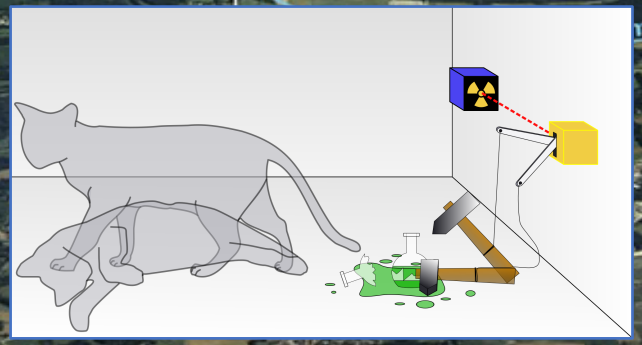
The Relativistic Quantum World

A lecture series on Relativity Theory and Quantum Mechanics

Marcel Merk



University of Maastricht, Sept 16 – Oct 14, 2020

The Relativistic Quantum World

Relativity	Sept. 16:	Lecture 1: The Principle of Relativity and the Speed of Light Lecture 2: Time Dilation and Lorentz Contraction
	Sept. 23:	Lecture 3: The Lorentz Transformation and Paradoxes Lecture 4: General Relativity and Gravitational Waves
Quantum Mechanics	Sept. 30:	Lecture 5: The Early Quantum Theory Lecture 6: Feynman's Double Slit Experiment
	Oct. 7:	Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat Lecture 8: Quantum Reality and the EPR Paradox
Standard Model	Oct. 14:	Lecture 9: The Standard Model and Antimatter Lecture 10: The Large Hadron Collider

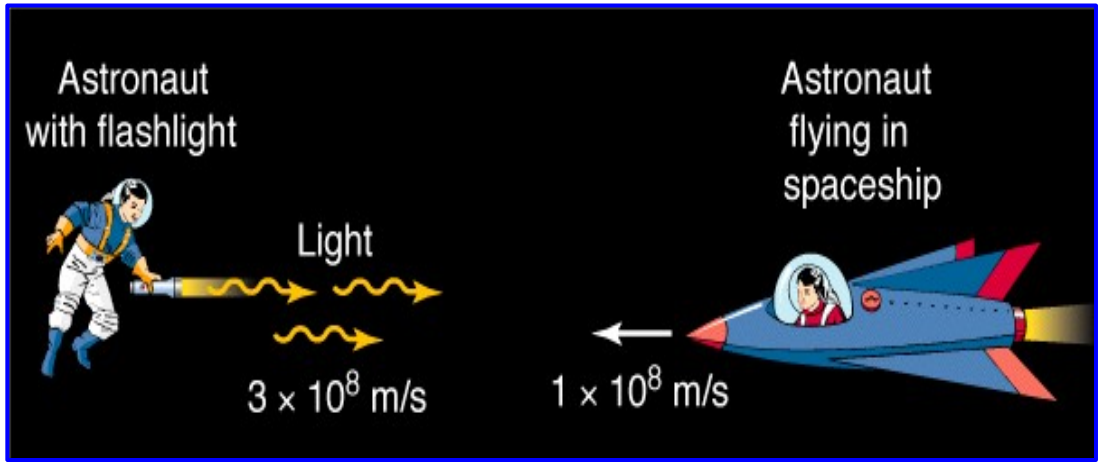
Lecture notes, written for this course, are available: www.nikhef.nl/~i93/Teaching/
Prerequisite for the course: High school level physics & mathematics.

Relativity Theory

Special Relativity

All observers moving in inertial frames:

- Have identical laws of physics,
- Observe the same speed of light: c.

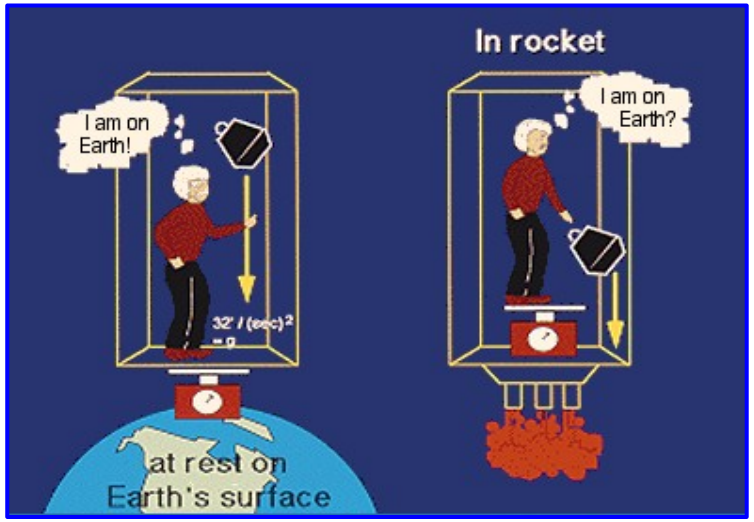


Consequences:

- Simultaneity is not the same for everyone,
- Distances shrink, time slows down at high speed,
- Velocities do not add-up as expected.

General Relativity

- A free falling person is also inertial frame,
- Acceleration and gravitation are equivalent: Inertial mass = gravitational mass

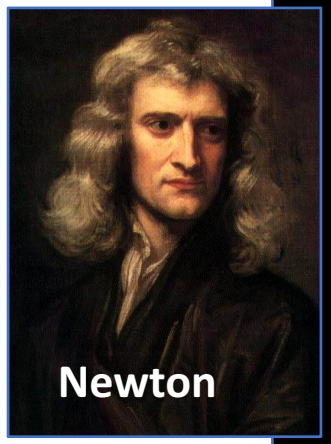


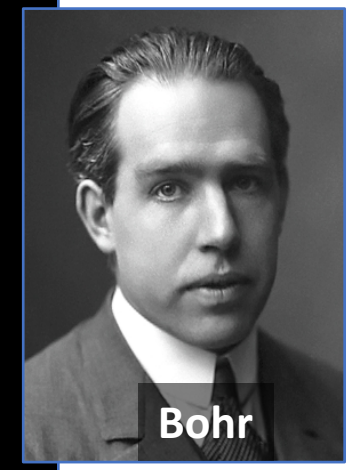
Consequences:

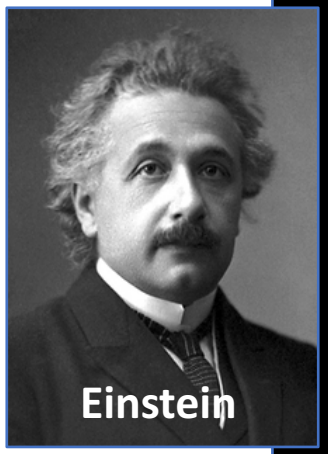
Space-time is curved:

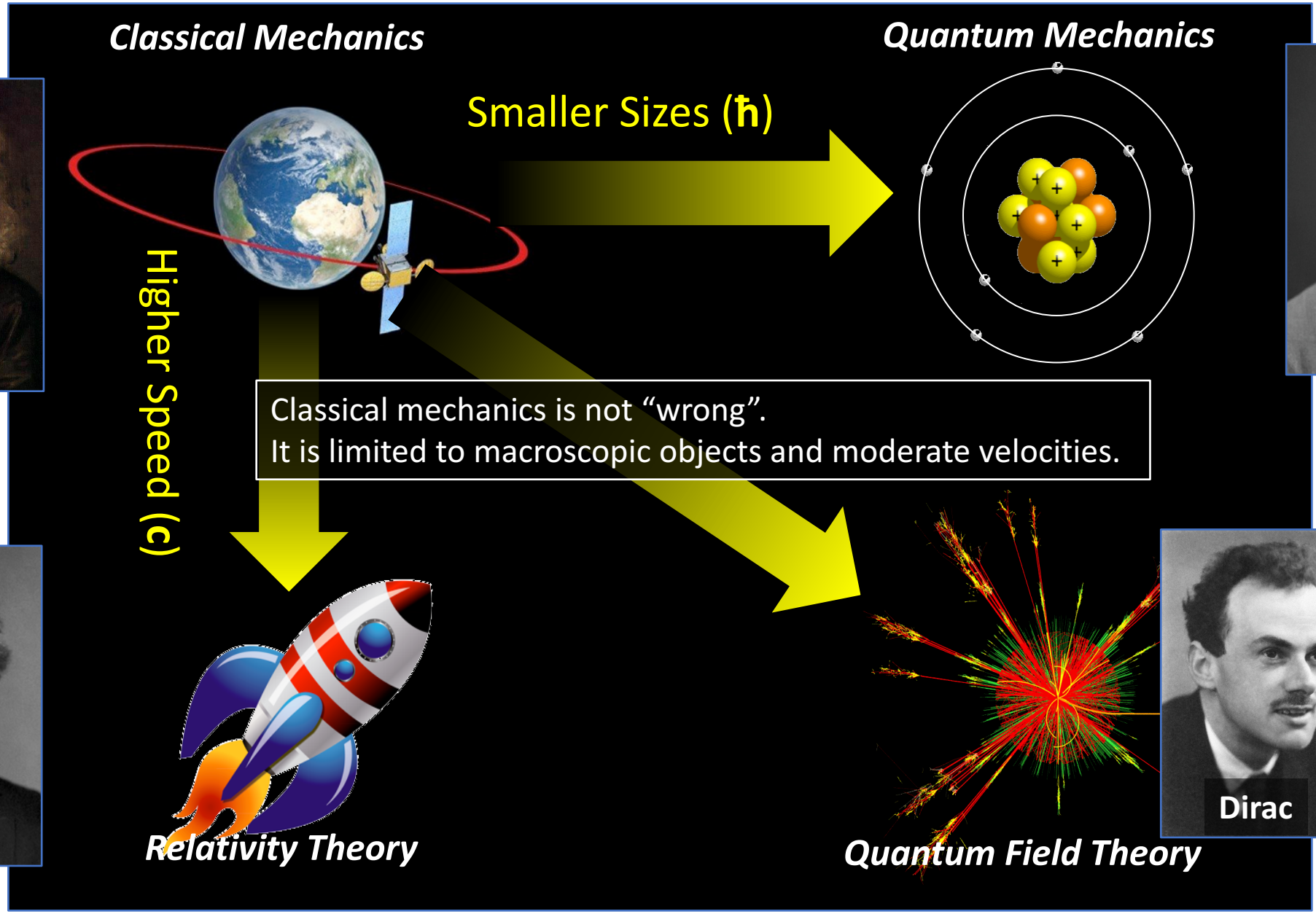
- Light bends around a massive object,
- Time slows down and space shrinks in gravitational fields,
- Gravitational radiation exists.

Relativity and Quantum Mechanics









Lecture 5

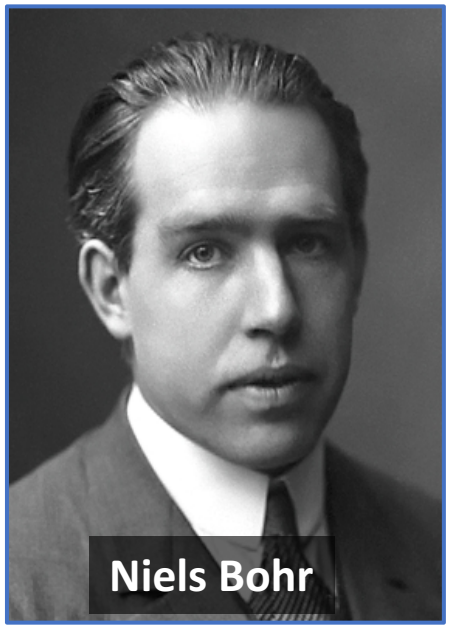
The Early Quantum Theory

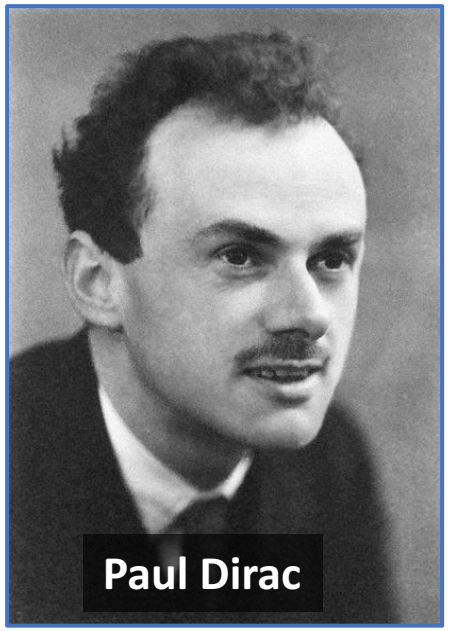
“If Quantum Mechanics hasn’t profoundly shocked you, you haven’t understood it yet.”
- Niels Bohr

“Gott würfelt nicht (God does not play dice).”
- Albert Einstein

”Einstein, stop telling God what to do!”
- Niels Bohr

Key Persons of Quantum Mechanics





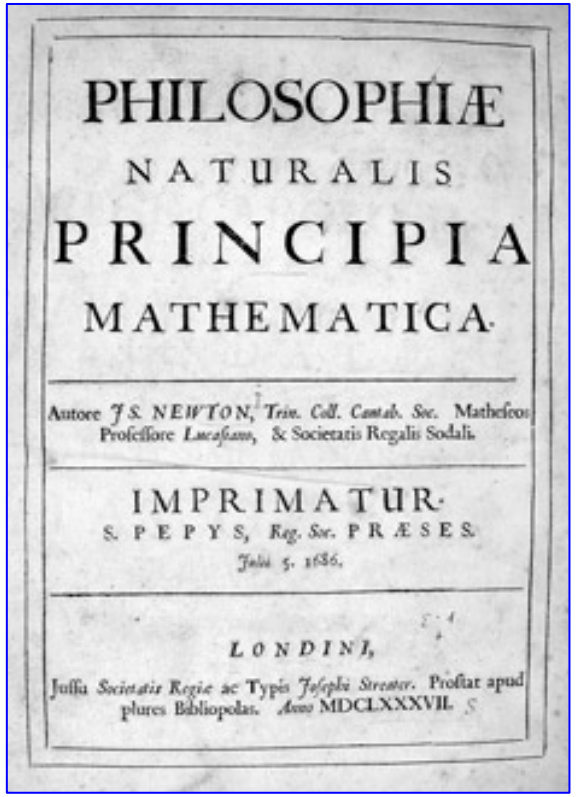
Niels Bohr:	Nestor of the ”Copenhagen Interpretation”
Erwin Schrödinger:	Inventor of the quantum mechanical wave equation
Werner Heisenberg:	Inventor of the uncertainty relation and “matrix mechanics”
Paul Dirac:	Inventor of relativistic wave equation: Antimatter!
Max Born:	Inventor of the probability interpretation of the wave function

We will focus of the Copenhagen Interpretation and work with the concept of Schrödinger’s wave-function: ψ

Deterministic Universe

Mechanics Laws of Newton:

1. The law of inertia: a body in rest moves with a constant speed
2. The law of force and acceleration: F= m a
3. The law: Action = - Reaction



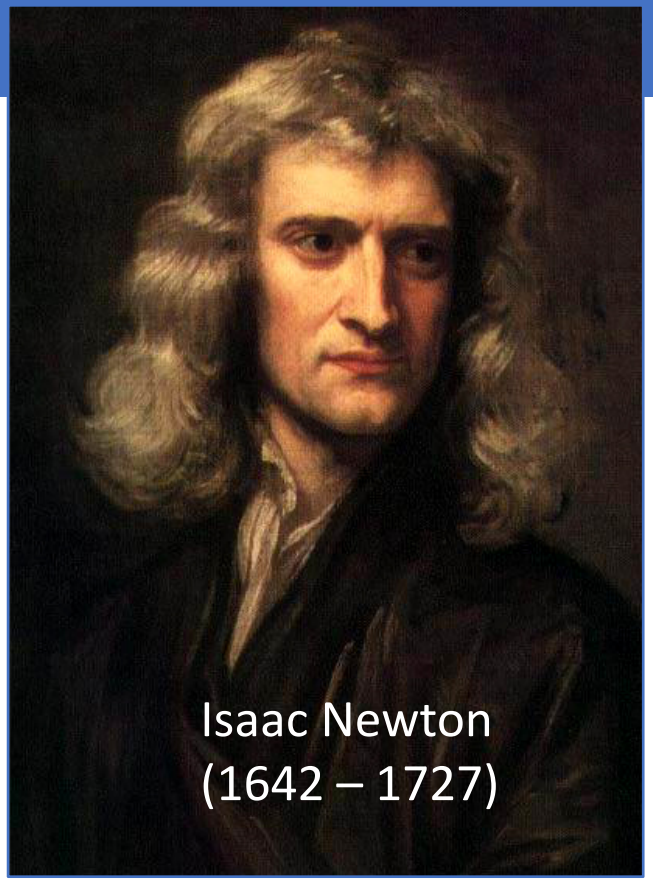
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore *IS. NEWTON*, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Justu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

"Principia" (1687)



Isaac Newton
(1642 – 1727)

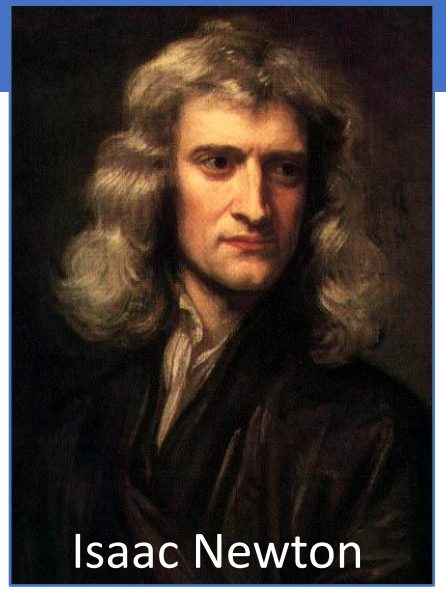
- Classical Mechanics leads to a deterministic universe.
  - From ***exact initial conditions future can be predicted***.
- Quantum mechanics introduces a fundamental element of chance in the laws of nature: Planck's constant: h.
  - ***Quantum mechanics only makes statistical predictions***.

The Nature of Light

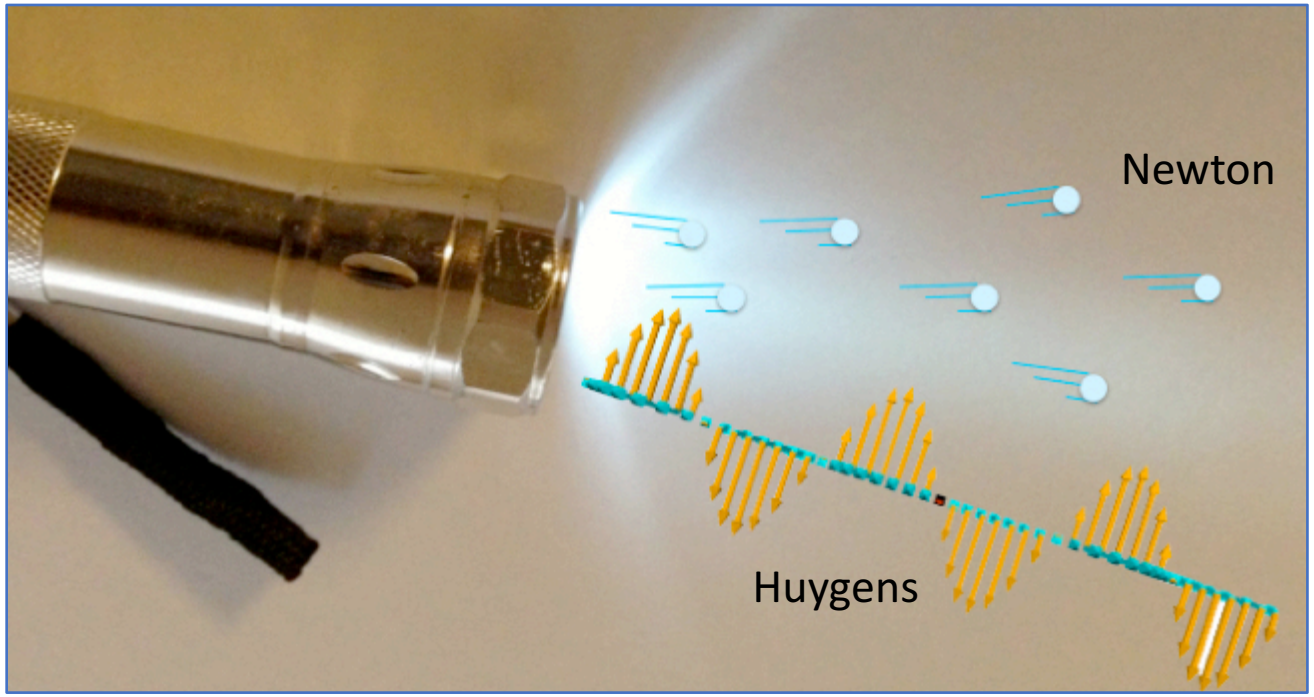
Isaac Newton (1642 – 1727):
Light is a stream of particles.

Christiaan Huygens (1629 – 1695):
Light consists of waves.

Thomas Young (1773 – 1829):
Interference observed: Light is waves!


Isaac Newton

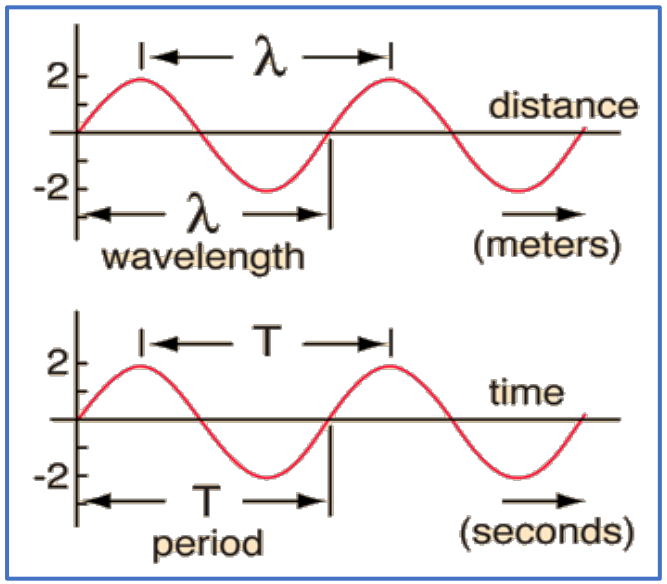

Christiaan Huygens



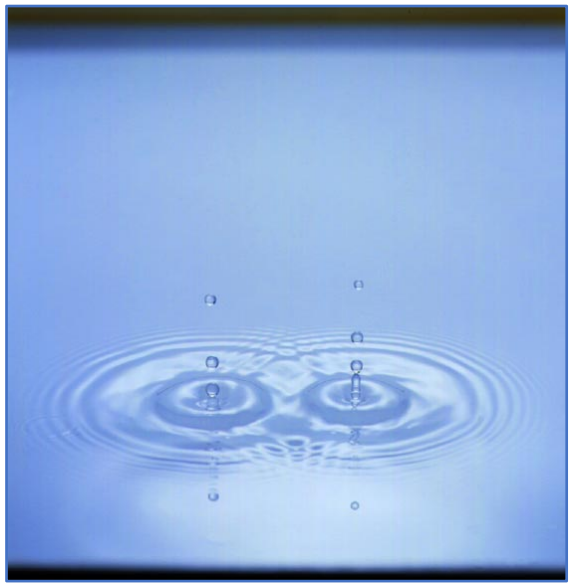

Thomas Young

Waves & Interference : water, sound, light

Principle of a wave



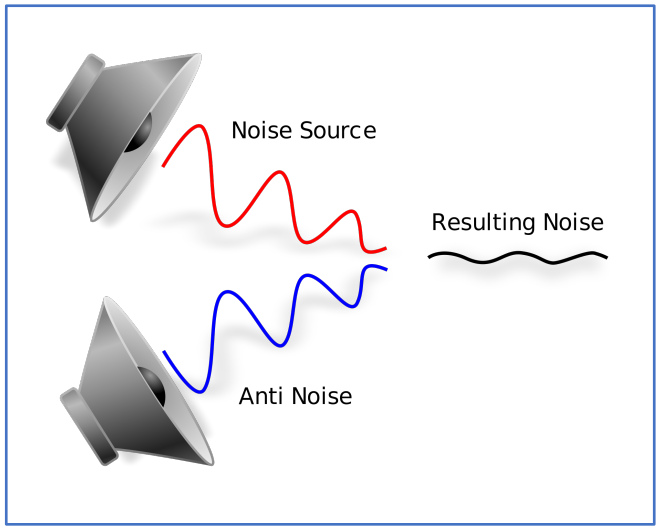
Water: Interference pattern:



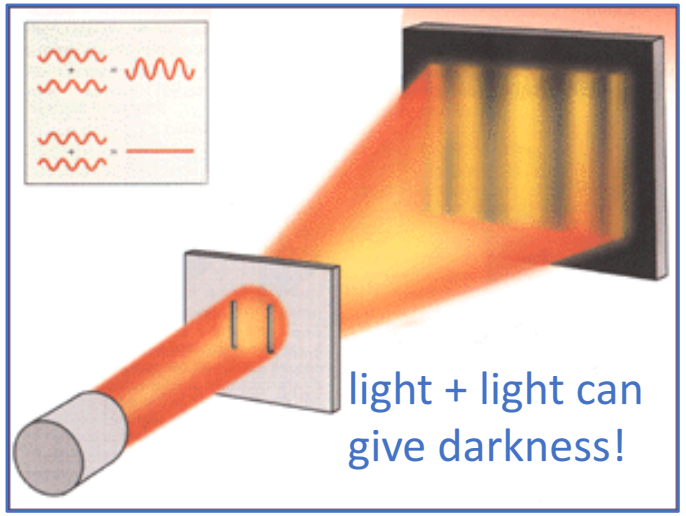
$$\lambda = v/f$$

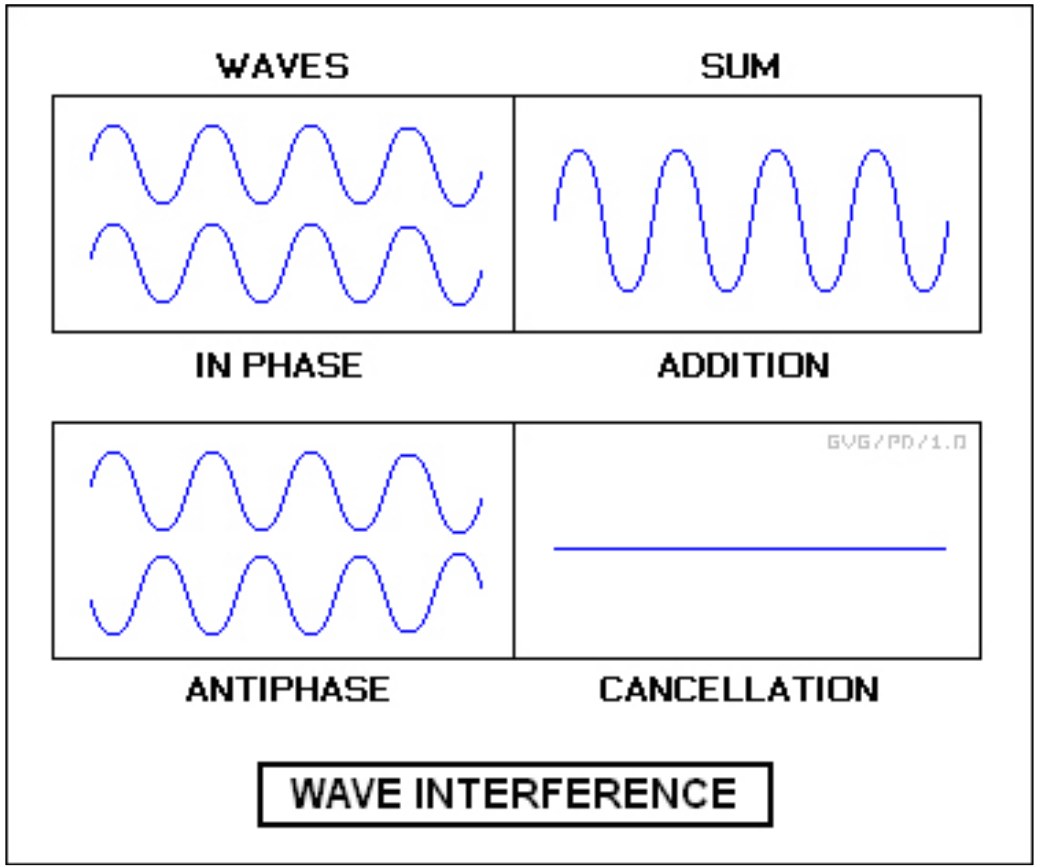
$$f = 1/T$$

Sound: Active noise cancellation:



Light: Thomas Young experiment:

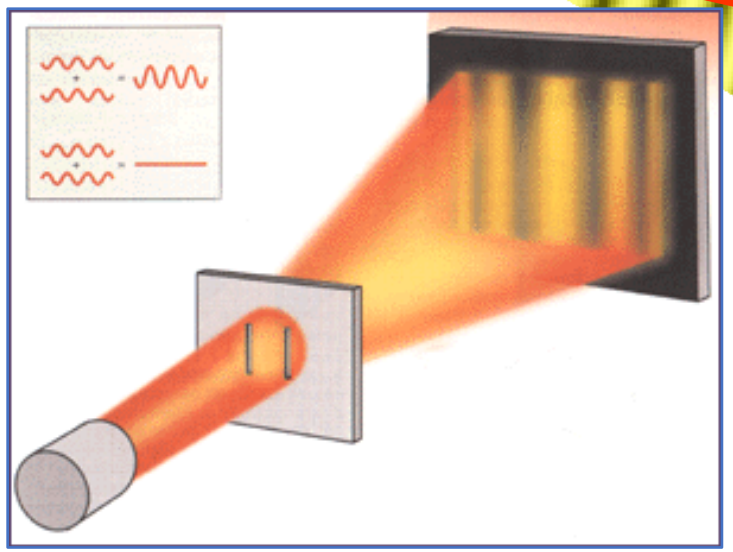


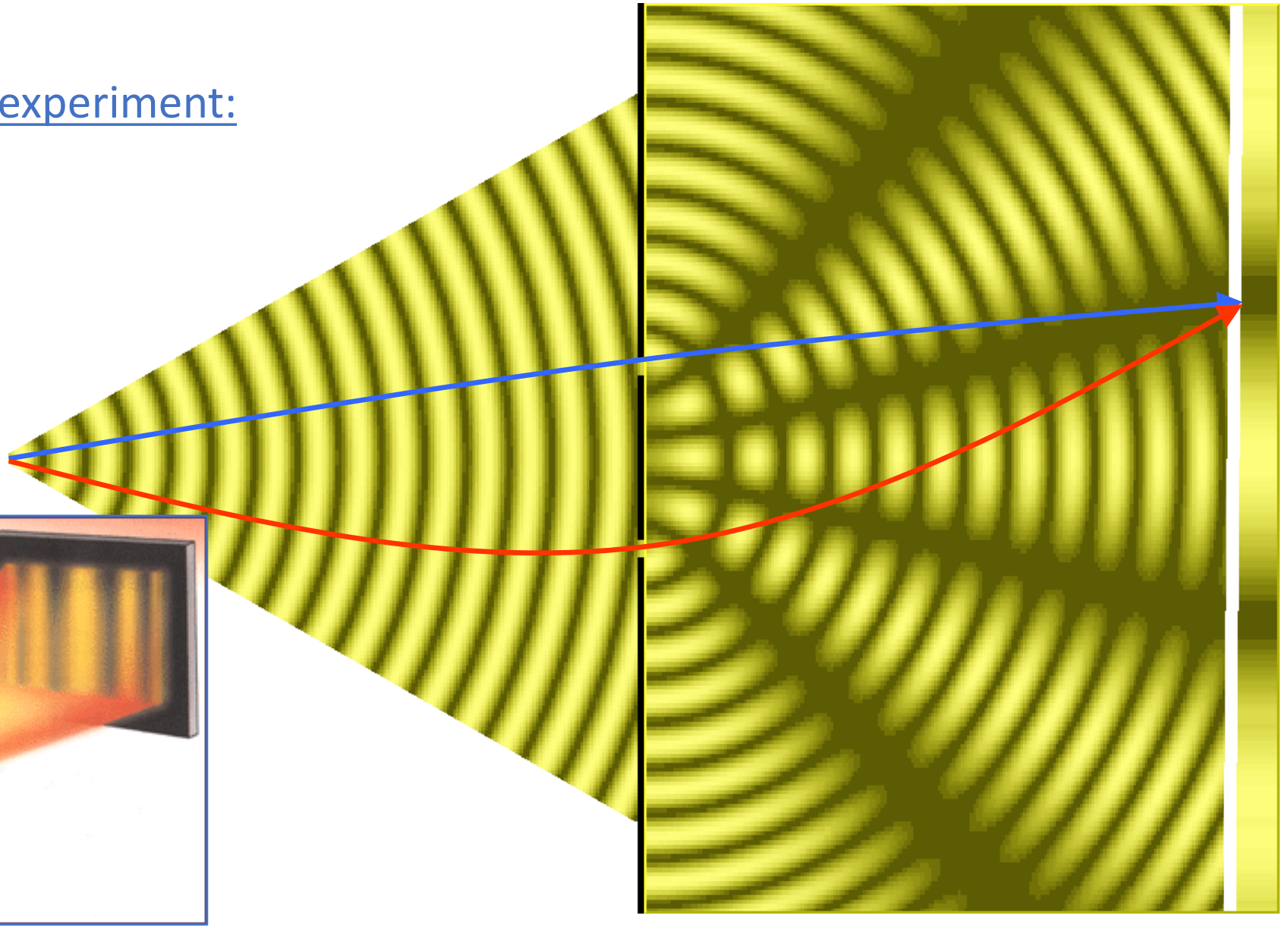


Interference with Water Waves

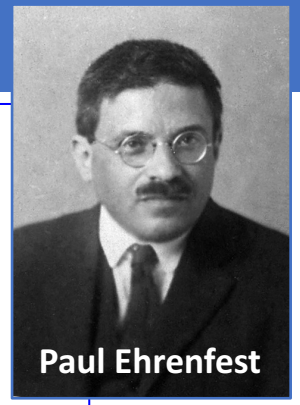
Interfering Waves

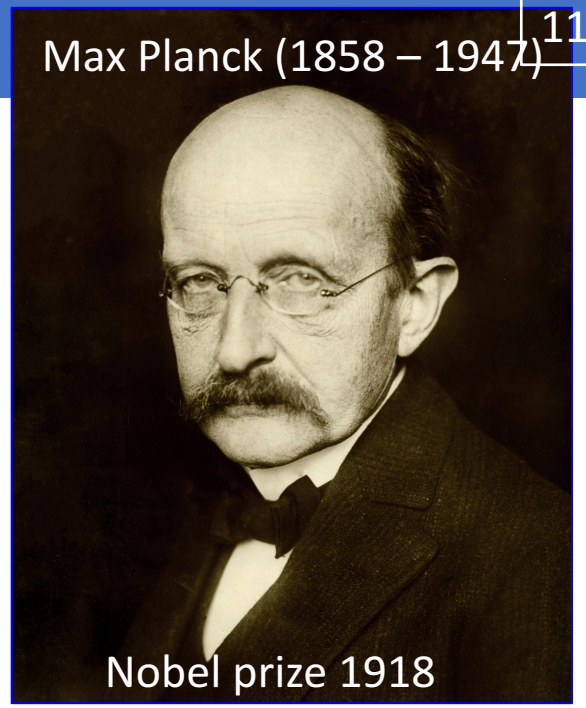
Double slit experiment:





Particle nature: Quantized Light


Paul Ehrenfest


Max Planck (1858 – 1947)

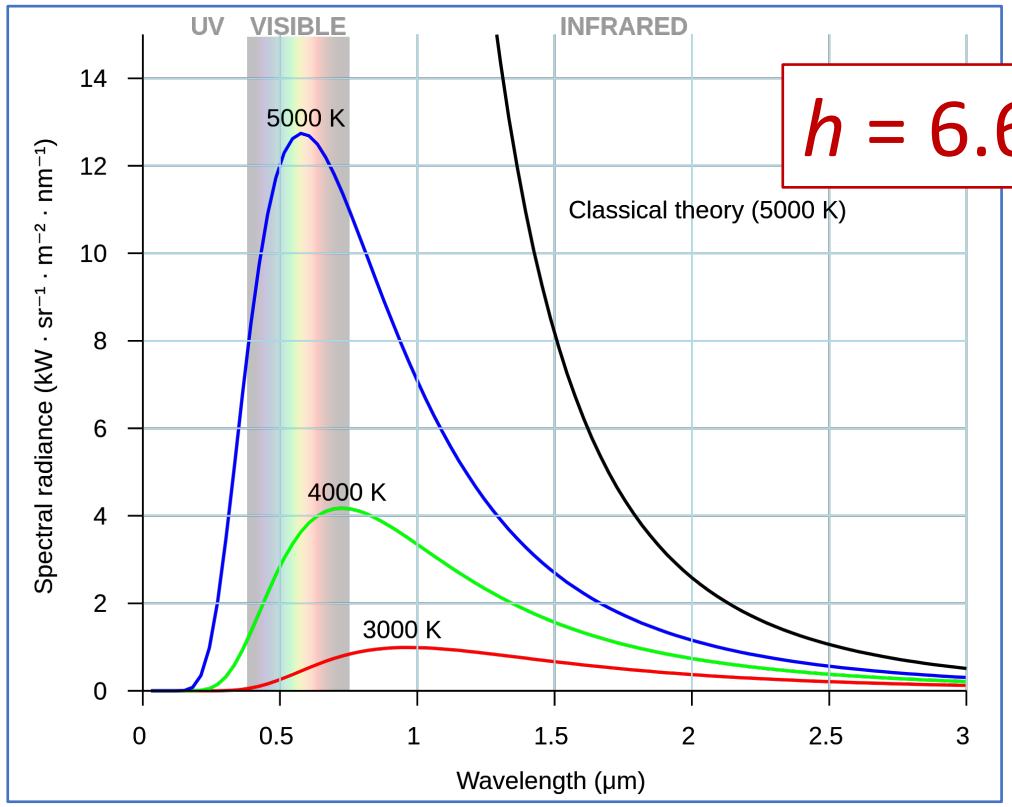
Nobel prize 1918

"UV catastrophe" in Black Body radiation spectrum:

If you heat a body it emits radiation. Classical thermodynamics predicts the amount of light at very short wavelength to be infinite!

Planck invented an ad-hoc solution:

For some reason material emitted light in "packages".



$h = 6.62 \times 10^{-34}$ Js

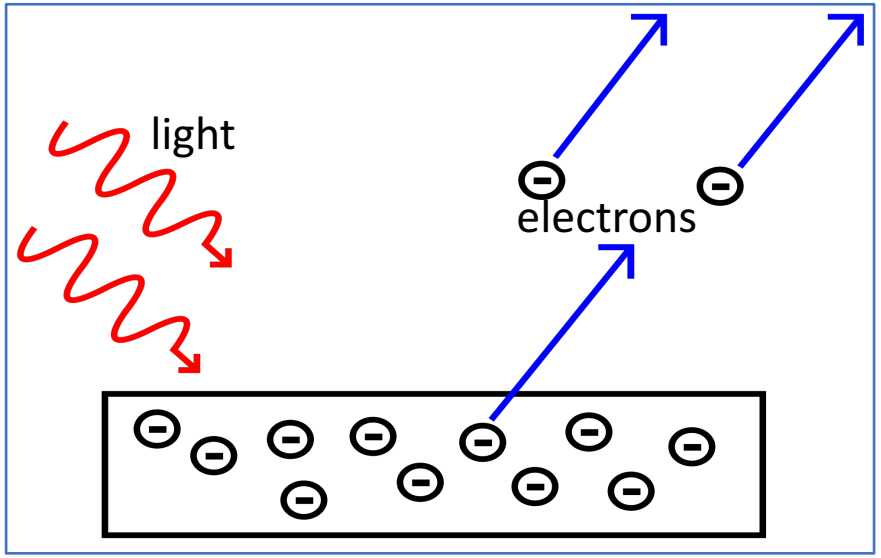
Classical theory:

There are more short wavelength "oscillation modes" of atoms than large wavelength "oscillation modes".

Quantum theory:

Light of high frequency (small wavelength) requires more energy: $E = hf$ (h = Planck's constant)

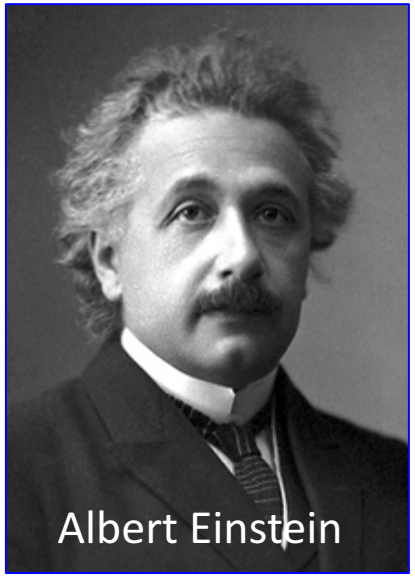
Photoelectric Effect



Photoelectric effect:

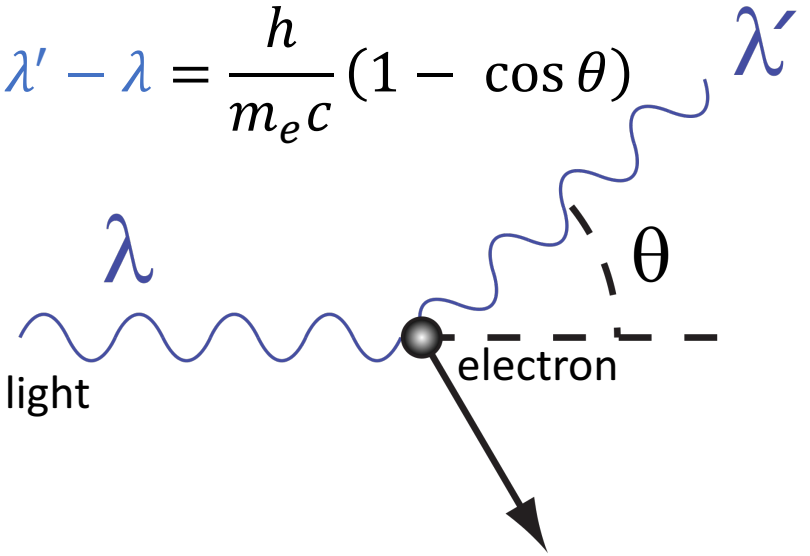
Light kicks out electron with ***E = h f***
(Independent on light intensity!)

Light ***consists*** of quanta.
(Nobelprize 1921)



Wave: $E = hf = hc/\lambda$ → $\lambda = hc/E$
Momentum: $p = mv = mc = E/c$ → $E = pc$
It follows that: $\lambda = h/p$

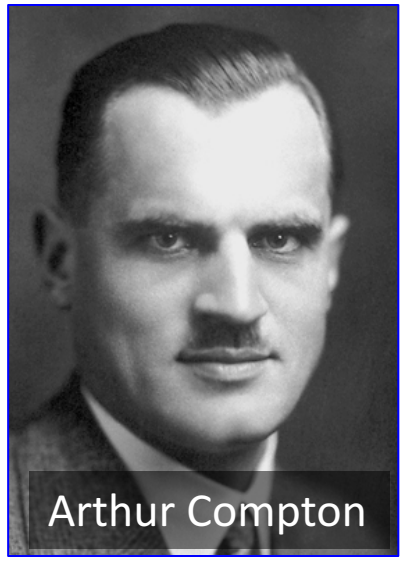
$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$



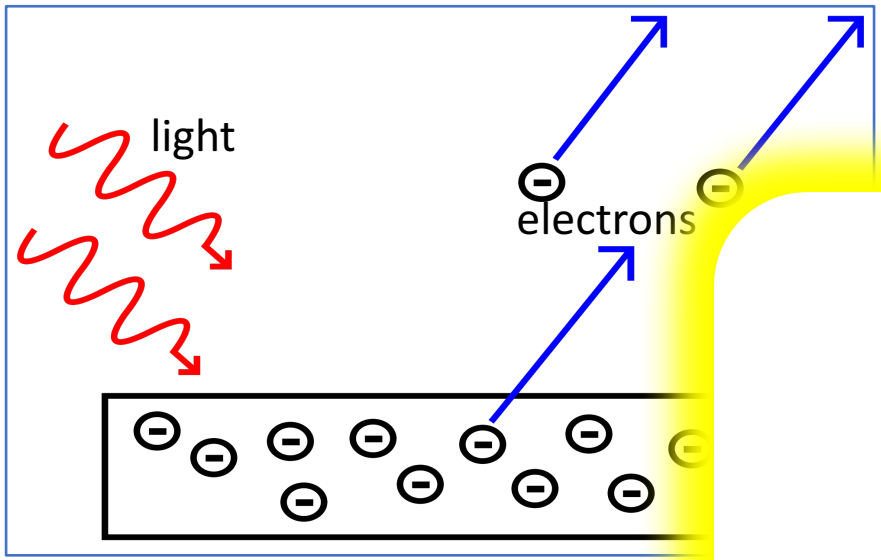
Compton Scattering:

"Playing billiards" with light quanta and electrons.

Light behaves as a particle with: ***λ = h / p***
(Nobelprize 1927)



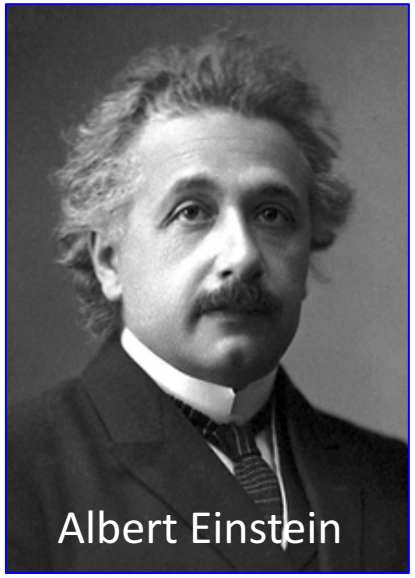
Photoelectric Effect



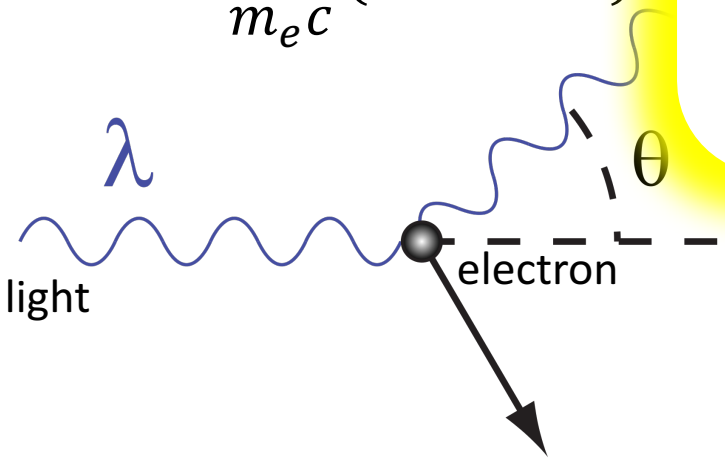
Photoelectric effect:
Light kicks out electron with ***E = h f***
(Independent on light intensity!)

LIGHT IS A
WAVE!

$\lambda = hc/E$
$E = pc$

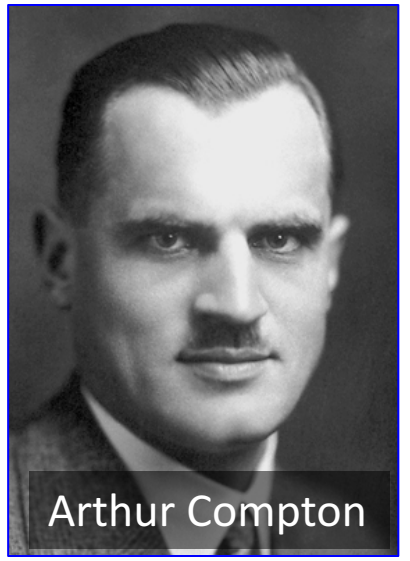

Albert Einstein

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$



"Playing billiards" with light quanta and electrons.

Light behaves as a particle with: ***λ = h / p***
(Nobelprize 1927)

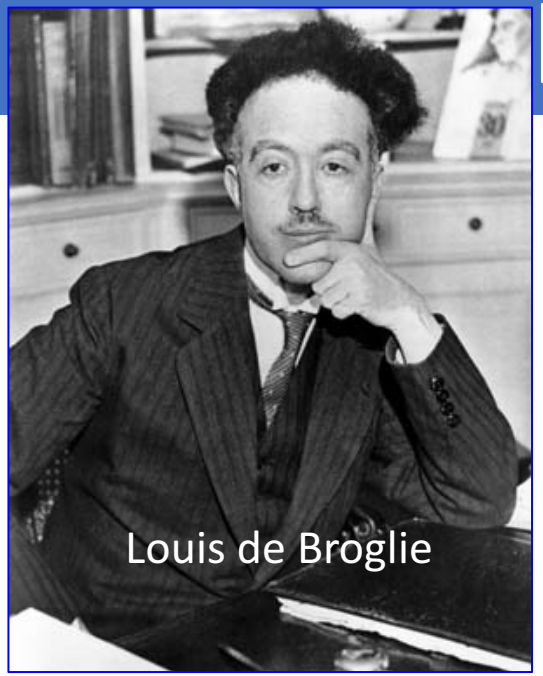

Arthur Compton

"Once and for all I want to know what I'm paying for. When the electric company tells me whether light is a wave or a particle I'll write my check."

Matter Waves

Louis de Broglie - PhD Thesis(!) 1924 (Nobel prize 1929):
If light are particles incorporated in a wave, it suggests that particles (electrons) “are carried” by waves.

Original idea: a physical wave ➔ Quantum mechanics: probability wave!

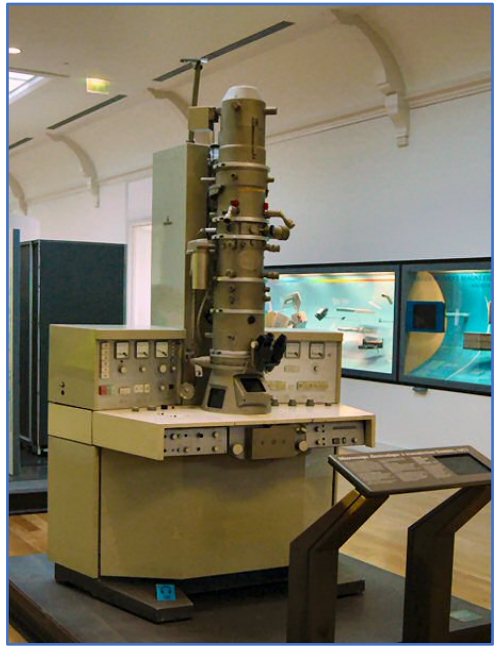

Louis de Broglie

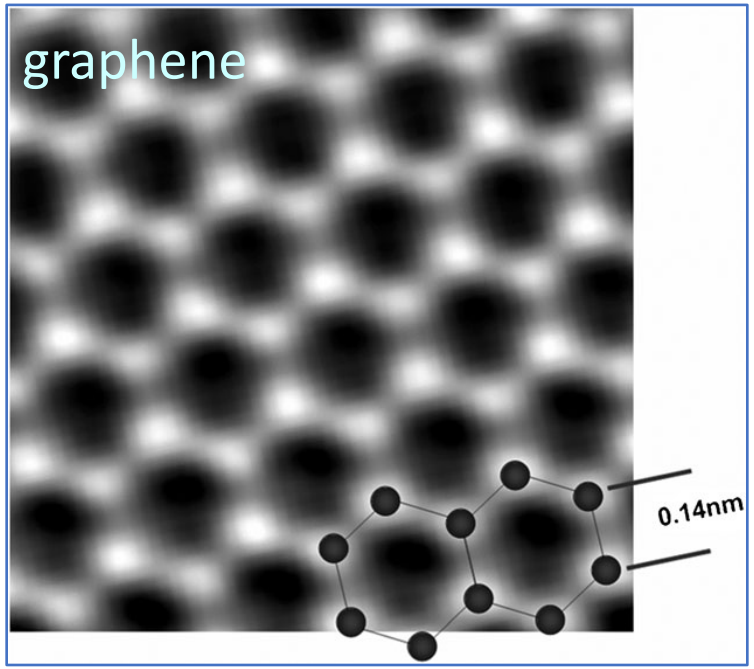
Particle wavelength: $\lambda = h/p \rightarrow \lambda = h/(mv)$

Wavelength visible light:
400 – 700 nm

Use $h = 6.62 \times 10^{-34}$ Js to calculate:

- Wavelength electron with v = 0.1 c:
  0.024 nm
- Wavelength of a fly (m = 0.01 gram, v = 10 m/s):
  0.00000000000000000062 nm





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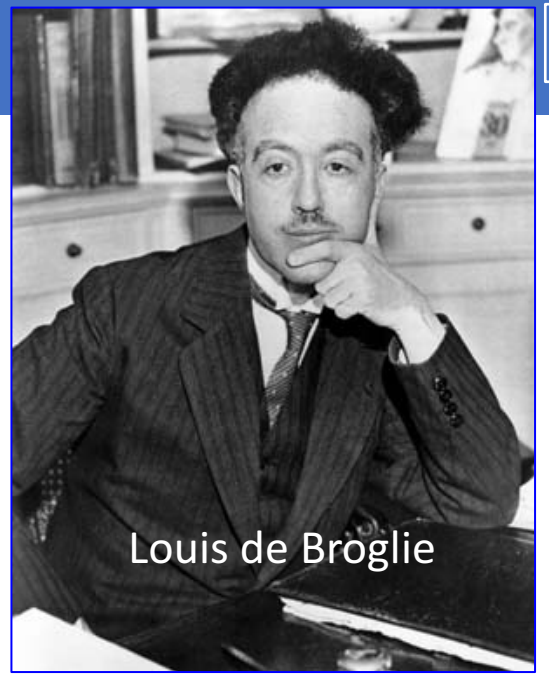
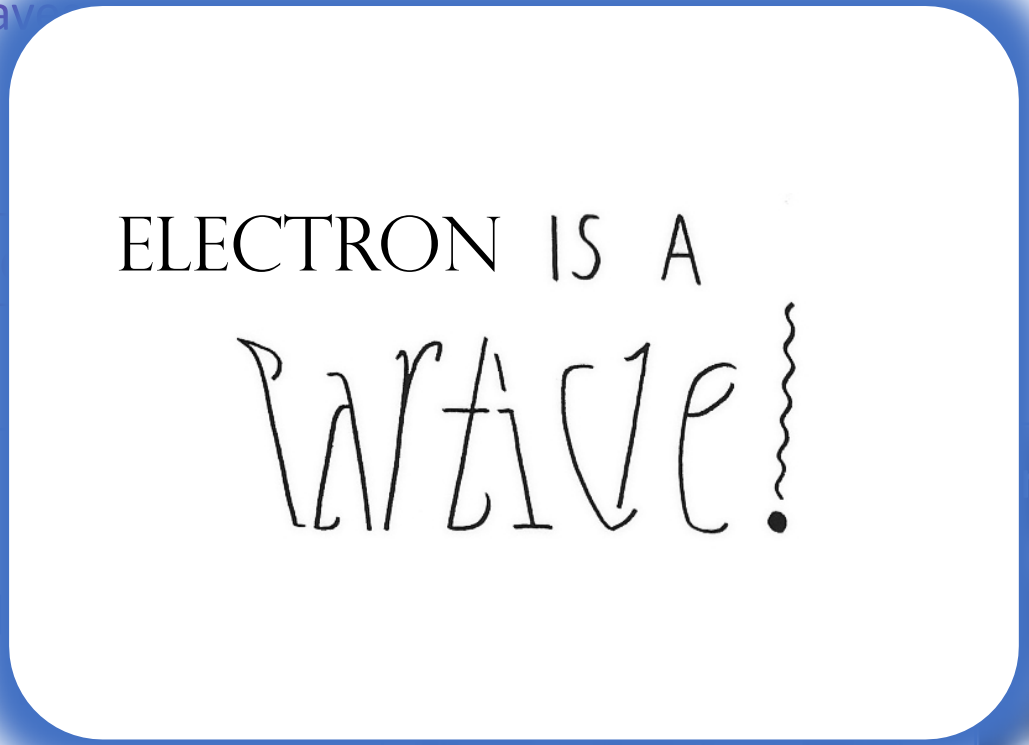
Particle wavel





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- Wavelength electron with
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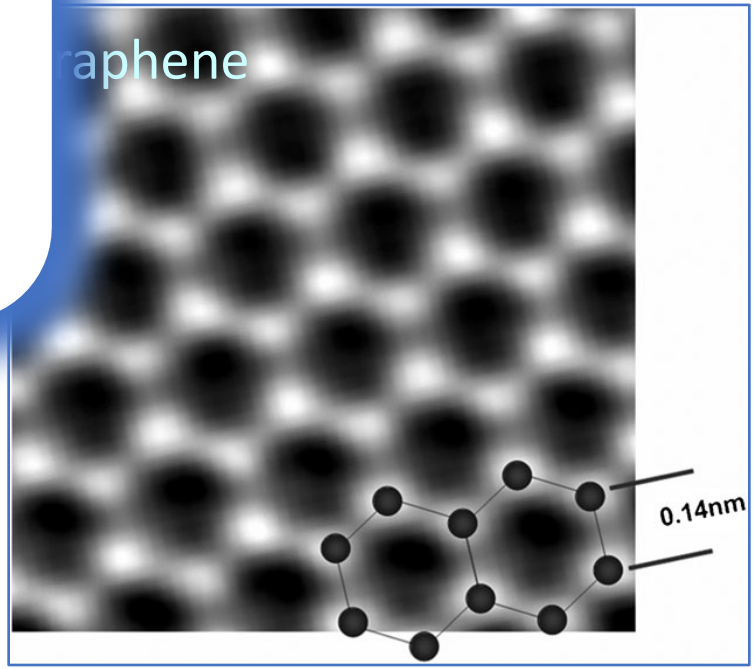
raphene




JUST KIDDING
WELL... YOU TOLD US TO DRAW WHAT WE SEE UNDER THE MICROSCOPE!
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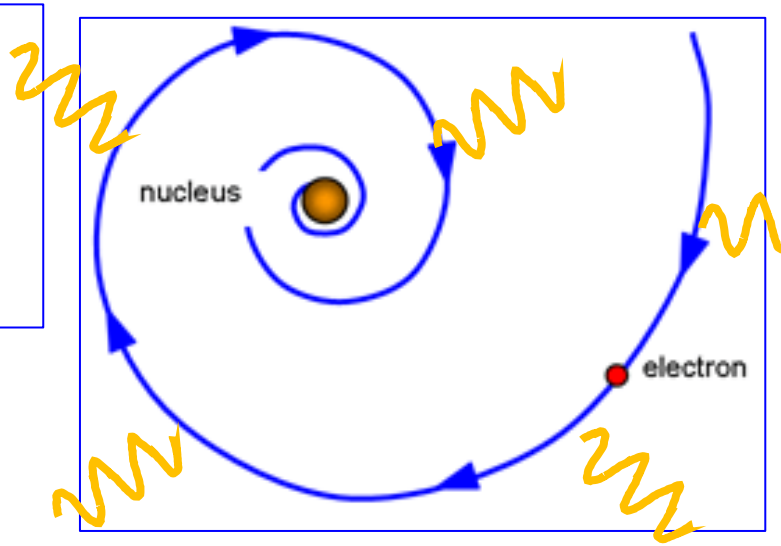
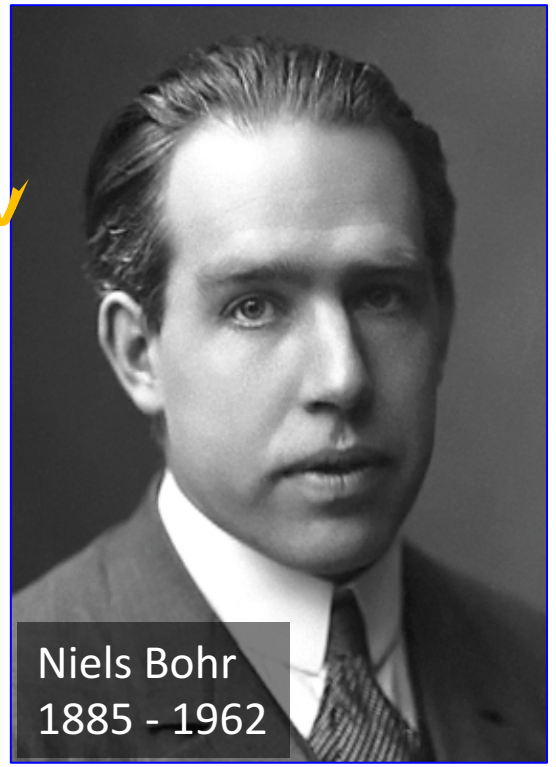
The Quantum Atom of Niels Bohr

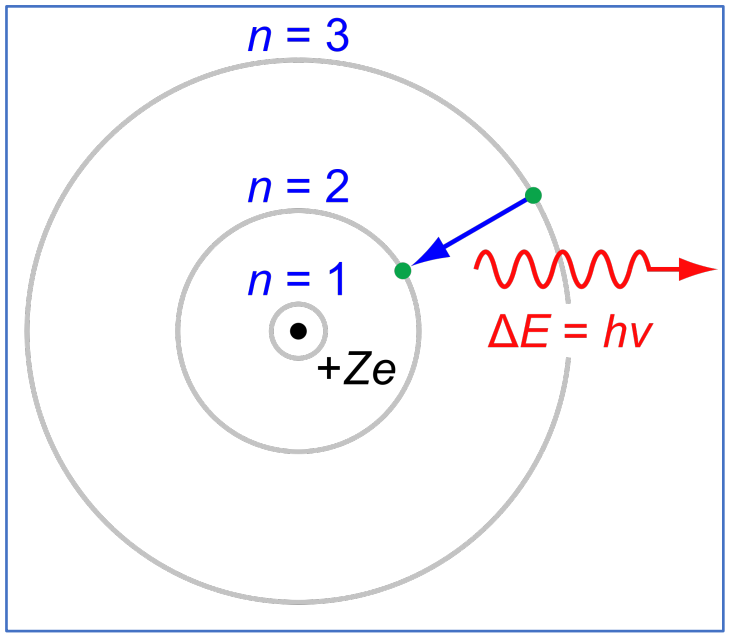
The classical Atom is unstable!

Expect: $t < 10^{-10}$ s

Niels Bohr: Atom is only stable for specific orbits: “energy levels”.



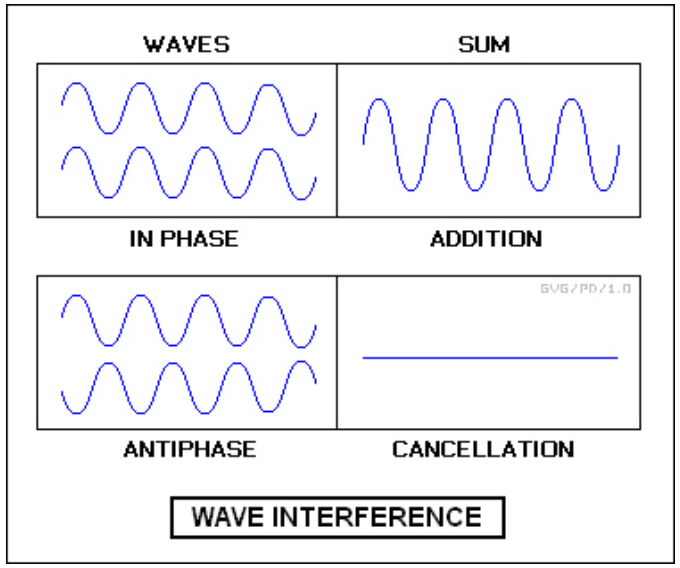

Niels Bohr
1885 - 1962

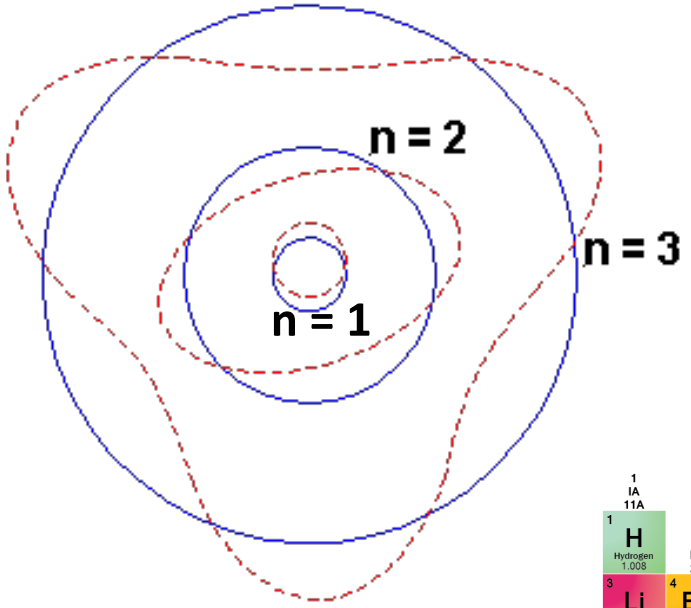


An electron can jump from a high to lower level by emitting a light quantum with corresponding energy difference.

Balmer spectrum of wavelengths:

Schrödinger: Bohr atom and de Broglie waves



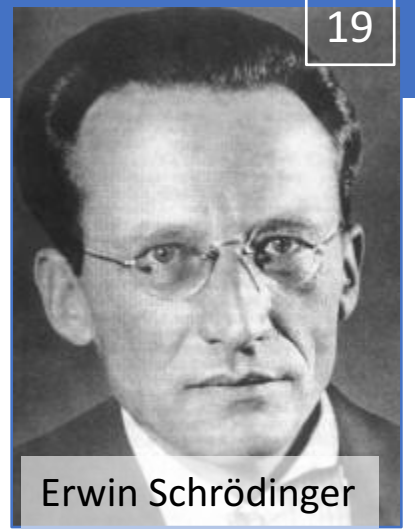


If orbit length "fits":

$2\pi r = n\lambda$ with $n = 1, 2, 3, \ldots$

The wave positively interferes with itself!

➔ Stable orbits!



de Broglie: $\lambda = h / p$

$L = r\ p$

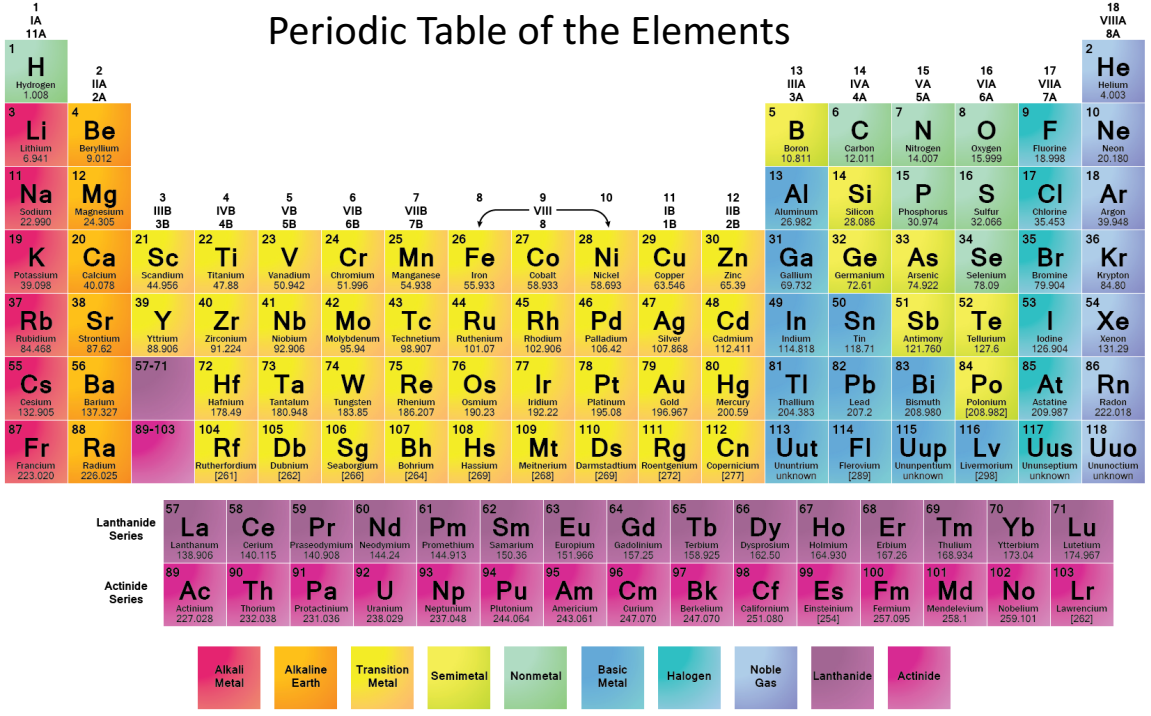
$L = r\ h/\lambda$

$L = r\ n\ h/(2\pi r)$

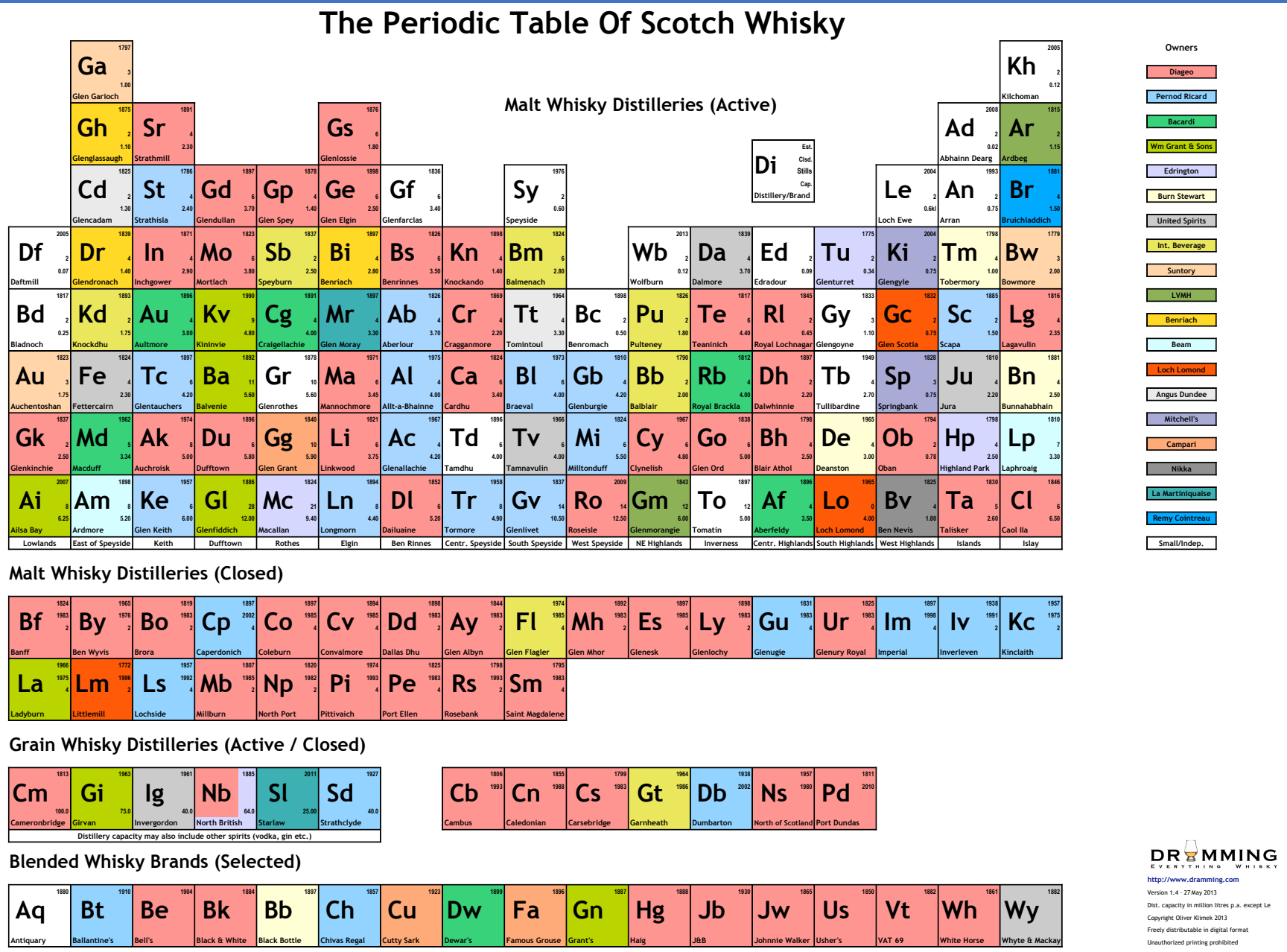
$L = n\ h/(2\pi) = n\ \hbar$

(L = angular momentum)

Energy levels explained
→ atom explained
Outer shell electrons
→ "chemistry explained"

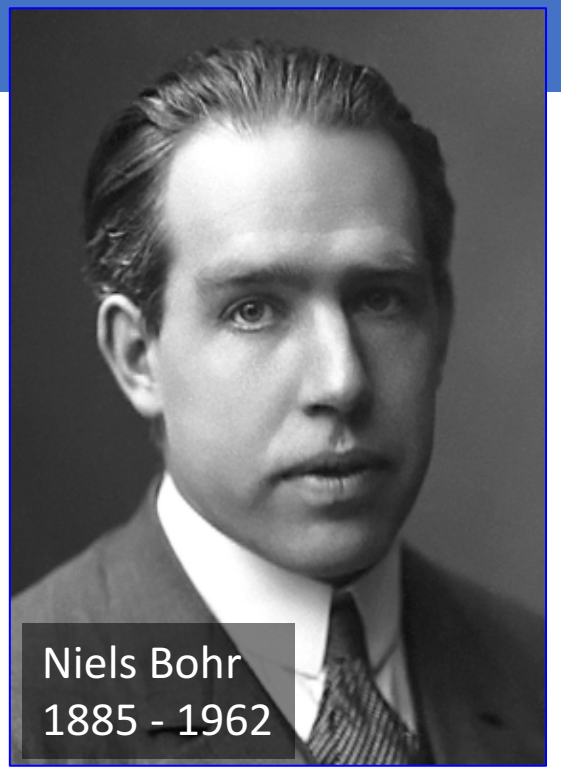


Not yet explained



Complementarity

Subatomic matter is not just waves and it is not just particles.
It is nothing we know from macroscopic world.


Niels Bohr
1885 - 1962

Copenhagen Interpretation (Niels Bohr)

One can observe wave characteristics or particle characteristics of quantum objects, ***never both*** at the same time.

Particle and Wave aspects of a physical object are ***complementary***.

Similarly one can never determine from a quantum object at the same time: ***energy and time, position and momentum*** (and more).

Heisenberg Uncertainty Relation

A measurement of a characteristic of quantum matter affects the object.
Heisenberg's "non-commuting" observables:

Energy and time:

$$E\,t - t\,E = i\,\hbar$$

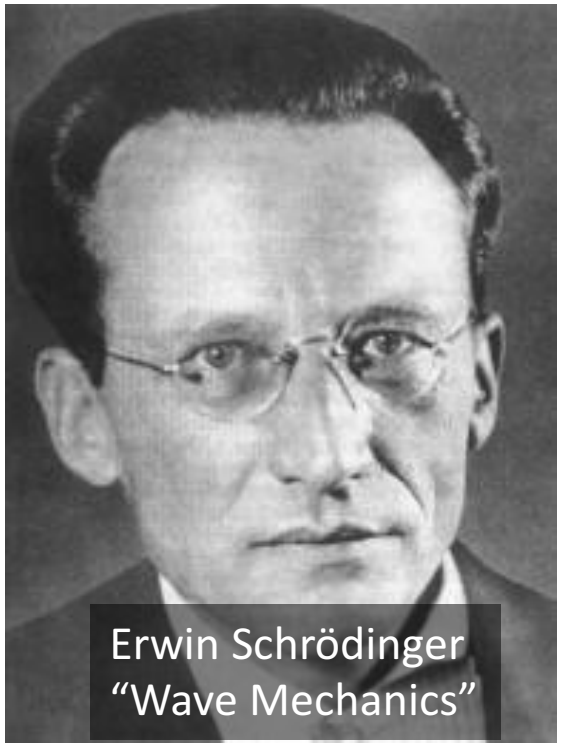
$$\Delta E\ \Delta t\ \geq \hbar / 2$$

Position and momentum:

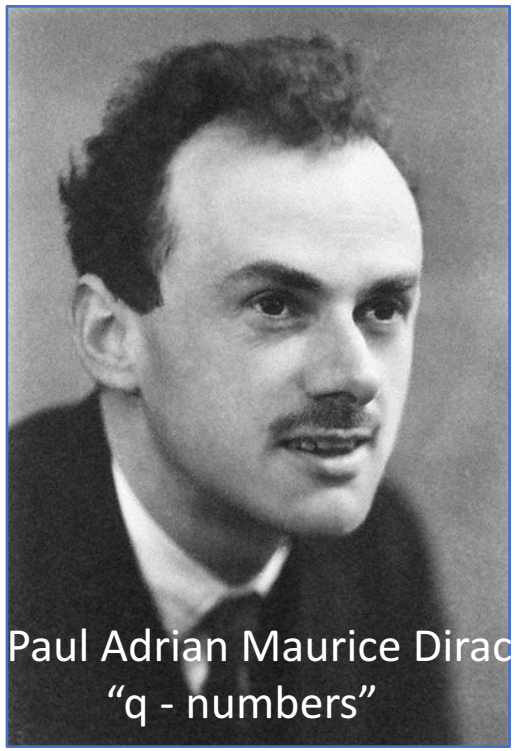
$$x\,p - p\,x = i\,\hbar$$

$$\Delta x\ \Delta p\ \geq \hbar / 2$$

Werner Heisenberg
"Matrix mechanics"



Erwin Schrödinger
"Wave Mechanics"



Paul Adrian Maurice Dirac
"q - numbers"

It is a fundamental aspect of nature.

Not related to limited technology!

Heisenberg Uncertainty Relation

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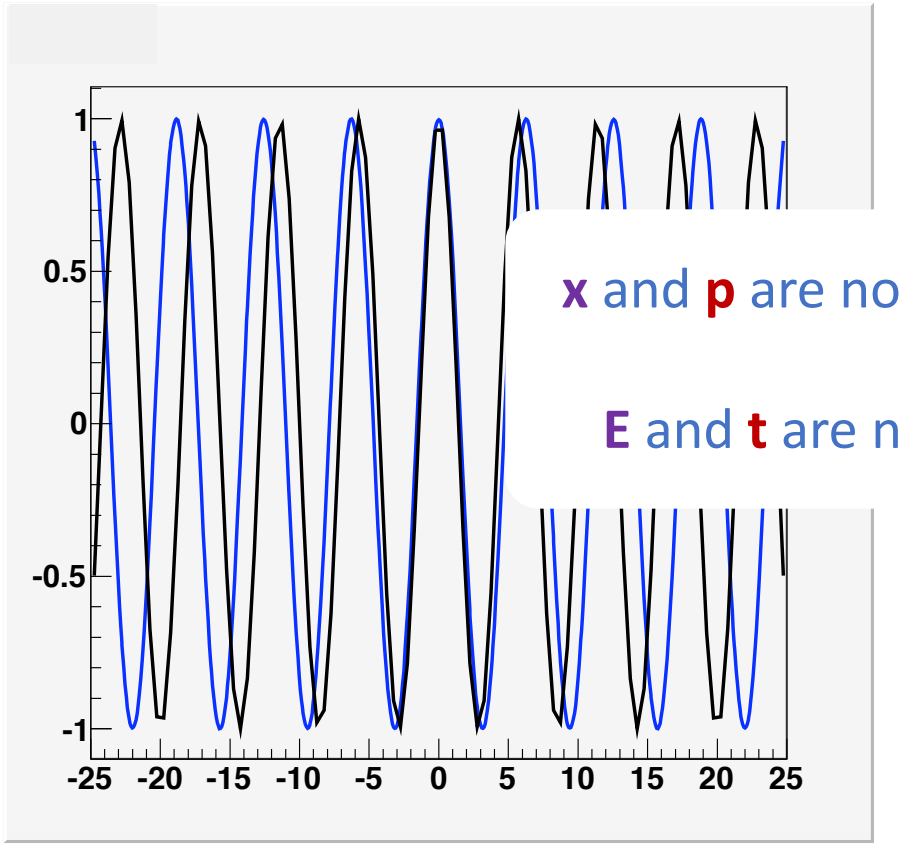
Not related to limited technology!

Waves and Uncertainty

Use the "wave-mechanics" picture of Schrödinger

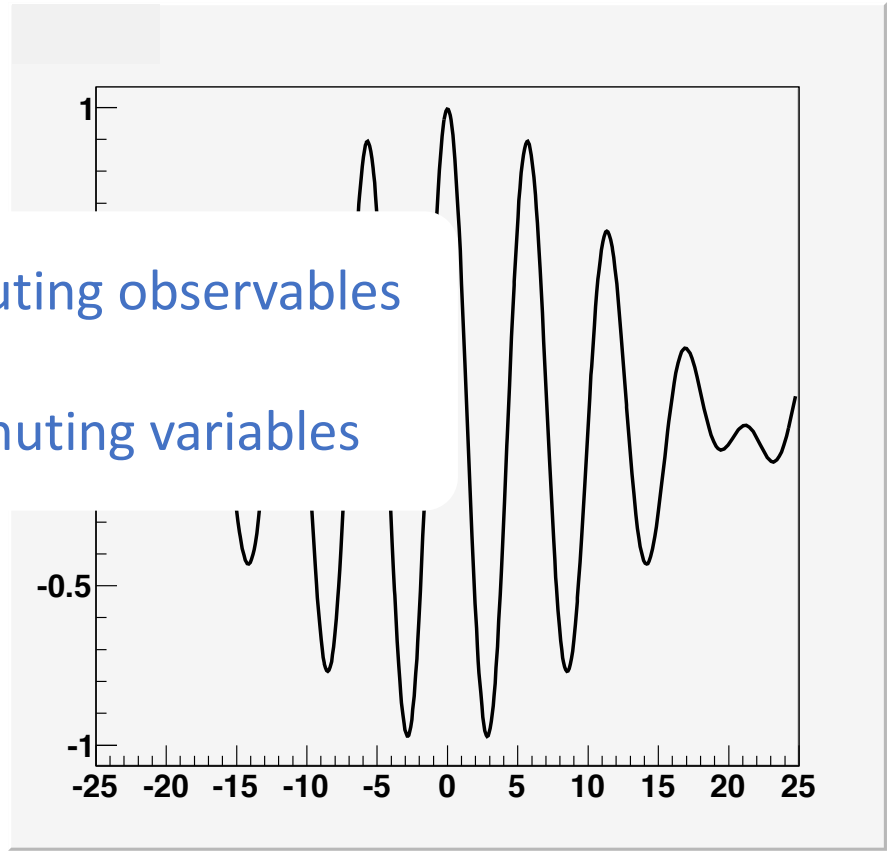
A wave has an exactly defined frequency.

Two waves: $p_1 = hf_1/c$, $p_2 = hf_2/c$



A particle has an exactly defined position.

Wave Packet: sum of black and blue wave



x and **p** are non-commuting observables

also

E and **t** are non-commuting variables

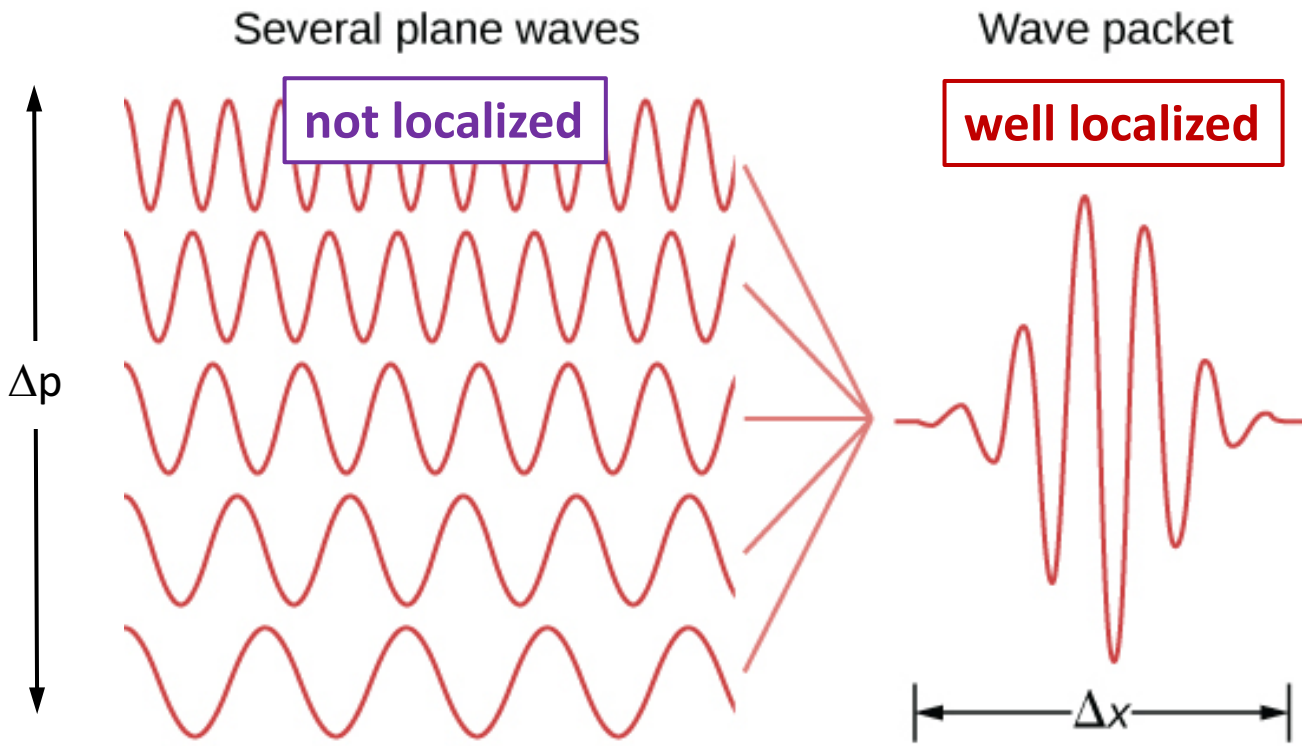
The more waves are added, the more the wave packet looks like a particle, or,
If we try to determine the position **x**, we destroy the momentum **p** and vice versa.

Waves and Particles

"Particle:"

Pure waves of different frequency, i.e. different momentum $p = hf/c$

Wave package, i.e. "particle"



Measure precise frequency → no position information.

Measure precise position → no momentum (frequency) information

The uncertainty relation at work

Shine a beam of light through a narrow slit which has a opening size Δx.
The light comes out over an undefined angle that corresponds to Δp_x.

$$\Delta x \, \Delta p_x \sim \hbar/2$$

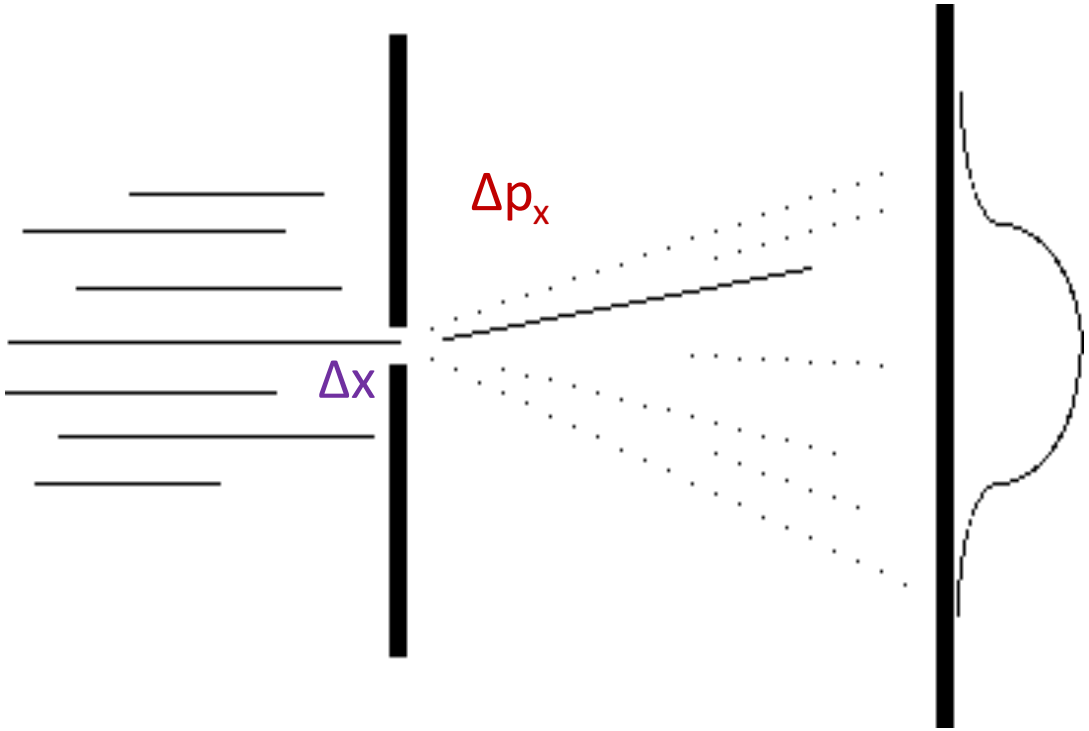
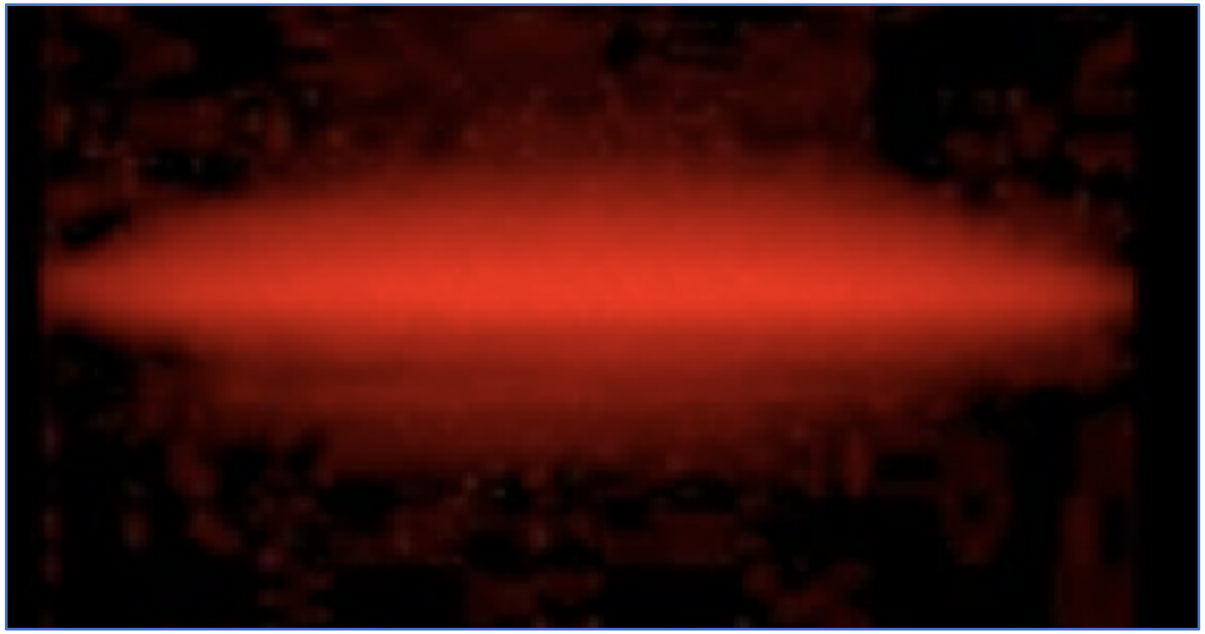
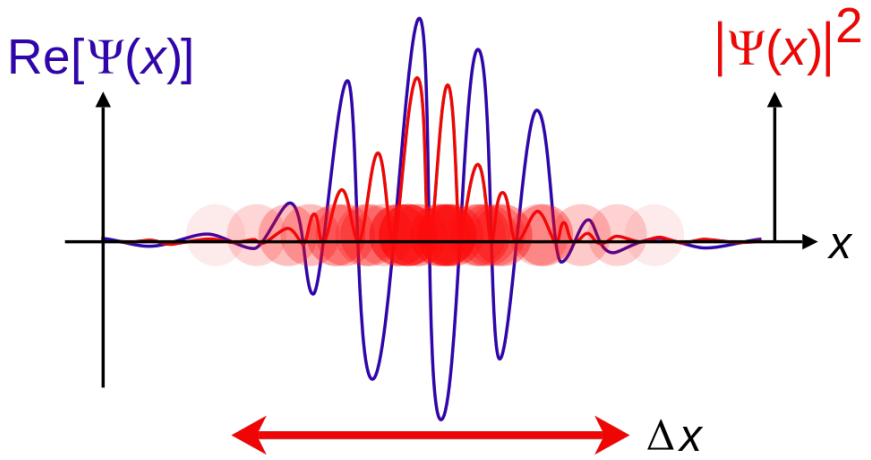


Image of a laser pointer after passing through a slit:

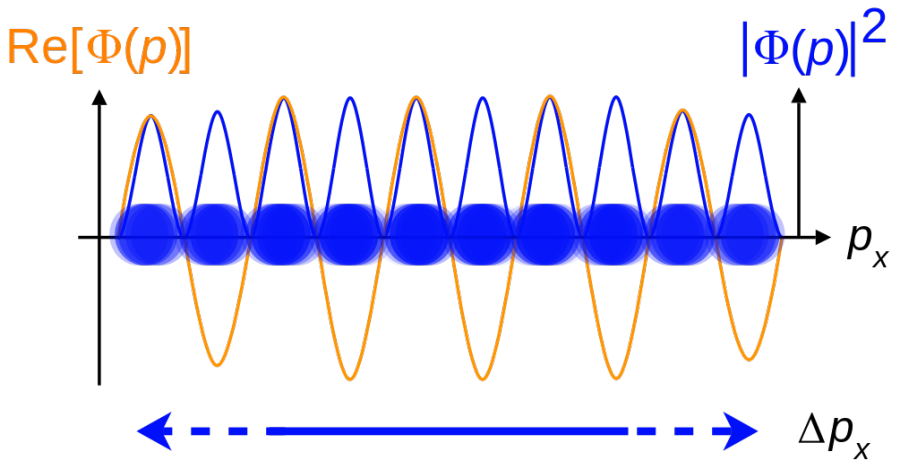


The wave function ψ

Position fairly known

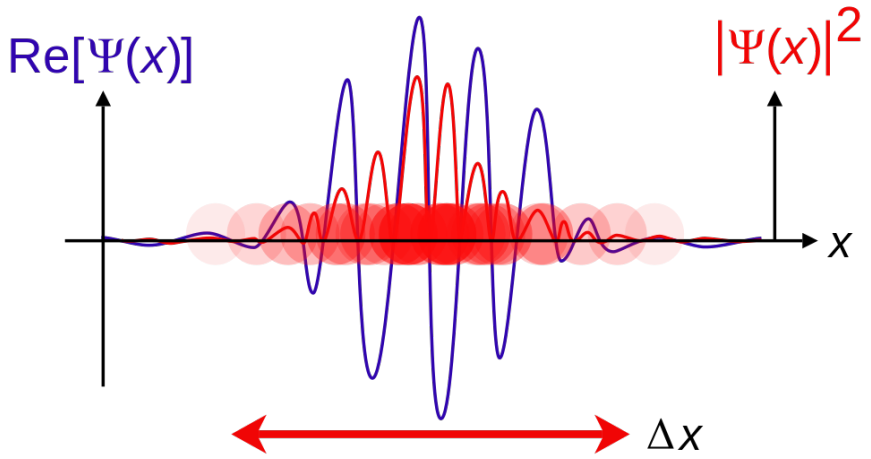


Momentum badly known

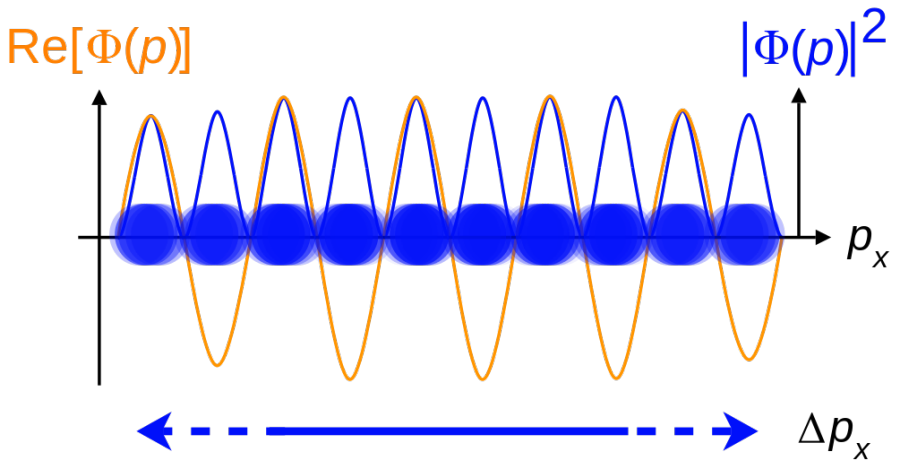


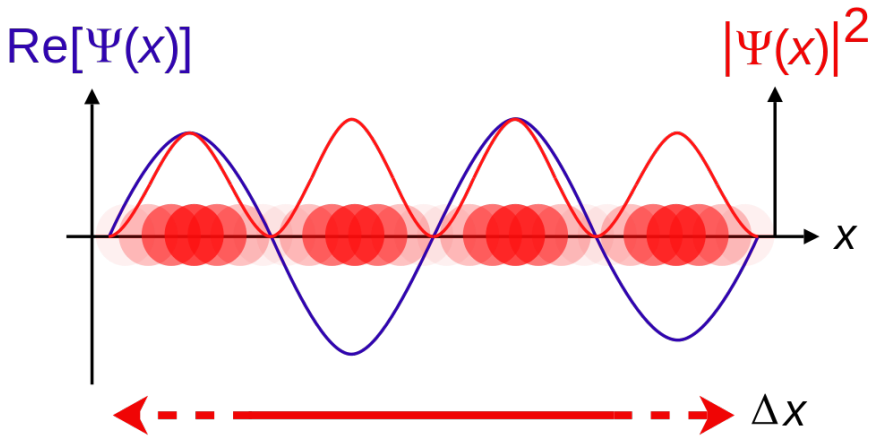
The wave function ψ

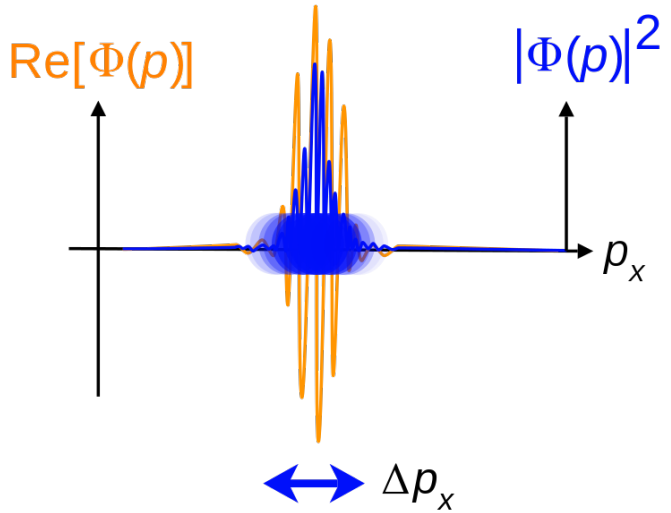
Position fairly known



Momentum badly known



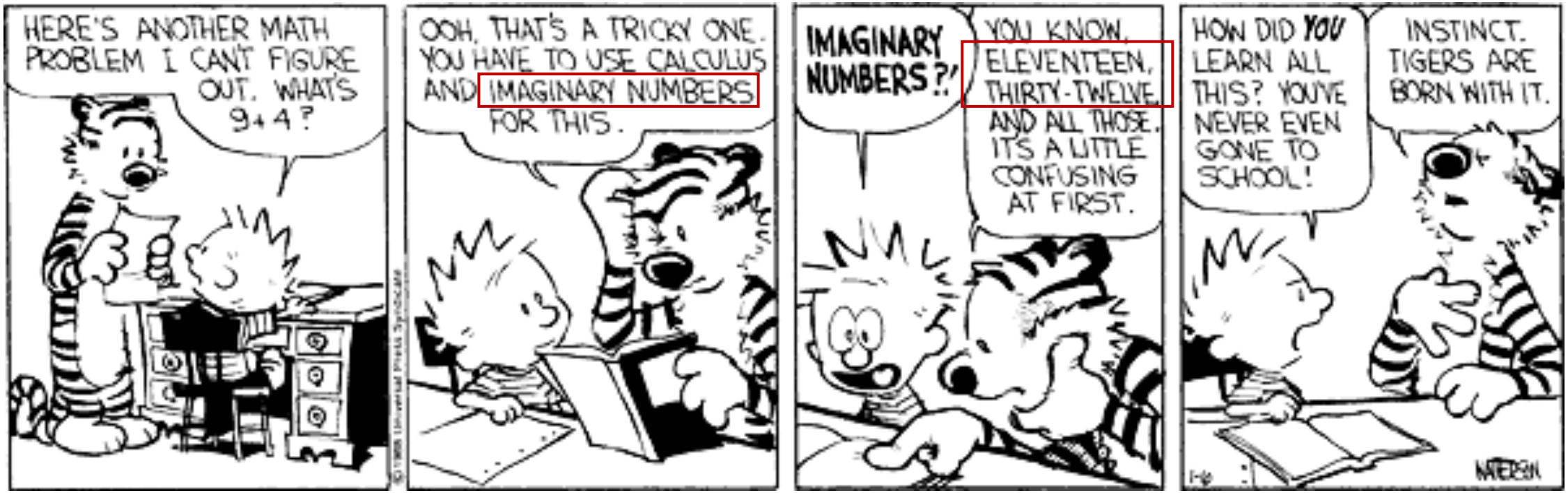




Position badly known

Momentum fairly known

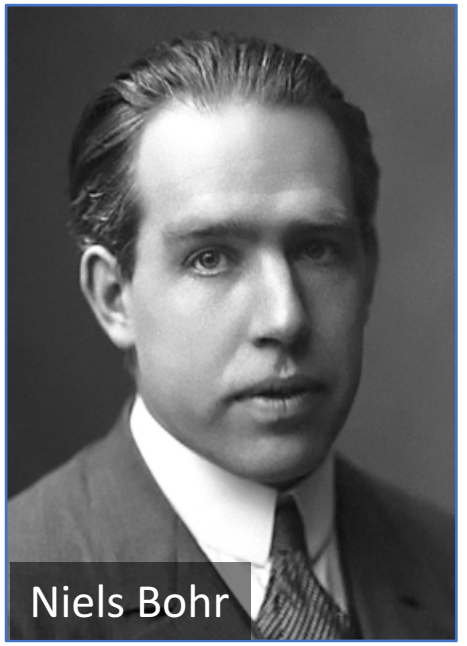
Imaginary Numbers



The Copenhagen Interpretation

The wave function ψ is not a real object. The only physical meaning is that its square gives the probability to find a particle at a position x and time t.

$$\text{Prob}(x,t) = |\psi(x,t)|^2 = \psi\,\psi^*$$


Niels Bohr

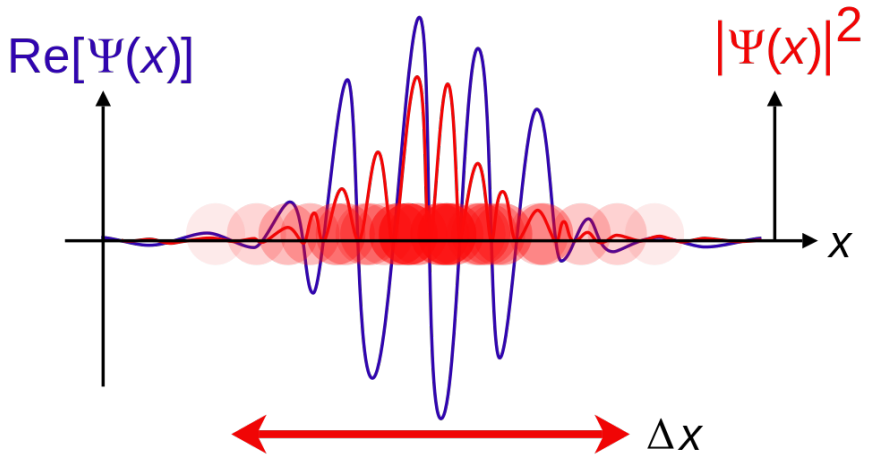

Max Born

Quantum mechanics allows only to calculate ***probabilities*** for possible outcomes of an experiment and is non-deterministic, contrary to classical theory.
Einstein: "Gott würfelt nicht."

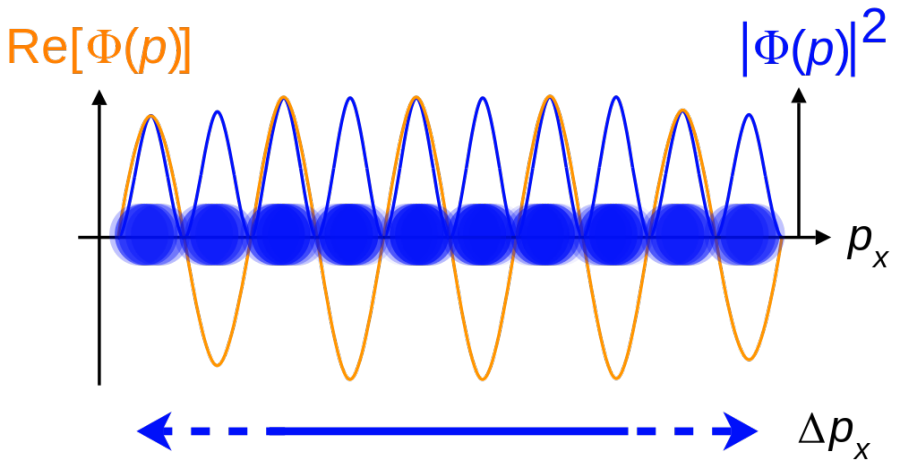
The mathematics for the ***probability*** of the quantum wave-function is the same as the mathematics of the ***intensity*** of a classical wave function.

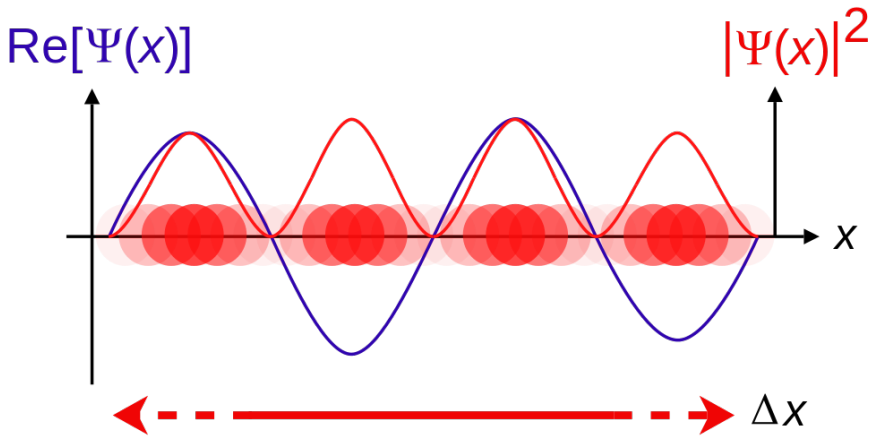
The wave function ψ

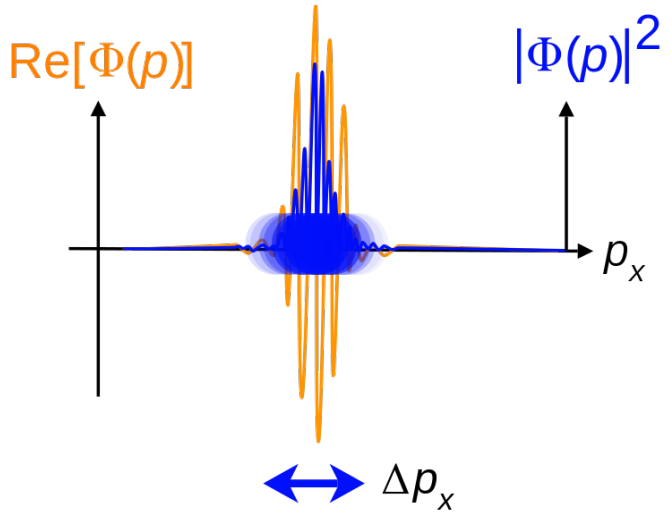
Position fairly known



Momentum badly known



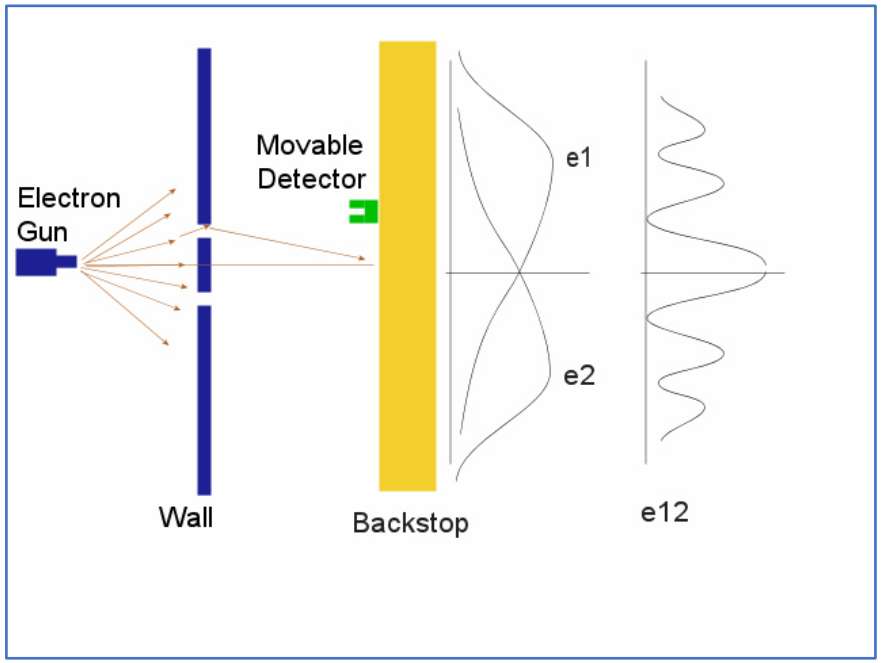


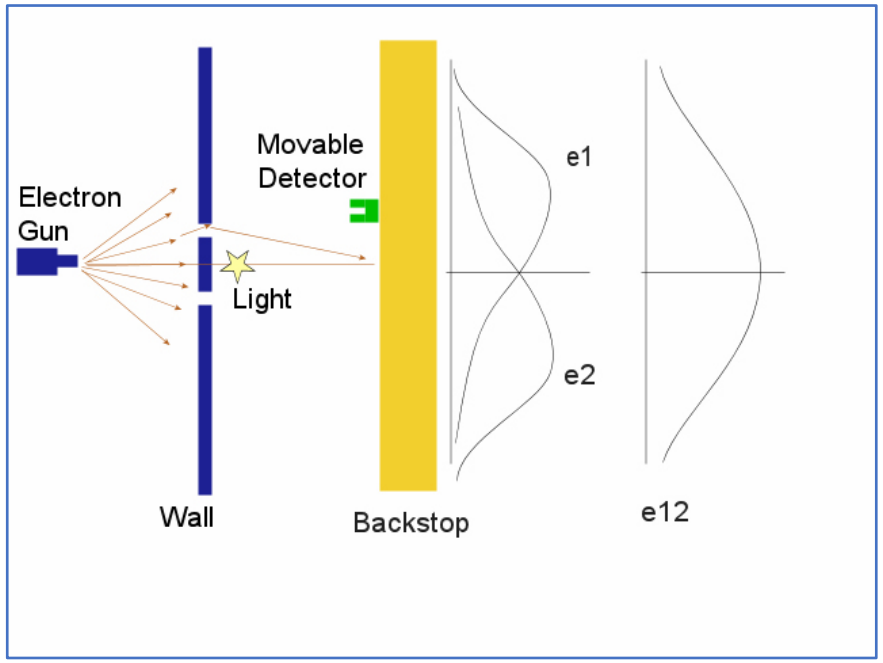


Position badly known

Momentum fairly known

Next Lecture: double slit experiment





The core of quantum mechanics illustrated by Feynman.
Einstein and Schrödinger did not like it.
Wheeler later took it to the extreme.
Even today people are debating its interpretation.