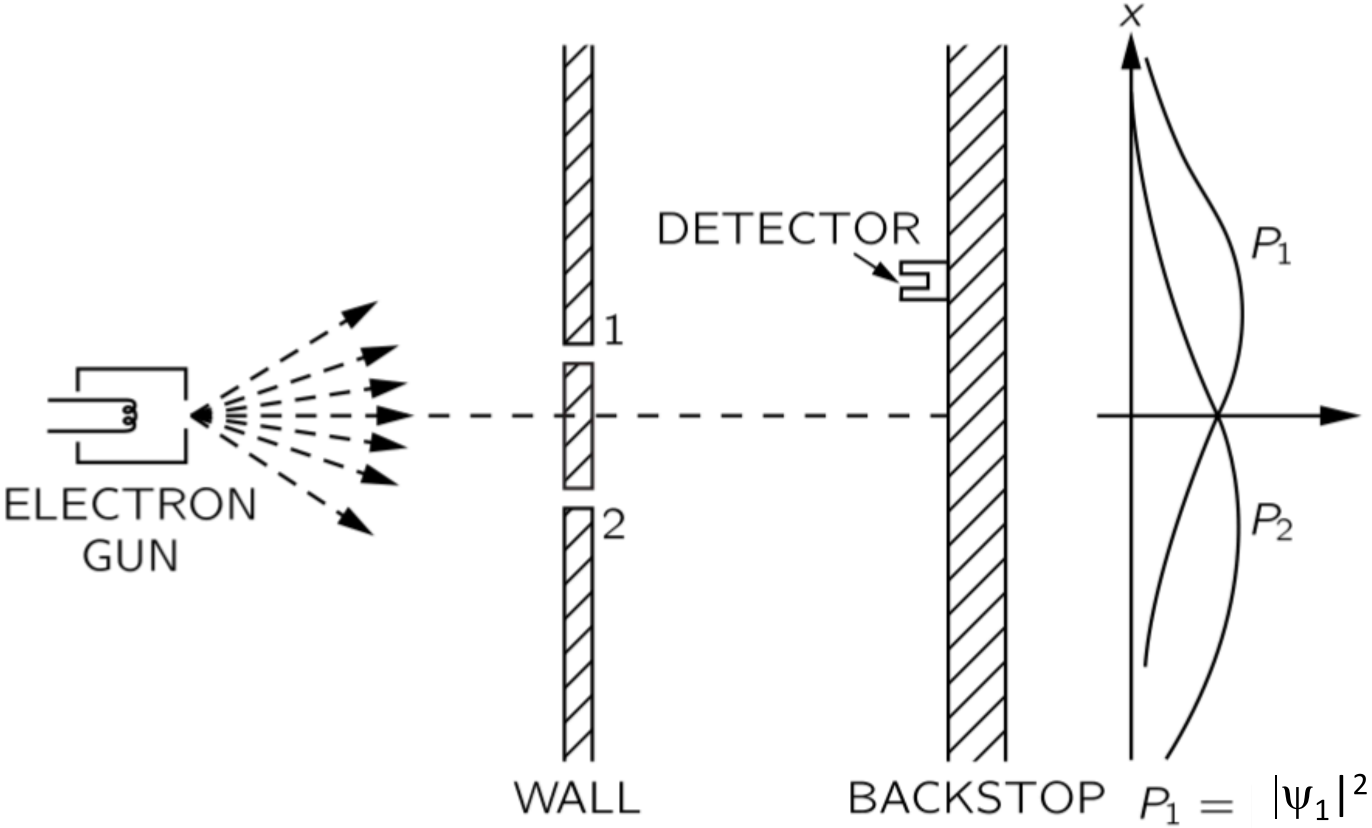

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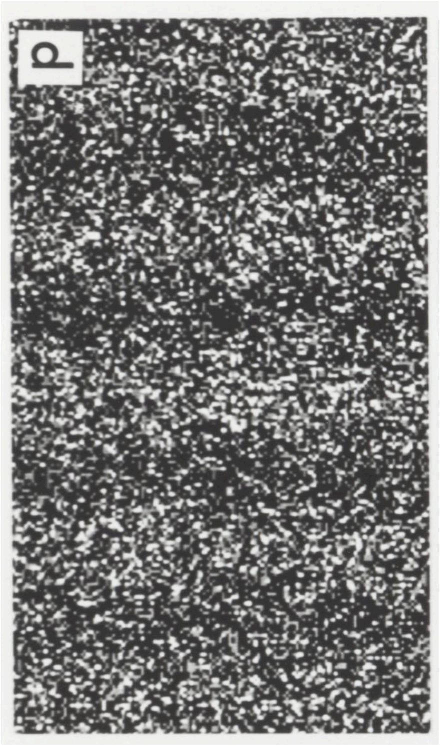

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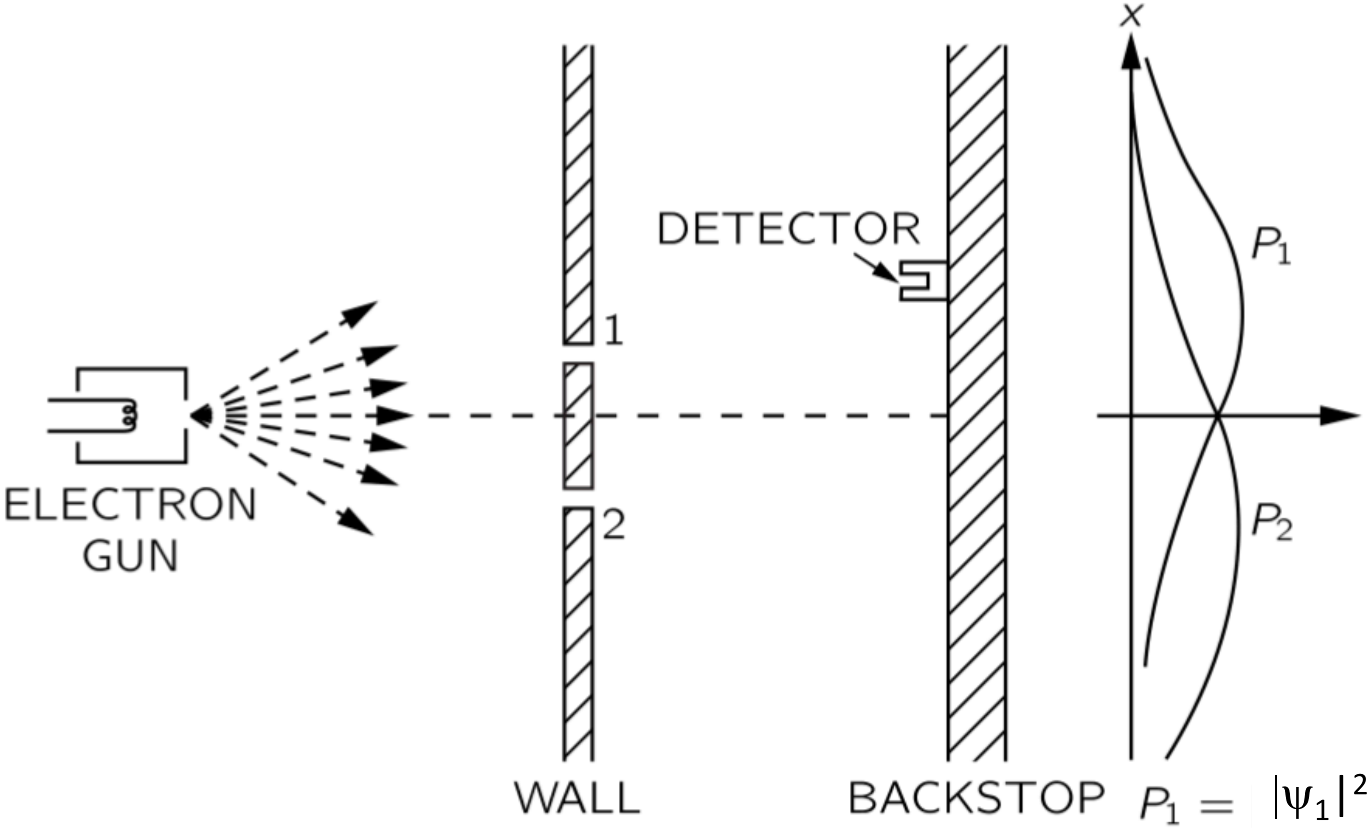

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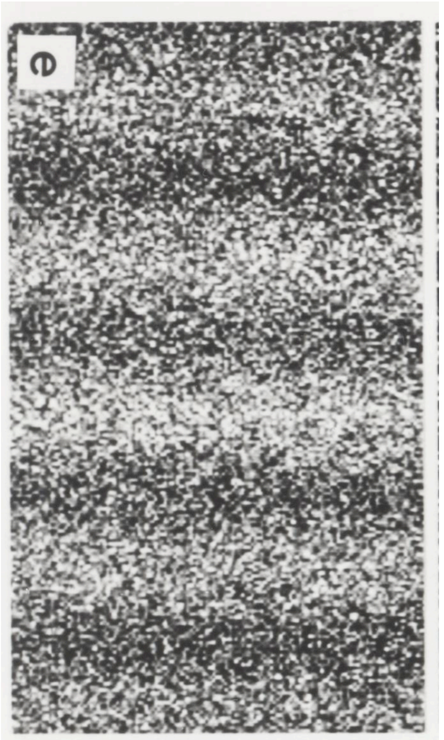
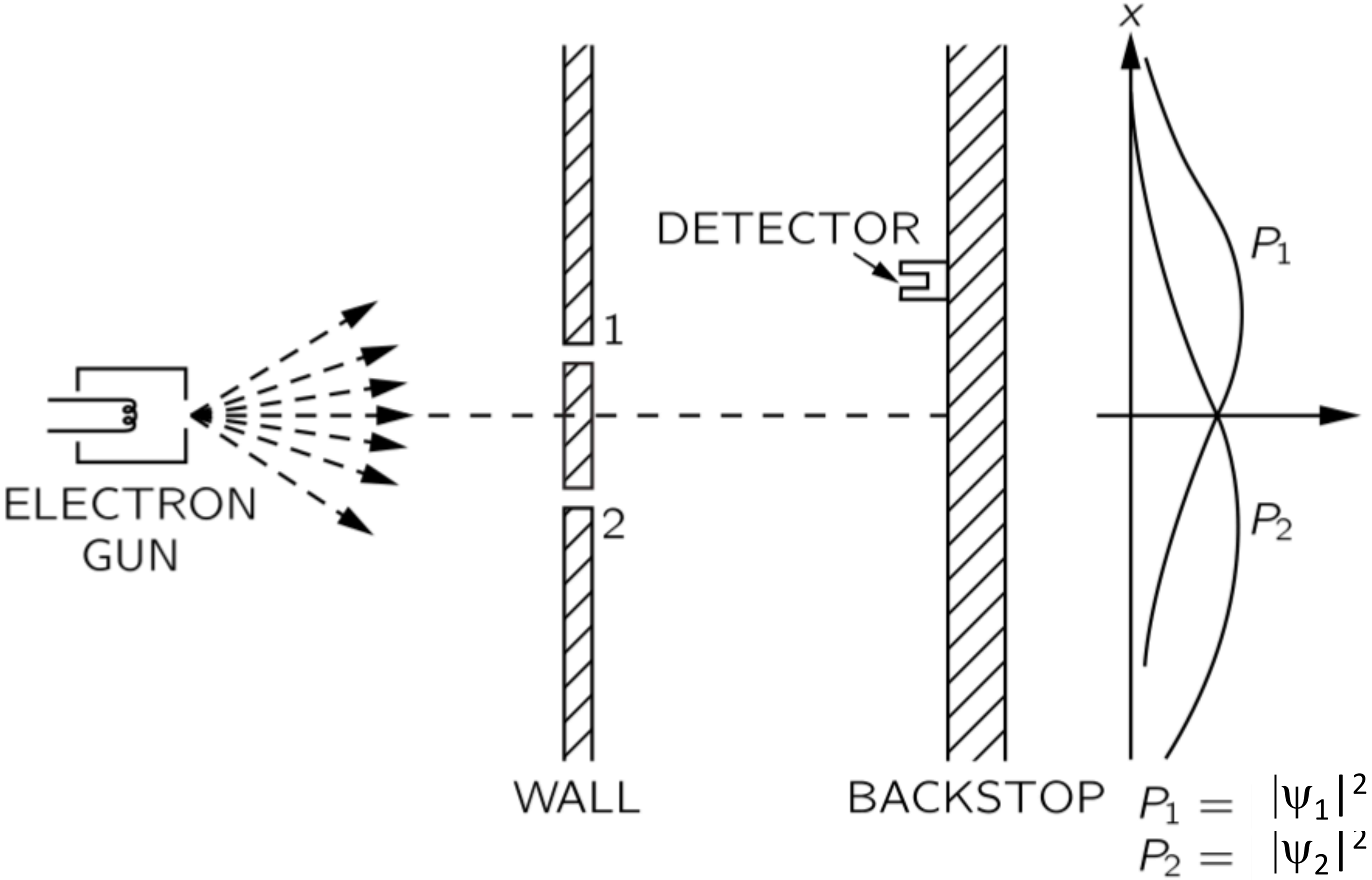
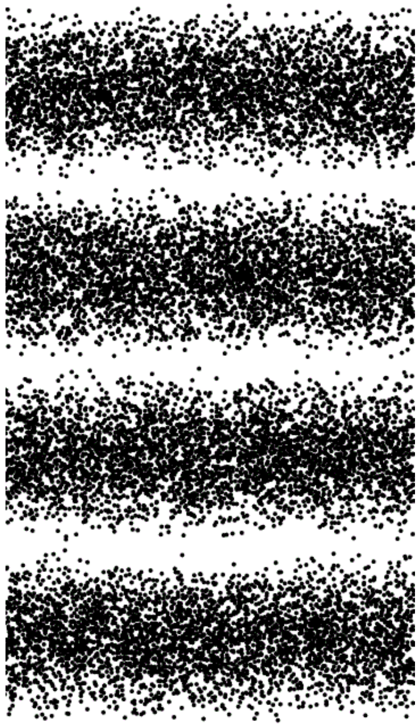


Case 3: Experiment with Electrons



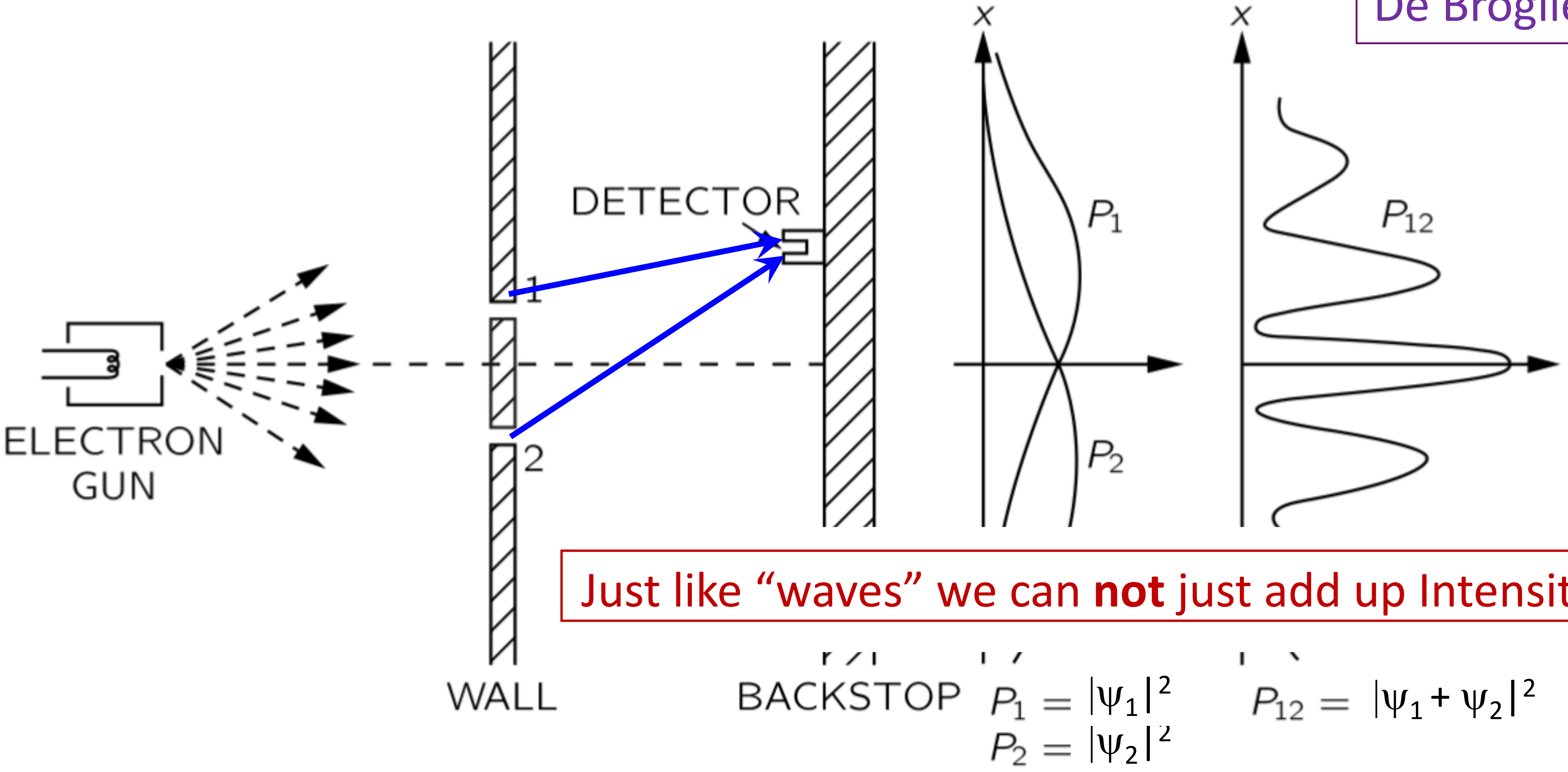


Case 3: Experiment with Electrons

An Interference pattern!

The electron wave function behaves exactly like classical waves.

De Broglie waves



Just like "waves" we can **not** just add up Intensities.

Add the wave amplitudes:

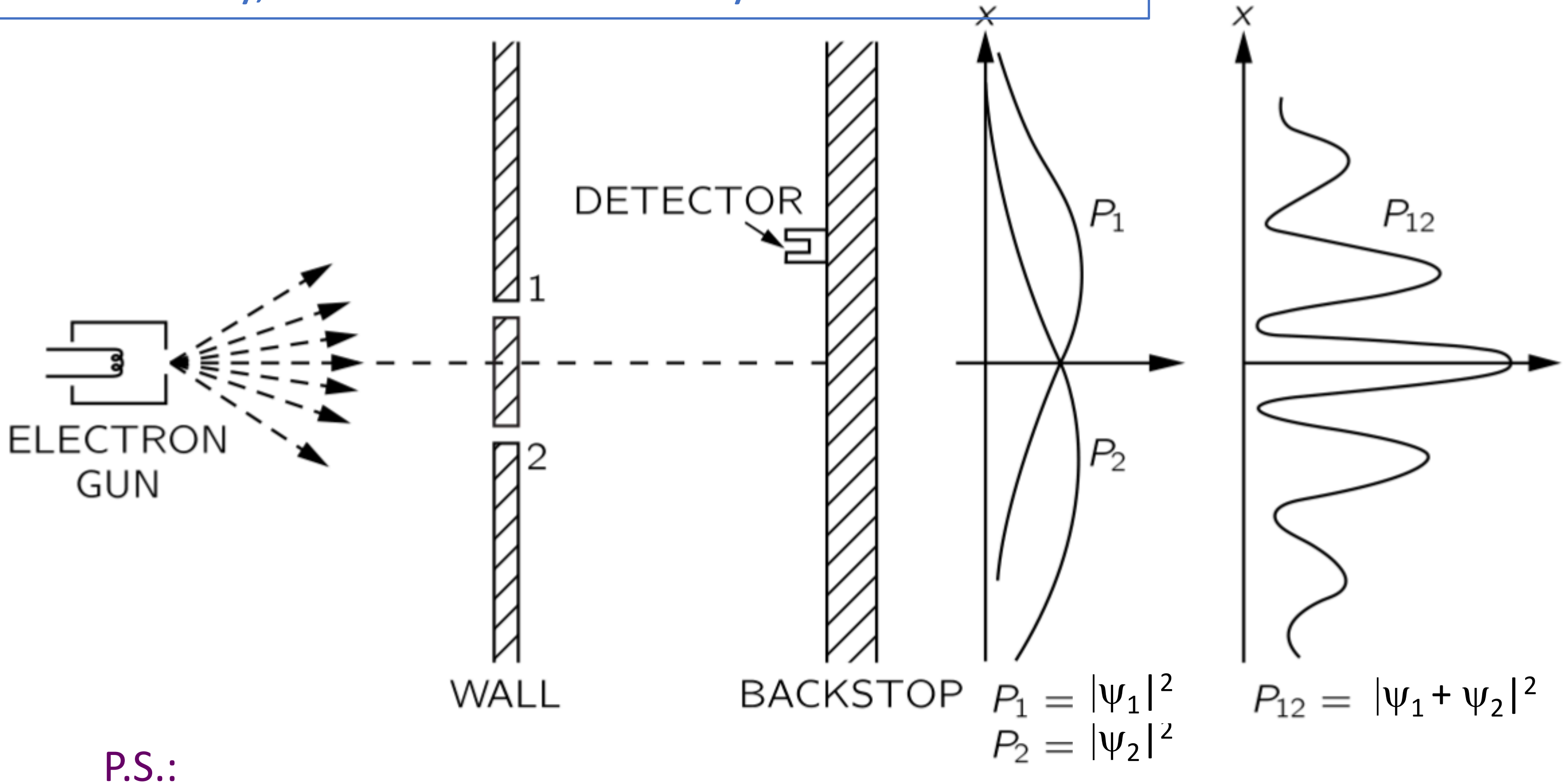
$\psi_{12} = \psi_1 + \psi_2$

The probability is the square of the sum:

$P_{12} = |\psi_{12}|^2 = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\psi_1\psi_2^*$

Case 3: Experiment with Electrons

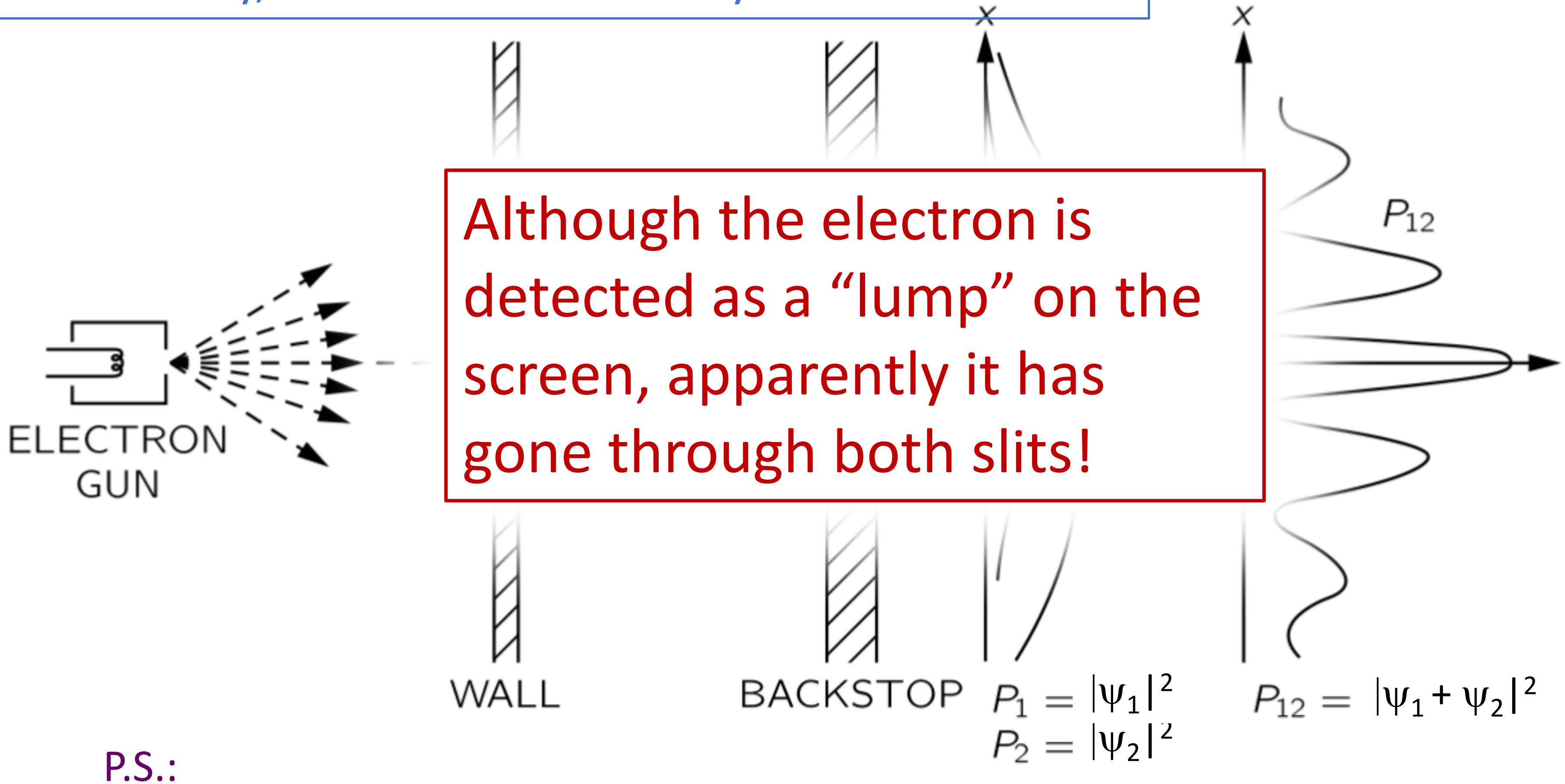
Perhaps the electrons interfere with each other.
Reduce the intensity, shoot electrons one by one: same result.



P.S.:
Classically, light behaves light waves. However, if you shoot light, photon per photon, it “comes in lumps”, just like electrons.
Quantum Mechanics: for photons it is the same story as for electrons.

Case 3: Experiment with Electrons

Perhaps the electrons interfere with each other.
Reduce the intensity, shoot electrons one by one: same result.



Although the electron is detected as a "lump" on the screen, apparently it has gone through both slits!

P.S.:
Classically, light behaves light waves. However, if you shoot light, photon per photon, it "comes in lumps", just like electrons.
Quantum Mechanics: for photons it is the same story as for electrons.

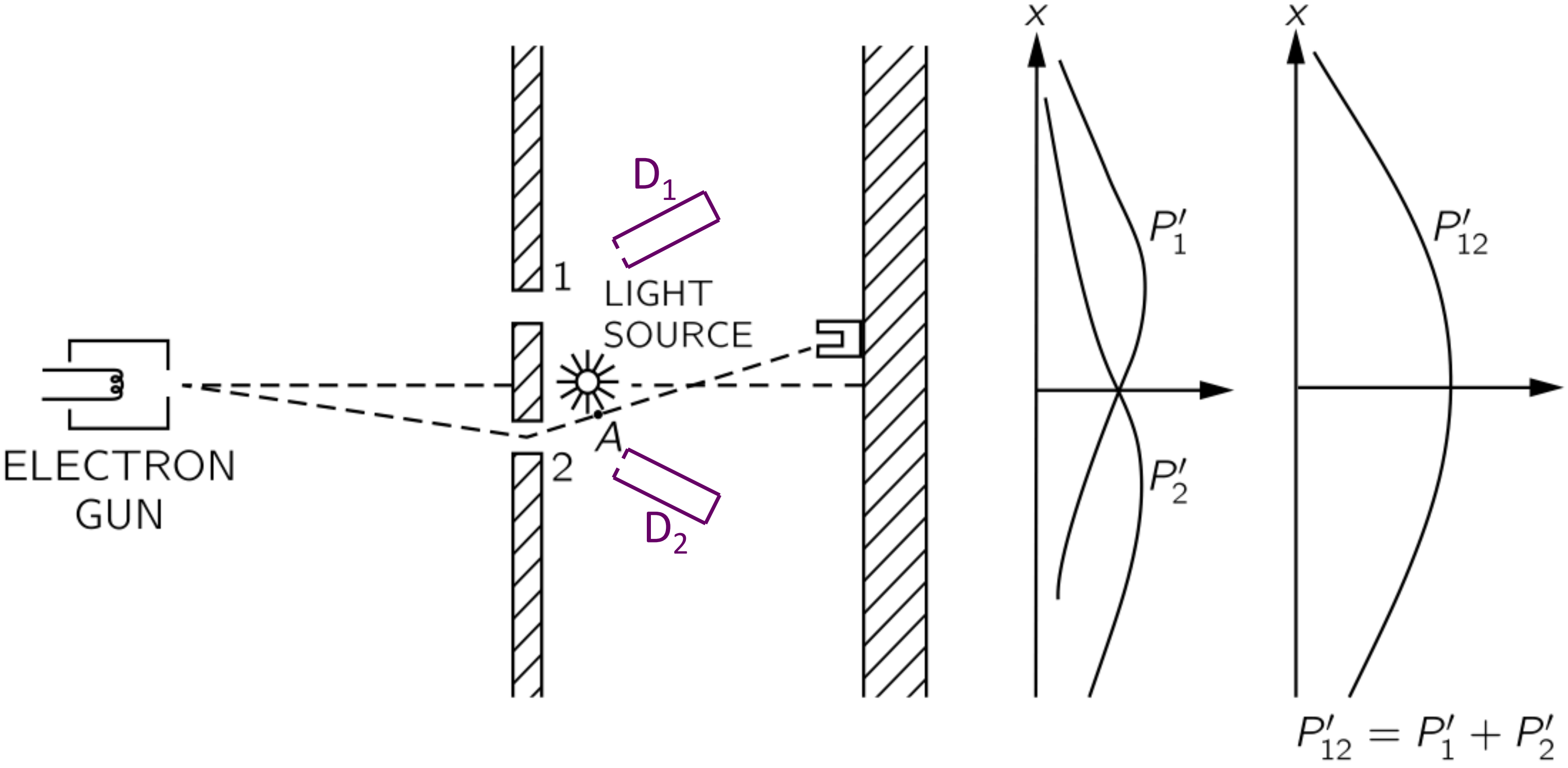
The Double Slit Experiment

Case 4:

A Different Experiment with Electrons

Case 4: Watch the Electrons

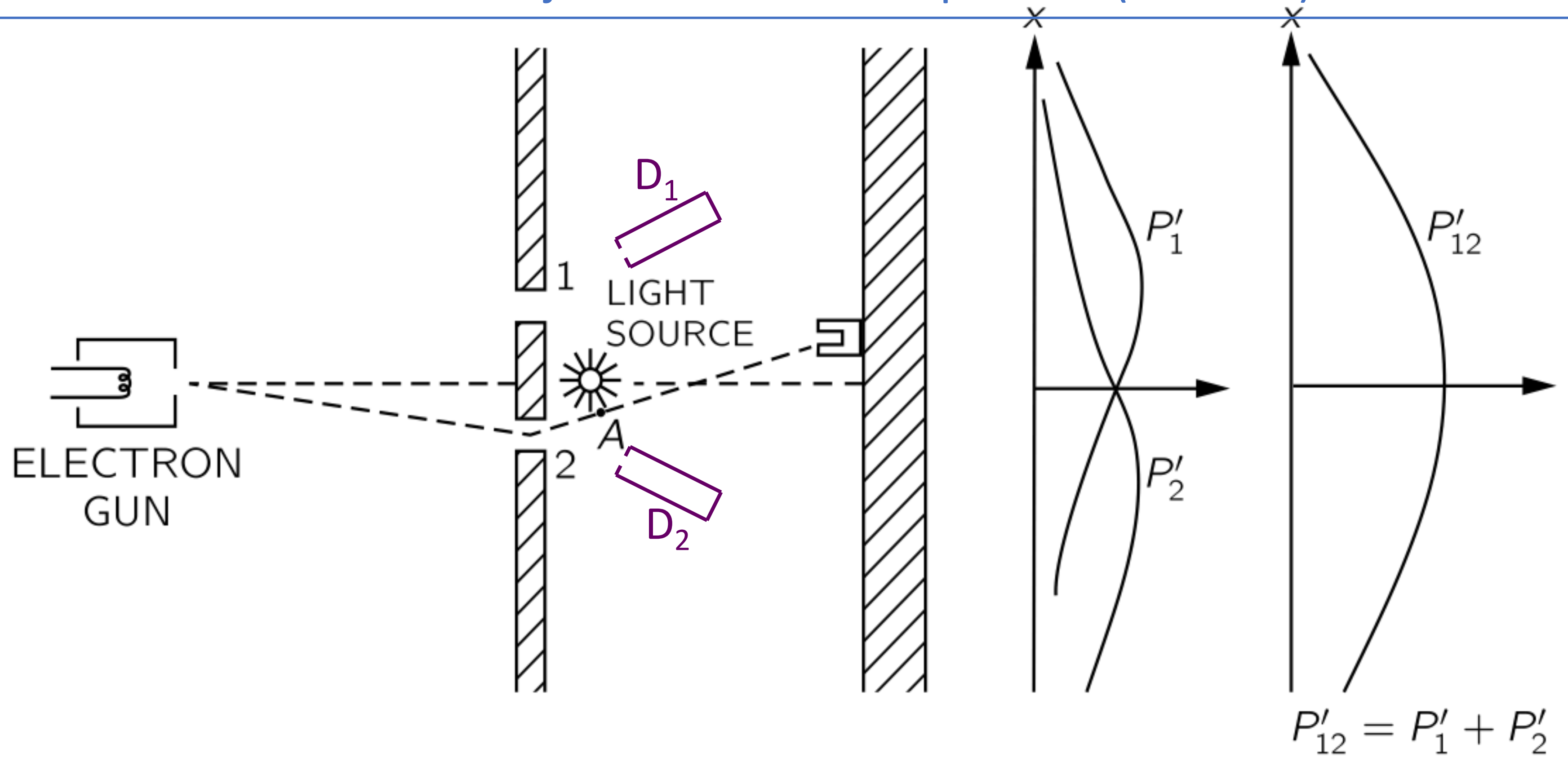
Let us try to out-smart the electron: just watch through which slit it goes!



D_1 and D_2 are two "microscopes" looking at the slits 1 and 2, respectively.

Case 4: Watch the Electrons

When we watch through which slit the electrons go, we destroy the interference!
Now the electron behaves just like a classical particle ("bullet").

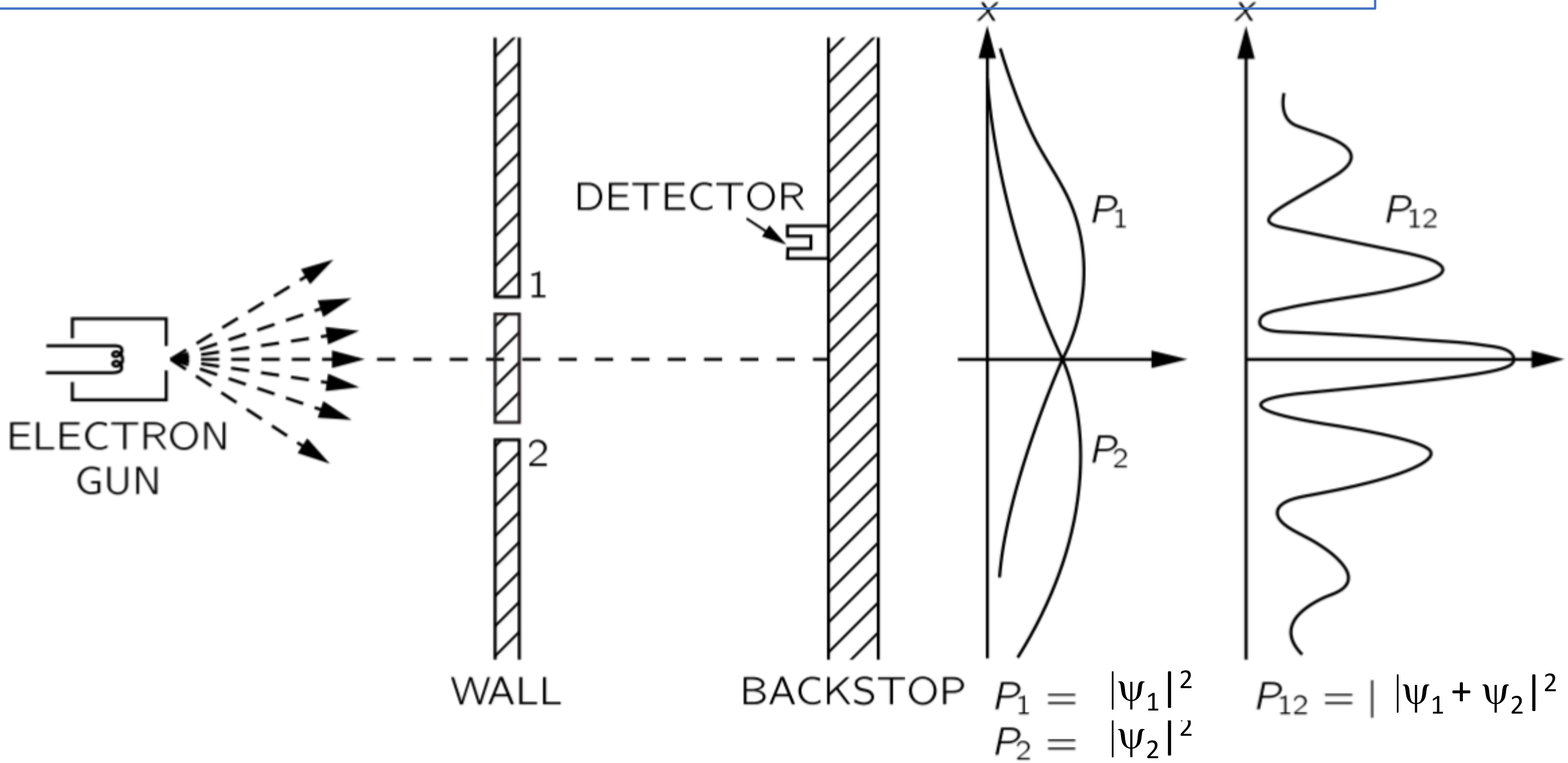


If you watch ***half the time***; you only get the interference for the cases you ***did not watch***.

It requires an observation to let the quantum wave function "collapse" into reality.
As long as no measurement is made the wave function keeps "all options open".

Case 3: Don't Watch the Electrons

When we don't watch through through which slit the electrons go, the electron is an object that interferes with itself!



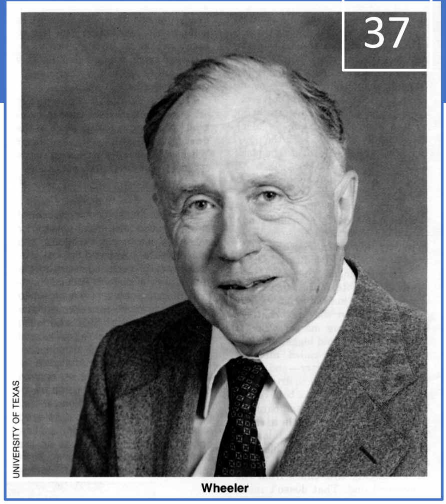
If you watch ***half the time***; you only get the interference for the cases you ***did not watch***.

It requires an observation to let the quantum wave function "collapse" into reality. As long as no measurement is made the wave function keeps "all options open".

Wave Particle Duality

Next lecture we will try to out-smart nature one step further…
… and face the consequences.

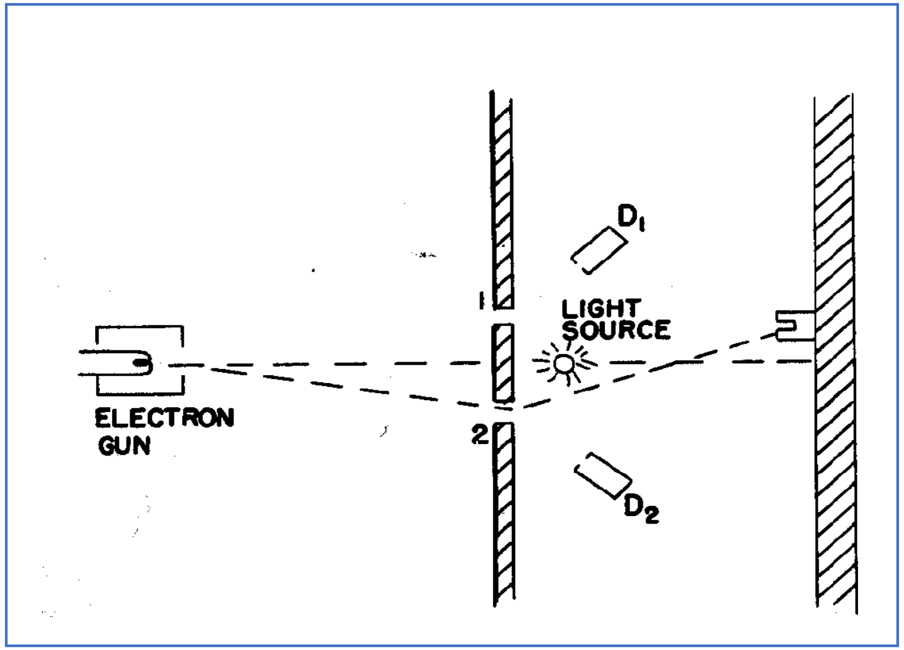
Next Lecture: Wheeler's Delayed Choice

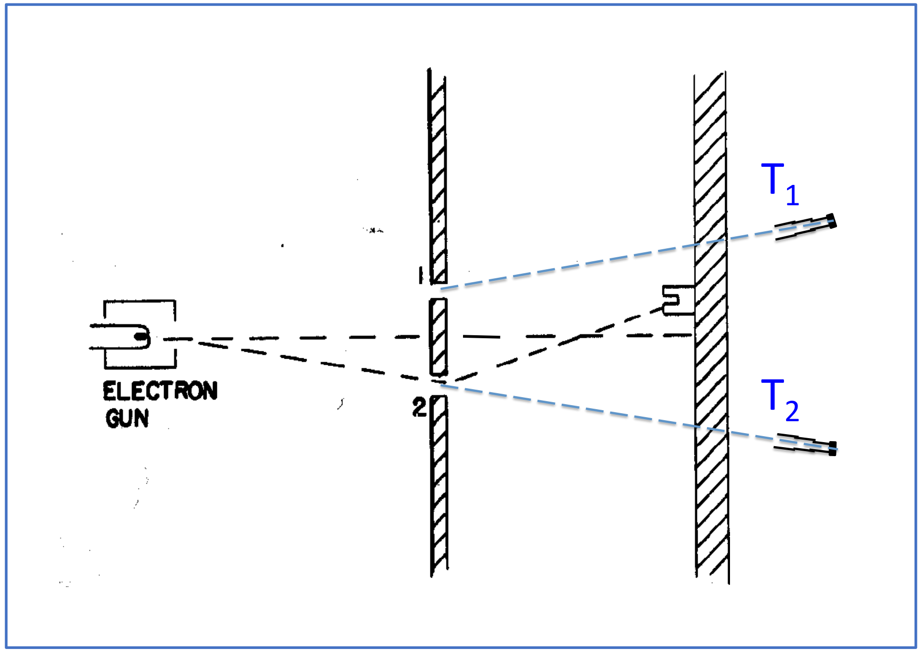


John Wheeler (1911 – 2008):
Famous for work on gravitation
(Black holes – quantum gravity)

Replace detectors D_1 and D_2 with telescopes T_1 and T_2 which are focused on slits 1 and 2

What happens if we ***afterwards would reconstruct*** whether the electron went through slit 1 or slit 2?





Try to out-smart nature one step further… and face the consequences: Schrödinger's cat.

Next Lecture...