# Exercises Particle Physics 2 #1

**Exercise 1:**
In classial mechanics the principle of least action states:

$$\delta S = \delta \int_{t1}^{t2} dt\, \mathcal{L}(q, \dot{q}) = 0$$

Show that the principle of least action results in the Eular Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial q} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right)$$

**Exercise 2:**

(a) Show that the field theory Euler Lagrange equations of the Lagrangian

$$\mathcal{L} = \mathcal{L}_{free} = \bar{\psi}(x)\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi(x)$$

lead to the Dirac equation:

$$(i\gamma^{\mu}\partial_{\mu} - m)\,\psi(x) = 0$$

and its adjoint.

(b) Show that the covariant Dirac Lagrangian

$$\mathcal{L}_{Dirac} = \bar{\psi}\,(i\gamma^{\mu}D_{\mu} - m)\,\psi$$

is invariant under local gauge transformations:

$$\psi(x) \rightarrow e^{iq\alpha(x)}\psi(x) \qquad ; \qquad \bar{\psi}(x) \rightarrow e^{-iq\alpha(x)}\bar{\psi}(x)$$

if simultaneously, $A_{\mu}$ transforms as:

$$A_{\mu}(x) \rightarrow A_{\mu}(x) - \partial_{\mu}\alpha(x)$$

**Exercise 3:**
Consider an infinitesimal SU(2) gauge transformation

$$G = 1 + \frac{i}{2}\,\vec{\tau}\cdot\vec{\alpha} \qquad\qquad |\alpha_i| << 1$$

Use the equation from the lecture

$$B'_{\mu} = GB_{\mu}G^{-1} + \frac{i}{g}\,(\partial_{\mu}G)\,G^{-1}$$

and also

$$B_\mu = \frac{1}{2}\,\vec{\tau}\cdot\vec{b}_\mu$$

to demonstrate that the fields transform as:

$$\vec{b}'_\mu = \vec{b}_\mu \;-\; \vec{\alpha}\times\vec{b}_\mu \;-\; \frac{1}{g}\partial_\mu\vec{\alpha}$$

Do this using the Pauli-matrix identity:

$$(\vec{\tau}\cdot\vec{a})\,(\vec{\tau}\cdot\vec{b}) = \vec{a}\cdot\vec{b} \;+\; i\vec{\tau}\cdot(\vec{a}\times\vec{b})$$

**Exercise 4:**

What do you think is the difference between an exact and a broken symmetry?
Can you make a (wild) guess what spontaneous symmetry breaking means?
Which symmetry is involved in the gauge theories below? Which of these gauge symmetries are exact? Why/Why not?

(a) U1(Q) symmetry

(b) SU2(u-d-flavour) symmetry

(c) SU3(u-d-s-flavour) symmetry

(d) SU6(u-d-s-c-b-t) symmetry

(e) SU3(colour) symmetry

(f) SU2(weak-isospin) symmetry

(f) SU5(Grand unified) symmetry

(g) SUSY