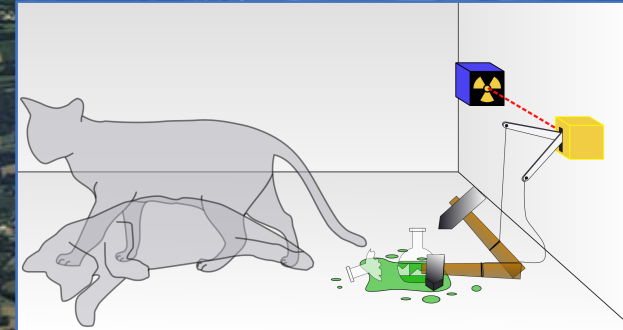
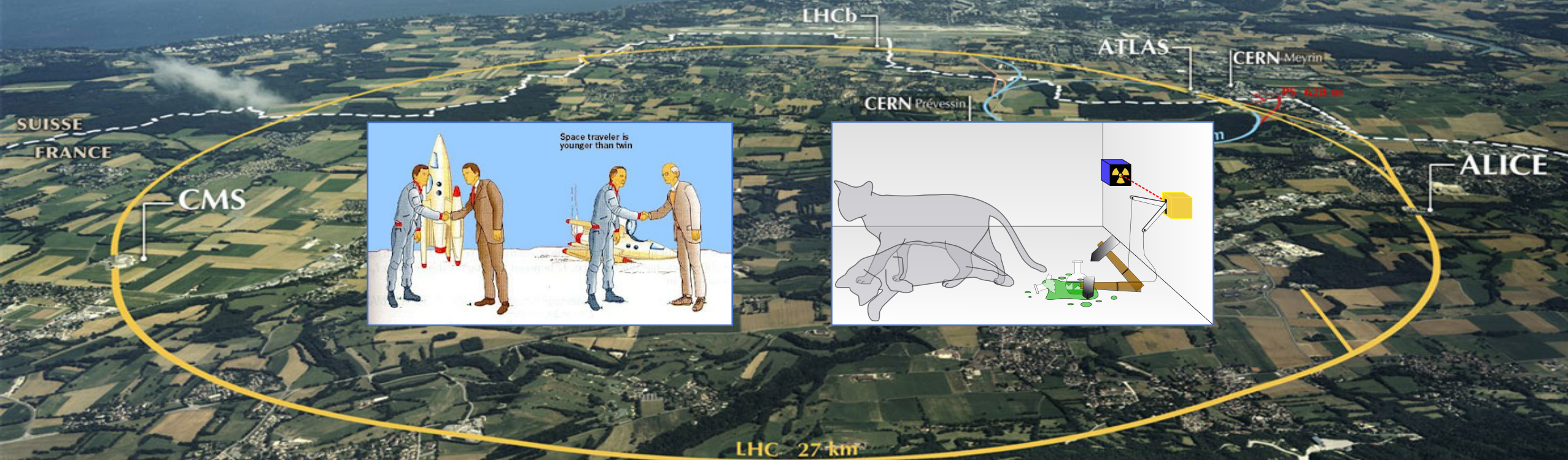


# The Relativistic Quantum World

A lecture series on Relativity Theory and Quantum Mechanics

Marcel Merk



University of Maastricht, Sept 16 – Oct 14, 2020



## Relativity

Sept. 16:

Lecture 1: The Principle of Relativity and the Speed of Light  
Lecture 2: Time Dilation and Lorentz Contraction

Sept. 23:

Lecture 3: The Lorentz Transformation and Paradoxes  
Lecture 4: General Relativity and Gravitational Waves

## Quantum Mechanics

Sept. 30:

Lecture 5: The Early Quantum Theory  
Lecture 6: Feynman's Double Slit Experiment

Oct. 7:

Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat  
Lecture 8: Quantum Reality and the EPR Paradox

## Standard Model

Oct. 14:

Lecture 9: The Standard Model and Antimatter  
Lecture 10: The Large Hadron Collider

Lecture notes, written for this course, are available: [www.nikhef.nl/~i93/Teaching/](http://www.nikhef.nl/~i93/Teaching/)  
Prerequisite for the course: High school level physics & mathematics.

## Lecture 4

# General Relativity and Gravitational Waves

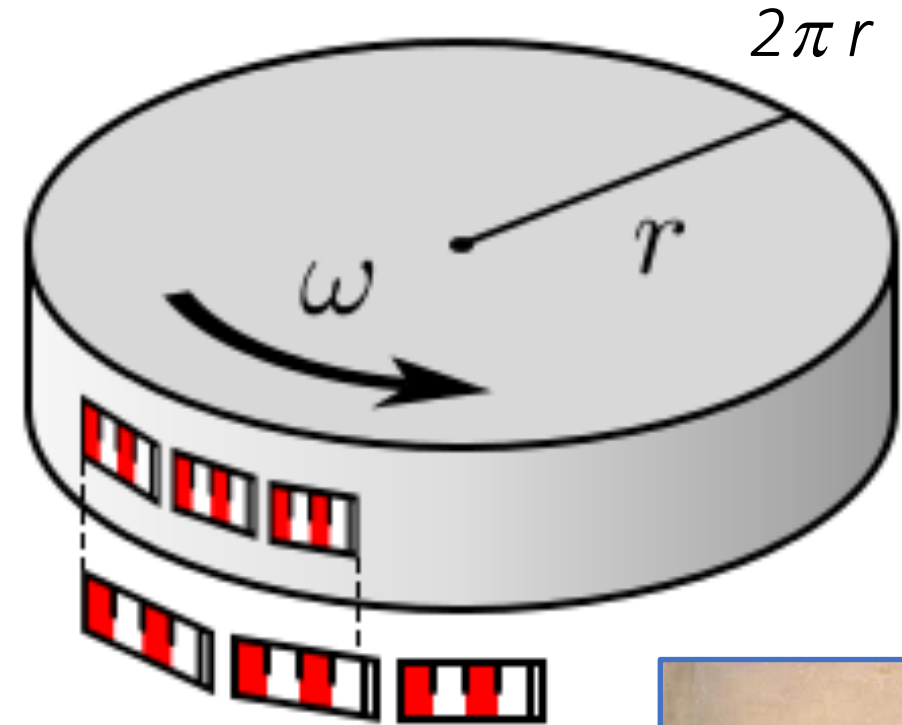
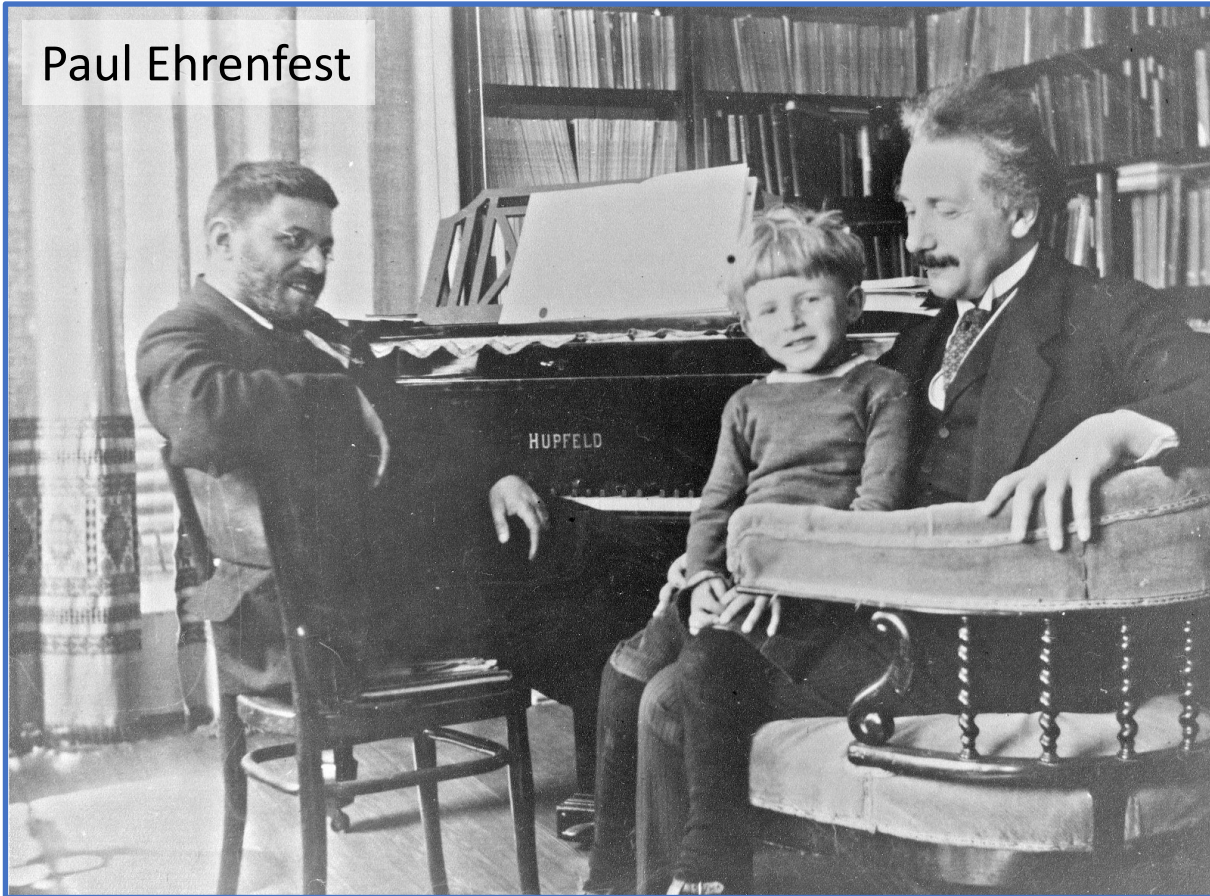
*“Do not worry about your difficulties in mathematics.  
I can assure you mine are still greater.”*  
- Albert Einstein

# Ehrenfest Paradox

3

Rotating disk with ruler on the edge:  
Circumference:  $C = 2 \pi r$

Paul Ehrenfest





Rotating disk with ruler on the edge:  
Circumference:  $C = 2 \pi r$

**Alice** stands next to the disk and sees  
*rulers on disk* Lorentz contracted:

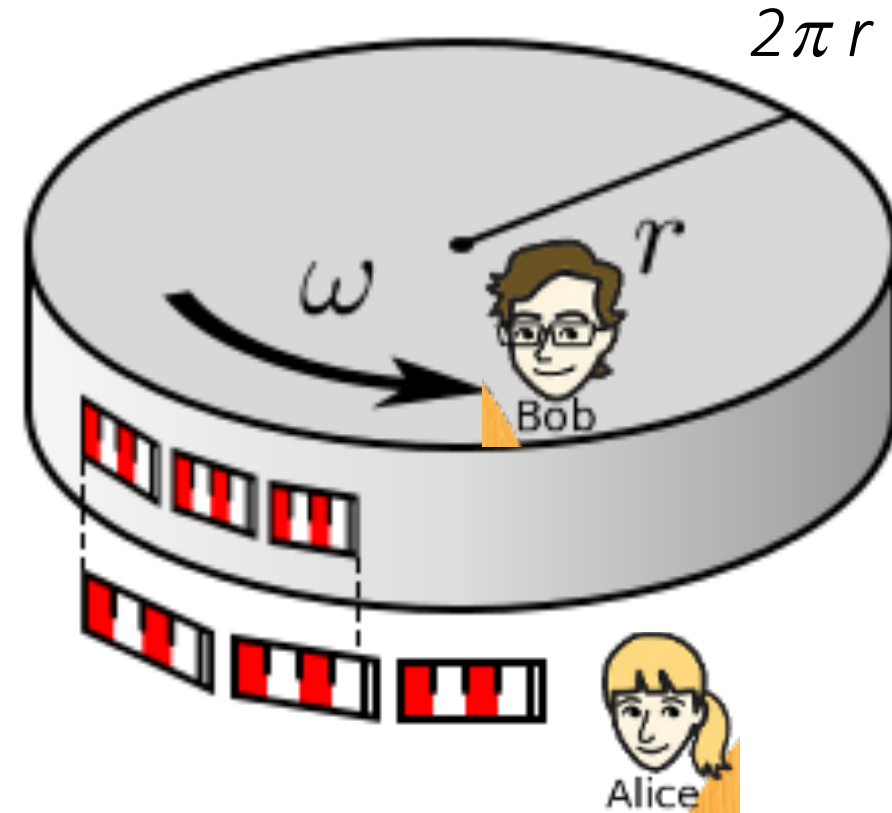
$$C = 2 \pi r / \gamma$$

→ Circumference is smaller!

**Bob** moves on the disk and sees  
*rulers next to disk* contracted:

$$C = 2 \pi r \cdot \gamma$$

→ Circumference is larger!

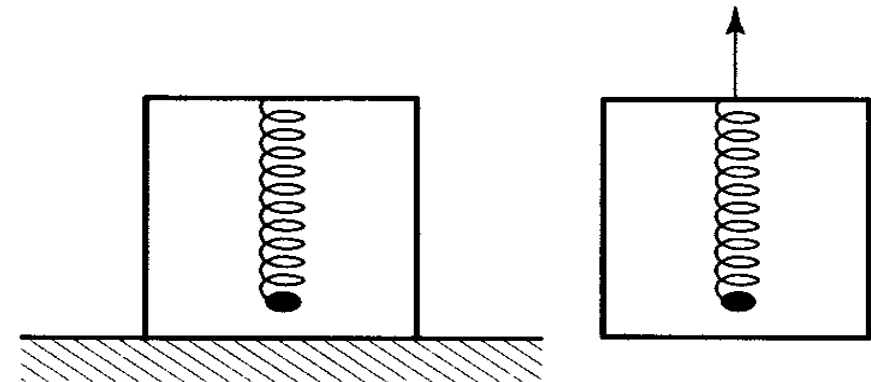
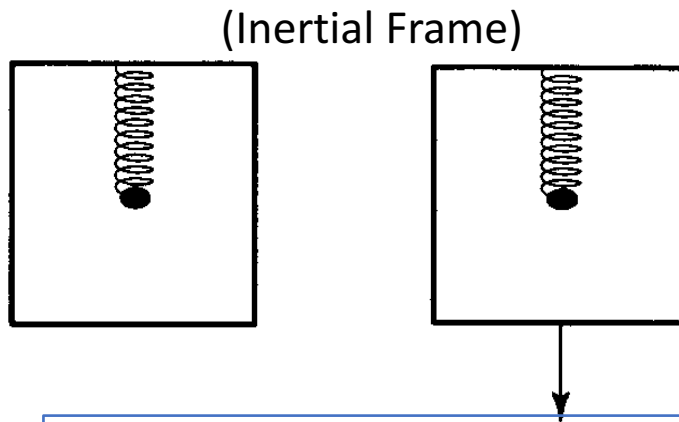
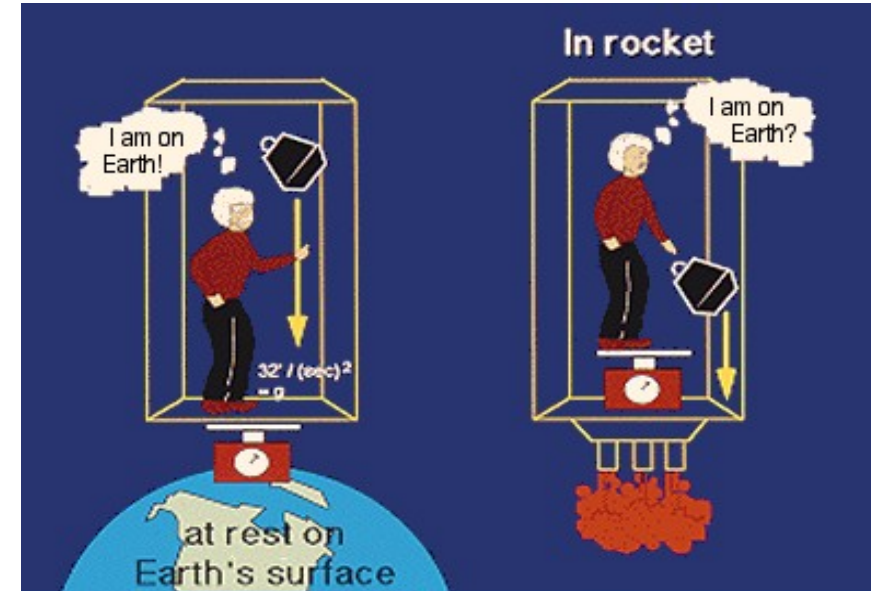


A rotating object is ***not an inertial*** frame:

- Postulate of relativity only worked for ***inertial frames***
- Need to adapt the postulates: special relativity → ***general relativity***

# The Equivalence Principle

5

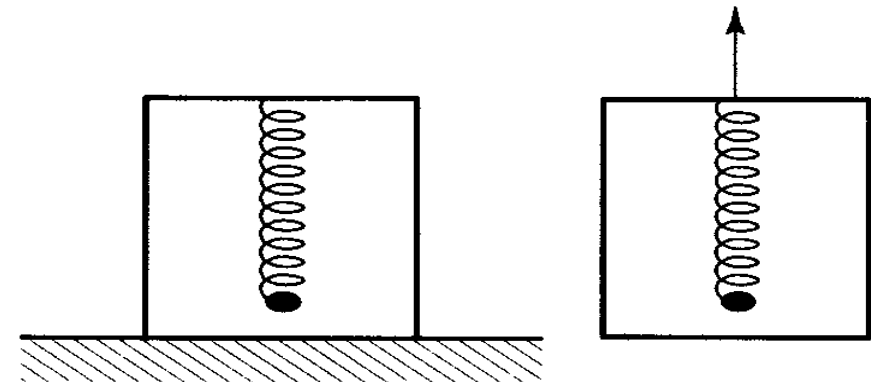
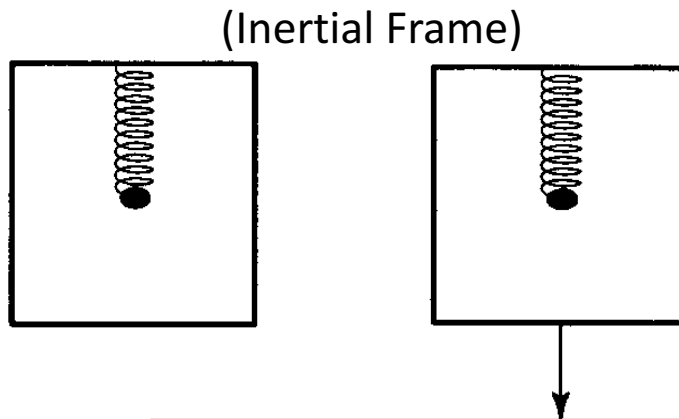
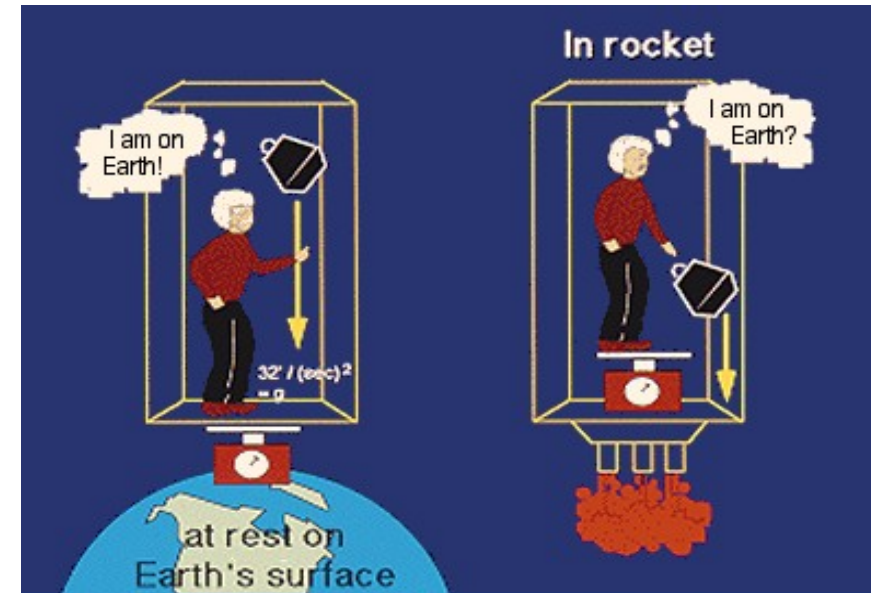


Einstein "happiest thought": there is no way to determine whether you are standing on the earth or accelerating upwards in a rocket in space!



# The Equivalence Principle

6

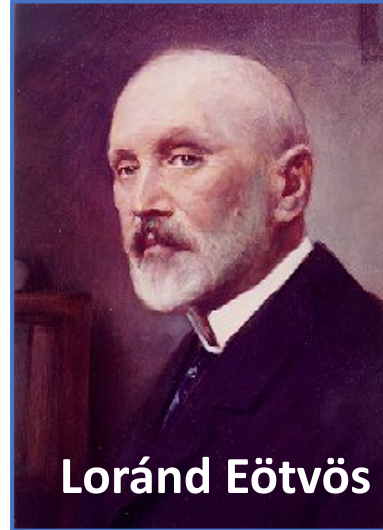
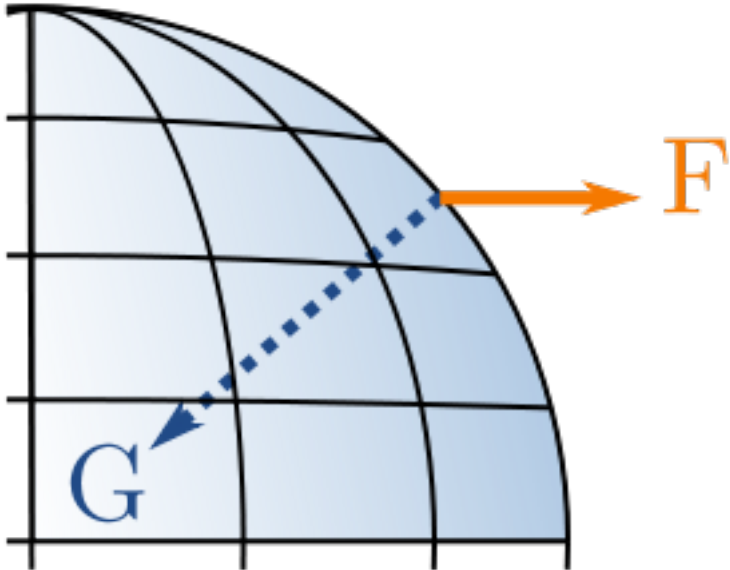


There is no difference between acceleration force and gravitation.  
***"Gravitational mass" = "Inertial mass"***

# The Eötvös Experiment

7

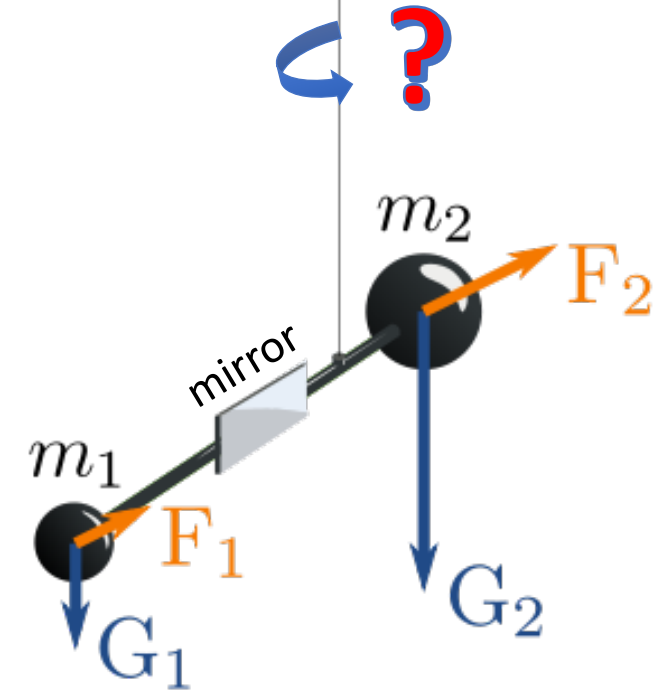
Direction of **gravity** and  
**centrifugal force** on earth



Loránd Eötvös

$$F = m_i a \text{ with } a = \frac{v^2}{R}$$
$$G = m_g g \text{ with } g = \frac{GM_{\oplus}}{R^2}$$
$$\rightarrow m_i = m_g$$

Small ( $m_1$ ) and big ( $m_2$ ) mass on  
a rod suspended by a thin fiber



Gravity force **G** depends on Newton's law of gravity: **gravitational mass**

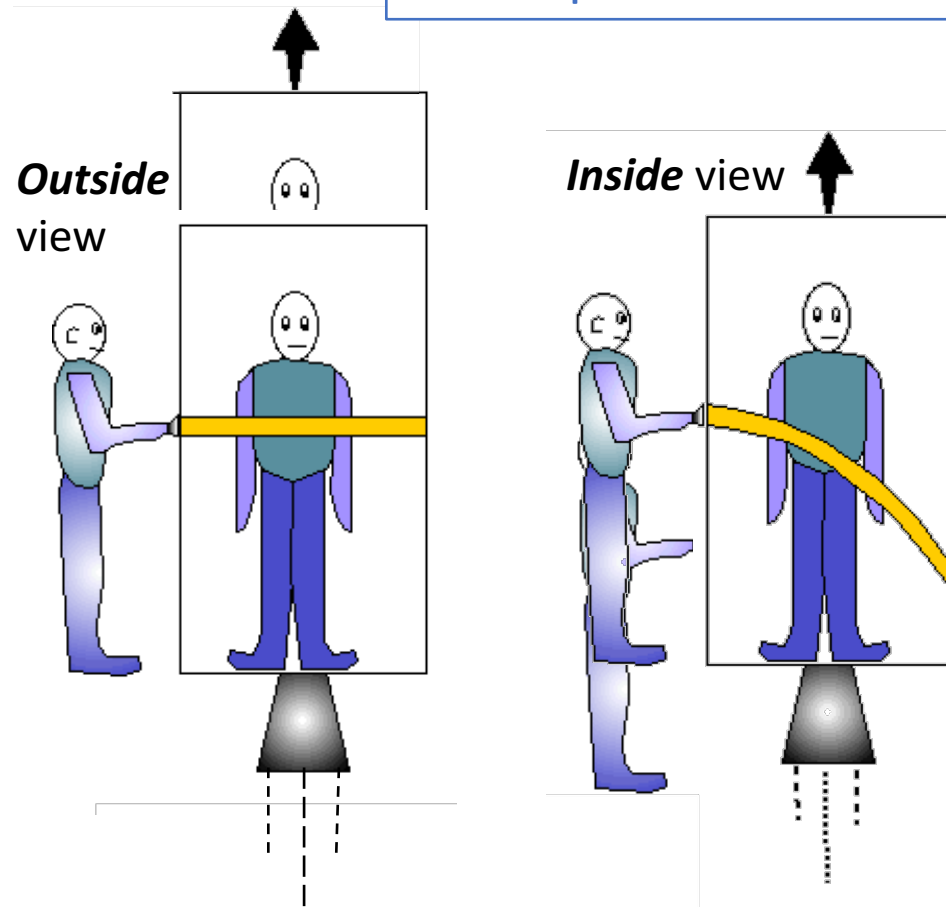
Centrifugal force **F** depends on Newton's law of motion inertial mass: **inertial mass**

The system did **not** rotate.  $\rightarrow F_1/F_2 = G_1/G_2$

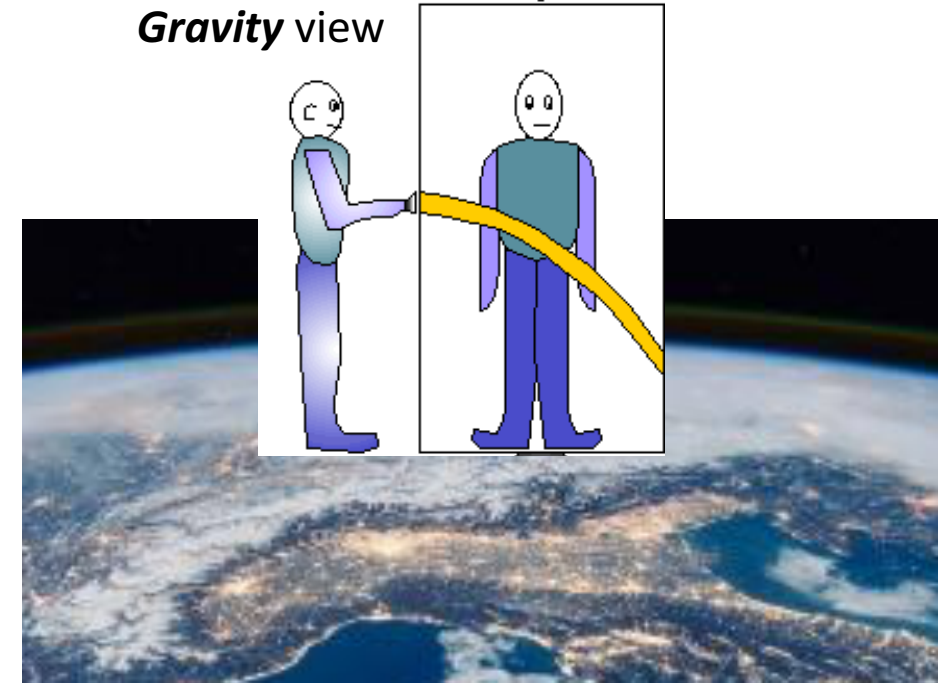
$\rightarrow$  Experimental proof that indeed gravitational mass is equivalent to inertial mass.



Consequence if acceleration and gravity are identical



A: Lightbeam in accelerating rocket

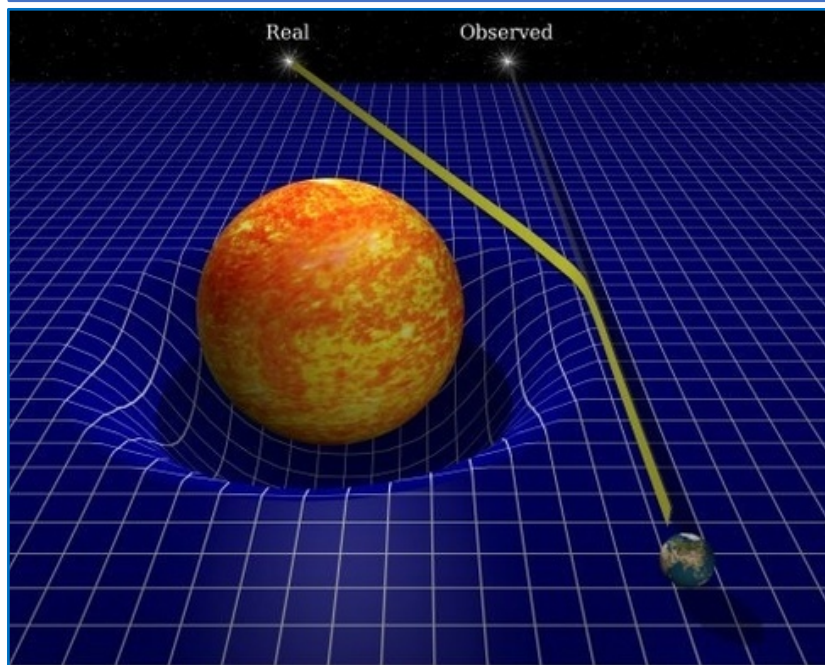
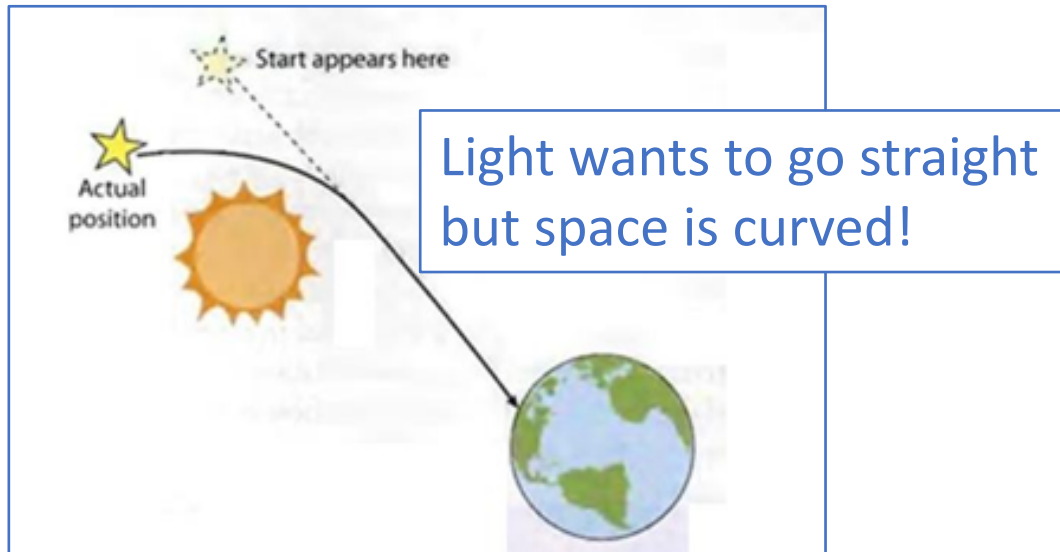


B: Lightbeam in gravitational field

**Prediction of Einstein: light beam bends under gravity!**

# Bending of light in gravitation field of the Sun

9

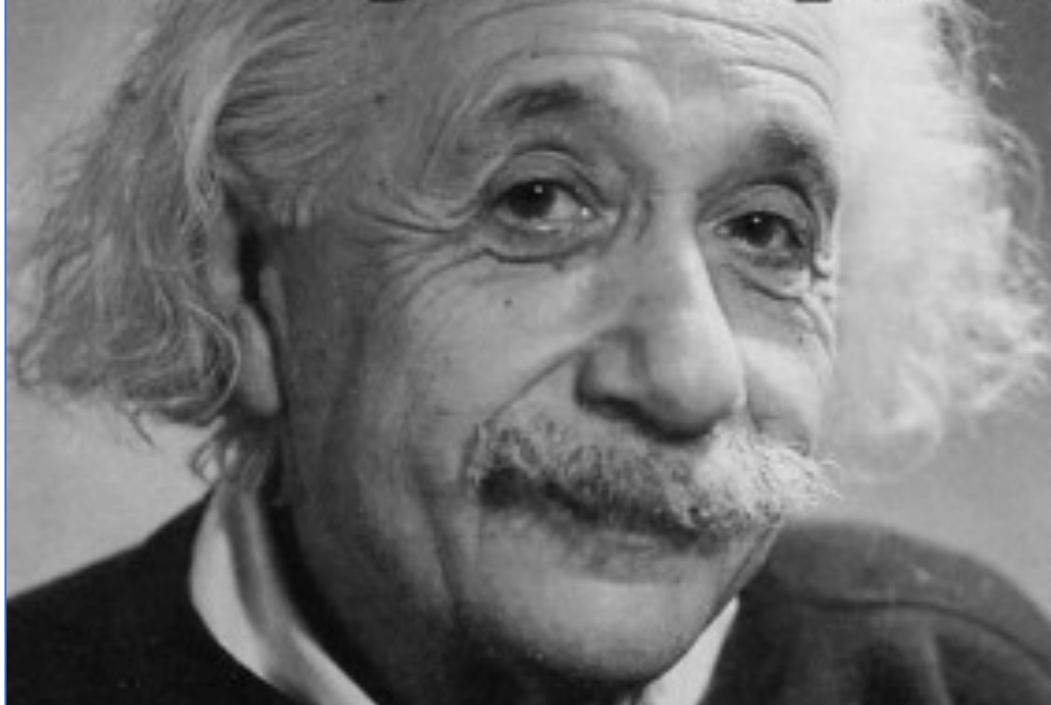


Confirmed during solar eclipse  
on November 10 1919!





**Einstein developped  
a theory about space**

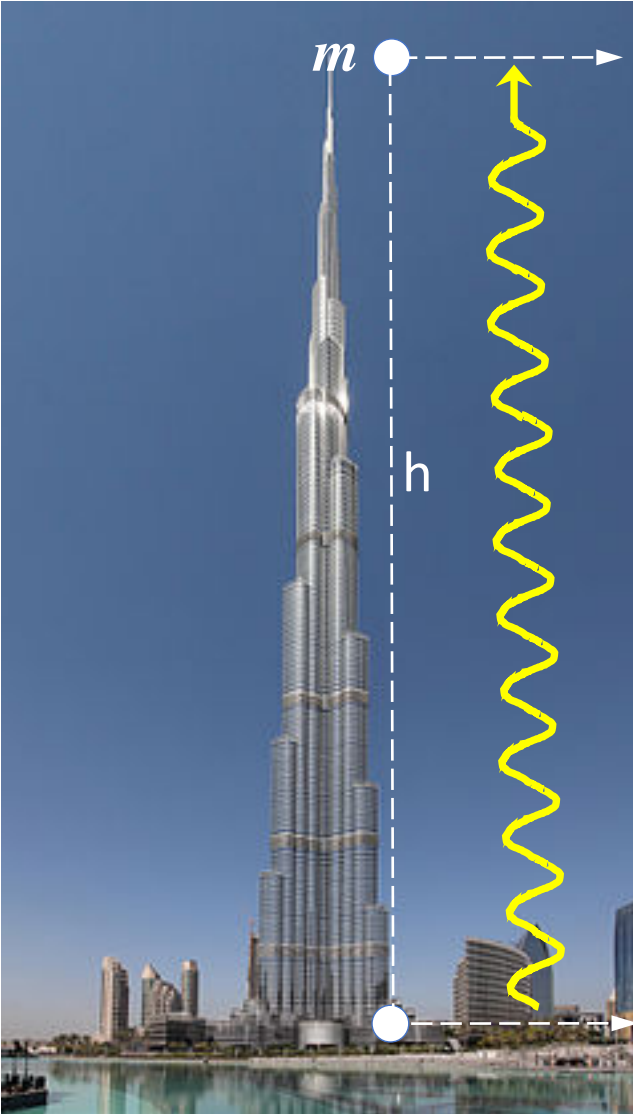


**And it was about time too**

# Einstein's thought experiment

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Particle with mass  $m$  falling from tower:



$$E = mc^2 \quad \Rightarrow \quad E = hf$$

From quantum mechanics we know that the energy of light is related to frequency (and wavelength):  $E = hf = hc/\lambda$

Perpetuum mobile?  $\Rightarrow$  No!

$$hf' > hf$$

Photon loses energy  $gh/c^2$  as it travels up the gravitational field!

$$\begin{aligned} E' &= mc^2 + \frac{1}{2}mv^2 = mc^2 + mgh \\ &= mc^2 (1 + gh/c^2) \quad \Rightarrow \quad E' = hf' \end{aligned}$$

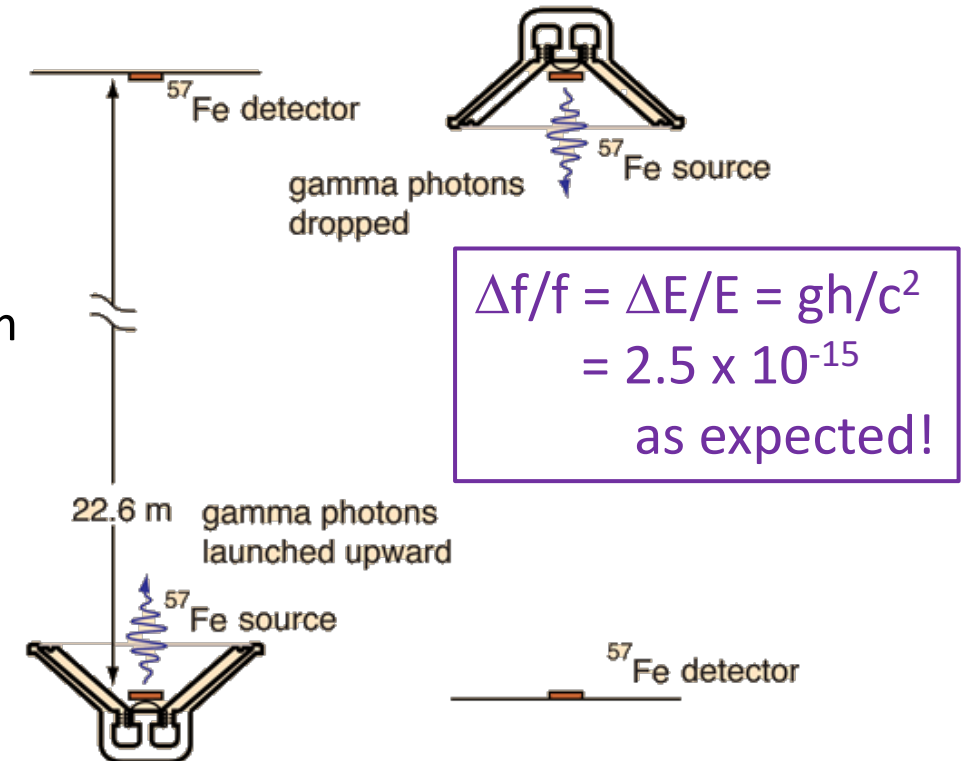
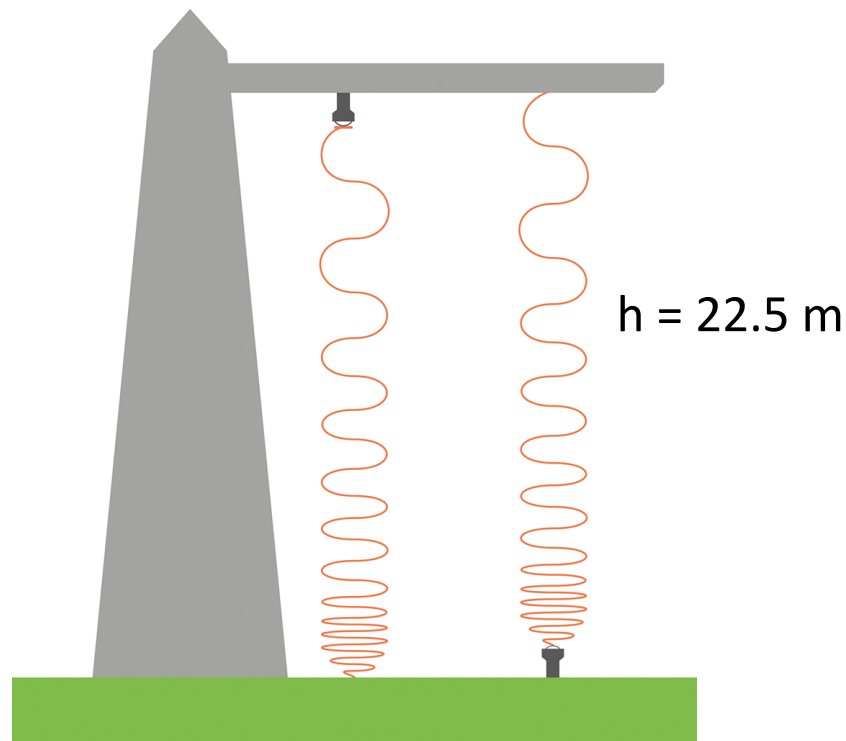
$$(E_{kin} = E_{pot})$$



# The Harvard Tower Experiment

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Harvard Tower Experiment (Pound-Rebka)  
at Jefferson lab in Harvard:  
Measure red-shift of photons in earth  
gravitational field.

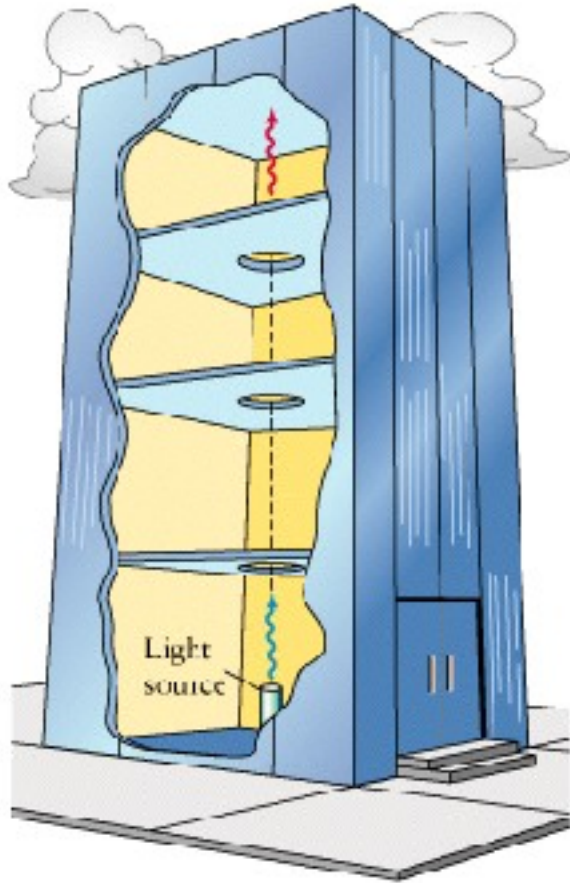


# Gravitational Time Dilation

13

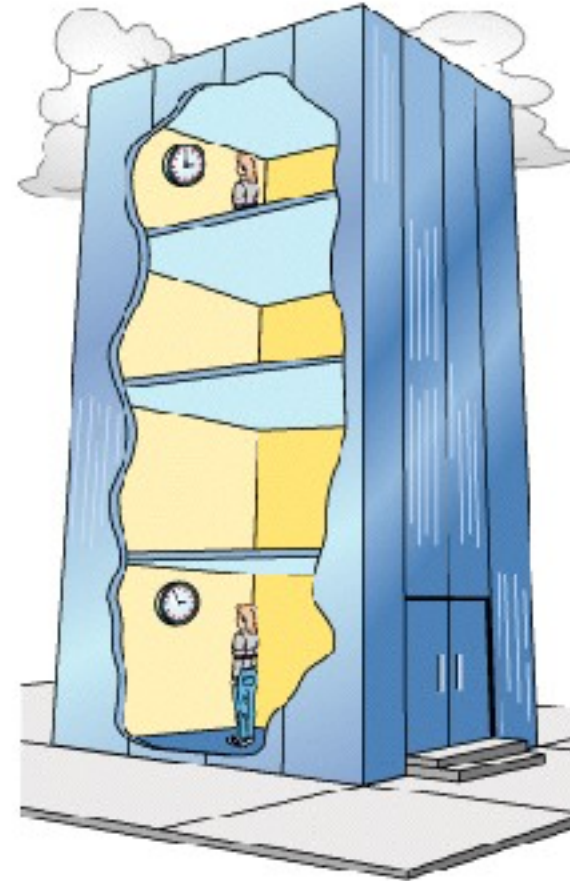
The photon loses energy as it climbs the gravitational field.

Longer  
wavelength



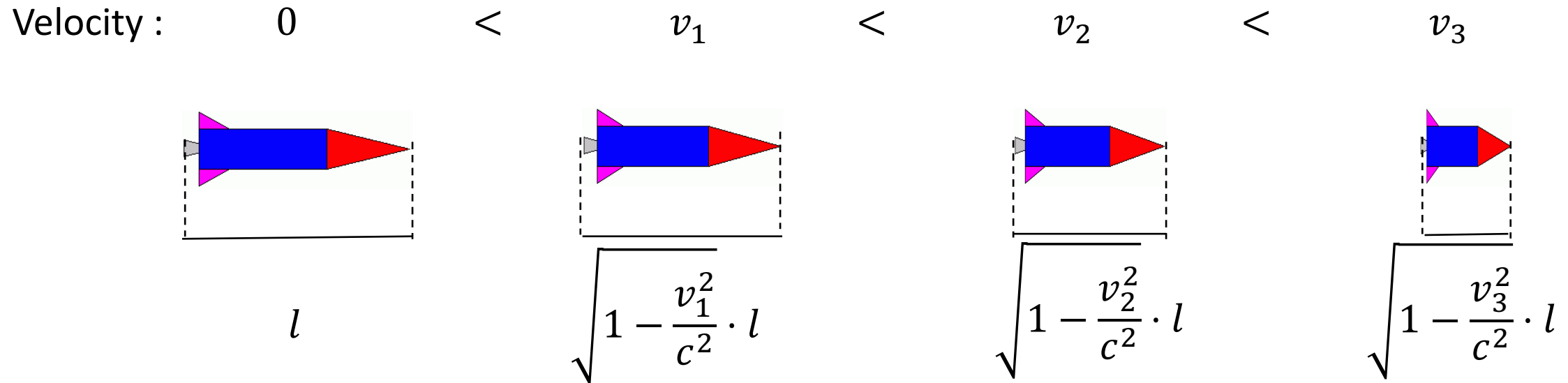
$$c = \lambda f$$

Lower  
frequency



→ Time ticks faster at higher altitude.

From special relativity we know that space contracts at high velocity



Space is seen to shrink further and further with increasing velocity!

$$1/\gamma = \sqrt{1 - \frac{v^2}{c^2}}$$



$$E_{pot} = G \frac{mM_{\oplus}}{R}$$



$$E_{kin} = E_{pot}$$

$$\frac{1}{2}mv^2 = G \frac{mM_{\oplus}}{R}$$



$$v^2 = 2 \frac{GM_{\oplus}}{R}$$



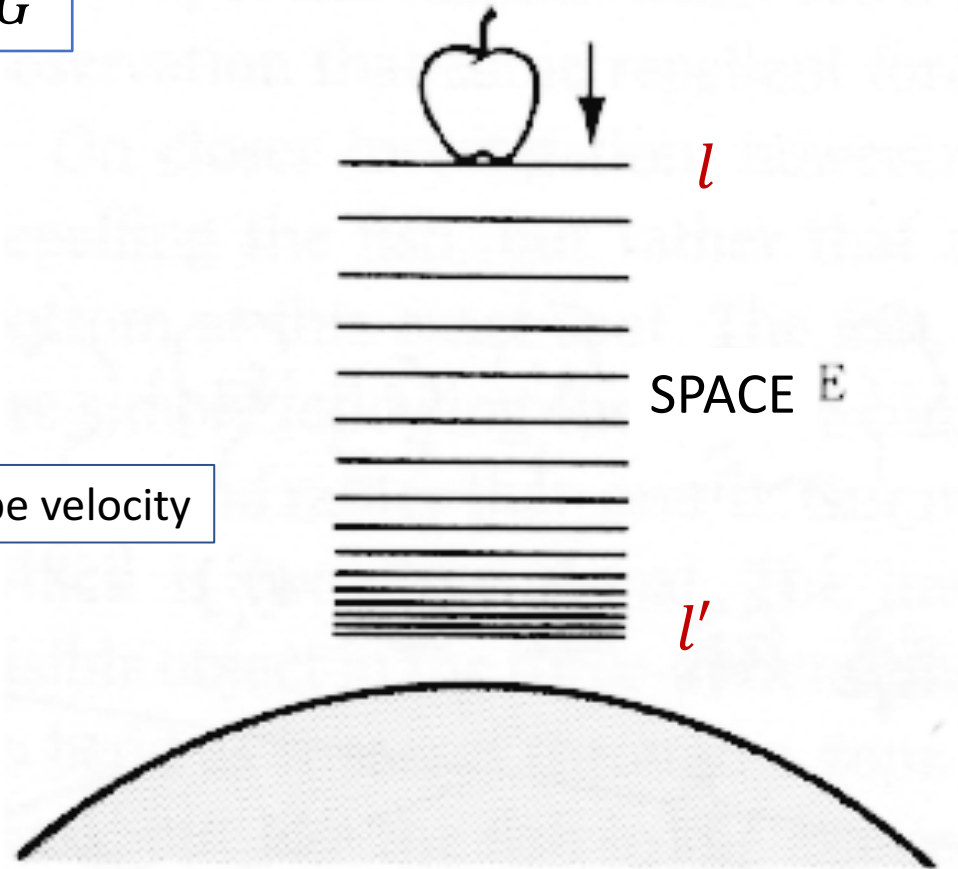
$$\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - 2 \frac{GM_{\oplus}}{Rc^2}}$$

Lorentz factor

Newton's  
Constant  $G$

$v$  = Escape velocity

Falling apple:



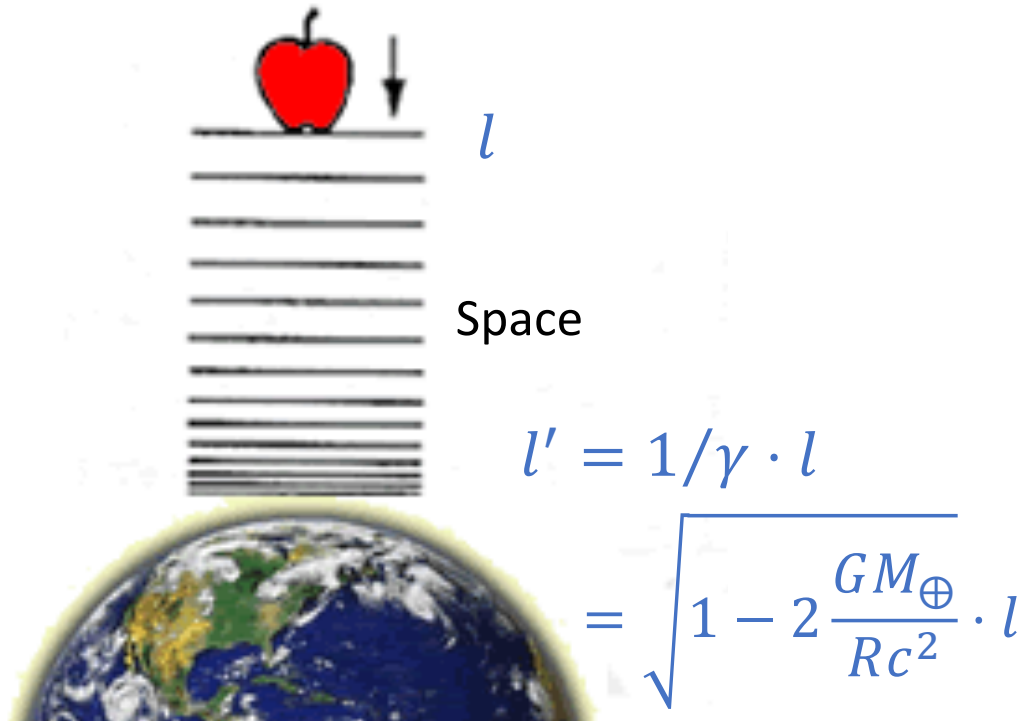
Compare to accelerating rocket:



Space shrinkage ("curvature"):

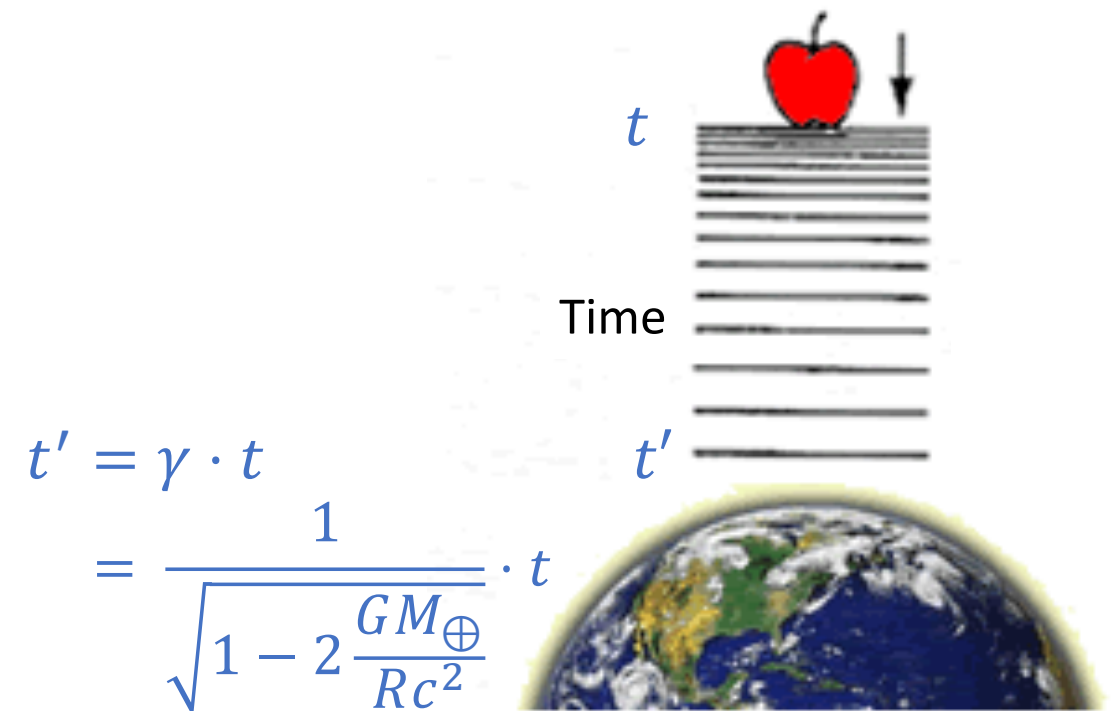
$$l' = \sqrt{1 - 2 \frac{GM_{\oplus}}{Rc^2}} \cdot l$$

A falling apple accelerates and units of space get more and more contracted:



*Space contracts near mass and dilates away from it.*

An apple falls into the gravitational field and time runs slower and slower:



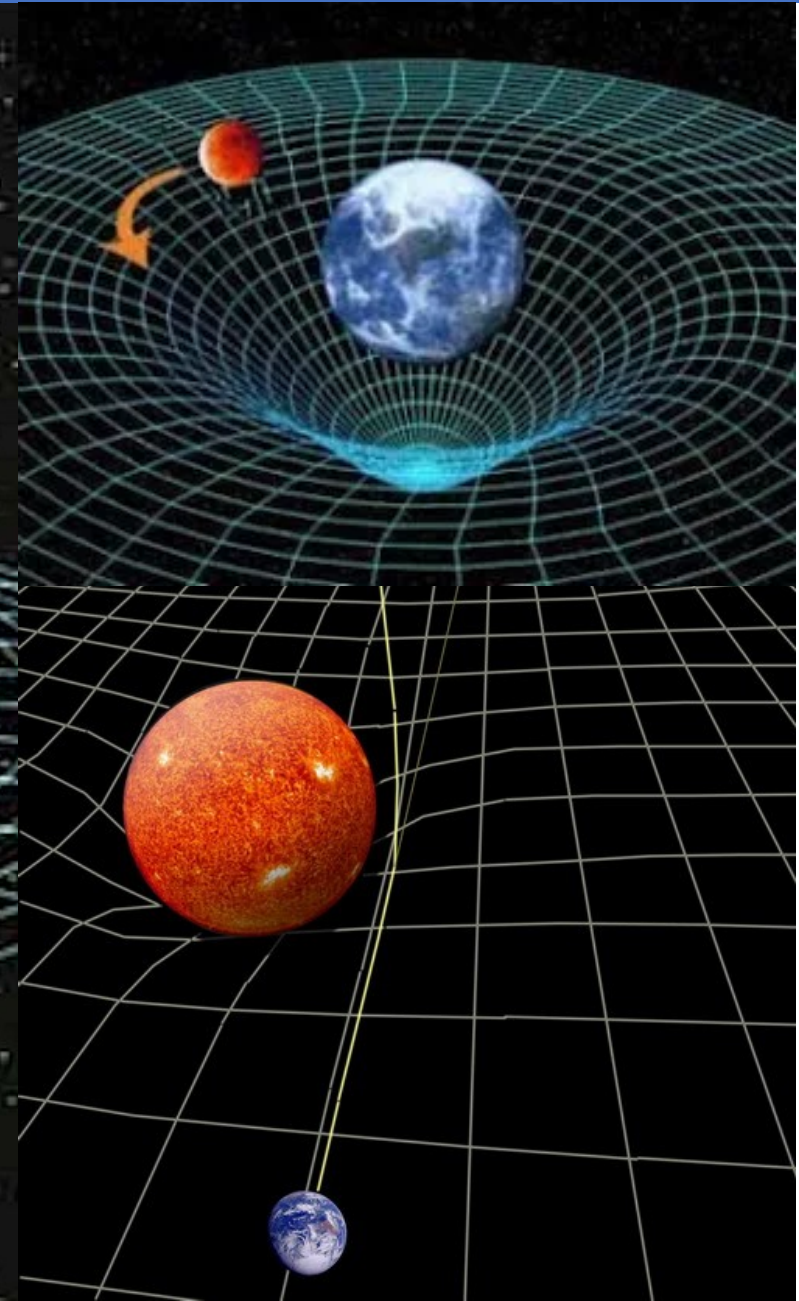
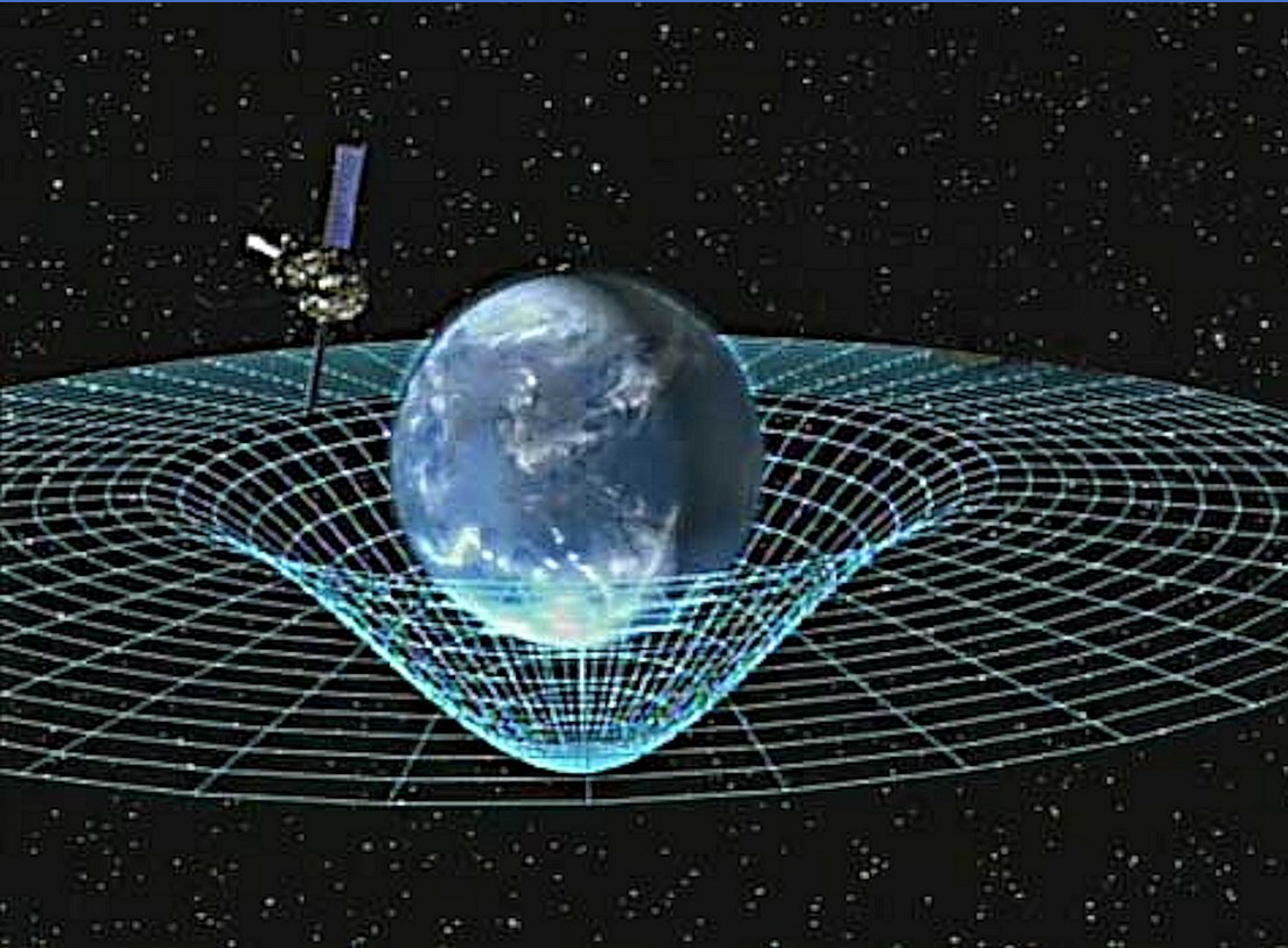
*Time slows near mass and speeds up away from it.*

***Space-time is curved in the presence of mass***



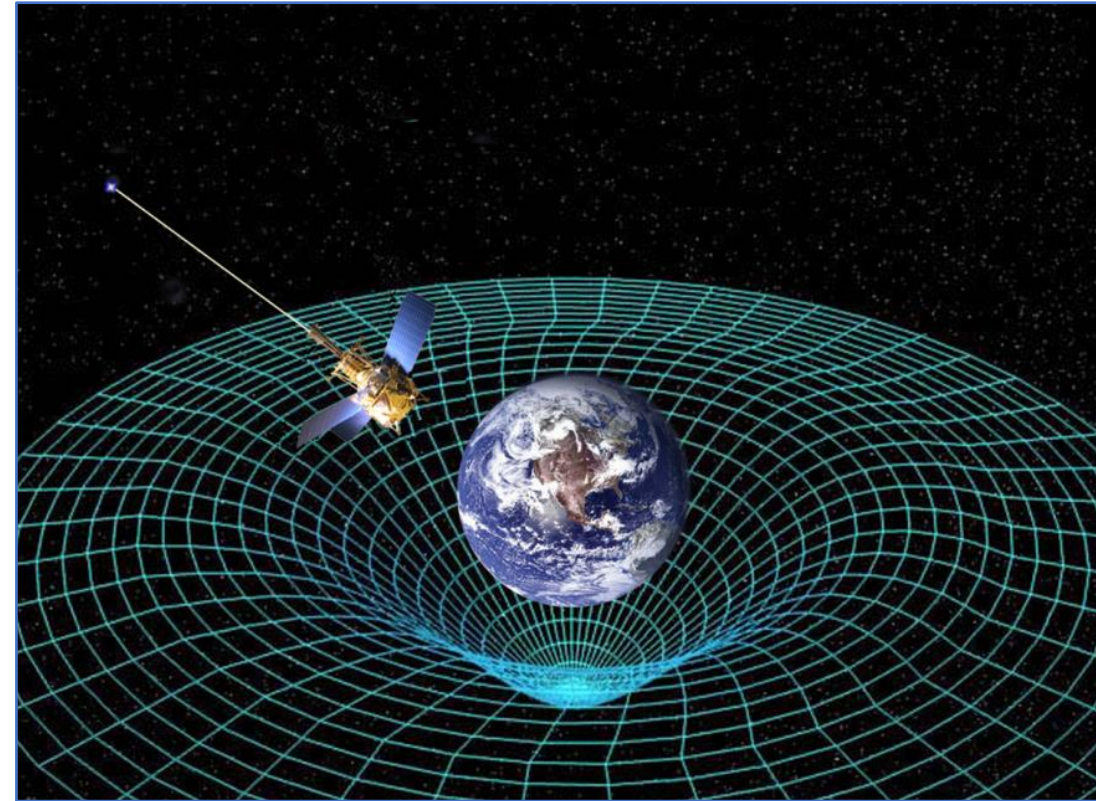
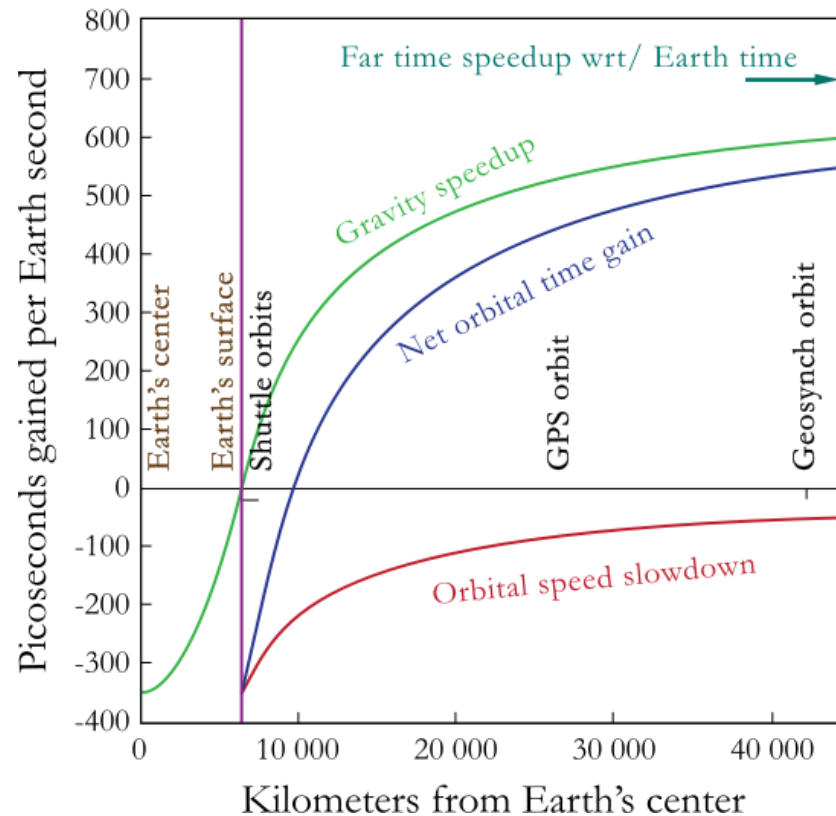
# Mass causes curvature in space-time

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## Time Dilation Effects on Earth



## Two effects:

- Time speeds up at the satellite in comparison to earth surface due to gravity
- Time slows down at the satellite due to high velocity compared to person on earth

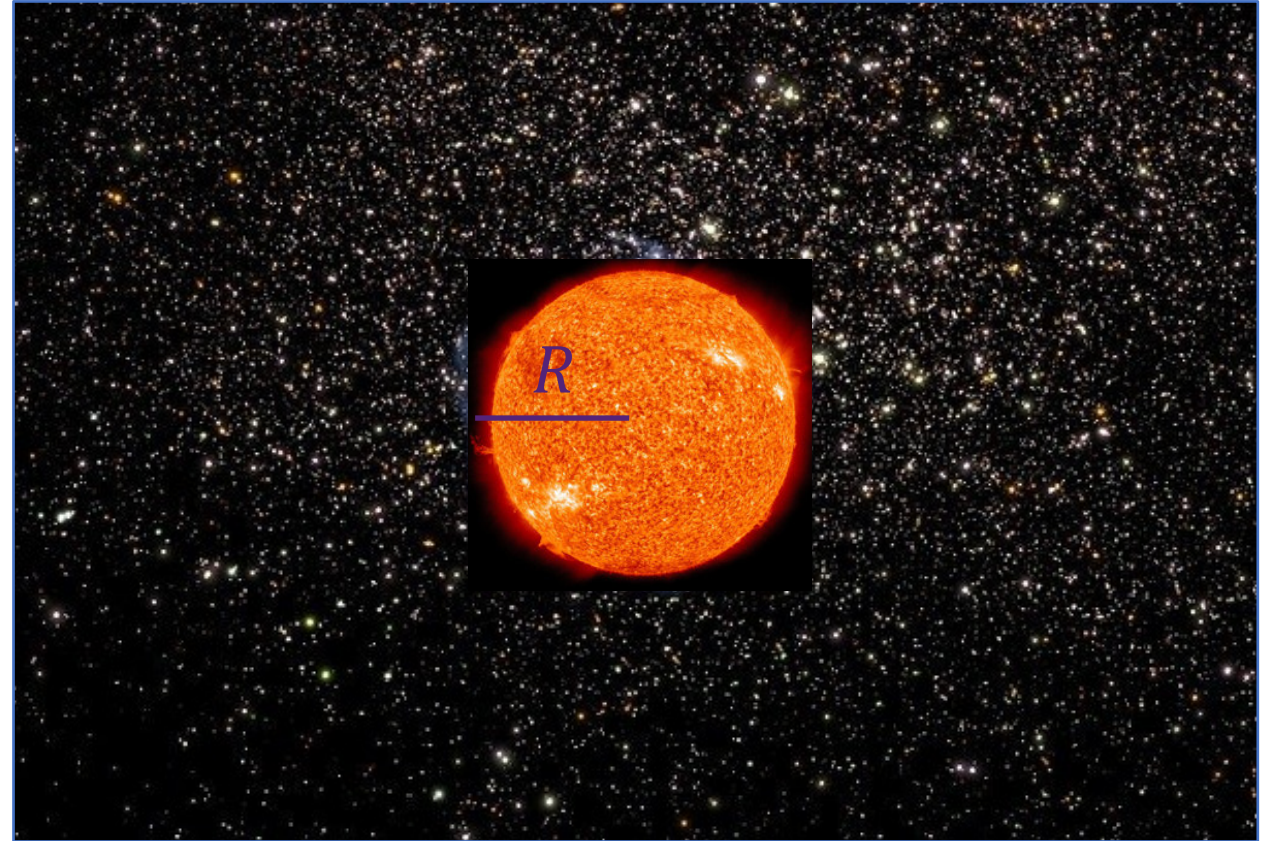
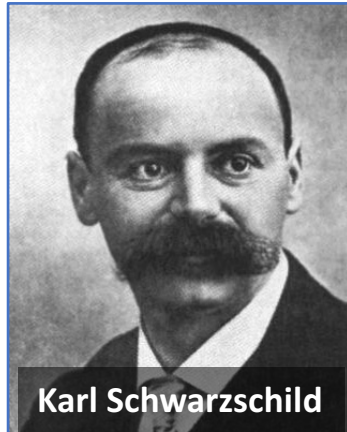
➔ Clocks in satellite and on earth de-synchronize with ~ 40 msec per day!

Gravitational time slowdown near a star with mass M:

$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{Rc^2}}$$

Schwartzschild radius:  $R_s = \frac{2GM}{c^2}$

$$\Delta t' = \Delta t \sqrt{1 - \frac{R_s}{R}}$$



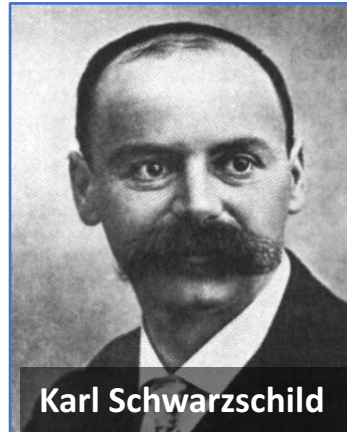


Gravitational time slowdown near a star with mass  $M$ :

$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{Rc^2}}$$

Schwartzschild radius:  $R_s = \frac{2GM}{c^2}$

$$\Delta t' = \Delta t \sqrt{1 - \frac{R_s}{R}}$$

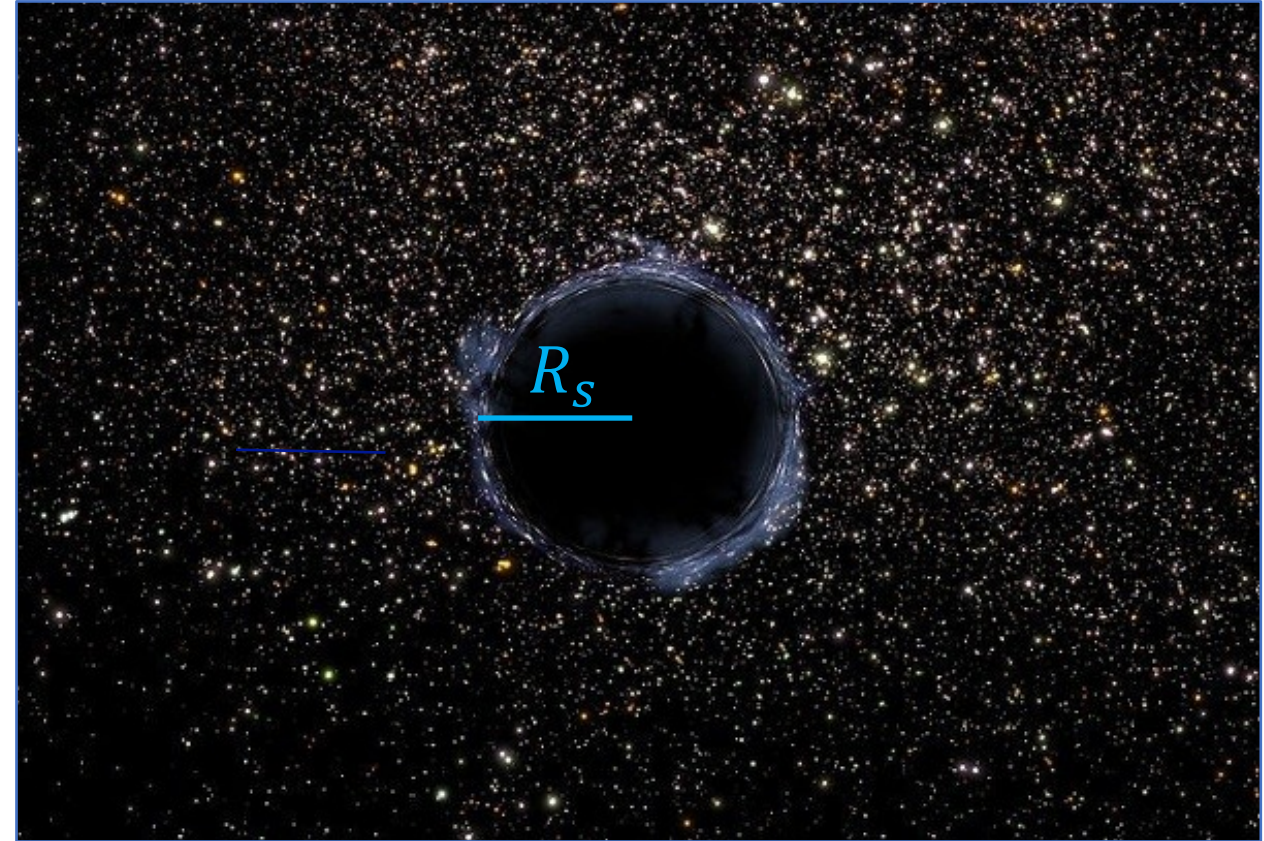


Time stand-still:

If  $R = R_s$  then  $\Delta t = 0$

Example our sun:  $G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2$  (Newton's gravitation constant)

$$M_{\text{sun}} = 2 \times 10^{30} \text{ kg}$$



(Time stands still at the horizon of a black-hole)

→  $R_s = 3 \text{ km}$  for a black hole

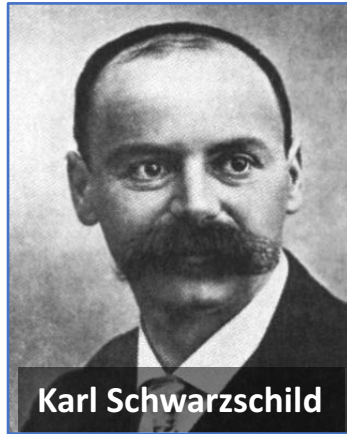


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$$\Delta t' = \Delta t \sqrt{1 - \frac{R_s}{R}}$$



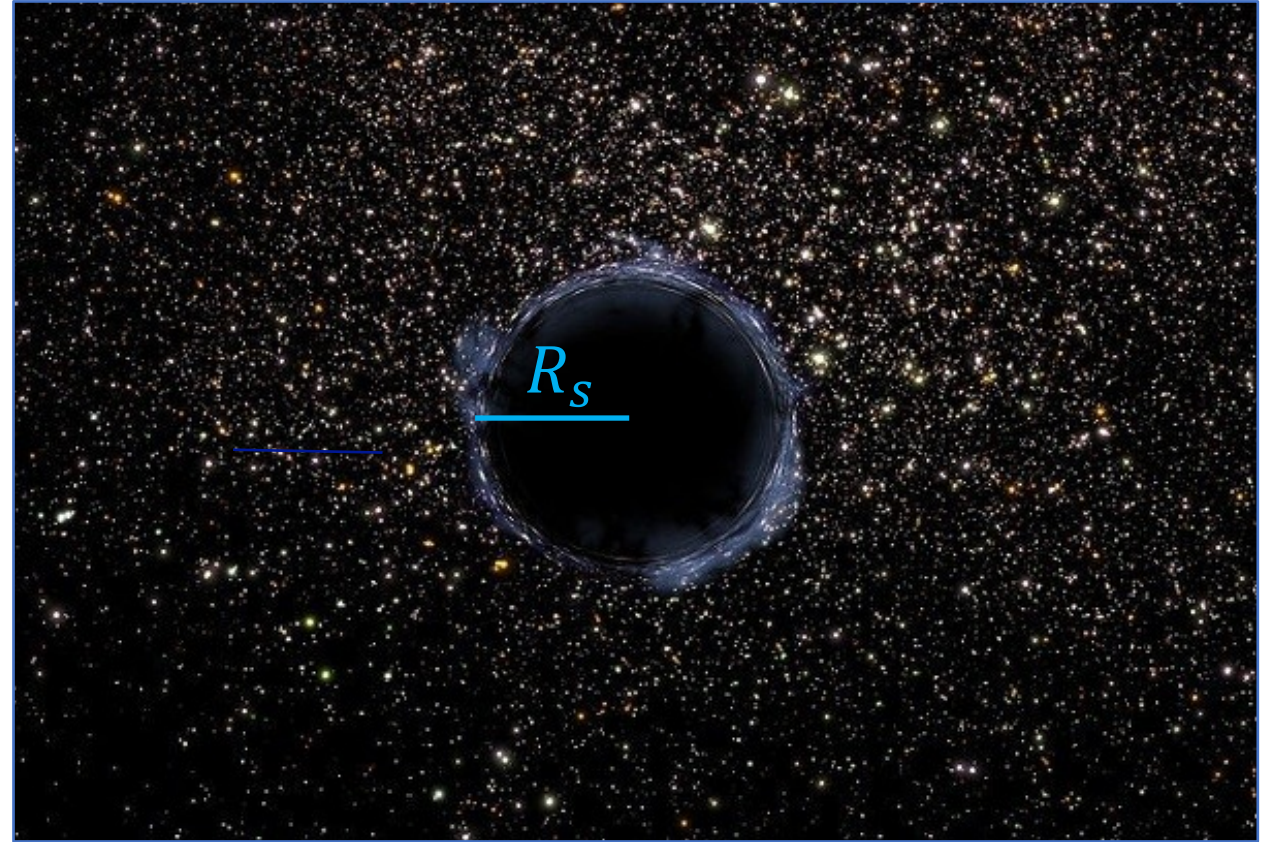
Time stand-still:

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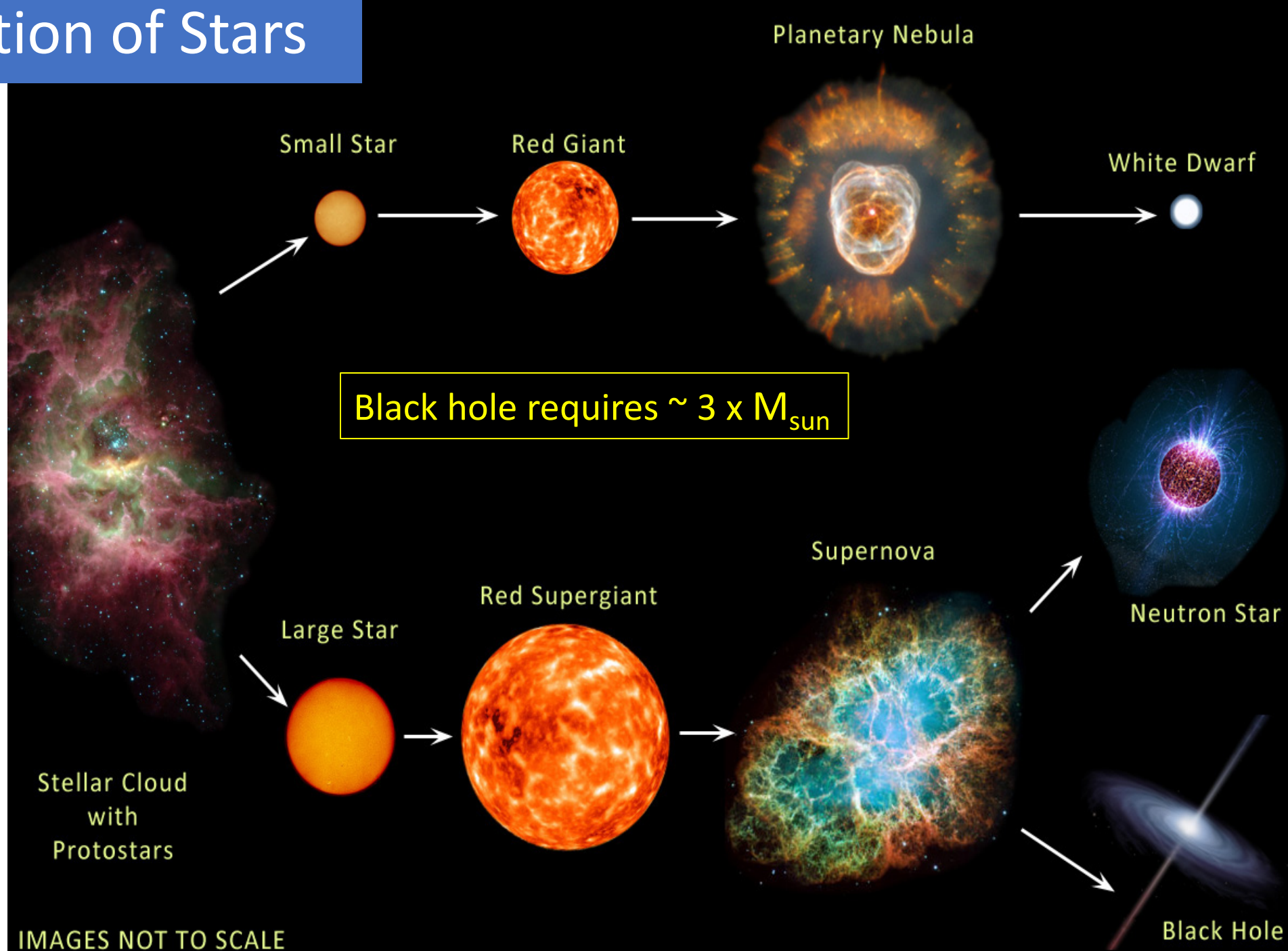
Example our earth:  $G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2$  (Newton's gravitation constant)

$$M_{\text{earth}} = 6 \times 10^{24} \text{ kg}$$

→  $R_s = 9 \text{ mm}$  for a black hole

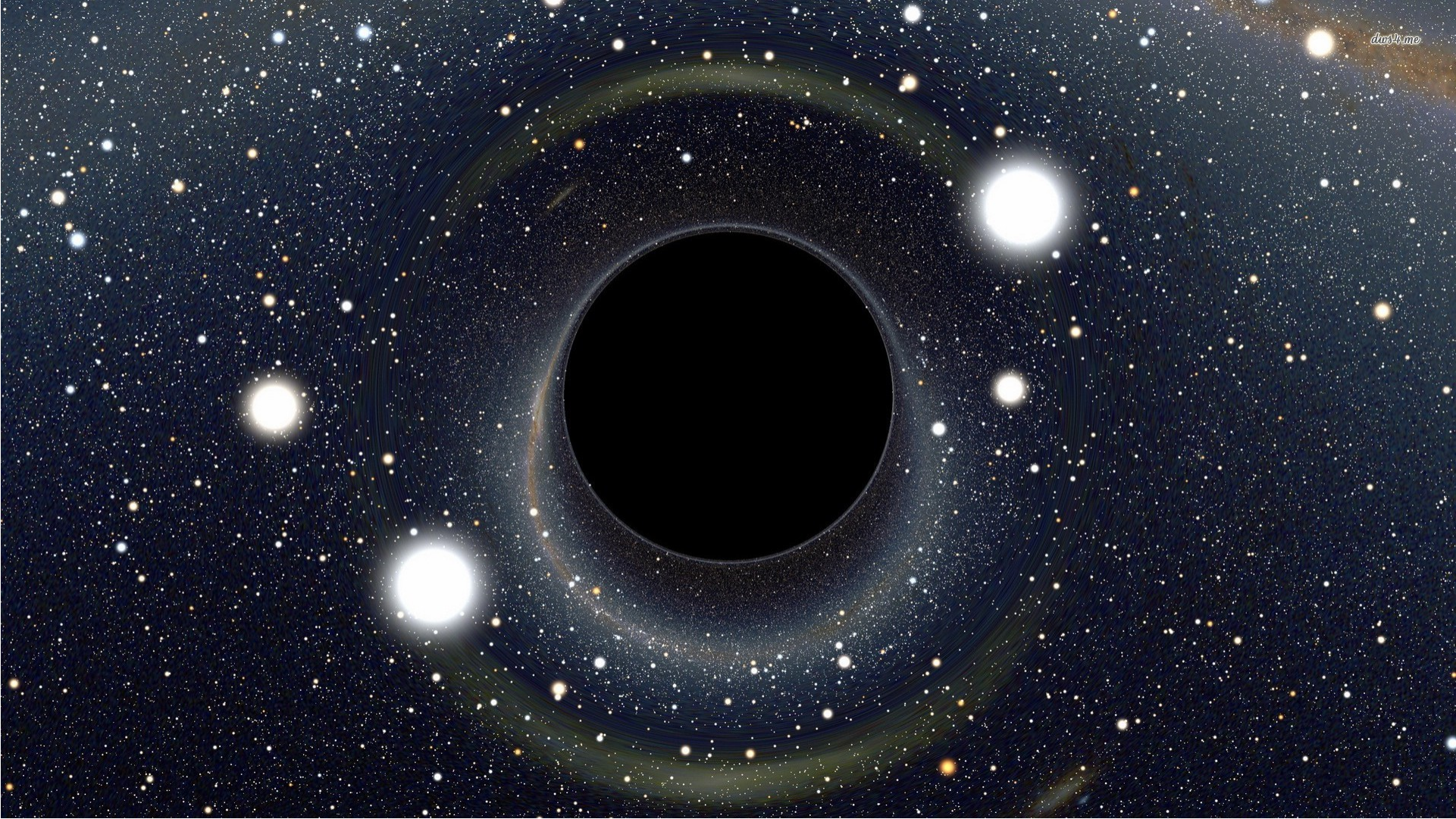


(Time stands still at the horizon of a black-hole)





# What is a black hole?

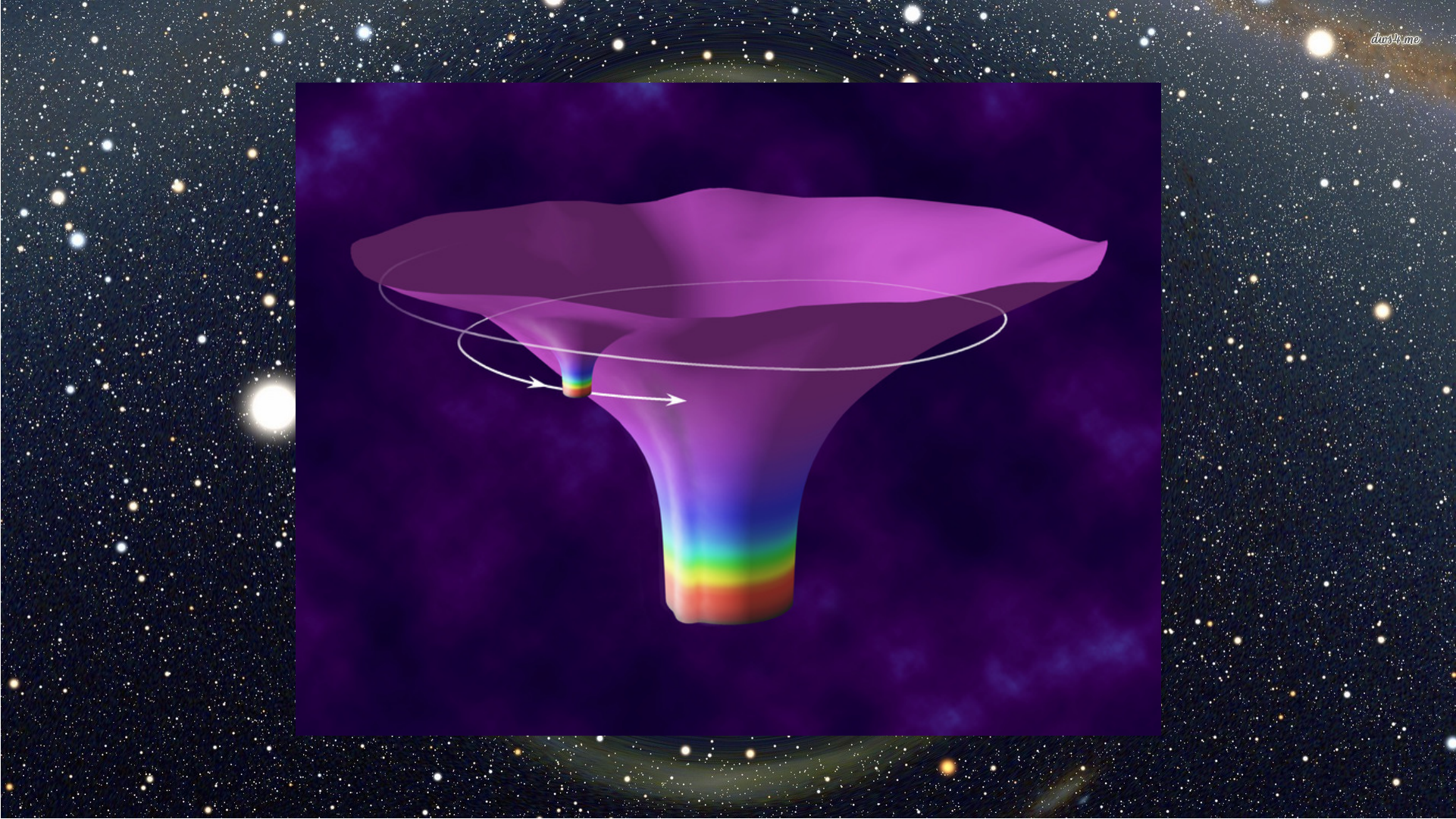


Purely curved space-time!



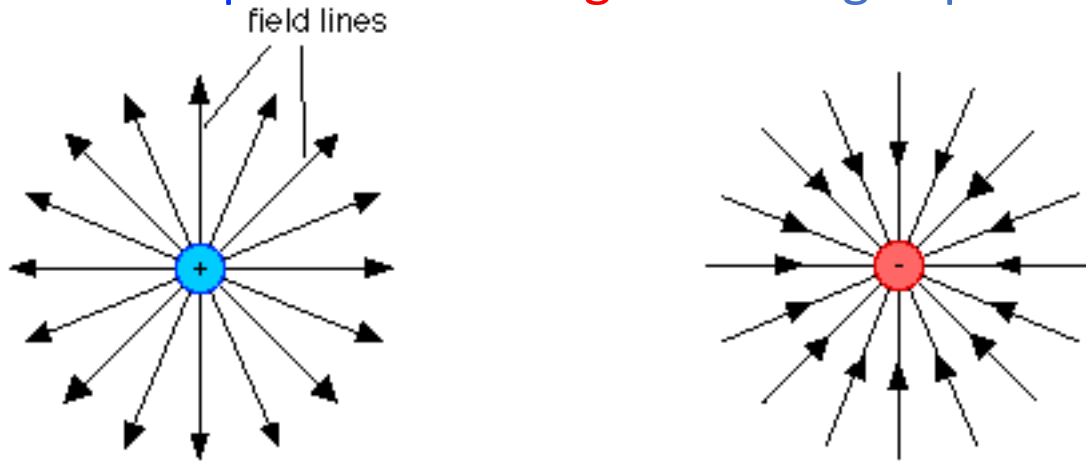
# What is a black hole?

24

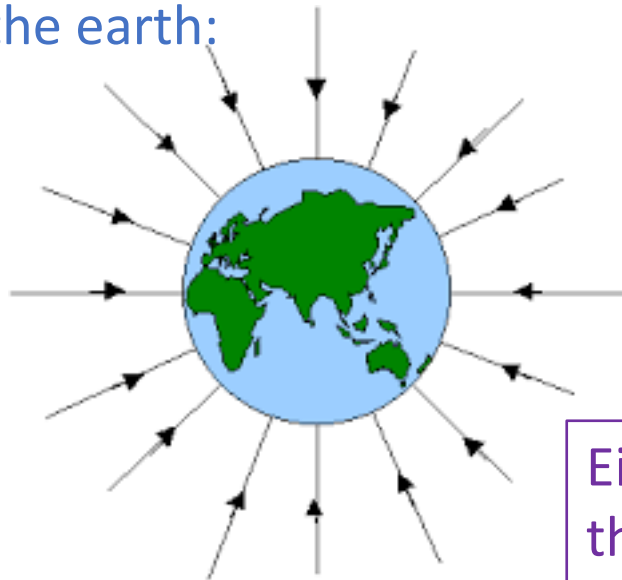


What happens when two black holes meet?

*Electric* field of **positive** and **negative** charged particle:



*Gravitational* field of the earth:

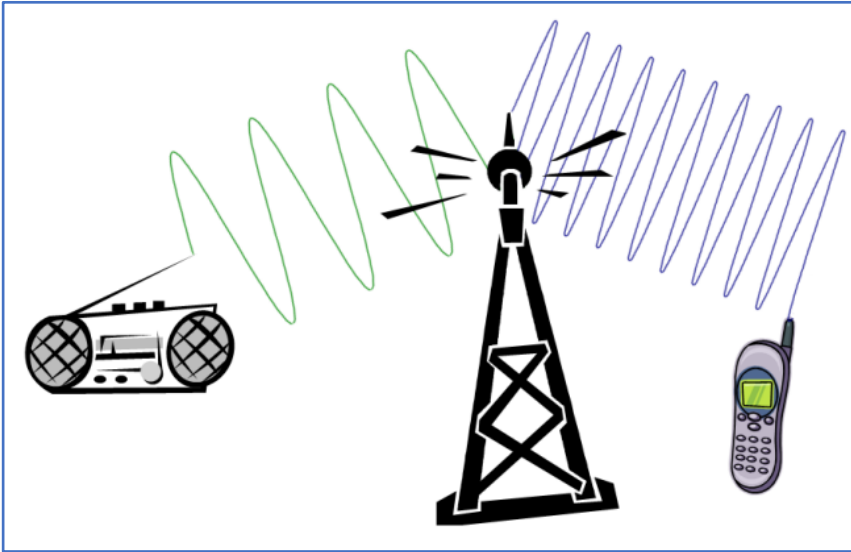


$$F_e = q E = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2}$$

$$F_g = m g = -G \frac{mM}{R^2}$$

Einstein spent most of his life looking for a unified theory of electromagnetism and general relativity.





## Electromagnetic waves:

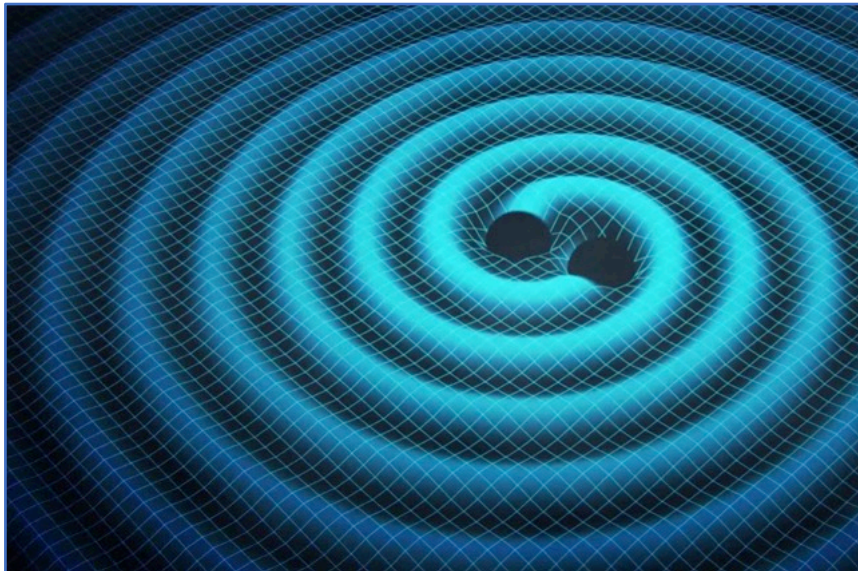
Caused by accelerating electric particles (electrons)  
eg.: radio-emission

Maxwell equations:

$$\partial_{\mu} F^{\mu\nu} = \frac{4\pi}{c} J^{\nu}$$

E and B fields

Electric charge  
and currents



## Gravitational Waves:

Caused by moving masses.

Requires very heavy masses → **black holes.**

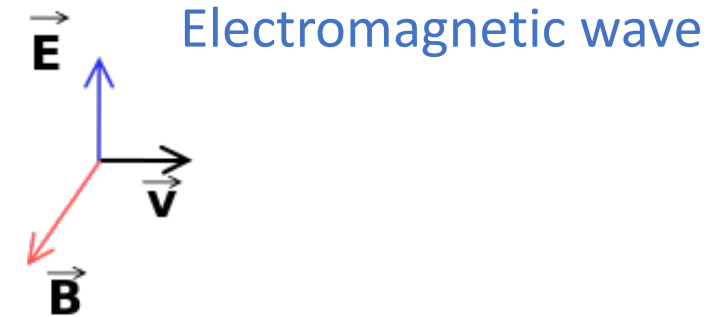
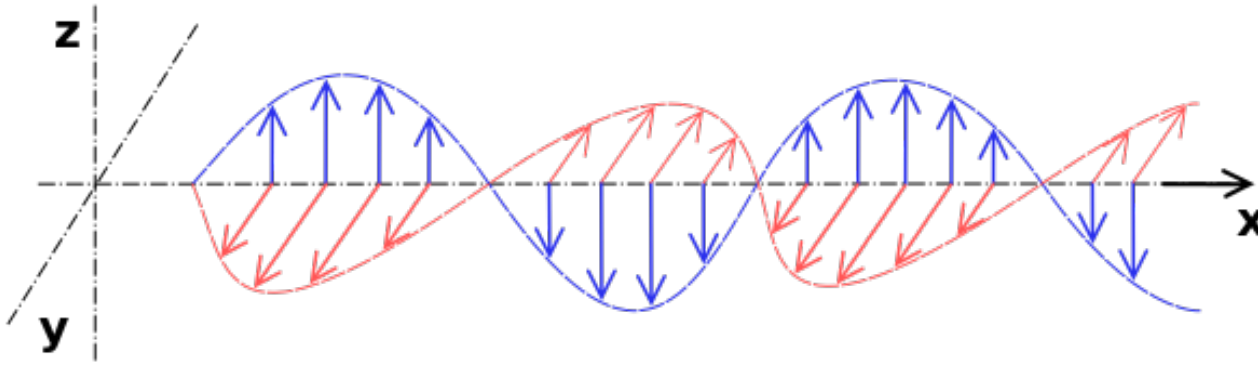
(Einstein thought these couldn't be observed)

Einstein equations:

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

Space-time fields

Mass and mass-flow



## Electromagnetic wave:

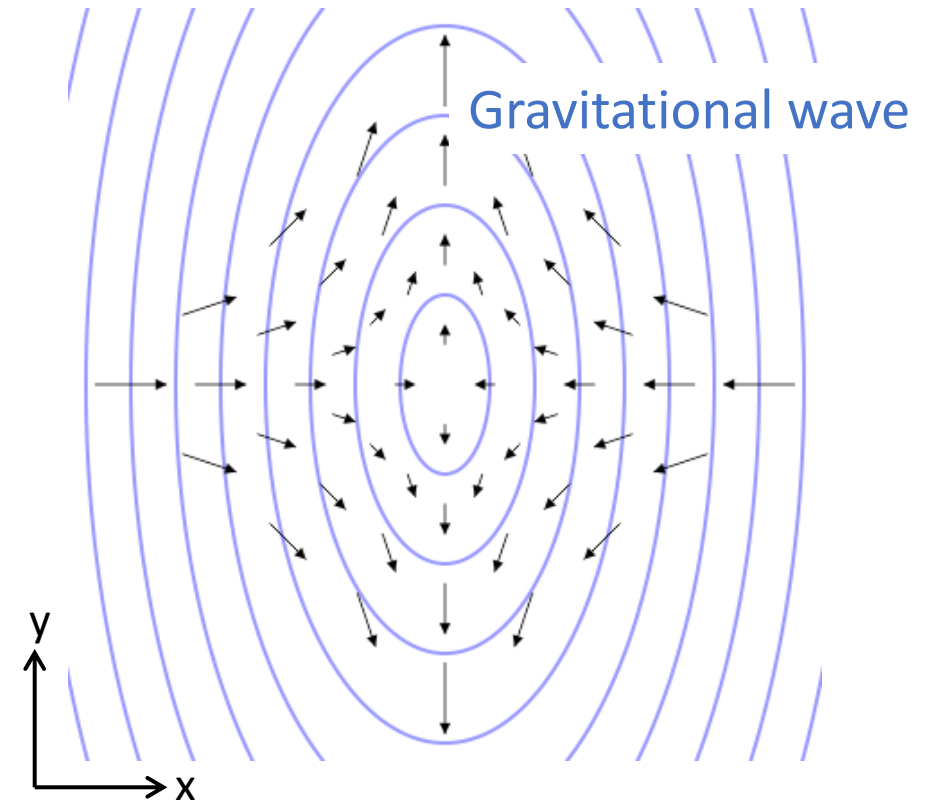
Changing electric and magnetic field propagating through space.

Caused by moving (accelerating!) electric charges.

## Gravitational wave:

Changing space-time field.

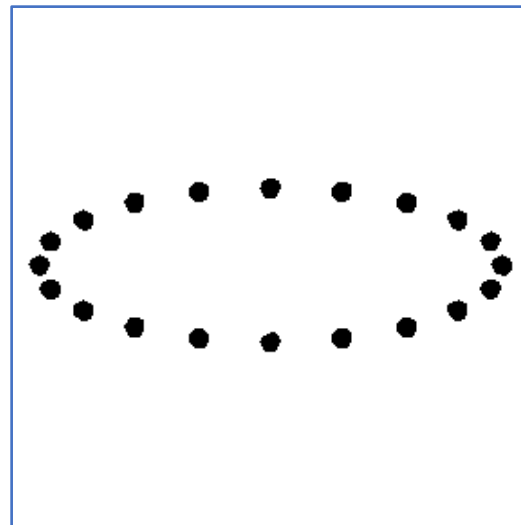
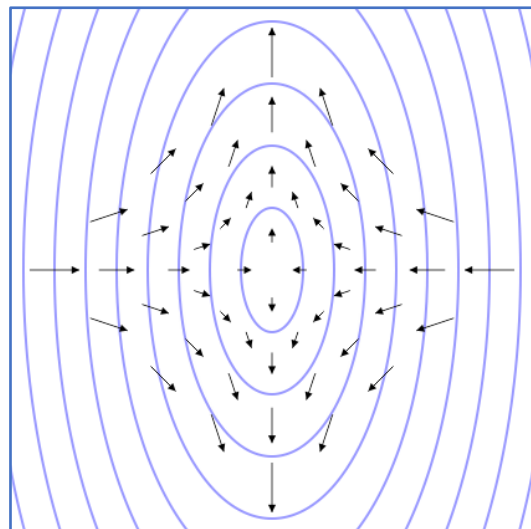
Caused by moving (accelerating!) masses.



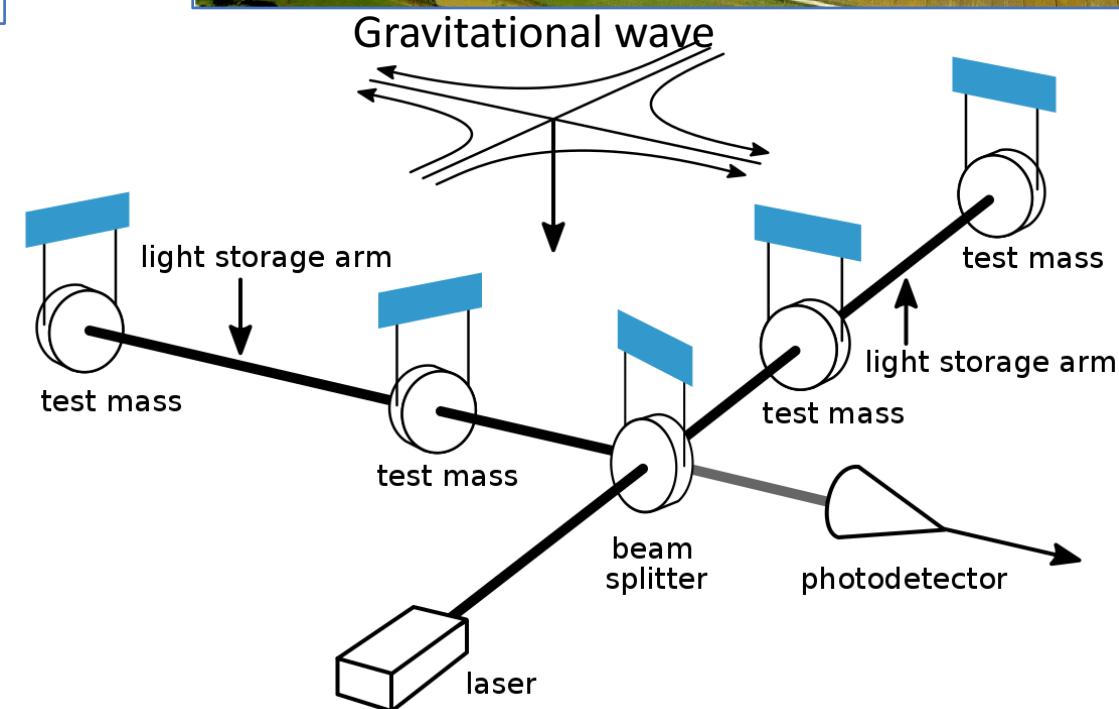
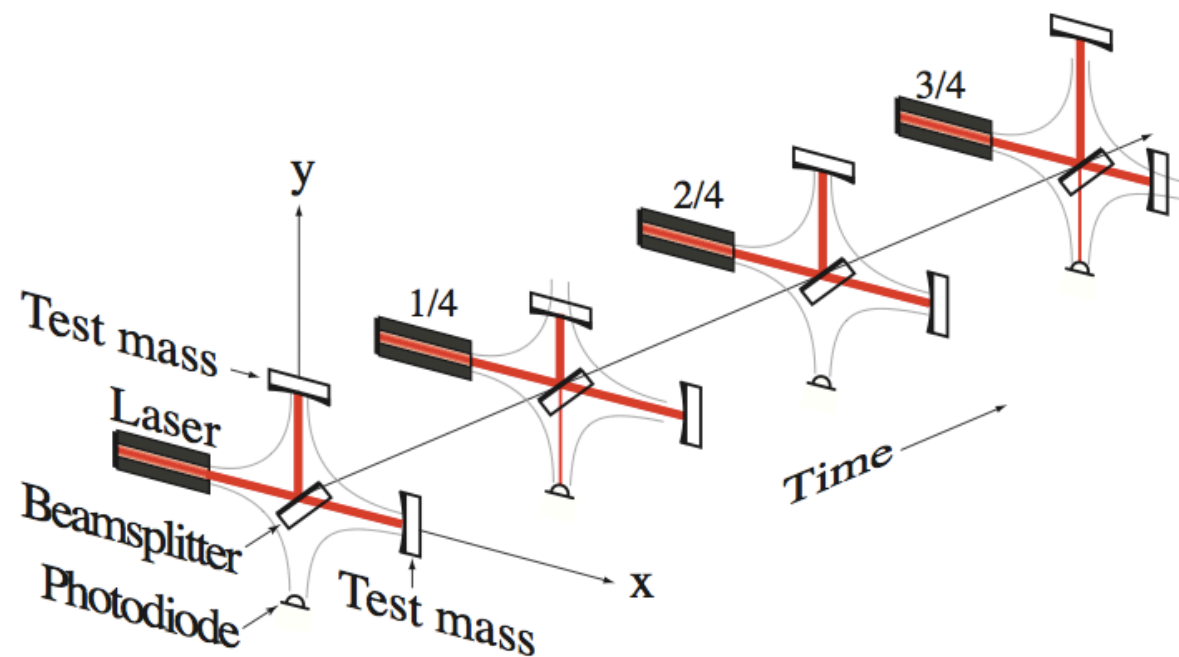


# Remember the interferometer!

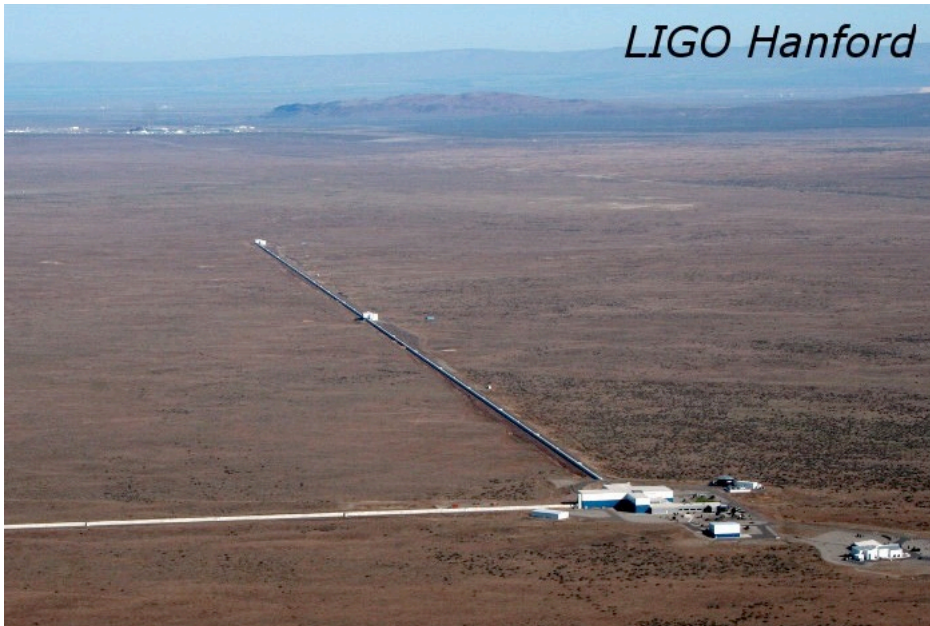
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Virgo experiment near Pisa

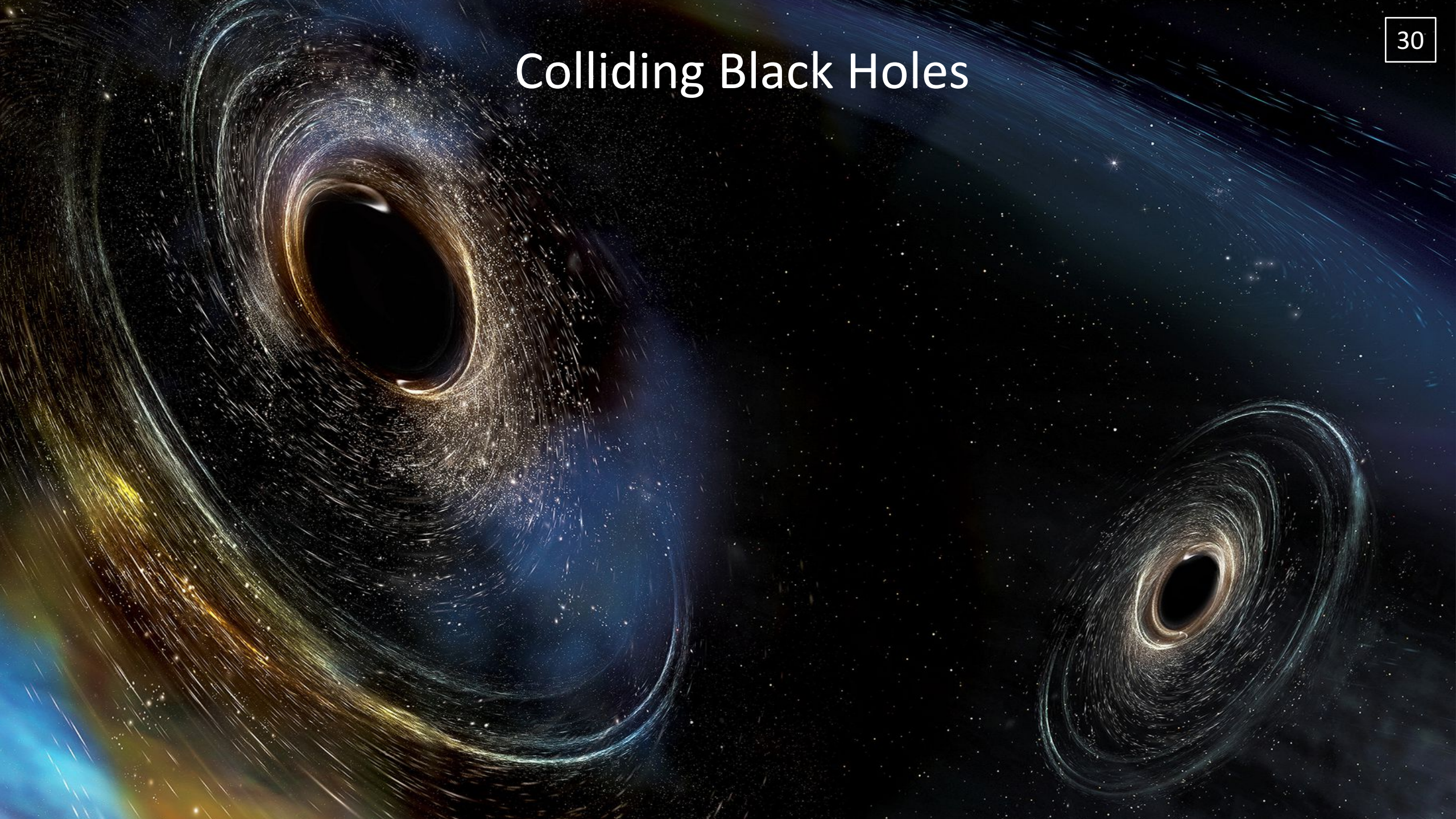




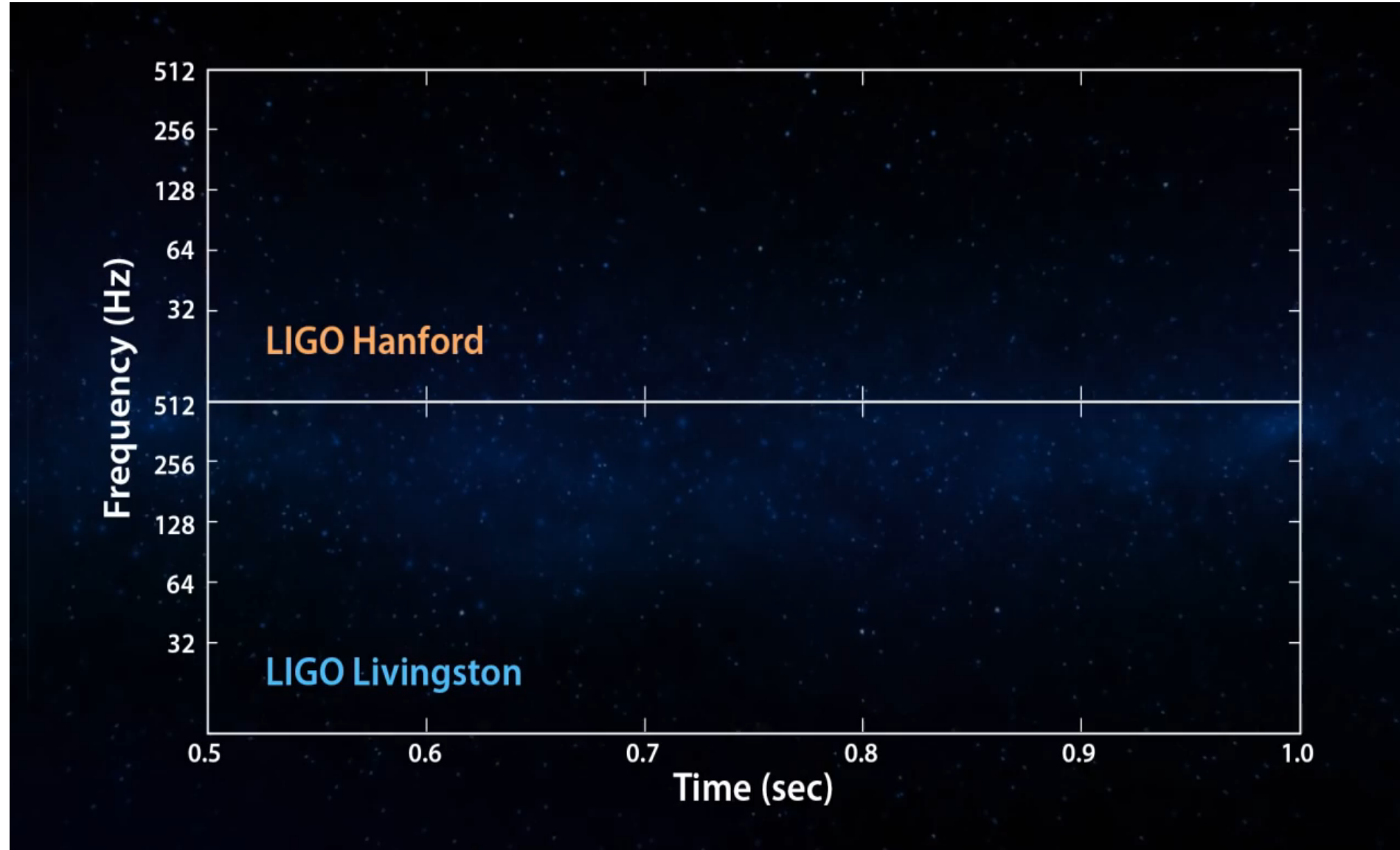




# Colliding Black Holes

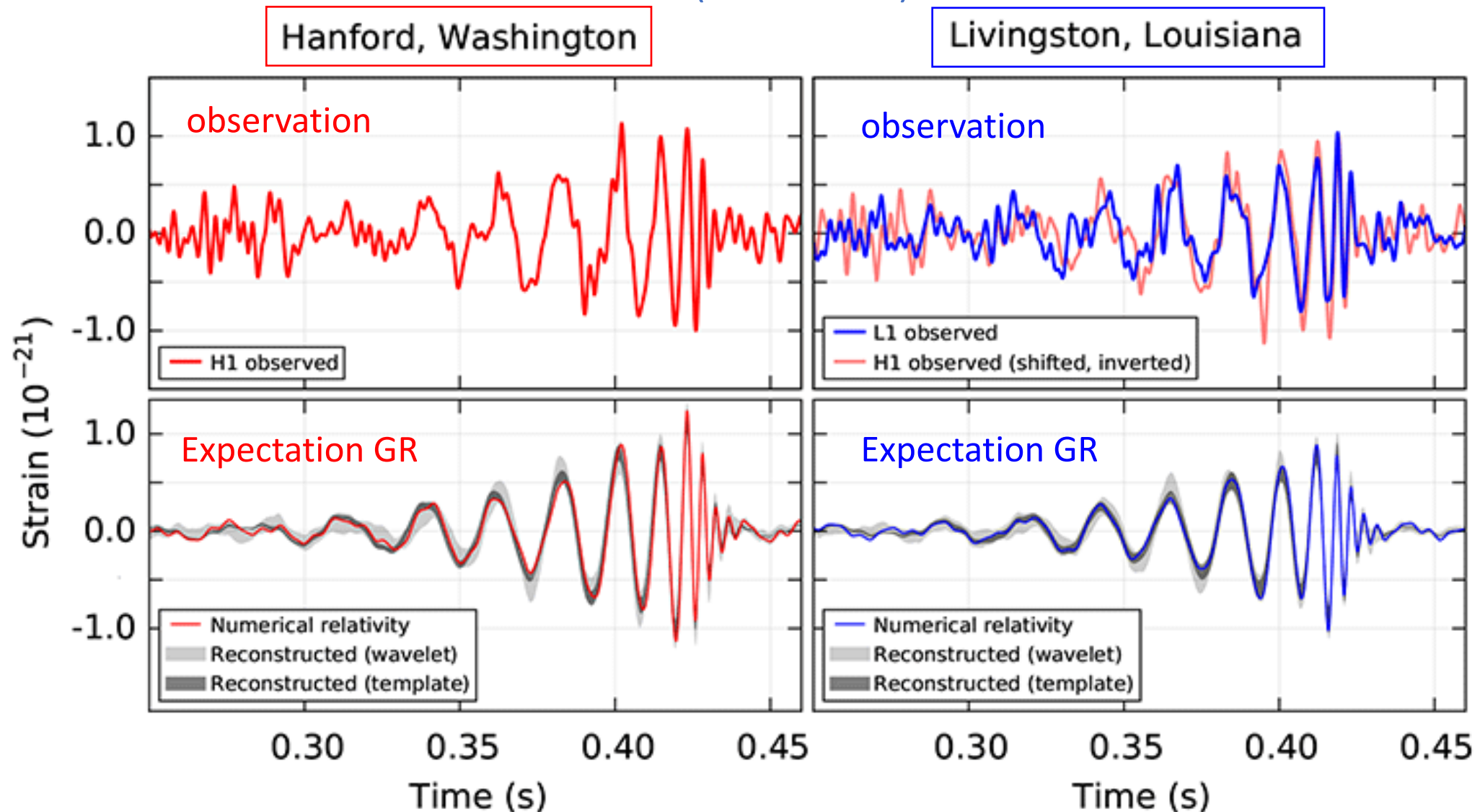






“Chirp” of colliding black holes at 1.3 billion lightyears distance

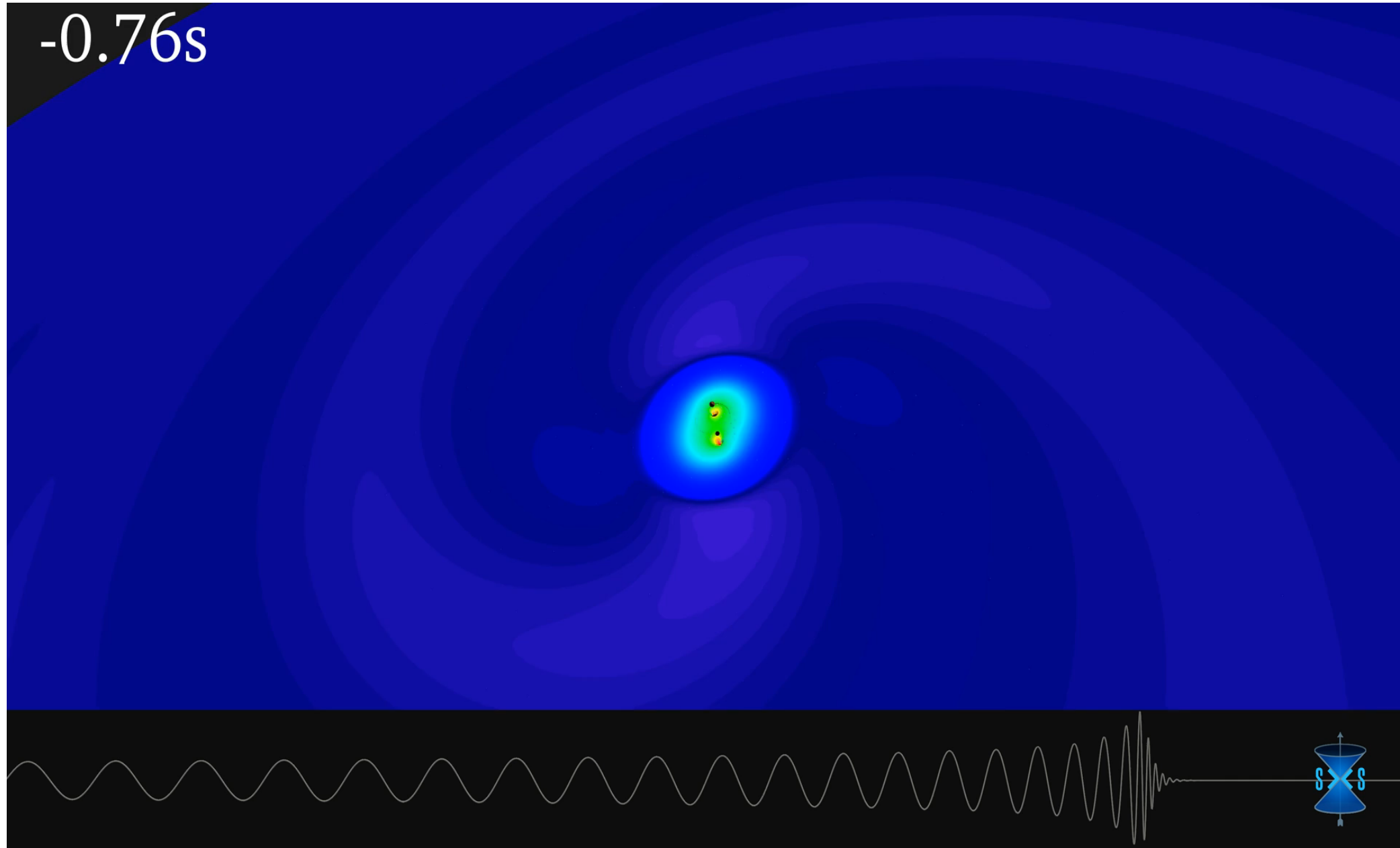
(GW150914)





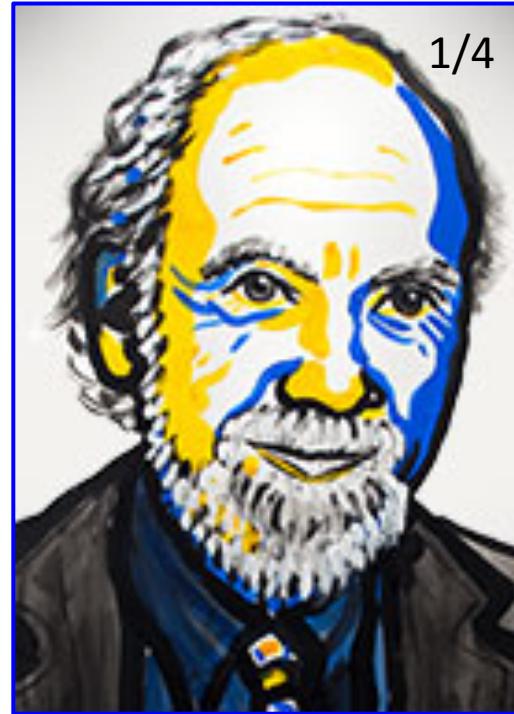


More energy was emitted in gravitational waves than all the visible (EM) energy of all stars in the universe!





Rainer Weiss



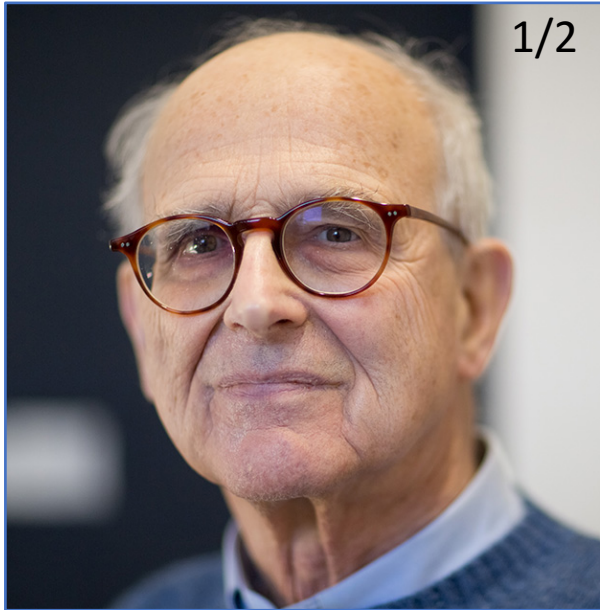
Barry C. Barish



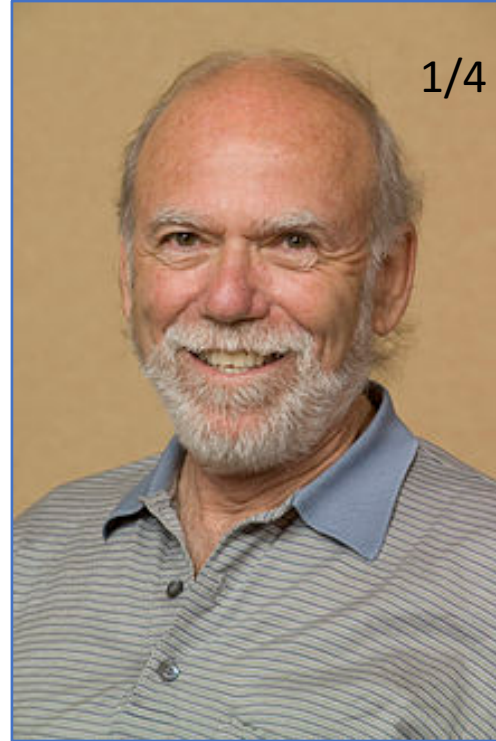
Kip S. Thorne

*“For decisive contributions to the LIGO detector and the observation of gravitational waves”*





Rainer Weiss



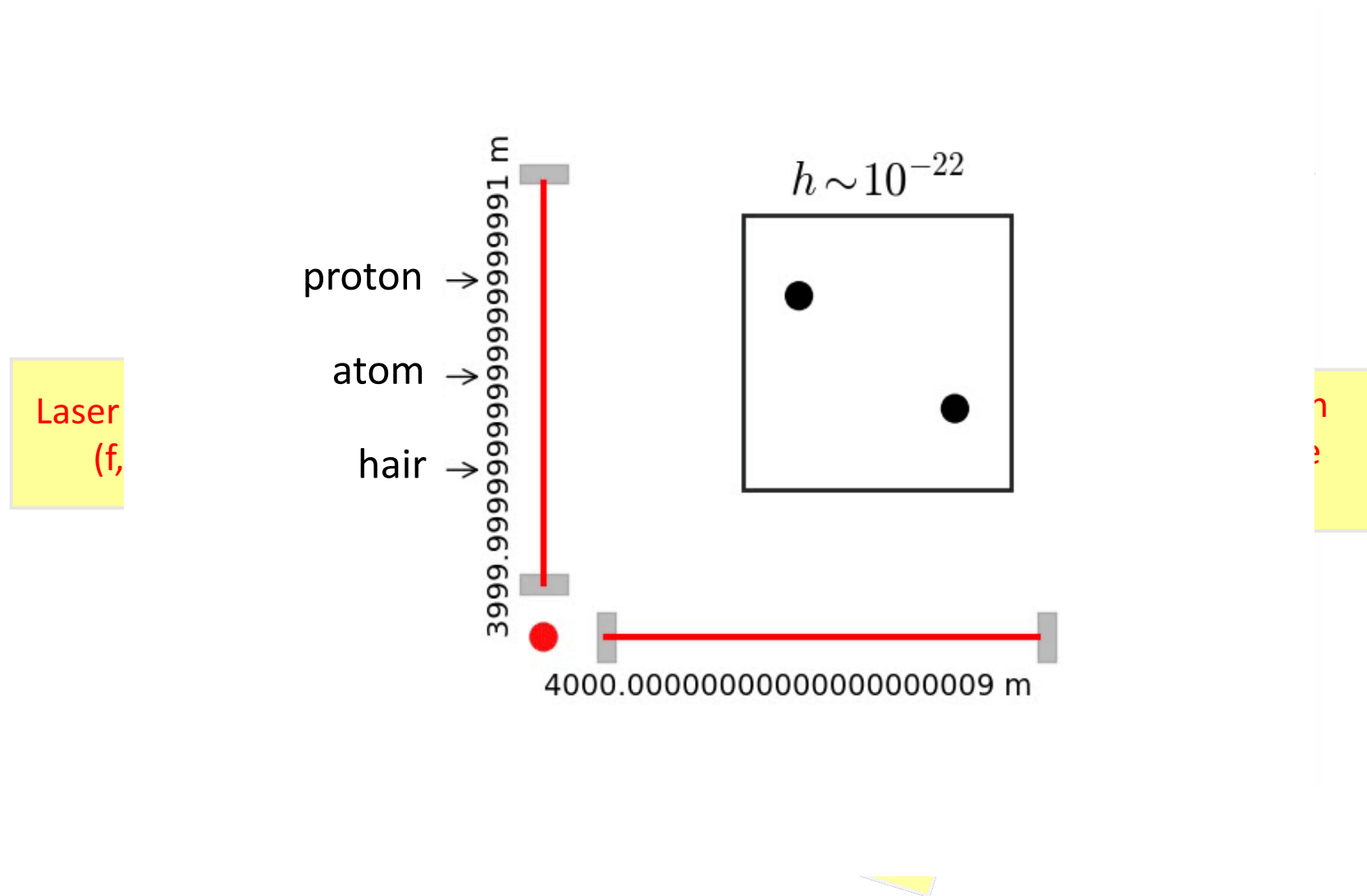
Barry C. Barish



Kip S. Thorne

***“For decisive contributions to the LIGO detector and the observation of gravitational waves”***

## 38





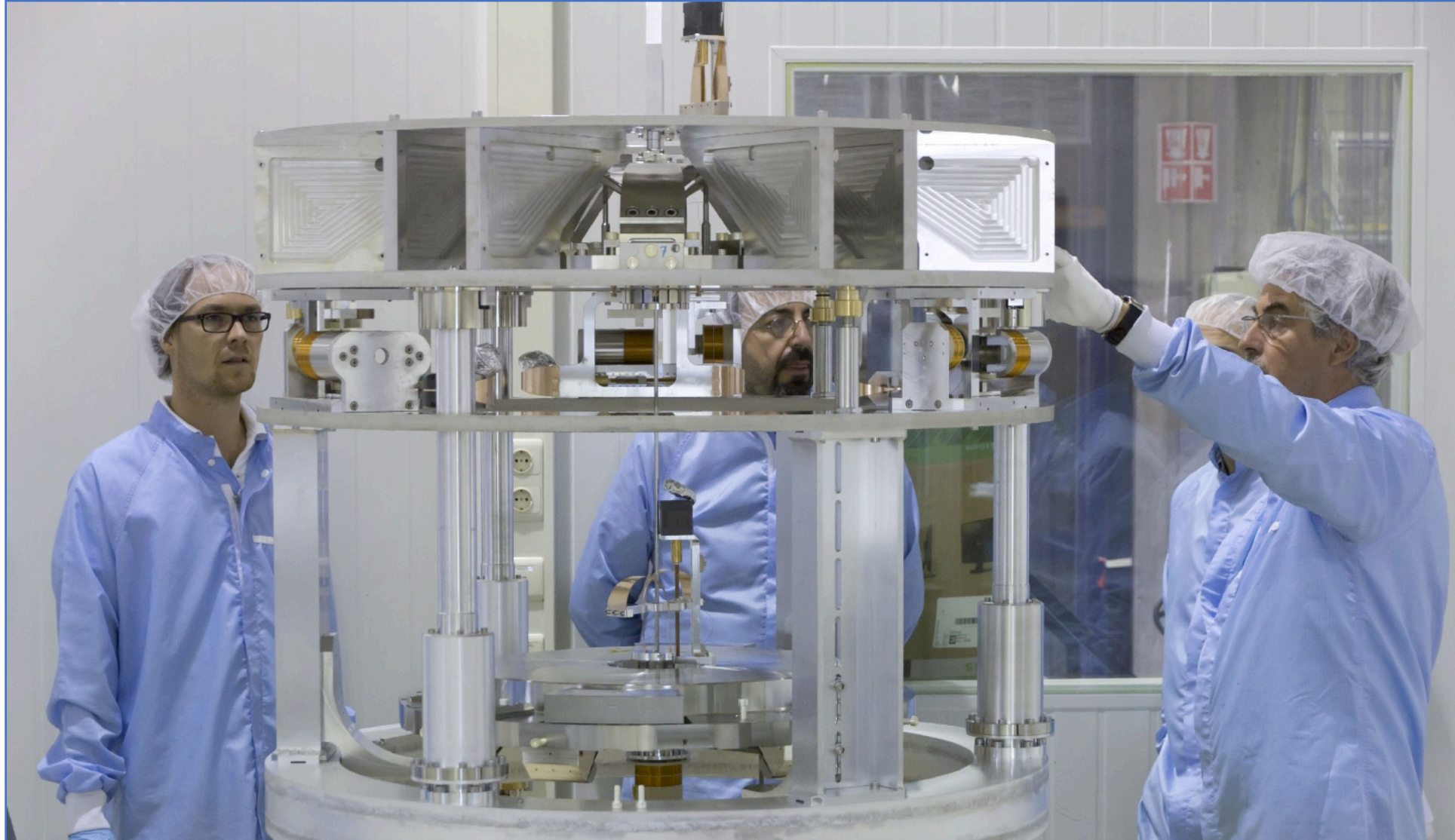
# The Virgo Experiment in Pisa







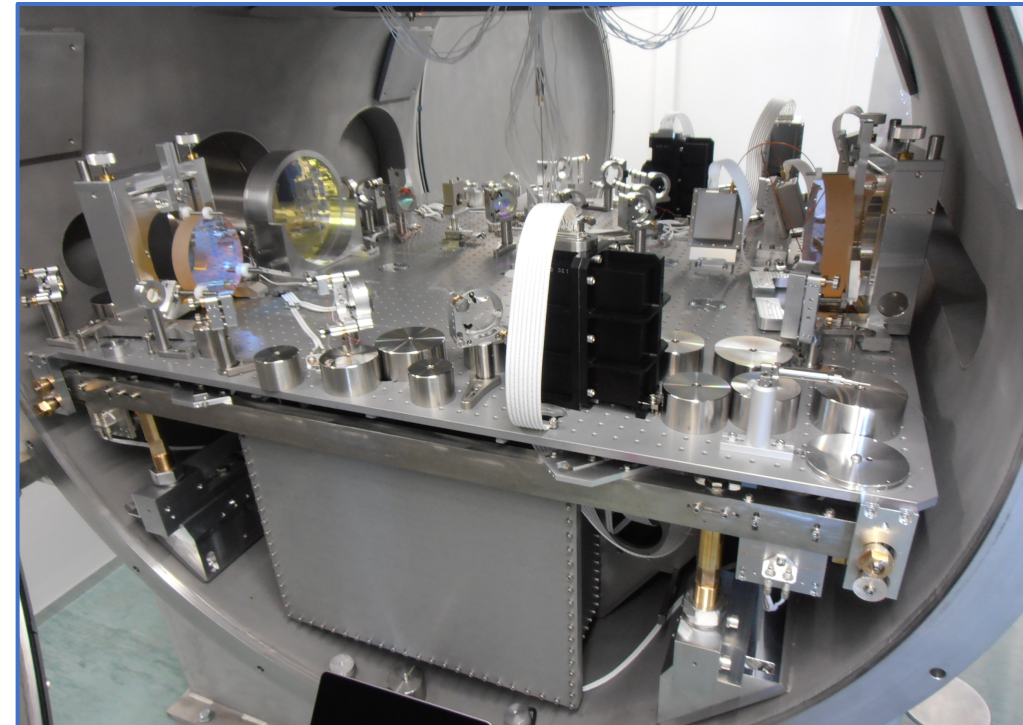
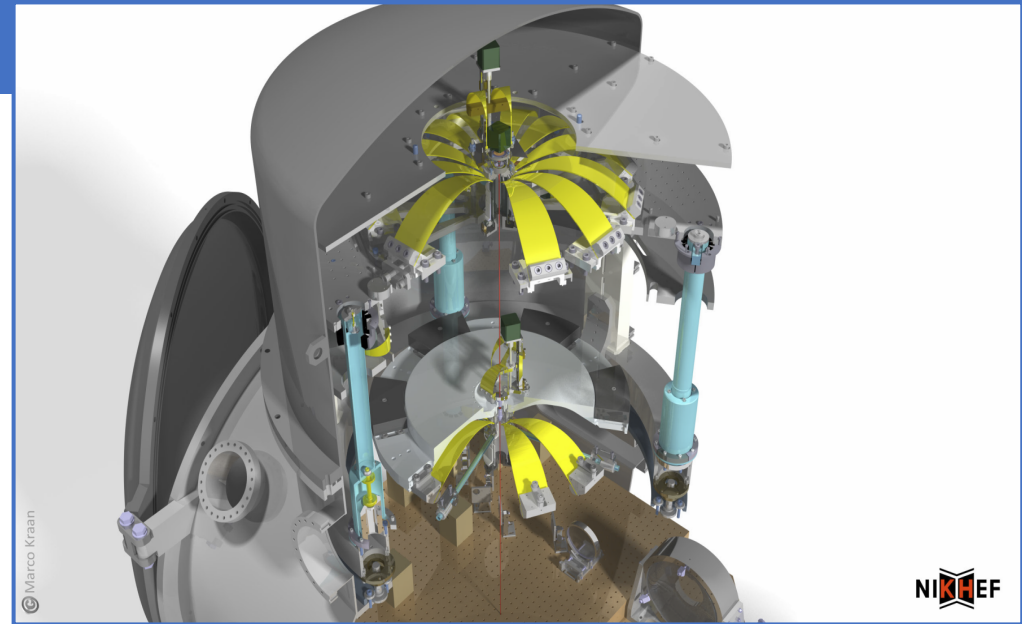
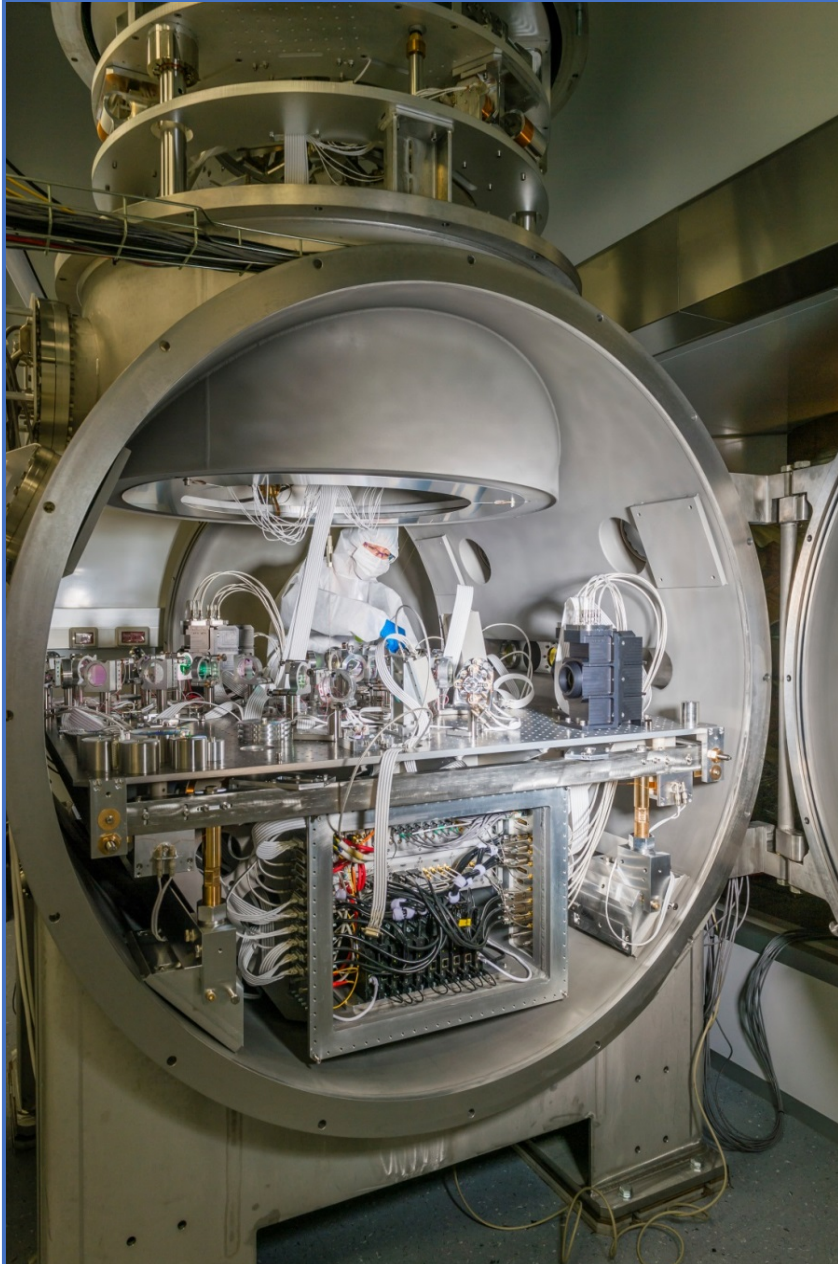
Largest vacuüm vessel in Europe: Pressure  $\sim 10^{-10}$  mbar



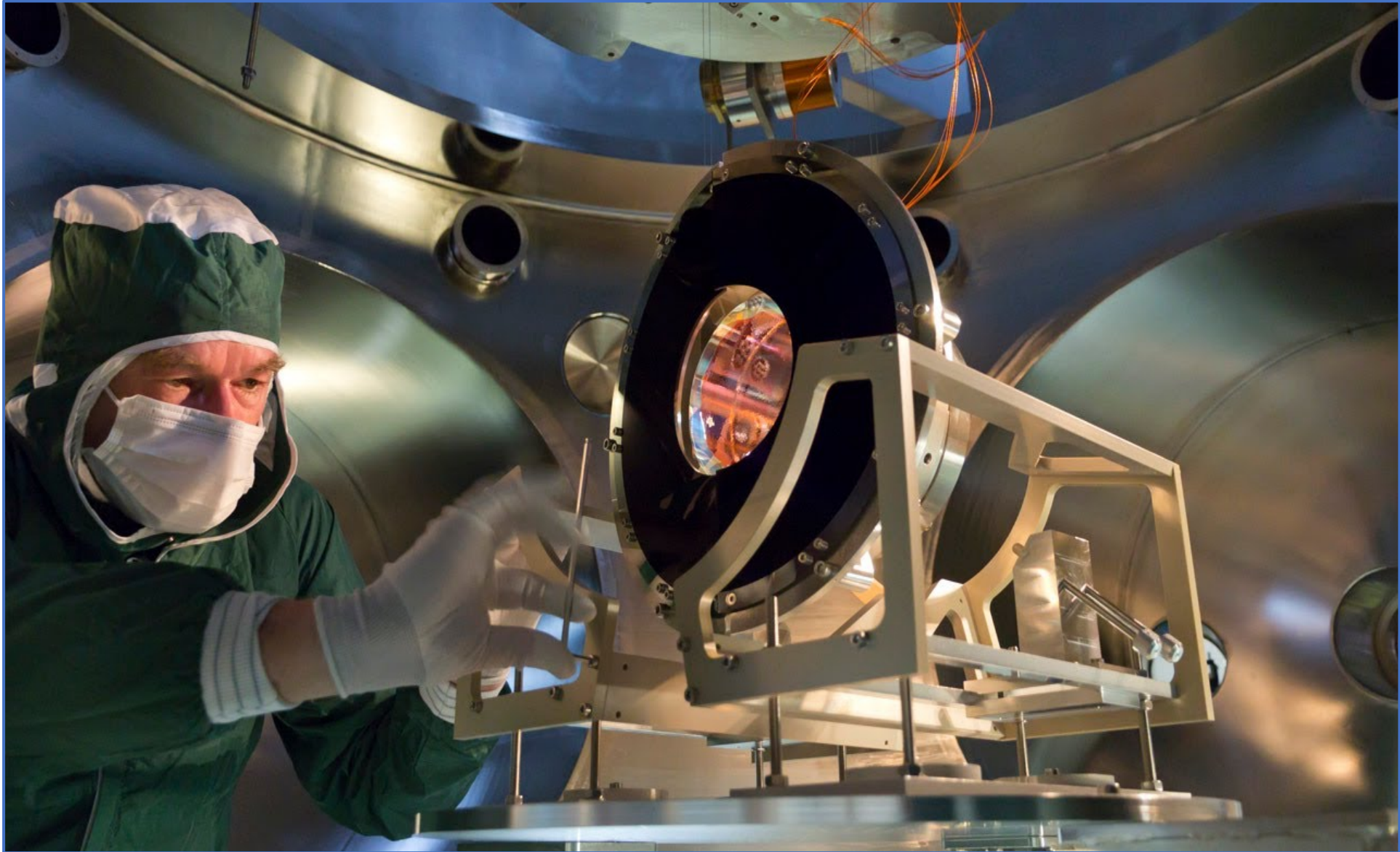


# Seismic Damping Table

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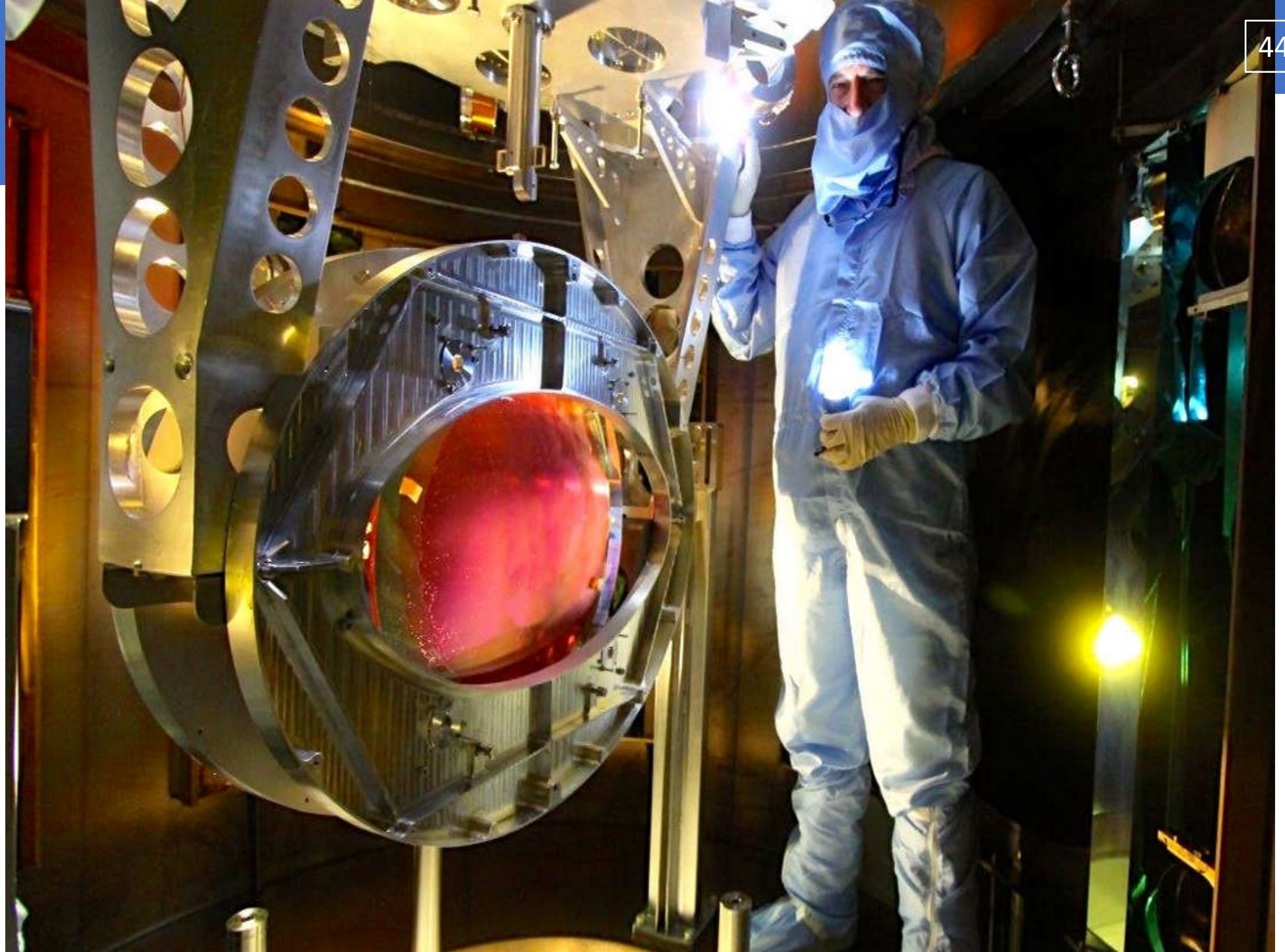




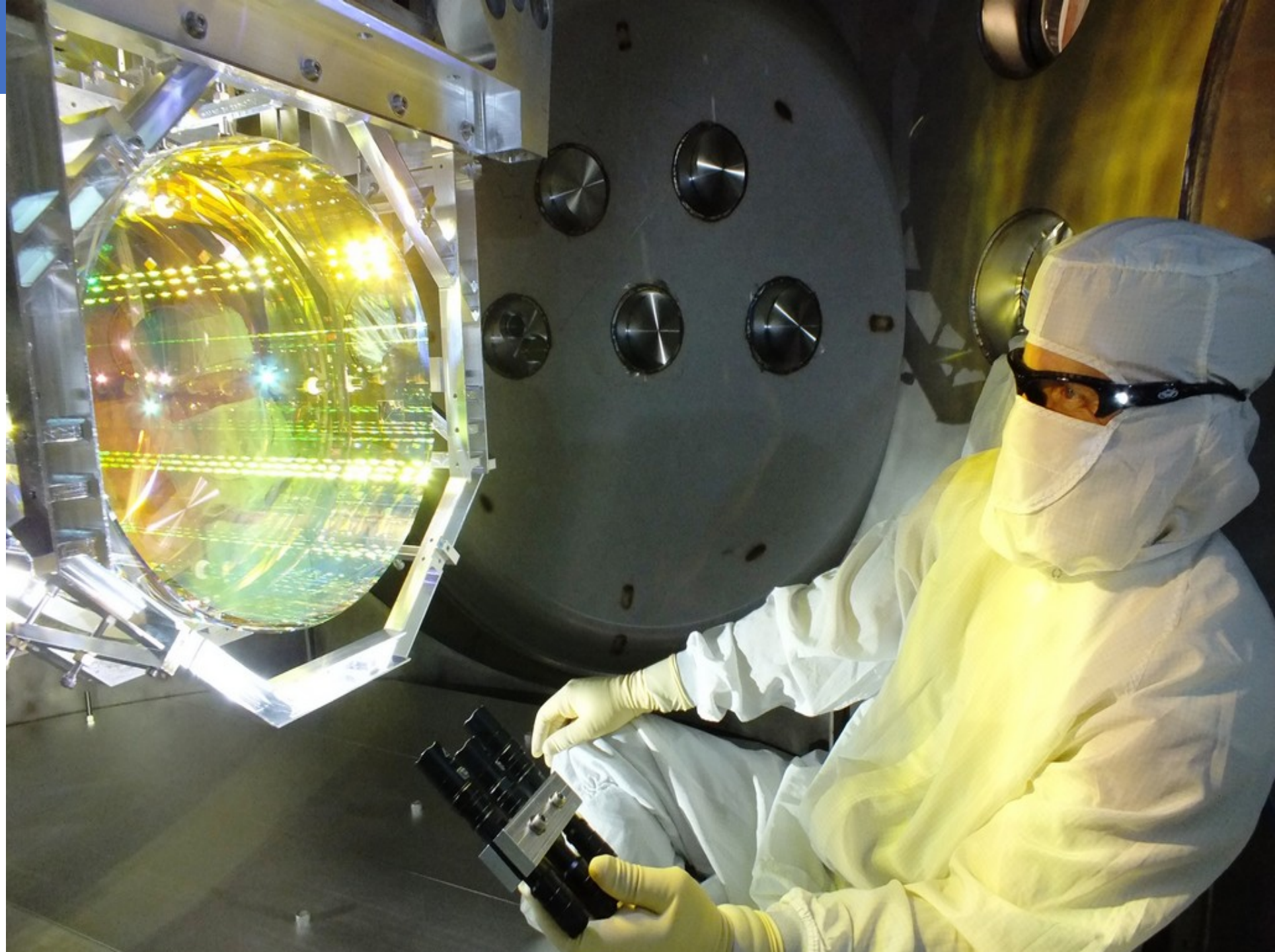




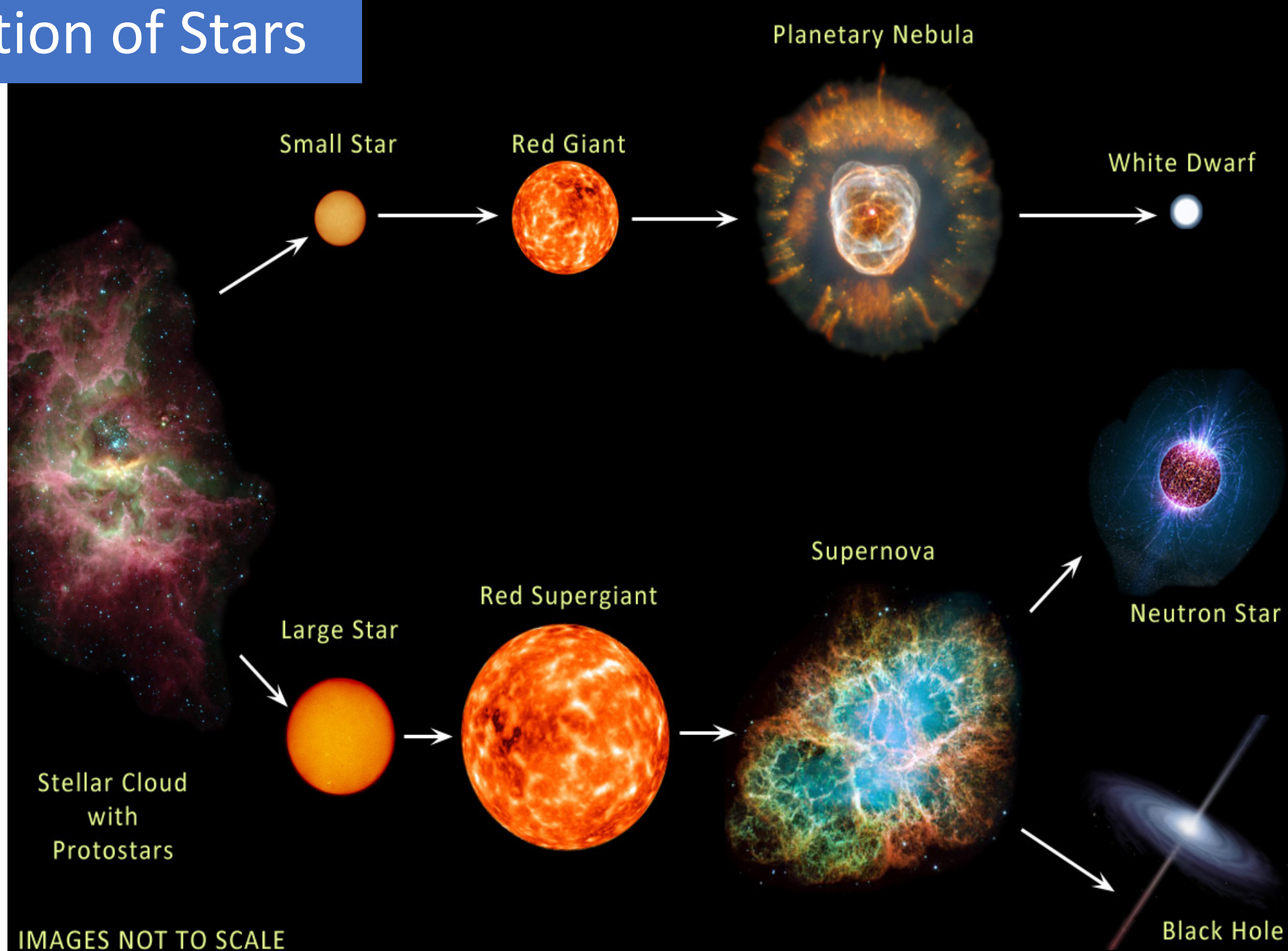
# Beam Splitter







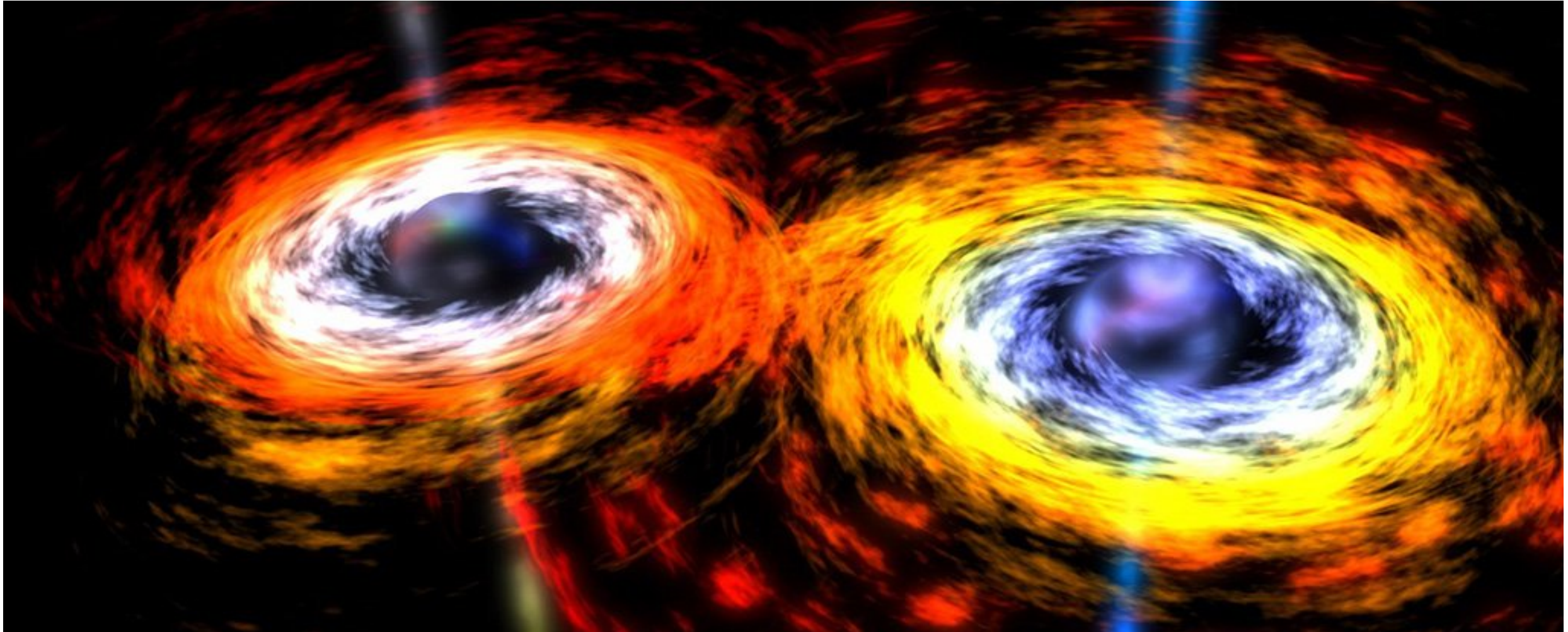




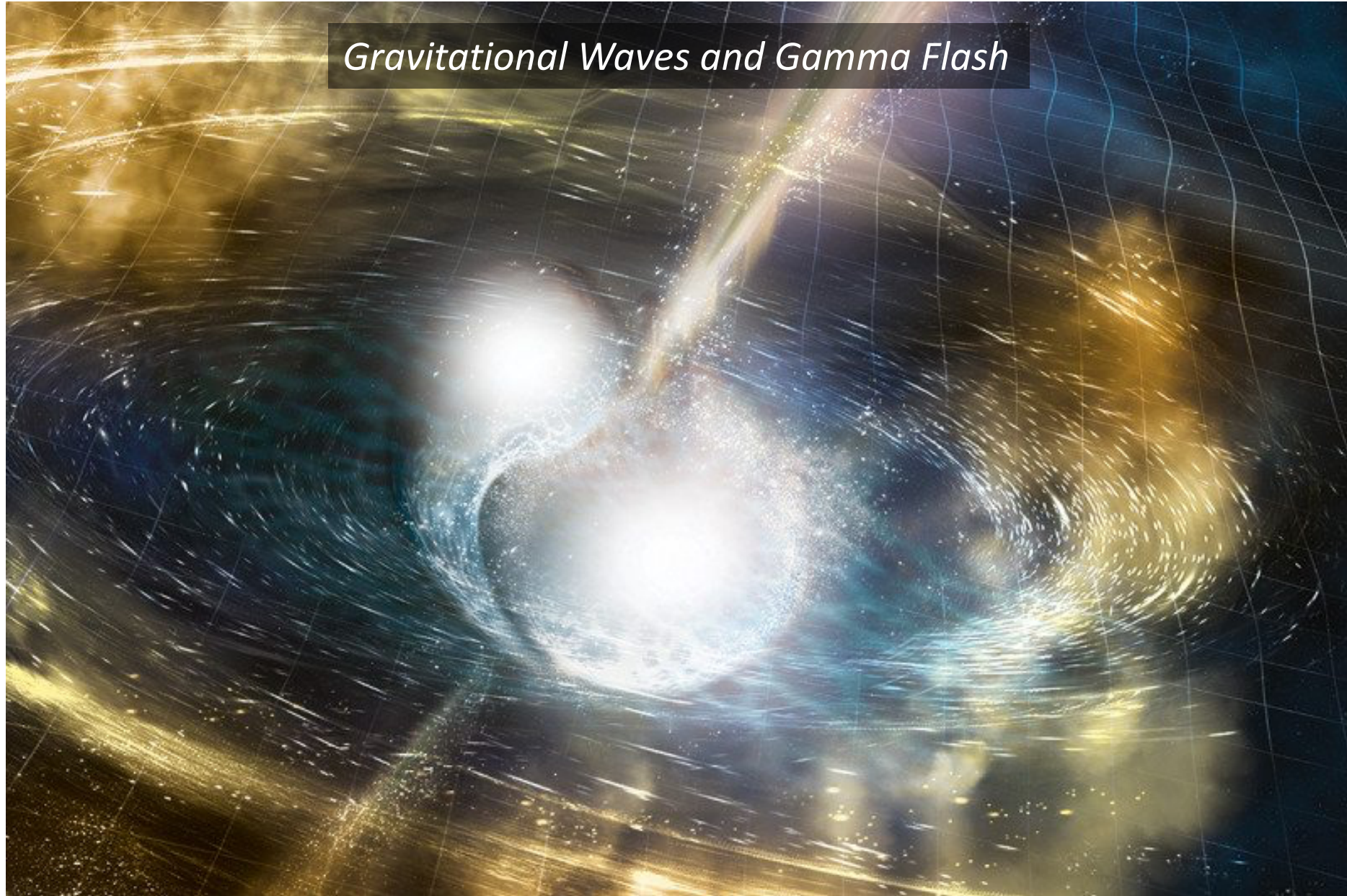




*Gravitational Waves and ...*









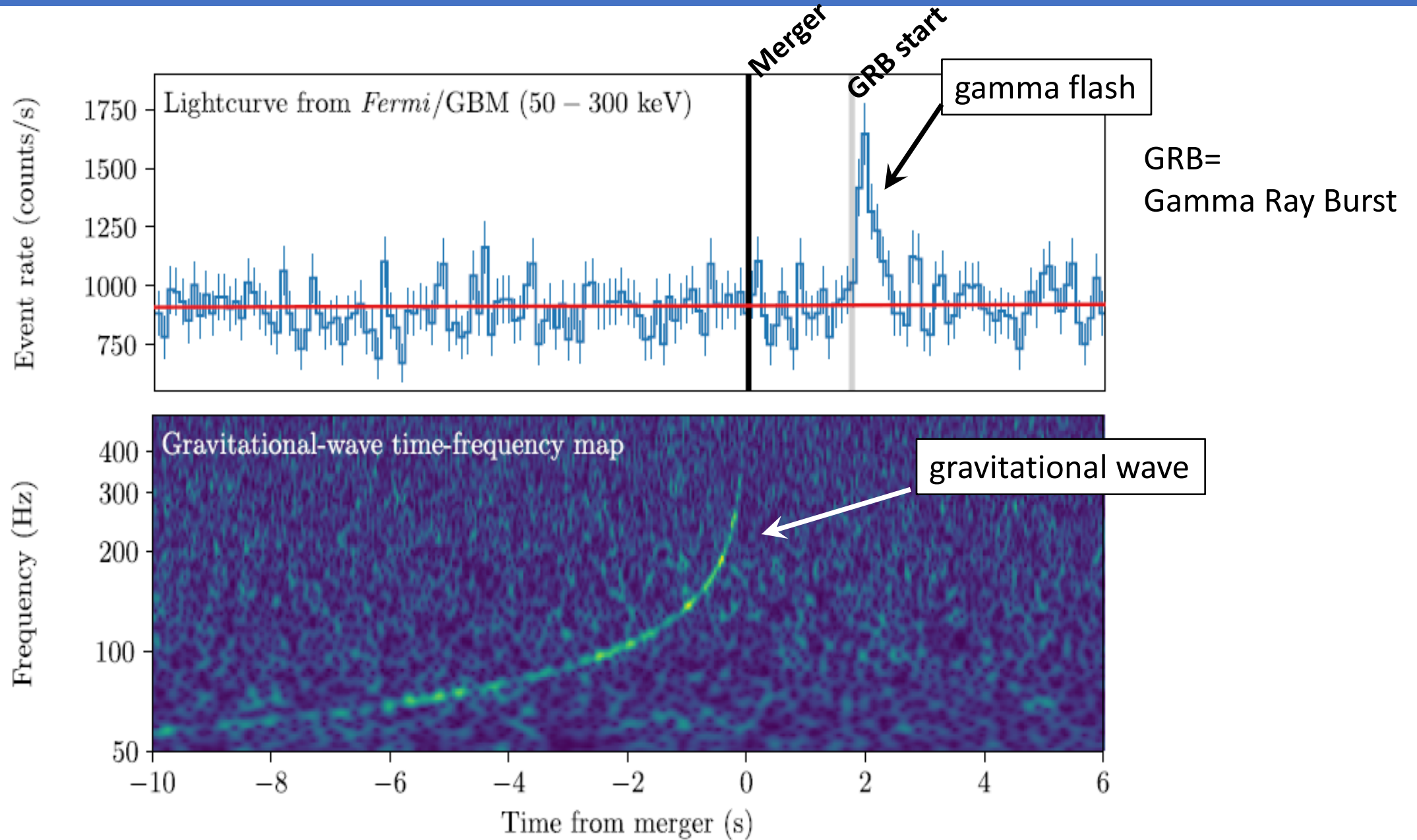
Gamma flash 1.7 sec later...

INTEGRAL

Fermi Space Telescope



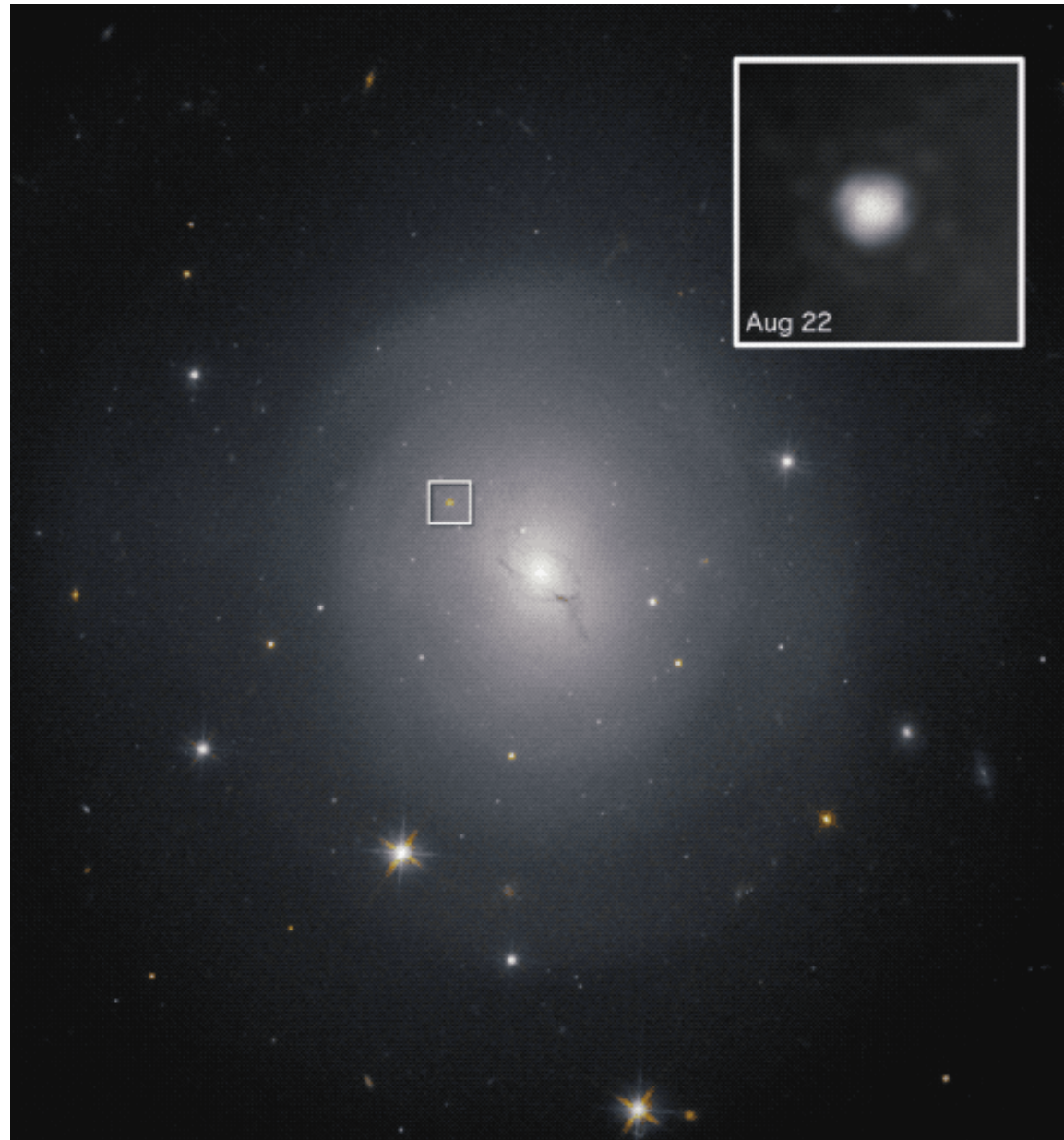








*Kilonova: production heavy elements: gold, platinum etc.*

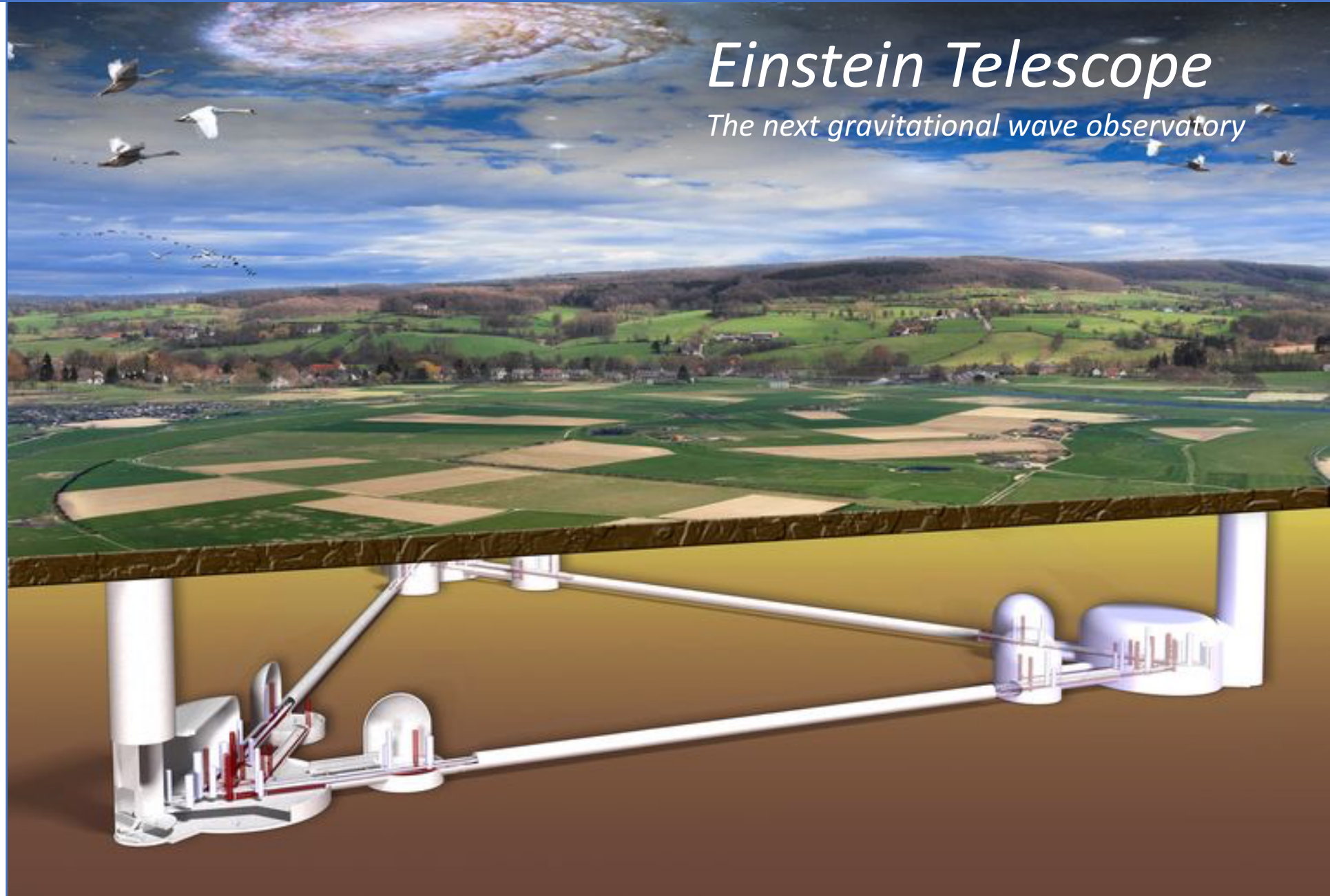




# Possible Future Facility...

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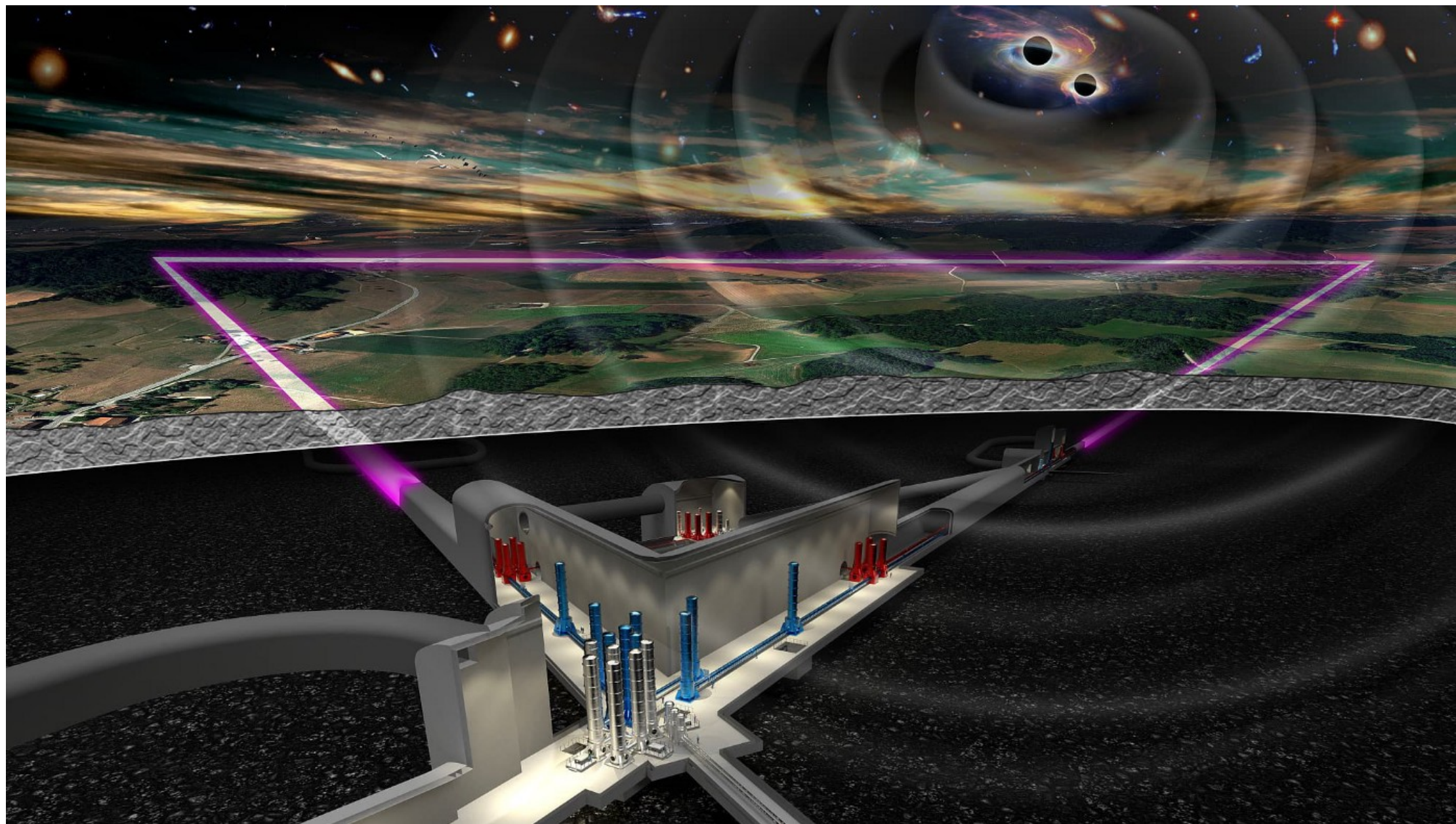


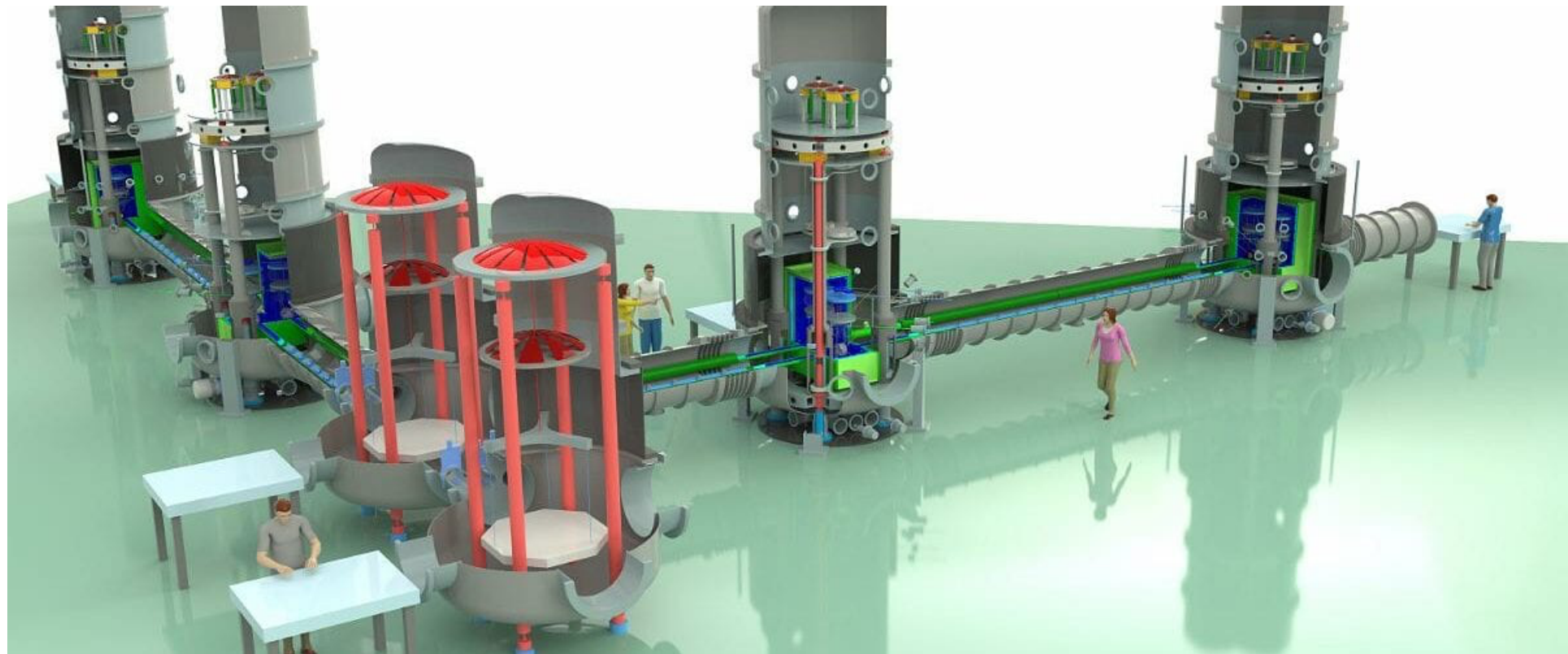




# Possible Future Facility...

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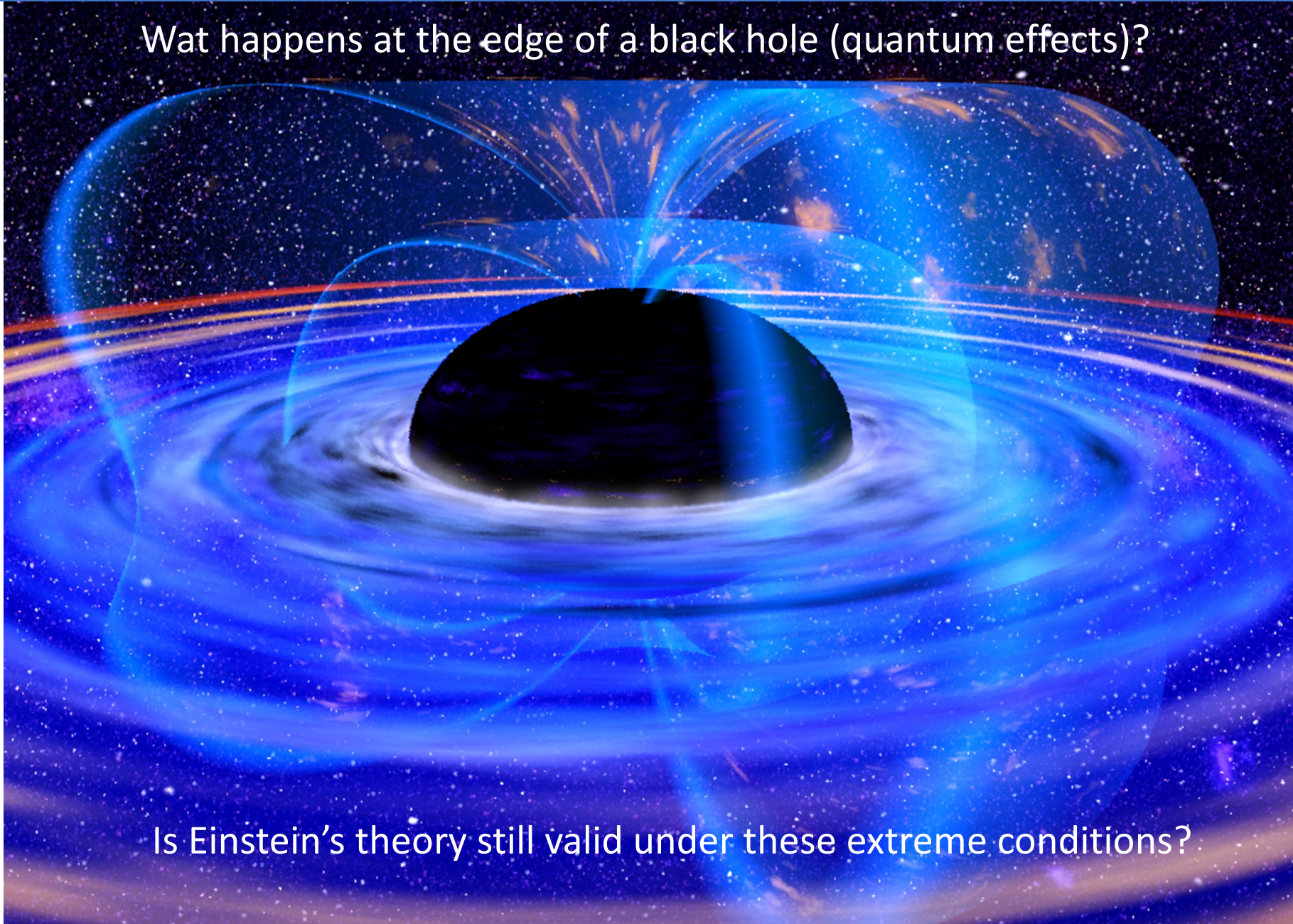
RWTH Aachen, UCL Louvain, Hasselt, Ghent, Antwerp, VUB Brussels, ULB Brussels, Liege, Radboud University Nijmegen, TU Eindhoven and Hamburg







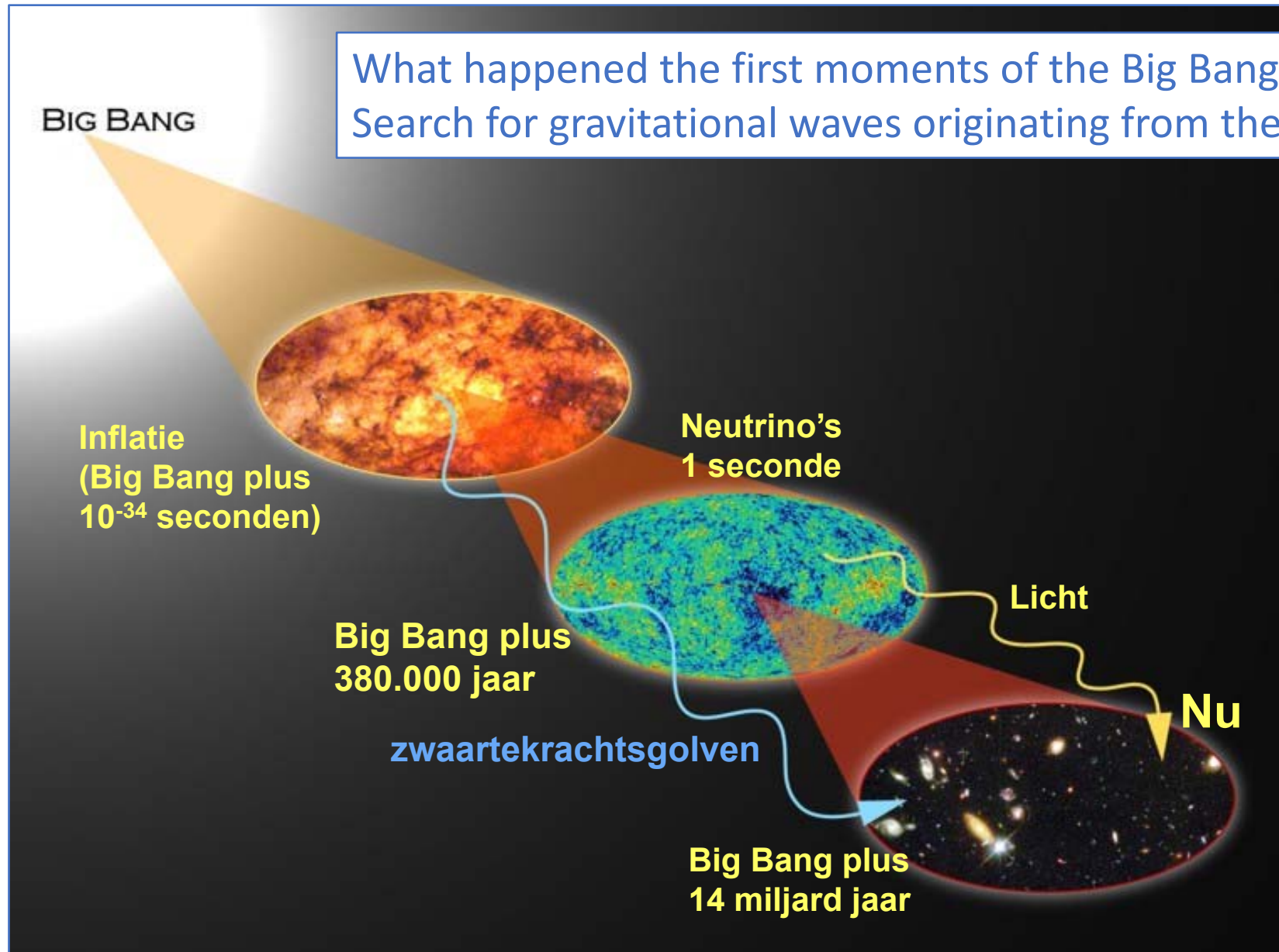
Wat happens at the edge of a black hole (quantum effects)?



Is Einstein's theory still valid under these extreme conditions?

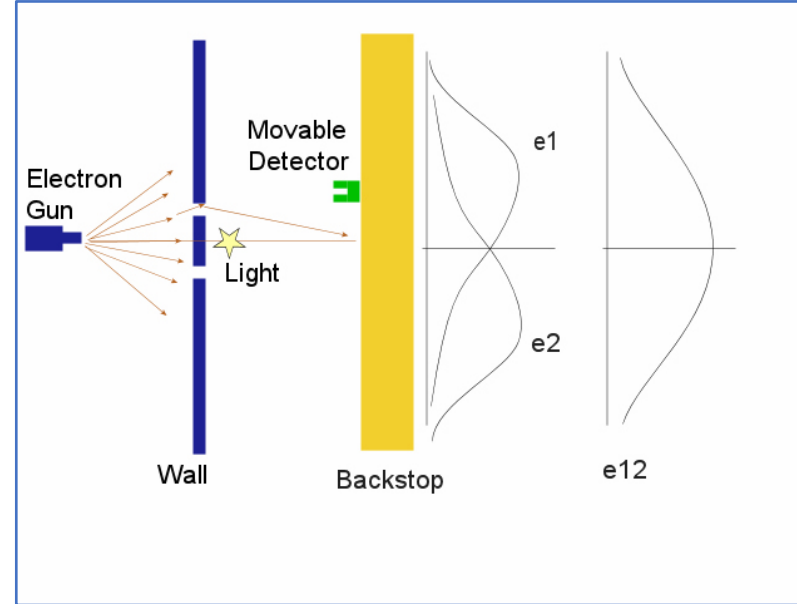
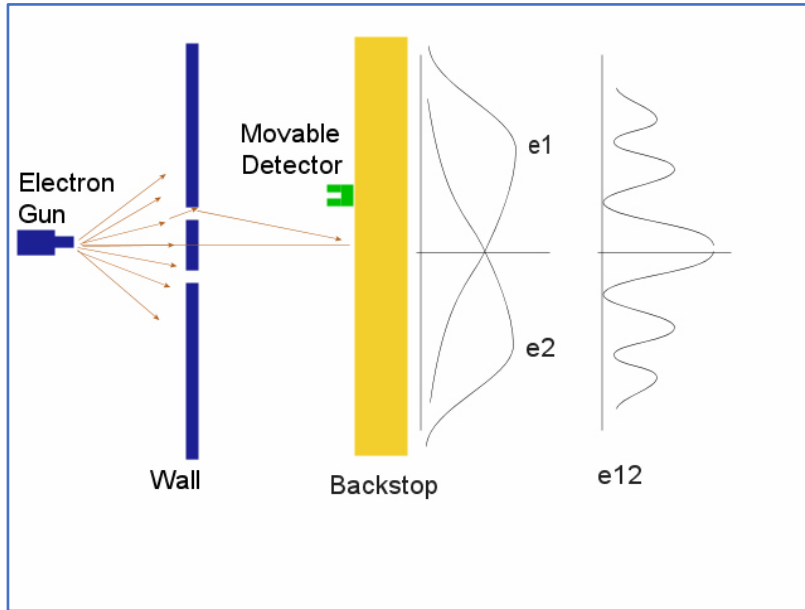


What happened the first moments of the Big Bang?  
Search for gravitational waves originating from the very first moment!



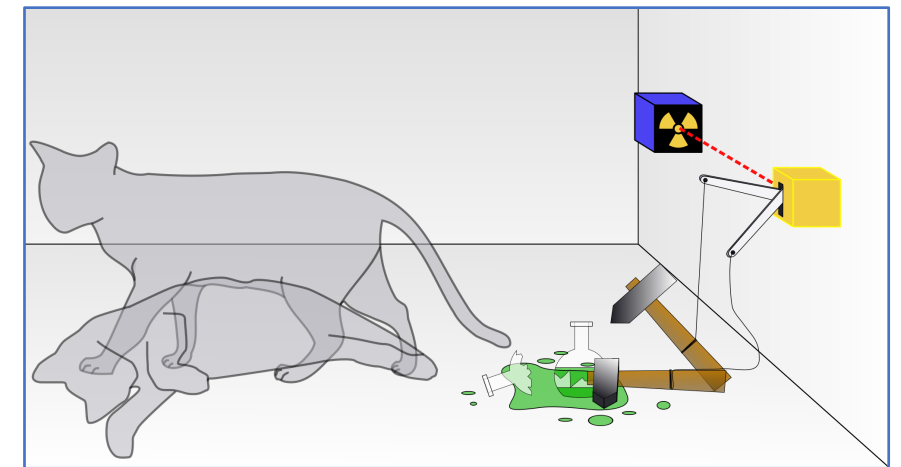
# Next week: Quantum Mechanics

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Quantum mechanics developed by Bohr and Heisenberg leads to "absurd" thought experiments of Feynman and Wheeler. Einstein and Schrödinger did not like it.

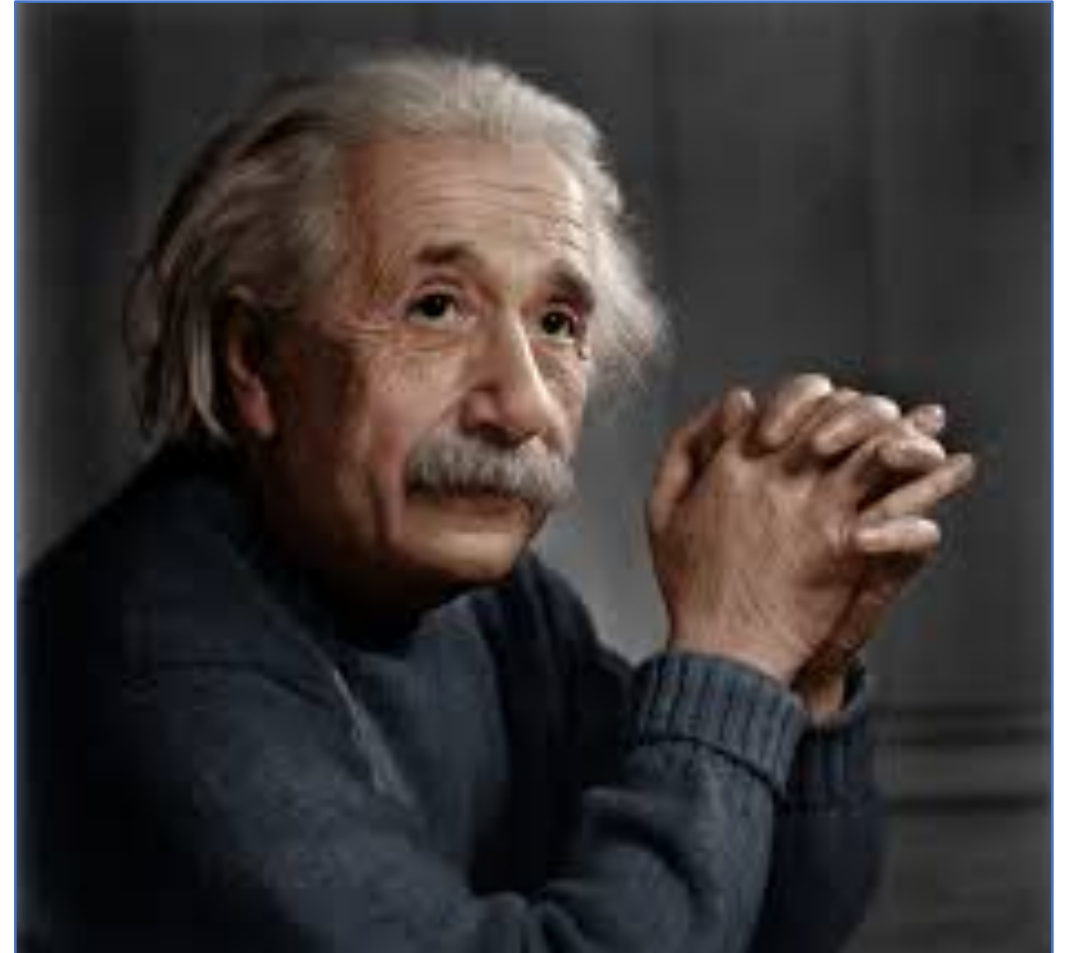
Even today people are debating its interpretation....



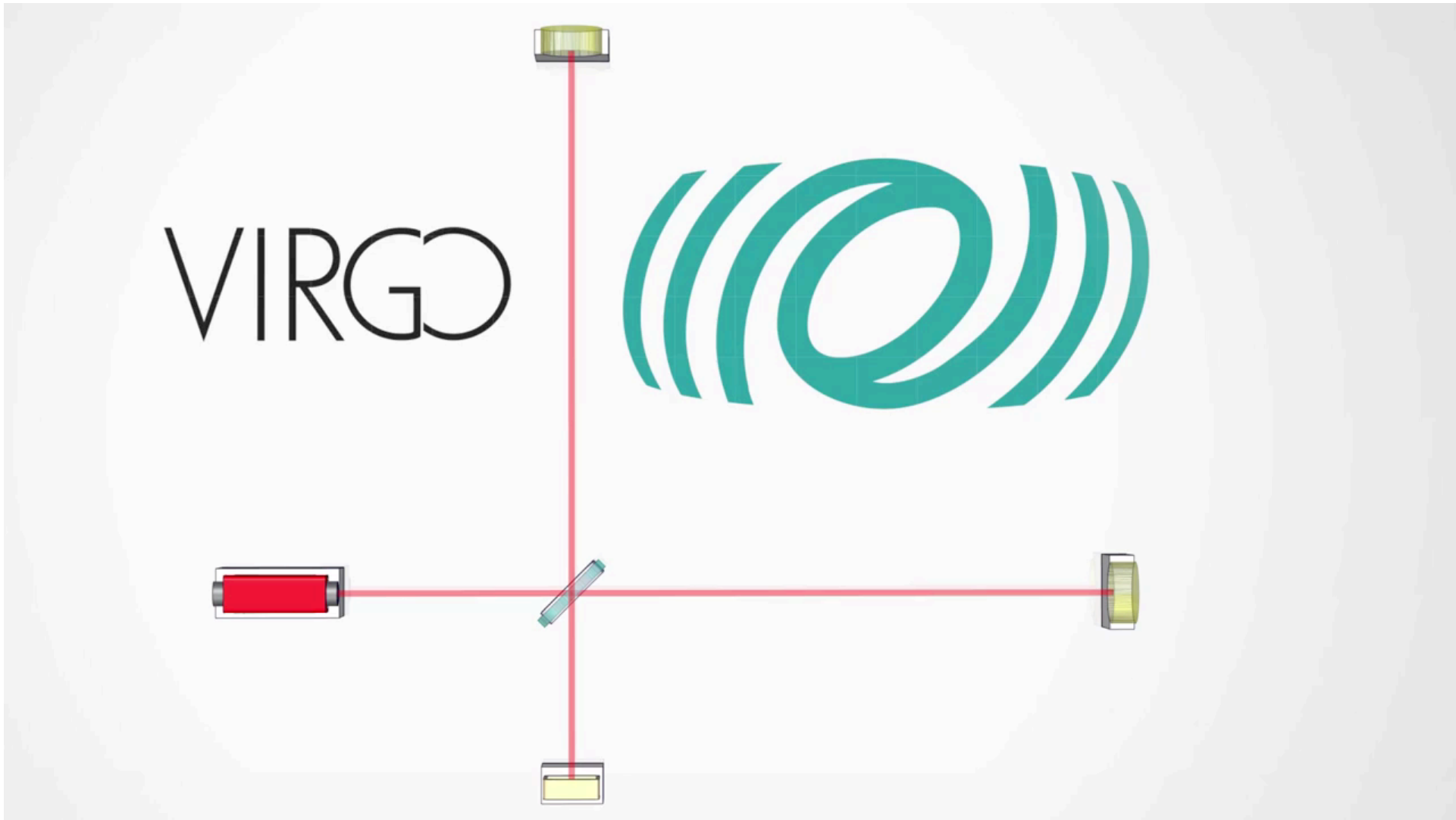


## Extra Slides

- “Imagination is more important than knowledge”
- “Education is what remains after one has forgotten what one has learned at school.”
- “I fear the day that technology will surpass our human interaction. The world will have a generation of idiots.”
- “A person who never made a mistake never tried anything new.”







# The Gravitational Wave Spectrum

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