The Relativistic Quantum World

A lecture series on Relativity Theory and Quantum Mechanics

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Lecture notes, written for this course, are available: [www.nikhef.nl/~i93/Teaching/](http://www.nikhef.nl/~i93/Teaching/)

Prerequisite for the course: High school level mathematics.

### Relativity

**Sept 14:**
- Lecture 1: The Principle of Relativity and the Speed of Light
- Lecture 2: Time Dilation and Lorentz Contraction

**Sept 21:**
- Lecture 3: The Lorentz Transformation and Paradoxes
- Lecture 4: General Relativity and Gravitational Waves

### Quantum Mechanics

**Sept 28:**
- Lecture 5: The Early Quantum Theory
- Lecture 6: Feynman’s Double Slit Experiment

**Oct 5:**
- Lecture 7: The Delayed Choice and Schrodinger’s Cat
- Lecture 8: Quantum Reality and the EPR Paradox

### Standard Model

**Oct 12:**
- Lecture 9: The Standard Model and Antimatter
- Lecture 10: The Large Hadron Collider
Lecture 8

Quantum Reality and EPR Paradox

“Philosophy is too important to leave to the philosophers.”
- John Archibald Wheeler

“When we measure something we are forcing an undetermined, undefined world to assume an experimental value. We are not measuring the world, we are creating it.”
- Niels Bohr

“If all of this is true, it means the end of physics.”
- Albert Einstein, in discussion with Niels Bohr
Einstein’s Final Objection

**Principle of locality:**

- An object is only directly influenced by its immediate surroundings.
- An action on a system at one point *cannot* have an *instantaneous* effect on another point.
- To have effect at a distance a field or particle (“signal”) must travel between the two points.
- Limit: the speed of light.
  - Otherwise trouble with causality (see relativity: “Bob dies before Alice actually shoots him?!”).

**Einstein:** Quantum mechanics is *not a local* theory, therefore: it is unreasonable!

The EPR discussion is the last of the Bohr – Einstein discussions. After receiving Bohr’s reply Einstein commented that QM is too much in contradiction with his scientific instinct.
The EPR Paradox
The EPR Paradox (1935)

**EPR** = Albert Einstein, Boris Podolsky, Nathan Rosen

**Bohr et al.: Quantum Mechanics:**
The wave function can be precisely calculated, but a measurement of *mutually exclusive quantities* is driven by pure chance.

**Einstein et al.: Local Reality:**
There must exist *hidden variables* (hidden to us) in which the outcome of the measurement is encoded such that effectively *it only looks as* if it is driven by chance.

**Local Realism vs Quantum Entanglement:**
EPR: What if the wave function is very large and a measurement at one end can influence the other end via some “unreasonable spooky interaction”.
Propose a measurement to test *quantum entanglement* of particles.
The EPR Paradox

Two particles produced with known total momentum $P_{\text{total}}$, and fly far away. Alice can not measure at the same time position ($x_1$) and momentum ($p_1$) of particle 1. Bob can not measure at the same time position ($x_2$) and momentum ($p_2$) of particle 2.

$$p_{\text{total}} = p_1 + p_2$$

But:

If Alice measures $p_1$, then automatically $p_2$ is known, since $p_1 + p_2 = p_{\text{total}}$
If Alice measures $x_1$, then $p_1$ is unknown and therefore also $p_2$ is unknown.

How can a decision of Alice to measure $x_1$ or $p_1$ affect the quantum state of Bob’s particle ($x_2$ or $p_2$) at the same time over a long distance?
Communication with speed faster than the speed of light? Contradiction with causality?
Is there “local realism” or “spooky action at a distance”? 
An EPR Experiment

Produce two particle with an opposite *spin quantum state*. Heisenberg uncertainty: an electron *cannot* have well defined *spin* at same time along two different directions, eg. *z* and *x*

Alice

1: z-Spin= +

2: z-Spin= –

Bob

1: x-Spin= +

2: x-Spin= –

After first measuring *z* than, the probability of *+x* vs *−x* = 50%-50%. After subsequently measuring eg. *+x*, the probability of *+z* vs *−z* = 50%-50% etc.!

Quantum wave function: total spin = 0.
If Alice measures spin of her particle along the *z*-direction, Then also Bob’s particle’s spin points (oppositely) along the *z*-direction!
An EPR Experiment

Produce two particle with an opposite spin quantum state. Heisenberg uncertainty: an electron cannot have well defined spin at same time along two different directions, eg. z and x.

But how does Bob’s particle know that Alice measures x-spin or z-spin?

1: z-Spin= +
2: z-Spin= –

1: x-Spin= +
2: x-Spin= –

Trick: if $A_z^+$ implies $B_z^-$, then alternatively: $B_x^-$ implies $A_x^+$

Does the measurement $A_z^+B_x^-$ means that we have determined both x and z spin according to $A_z^+A_x^+$?!

⇒ Local realism: yes!
⇒ QM: No! (The first measurement “collapses” the wave function: coherence is lost.)

Either the particles are linked because of some hidden variable (local reality) or they are QM “entangled” until a measurement “collapses” the wave function.
EPR experiment with photons. Testing the **Bell inequality** (1964).

Correlation test, count:  
\[ E(a,b) = \frac{N(+,+) - N(+,-) - N(-,+) + N(+,-)}{N(+,+) + N(+,-) + N(-,+) + N(+,-)} \]

**Polarizer settings:**  
- \( a = 0^\circ \) or \( a' = 45^\circ \)  
- \( b = 22.5^\circ \) or \( b' = 67.5^\circ \)

Determine:  
\[ S = E(a,b) - E(a,b') + E(a',b) + E(a',b') \]

- Local reality (hidden var’s):  
  \[ S \leq 2.0 \]
- Quantum Mechanics:  
  \[ S = 2.7 \]
EPR experiment with photons. Testing the Bell inequality (1964).

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\[ E(a,b) = \frac{N(+,+) - N(+,-) - N(-,+)}{N(+,+) + N(+,-) + N(-,+)} \]

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EPR experiment with photons. Testing the **Bell inequality** (1964).

\[ E(a, b) = \int A(a, \lambda)B(b, \lambda)\rho(\lambda)d\lambda \]

where \( A \) and \( B \) are the average values of the outcomes. Since the possible values of \( A \) and \( B \) are \(-1, 0, +1\), it follows that:

\[
|A| \leq 1 \quad |B| \leq 1
\]

Then, if \( a, a', b \) and \( b' \) are alternative settings for the detectors,

\[
E(a, b) - E(a, b') = \int [A(a, \lambda)B(b, \lambda) - A(a, \lambda)B(b', \lambda)] \rho(\lambda)d\lambda
\]

\[ = \int A(a, \lambda)B(b, \lambda) [1 \pm A(a', \lambda)B(b', \lambda)] \rho(\lambda)d\lambda - \int A(a, \lambda)B(b', \lambda) [1 \pm A(a', \lambda)B(b, \lambda)] \rho(\lambda)d\lambda \]

Taking absolute values of both sides, and applying the **triangle inequality** to the right-hand side, we obtain

\[
|E(a, b) - E(a, b')| \leq \left| \int A(a, \lambda)B(b, \lambda) [1 \pm A(a', \lambda)B(b', \lambda)] \rho(\lambda)d\lambda \right| + \left| \int A(a, \lambda)B(b', \lambda) [1 \pm A(a', \lambda)B(b, \lambda)] \rho(\lambda)d\lambda \right|
\]

We use the fact that \( [1 \pm A(a', \lambda)B(b', \lambda)] \rho(\lambda) \) and \( [1 \pm A(a', \lambda)B(b, \lambda)] \rho(\lambda) \) are both non-negative to rewrite the right-hand side of this as

\[
\int |A(a, \lambda)B(b, \lambda)| [1 \pm A(a', \lambda)B(b', \lambda)] \rho(\lambda)d\lambda + \int |A(a, \lambda)B(b', \lambda)| [1 \pm A(a', \lambda)B(b, \lambda)] \rho(\lambda)d\lambda
\]

By (4), this must be less than or equal to

\[
\int [1 \pm A(a', \lambda)B(b', \lambda)] \rho(\lambda)d\lambda + \int [1 \pm A(a', \lambda)B(b, \lambda)] \rho(\lambda)d\lambda
\]

which, using the fact that the integral of \( \rho(\lambda) \) is 1, is equal to

\[
2 \pm \left[ \int A(a', \lambda)B(b', \lambda)\rho(\lambda)d\lambda + \int A(a', \lambda)B(b, \lambda)\rho(\lambda)d\lambda \right]
\]

which is equal to \( 2 \pm [E(a', b') + E(a', b)] \).

Putting this together with the left-hand side, we have:

\[
|E(a, b) - E(a', b')| \leq 2 \pm [E(a', b') + E(a', b)]
\]

which means that the left-hand side is less than or equal to both \( 2 + [E(a', b') + E(a', b)] \) and \( 2 - [E(a', b') + E(a', b)] \). That is:

\[
|E(a, b) - E(a, b')| \leq 2 - |E(a', b') + E(a', b)|
\]

from which we obtain

\[
2 \geq |E(a, b) - E(a, b')| + |E(a', b') + E(a', b)| \geq |E(a, b) - E(a, b') + E(a', b') + E(a', b)|
\]

(by the **triangle inequality** again), which is the CHSH inequality.
EPR experiment with photons. Testing the *Bell inequality* (1964).

Correlation test, count:  
\[ E(a,b) = \frac{N(\text{+},\text{+}) - N(\text{+},\text{-}) - N(\text{-},\text{+}) + N(\text{-},\text{-})}{N(\text{+},\text{+}) + N(\text{+},\text{-}) + N(\text{-},\text{+}) + N(\text{-},\text{-})} \]

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- Local reality (hidden var’s):  
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- Quantum Mechanics:  
  \[ S = 2.7 \]
- Result:  
  \[ S = 2.697 \pm 0.015 \]

Observations agree with quantum mechanics and not with local reality!
Alain Aspect 1982 – EPR with photons!

EPR experiment with photons. Testing the **Bell inequality** (1964).

Correlation test, count: \[ E(a,b) = \frac{N(+,+) - N(+,-) - N(-,+) + N(-,-)}{N(+,+) + N(+,-) + N(-,+) + N(-,-)} \]

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There were two “loopholes” (comments of critics):

1. **Locality loophole:**
   - The particles and detectors were so close to each other that *in principle* they could have communicated with each other during the Bell test.

2. **“Detection loophole”:**
   - The detectors only measured *some* of the entangled particles, and they could be a *non-representative* selection of all.

Determine: \( S = E(a,b) - E(a,b') + E(a',b) + E(a',b') \)

- Local reality (hidden var’s): \( S \leq 2.0 \)
- Quantum Mechanics: \( S = 2.7 \)
- **Result:** \( S = 2.697 \pm 0.015 \)

**Observations agree with quantum mechanics and not with local reality!**
1. Put the detectors far away.
2. Make sure detection efficiency is high.
Ronald Hanson and his group performed the first EPR experiment without loopholes.

- Measurement of photons that are entangled with electron spins.
- Quantum entanglement again passes the test.
- No hidden variables!
The Wave function.
- \( \psi(x,t) \) contains all information of a system (eg. electron).
- Wave function is solution of equation that includes the fundamental laws of physics: all types of matter particles and their interactions (see next lecture).

Copenhagen Interpretation.
- **No physical interpretation** for the wave function.
- As long as **no measurement** on an electron is done the wave-function includes all possible outcomes.
  “Nature tries everything”.
- When a measurement is done, nature realizes one of the possibilities by the **collapse of the wavefunction** (particle or wave, \( x \) or \( p, \sigma_x \) or \( \sigma_z \)) according to probabilistic laws.
  “Nothing exists until it is measured”.

The Measurement Problem.
- **But what is a measurement?** Is it an irreversible process? Does it require consciousness?
- There are many interpretations apart from the Copenhagen Interpretation.
  – Objective collapse theory, consciousness causes collapse, pilot-wave, many worlds, many minds, participatory anthropic principle, quantum information (“it from bit”), ...
Hugh Everett (PhD Student of John Wheeler) formulated the Many Worlds Interpretation of quantum mechanics in 1957.

The wave function does not collapse, but at each quantum measurement both states continue to exist in a decoupled world.

**Multiverse:**
Very large tree of quantum worlds for each quantum decision. The total wave function of complete multiverse is deterministic.

Triggered science fiction stories with “parallel universes”.

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[Image of Hugh Everett]
THE MANY-WORLDS INTERPRETATION OF QUANTUM MECHANICS STATES THAT FOR EACH OUTCOME OF A MEASUREMENT, AN ENTIRELY SEPARATE UNIVERSE IS CREATED WHERE THE OPPOSITE EVENT OCCURRED.

THE THEORY AVOIDS THE MAJOR WEAKNESS OF THE COPENHAGEN INTERPRETATION, WHICH MAKES NO FURTHER COMMENT ON THE NATURE OF A MEASUREMENT DESPITE ITS KEY ROLE FOR THE COLLAPSE OF THE WAVE FUNCTION.

THE BALL WAS STILL OUT!

NOT IN THIS UNIVERSE!
Many Worlds test

- Incredibly many alternative versions of us exist in the multiverse.
- To prove it:
  - Try shooting yourself with 50%-50% quantum probability in Russian roulette.
  - Repeat it 50 times.
  - In many worlds survival will happen.
  - You only need the luck to be living in the correct universe.

Quantum Russian roulette

Max Tegmark's thought experiment to test the *many-worlds hypothesis* involves a machine gun controlled by an atom's spin.

- **Atom**: Spin is both up and down.
- **Measure Spin**: If spin is up, a bullet is fired; if spin is down, a blank is fired.
- **Machine Gun**: Controlled by atom spin.
- **Copenhagen Interpretation**: 50% chance of survival after each round is fired.
- **Chance of survival after 50 rounds fired**: 1 in 1,000,000,000,000,000,000

**Many Worlds Interpretation**: All possible outcomes happen, each in a parallel reality.

100% in one reality, dead in countless others.
Many Worlds test

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For me the Many World Interpretation is a very far-fetched (if not crazy) view of our existence. But it is very difficult to prove it wrong!

Most physicists consider such interpretation outside physics.
Application 1: Quantum Cryptography

Alice sends a secret message to Bob and prevents Eve to eavesdrop.

First idea by Stephen Wiesner (1970s)
Worked out by Bennet (IBM) and Brassard (1980s) → BB84 protocol

Quantum Key Distribution (QKD):
1. Public Channel (Internet, email):
   send an encrypted message.
2. Quantum Channel (Laser + fiber optics)
   send key to decode the public message
3. Eve cannot secretly eavesdrop. She destroys quantum information and is detected.

Physicsworld.com Sept 2, 2013
“Quantum cryptography coming to mobile phones”
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Application 2: Quantum Computer

Idea: Yuri Manin and Richard Feynman:
Use superposition and entanglement of quantum states to make a super-fast computer.

Normal computer: bits are either 0 or 1
Quantum computer: qubits are coherent super-positions of states 0 and 1 at the same time. (Eg. Electron spin up and spin down)

Compute with quantum logic.
With 2 bits it can do 4 calculations simultaneously.
With 3 bits 8 calculations, with n bits $2^n$!

Qubit Technologies:
Electron spin, Photon polarization, Nuclear spin, quantum dots, ...

Difficulty: prevent “decoherence”.
**Application 2: Quantum Computer**

**“Hardware” technological difficulty:**
- Prevent “decoherence”
- 2011: “D-wave systems” claim quantum computer of 128 qubits. (Not generally accepted that is a real QC.)
- 2017: IBM announces most powerful quantum computer 17 qubits.

**Software technological difficulty:**
- Prepare system in known state
- Let it evolve according to the algorithm into large simultaneous state.
- Correct solution results from constructive interference of states (think double slit)
- Only few algorithms exist:
  - Shor factorization
  - Grover’s search algorithm
- A science in itself!
Quantum reality differs from the classical world.

Einstein brought a revolutionary way of thinking with relativity theory, but could not accept the revolution of quantum mechanics.

Bohr never managed to convince Einstein.

Einstein's objections have been disproven in many tests while then quantum view is always confirmed.

The Copenhagen interpretation does not provide a meaning for what the wave function is and what the role of the observer (i.e. a measurement) is.

**Philosophical:**
Would the universe exist if there would be no “observers” to see it?
Is the universe perhaps created by acts of observation?
Further food for thought

Relativity theory:
The finite speed of light means that there is no sharp separation between space and time. (Think of different observers)
Universal constant: \( c = 300,000 \text{ km/s} \)

Quantum Mechanics:
The finite value of the quantum of action means that there is no sharp separation between a system and an observer
Universal constant: \( \hbar = 6.6262 \times 10^{-34} \text{ Js} \)

John Wheeler:
“Bohr’s principle of complementarity is the most revolutionary scientific concept of the century.”
Next week:
- Quantum Field Theory and Antimatter
- The Standard Model
- The Large Hadron Collider
- The Origin of Mass: Higgs
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“for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which was recently confirmed through the discovery of the predicted fundamental particle, by the Atlas and CMS experiments at CERN’s Large Hadron Collider.”
Perhaps time for...