Exercises Particle Physics 2 #3

Exercise 8:

(a) Show how the unpolarised cross section formula for the process $e^+e^- \to Z, \gamma \to \mu^+\mu^-$ can be obtained from the expression of the helicity cross sections in the lecture:

$$\frac{d\sigma}{d\Omega}\left(e^-_{L/R}e^+_{R/L} \to \mu^-_{L/R}\mu^+_{R/L}\right) = \frac{\alpha^2}{4s}\left(1 \pm \cos\theta\right)^2 \left|1 + r\, C^e_{L/R}C^\mu_{L/R}\right|^2$$

(b) Show, using the expression of r from the lecture, that close to the peak of the Z-lineshape the expression

$$Re(r) = \left(1 - \frac{s_0}{s}\right)\, |r|^2$$

with $s_0 = M_z^2 - \Gamma_z^2/4$ holds.

(c) Show also that at the peak:

$$\sigma_{peak} \approx \frac{12\pi}{M_z^2}\frac{\Gamma_e\Gamma_\mu}{\Gamma_Z}$$

(d) Calculate the relative contribution of the Z-exchange and the γ exchange to the cross section at the Z peak.
Use $\sin^2\theta_W = 0.23$, $M_z = 91\,GeV$ and $\Gamma_Z = 2.5\,GeV$.

(e) The actual line shape of the Z-boson is not a pure Breit Wigner, but it is asymmetrical: at the high $\sqrt{s}$ side of the peak the cross section is higher then expected from the formula derived in the lectures.
Can you think of a reason why this would be the case?

(f) The number of light neutrino generations is determined from the "invisible width" of the Z-boson as follows:

$$N_\nu = \frac{\Gamma_Z - 3\Gamma_l - \Gamma_{had}}{\Gamma_\nu}$$

Can you think of another way to determine the decay rate of $Z \to \nu\bar{\nu}$ directly?
Do you think this method is more precise or less precise?