Particle Physics 1
Introduction Lecture 1

Fall 2017

Lecturers: Wouter Hulsbergen, Marcel Merk, Ivo van Vulpen

Exercise classes: Laurent du Four, Maarten van Veghel

What is Nikhef?

1. Dutch National Institute for Subatomic Physics
   - Funded by NWO: the Netherlands Organisation for Scientific Research
   - NWO-i has 9 institutes:
     - AMOLF, ARCNL, CWI, Differ, Nikhef, NIOZ, NSCR, SRON
   - NWO is part of ministry of education, culture and science

2. Collaboration of NWO-i institute Nikhef and particle physics departments of 5 universities:
   - UvA, VU, UU, RUN, RUG
   - Director: Stan Bentvelsen
   - Coordinates all particle physics research in NL

Nikhef Activities

LHC Collider Physics

- Atlas: general purpose detector ("Higgs", "SUSY", "dark matter", etc)
  - ➢ *Contact: Wouter Verkerke*
- LHCb: heavy flavour physics ("CP violation", "rare decays", etc)
  - ➢ *Contact: Marcel Merk*
- Alice: quark gluon plasma
  - ➢ *Contact: Raimond Snellings*

AstroParticle Physics

- Km3NeT: Cosmic neutrinos
  - ➢ *Contact: Paul de Jong*
- Auger: high energy cosmic rays
  - ➢ *Contact: Charles Timmermans*
- Xenon: dark matter
  - ➢ *Contact: Patrick Decowski*
- Virgo/LIGO: grav waves
  - ➢ *Contact: Frank Linde*

Future Acitivities (preparations)

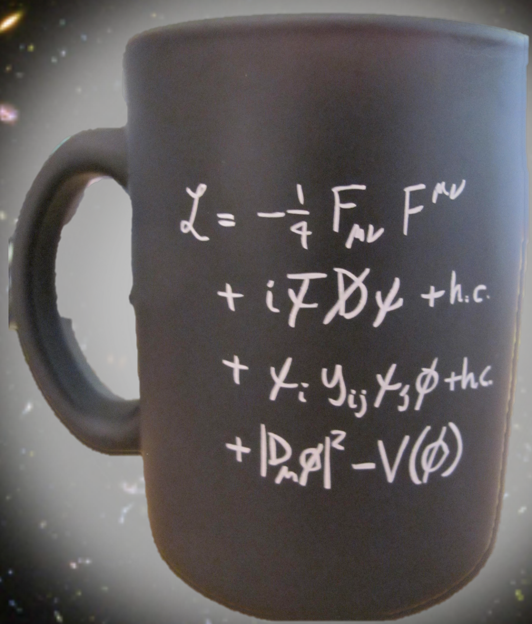
- LEPCOL: e^+e^- collider
  - ➢ *Contact: Peter Kluit*
- DUNE: (Deep Underground Neutrino Experiment)
  - ➢ *Contact: Paul de Jong*
- Einstein Telescope (Grav Waves)
  - ➢ *Contact: Frank Linde*

General Research

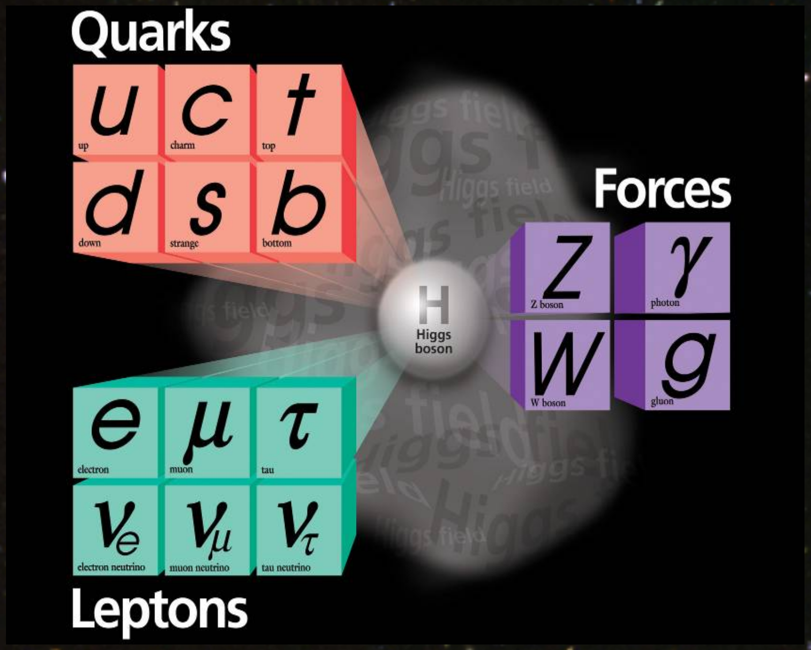
- Theory: QCD, Flavour, early universe
  - ➢ *Contact: Eric Laenen*
- Detector Research & Development
  - ➢ *Contact: Niels van Bakel*
- Grid computing
  - ➢ *Contact Jeff Templon*

Theory: The Standard Model

"The formula"



"Nature's building blocks"



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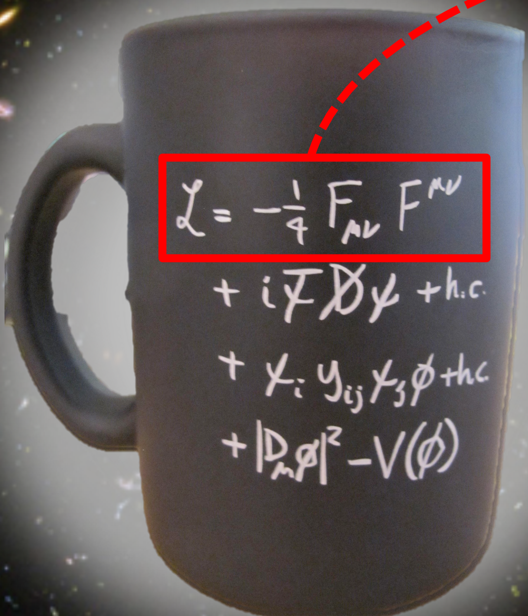

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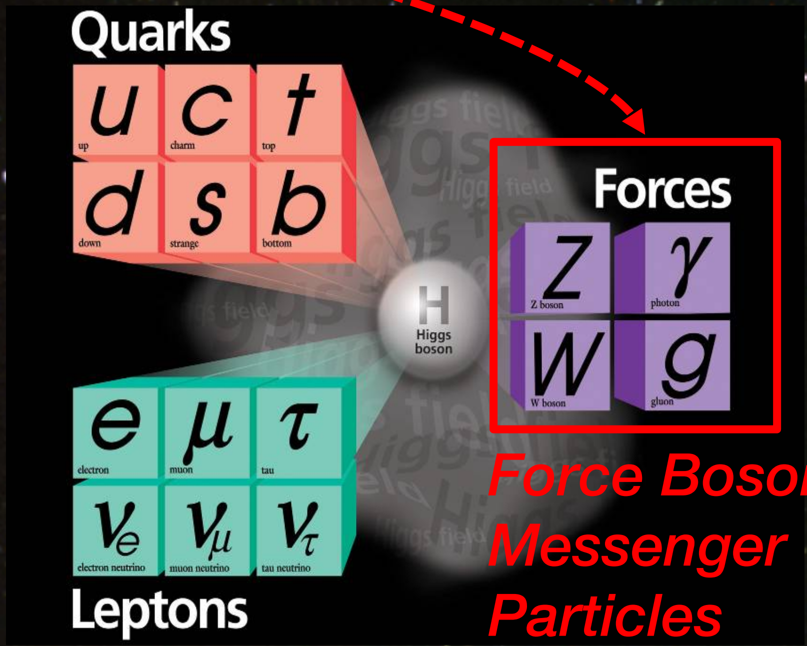
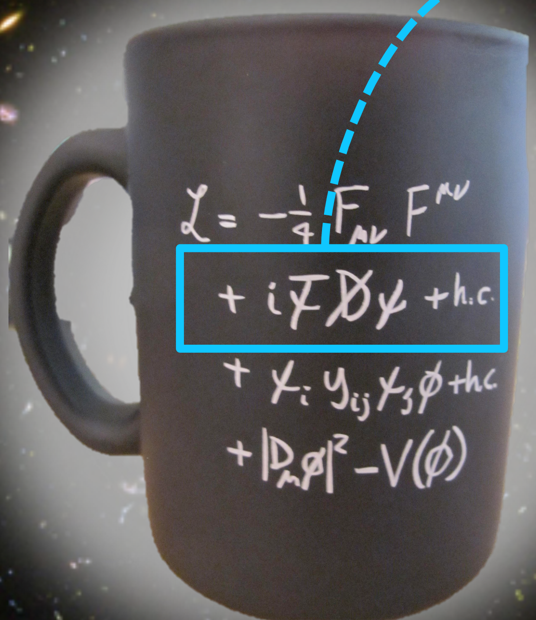


Force Bosons: Messenger Particles

Theory: The Standard Model

"The formula"



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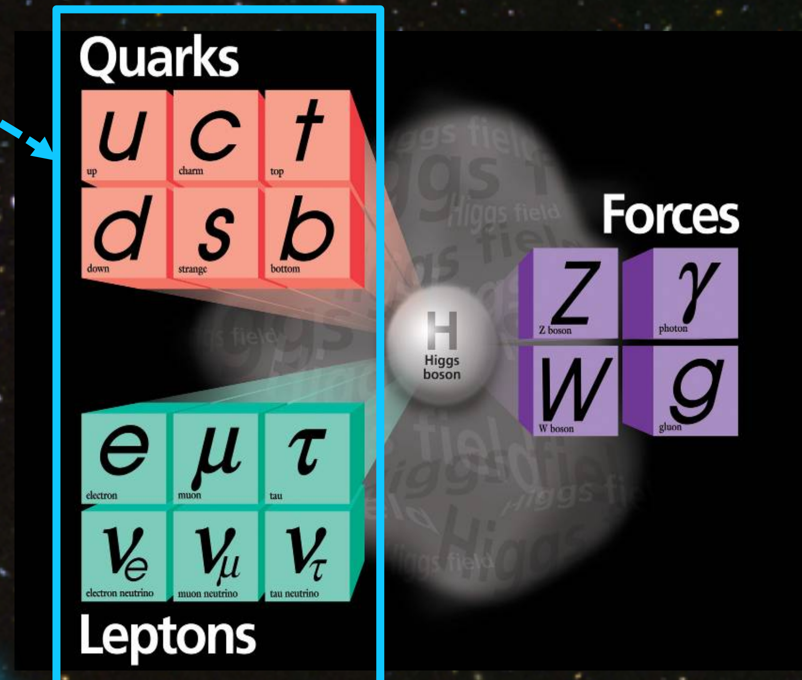
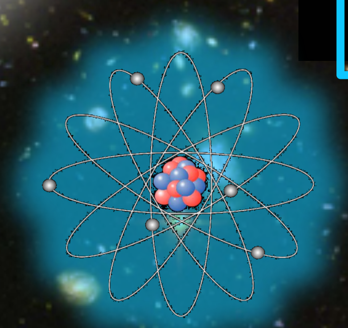
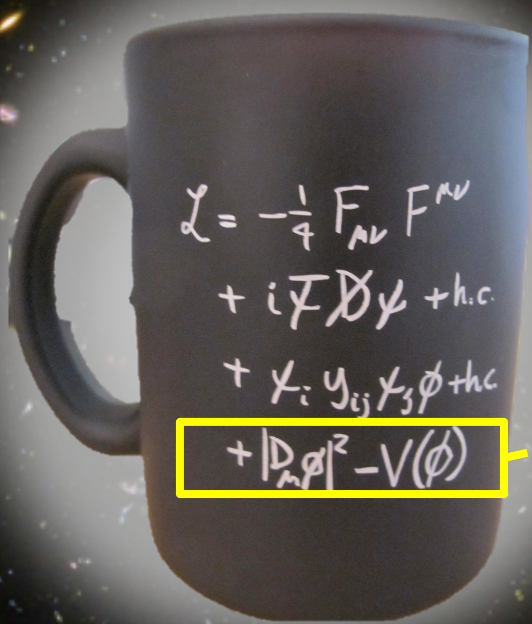




Matter Fermions

Theory: The Standard Model

"The formula"



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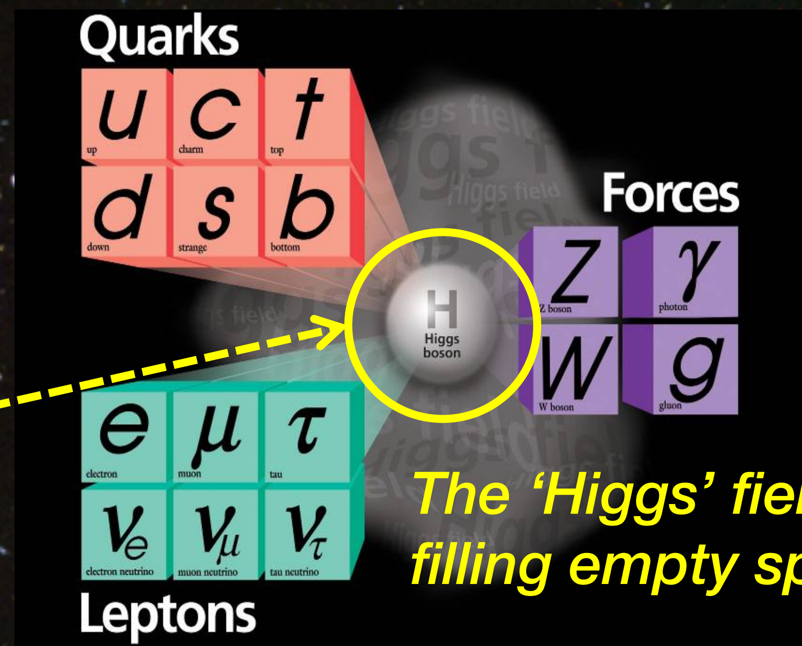


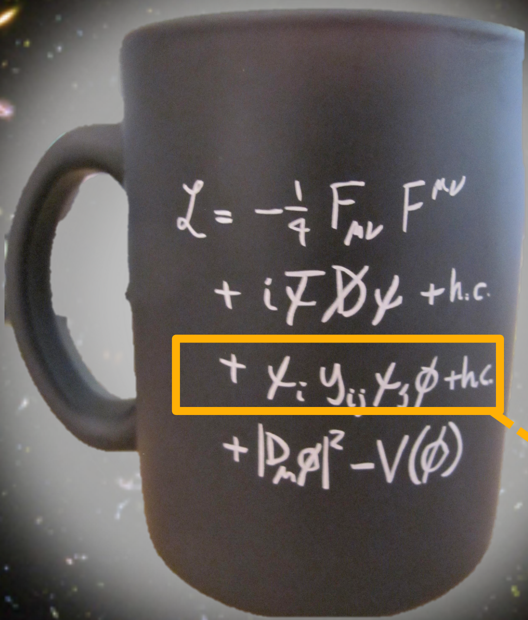
The 'Higgs' field filling empty space

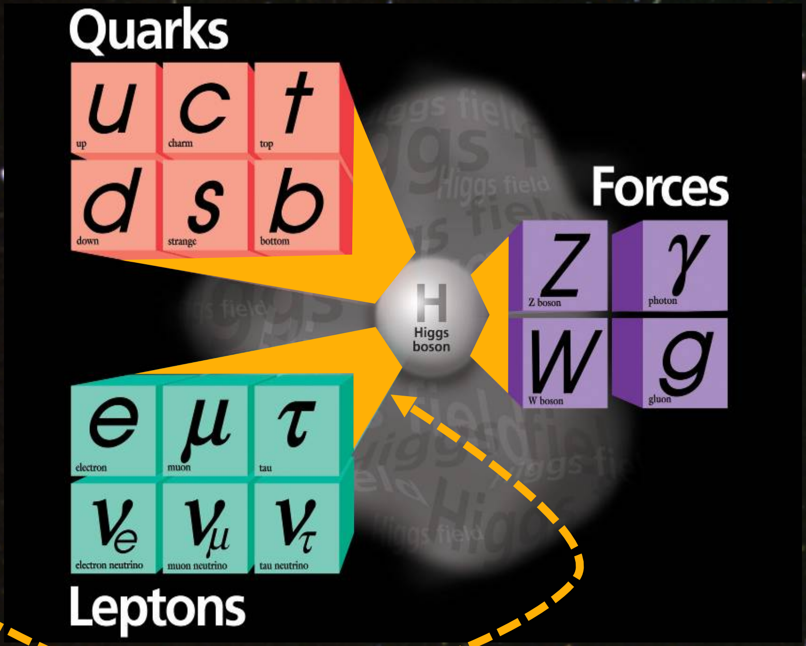
Note: the vacuum is not empty!

Theory: The Standard Model

"The formula"

"Nature's building blocks"



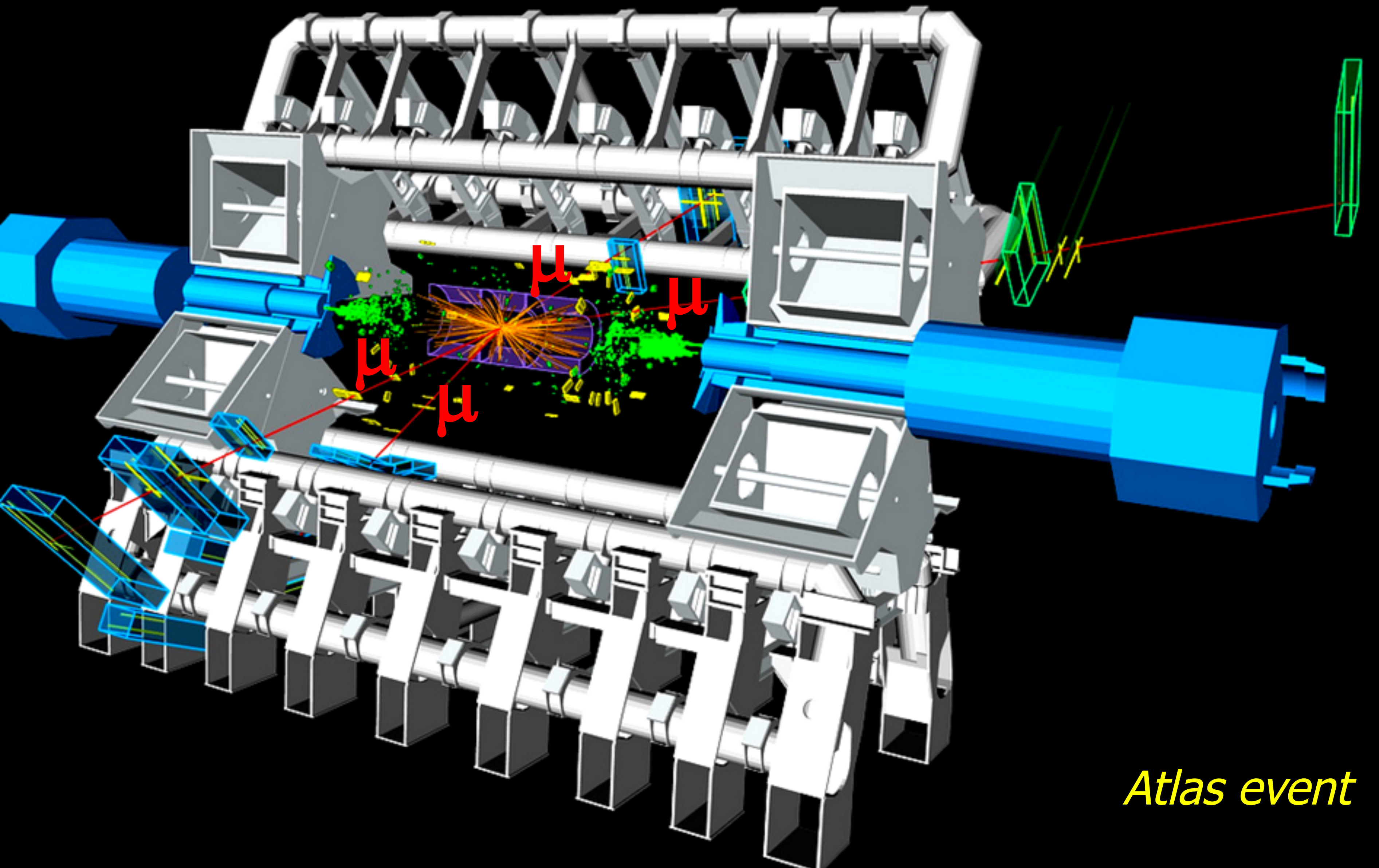


Interaction Higgs with fermions ("origin of mass")

→ Theorie formulated 50 years before Higgs discovery! ←

Atlas/CMS

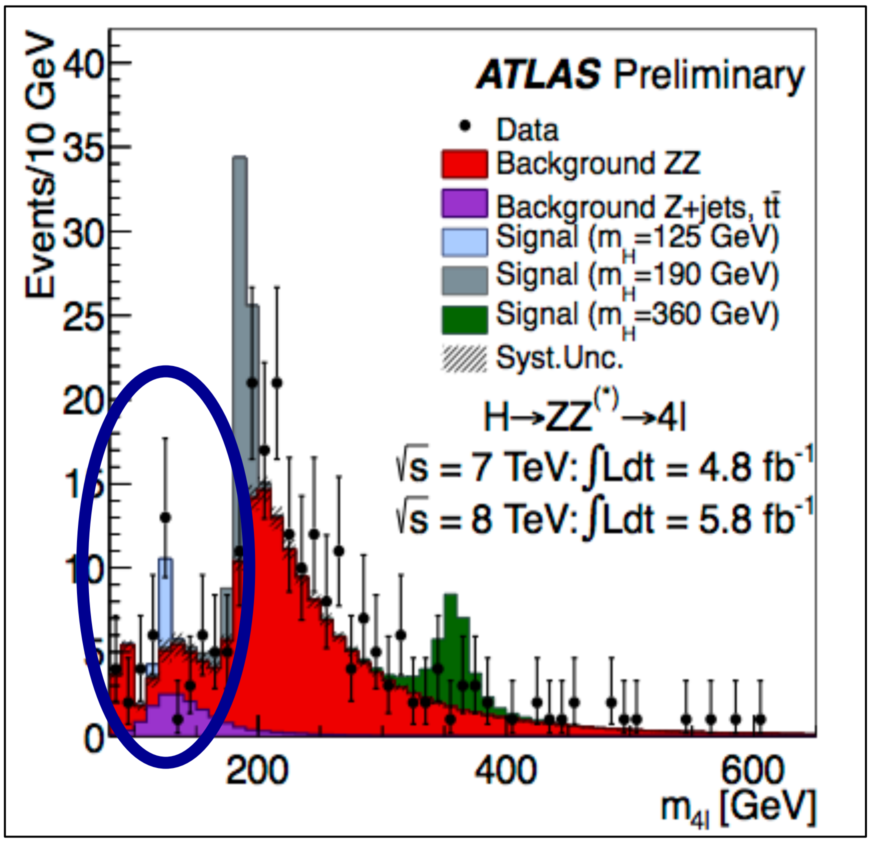
pp→Higgs→ZZ→μμμμ?



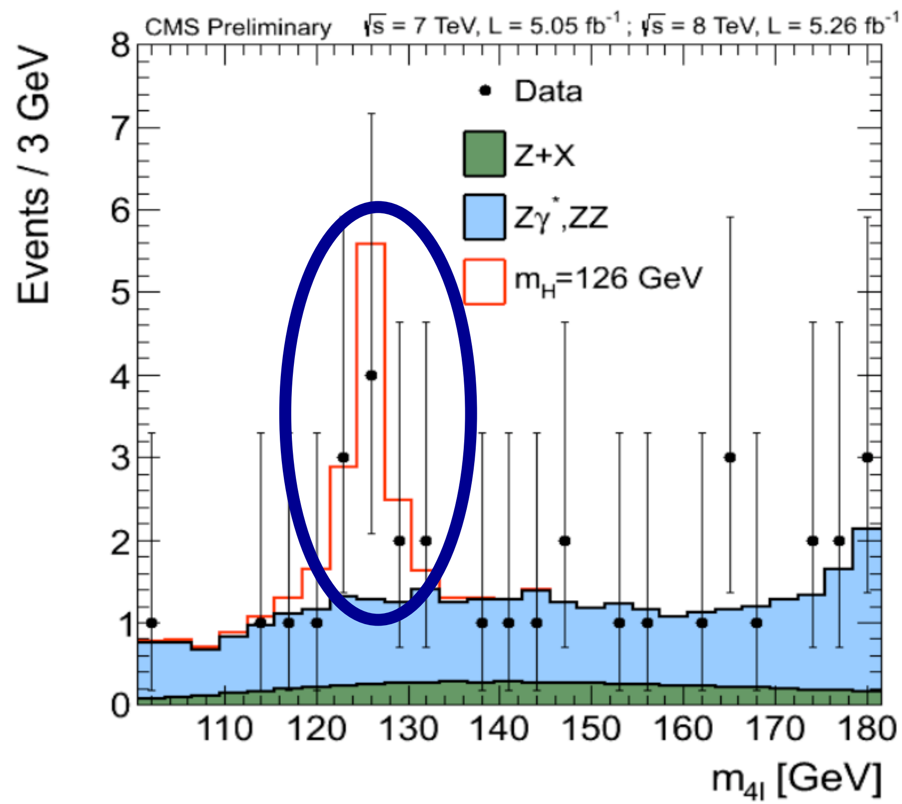
Atlas event

Higgs →ZZ→μμμμ

Atlas:



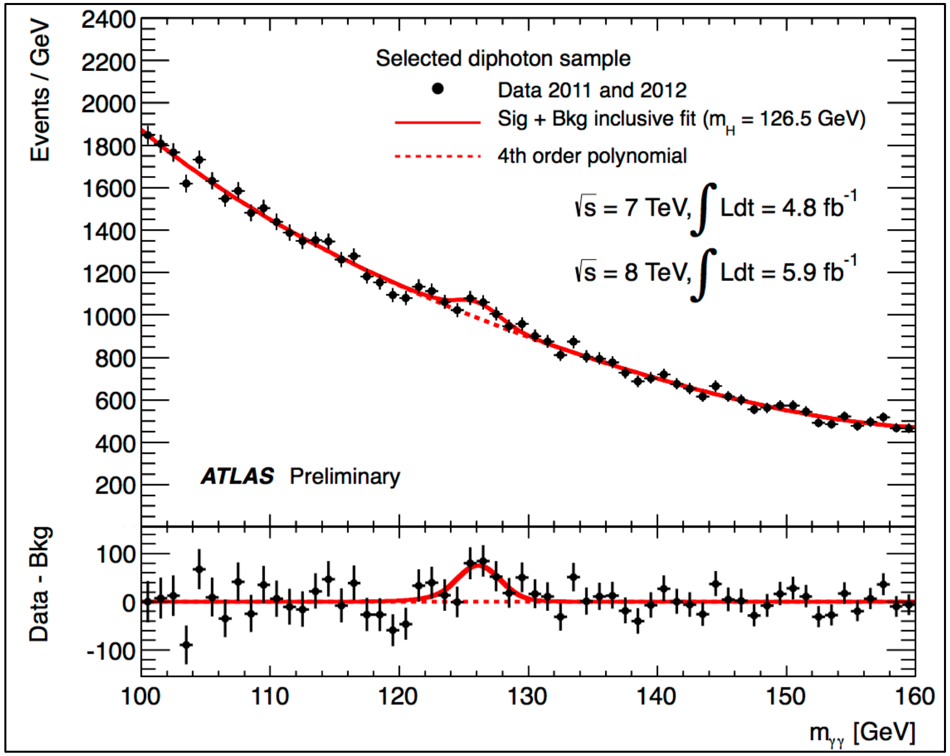
CMS:

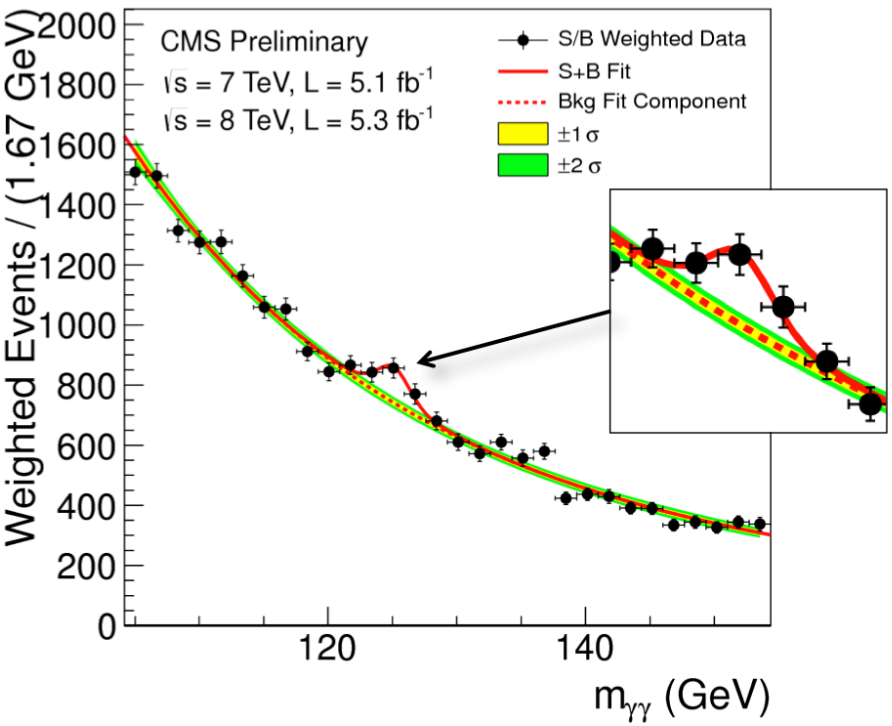


Higgs →γγ

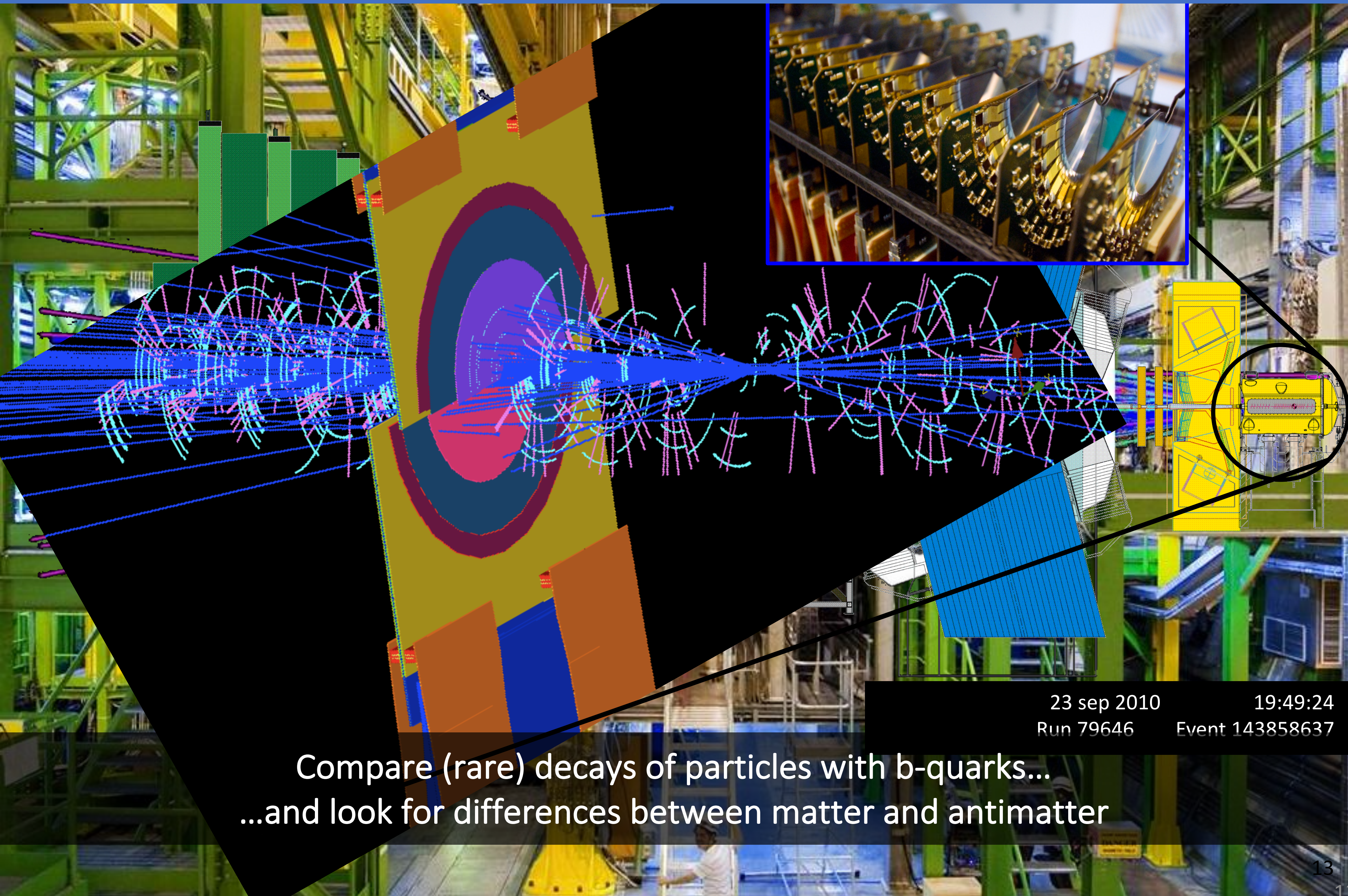
Atlas:

CMS:





LHCb

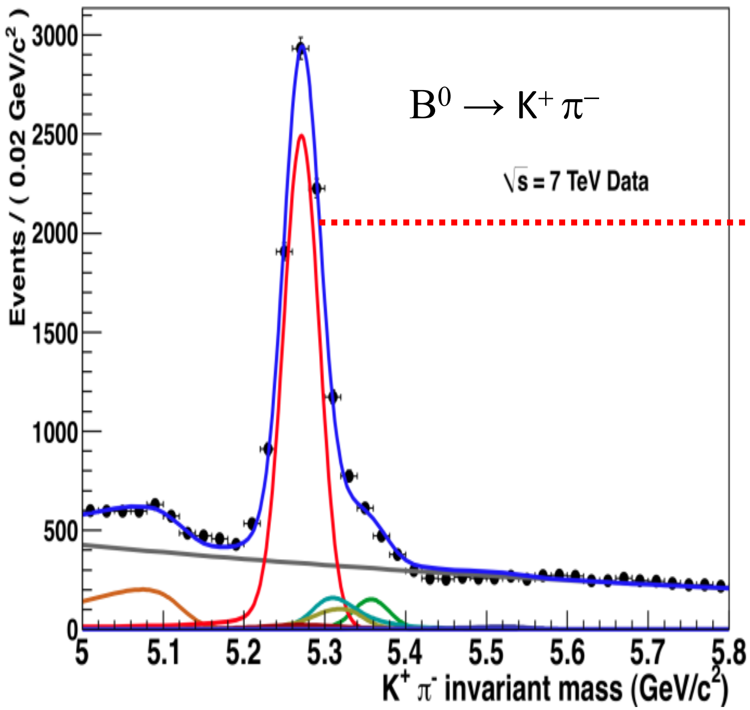


Compare (rare) decays of particles with b-quarks…
…and look for differences between matter and antimatter

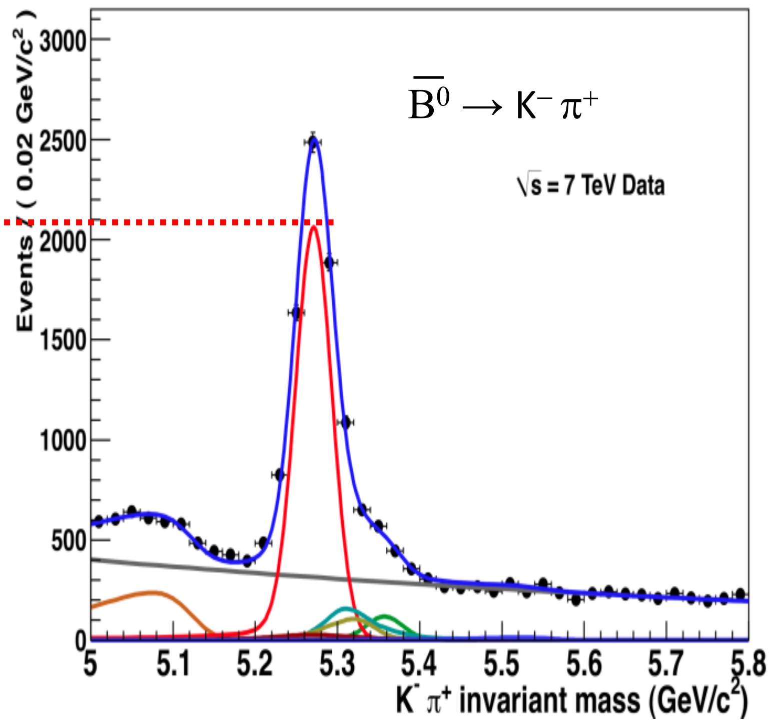
LHCb: B-mesons: matter vs antimatter

B^0 decays to K^+ and π^- ***anti-B0 decays to*** K^- **en** π^+

Matter process

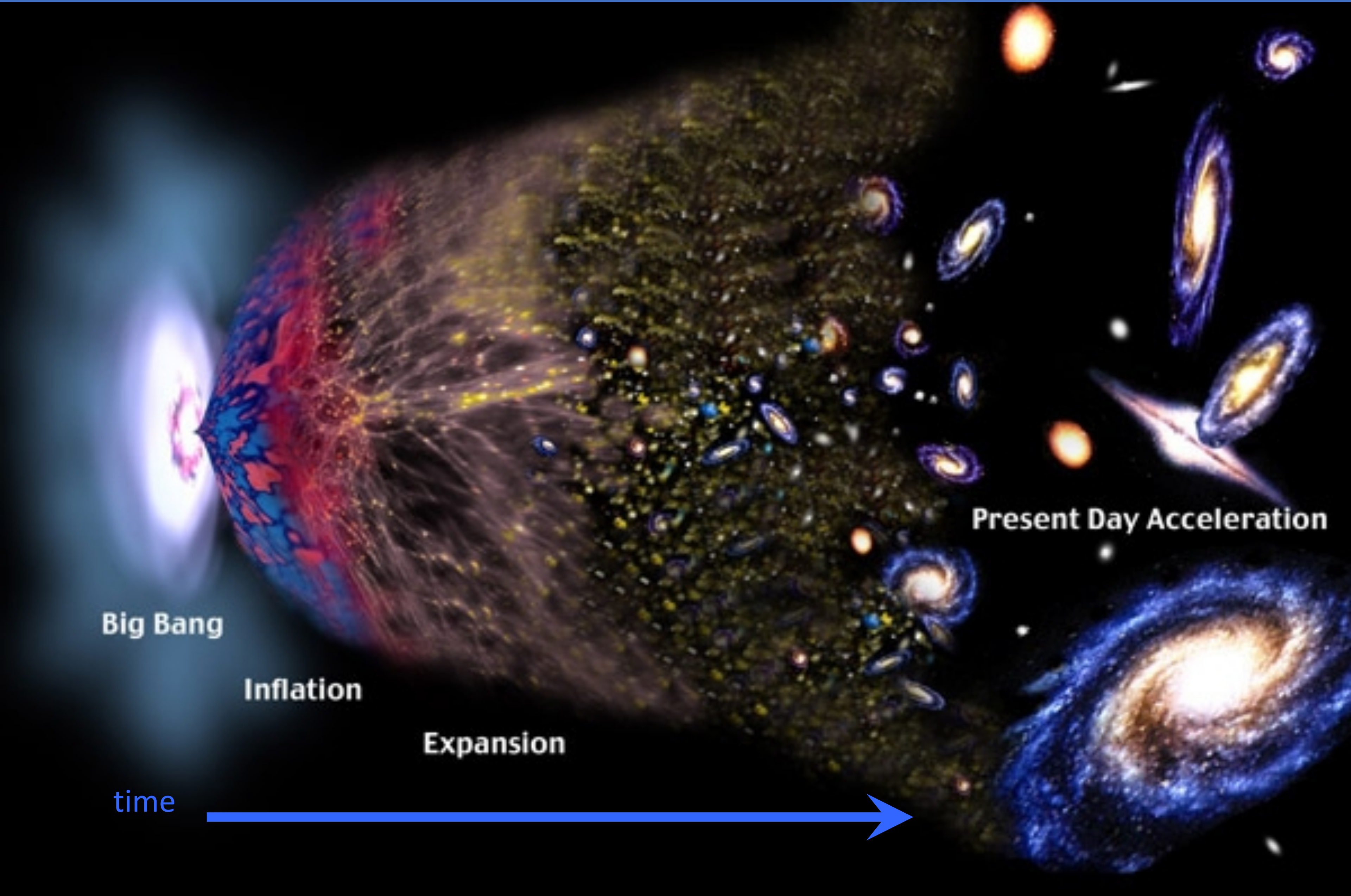


Antimatter process

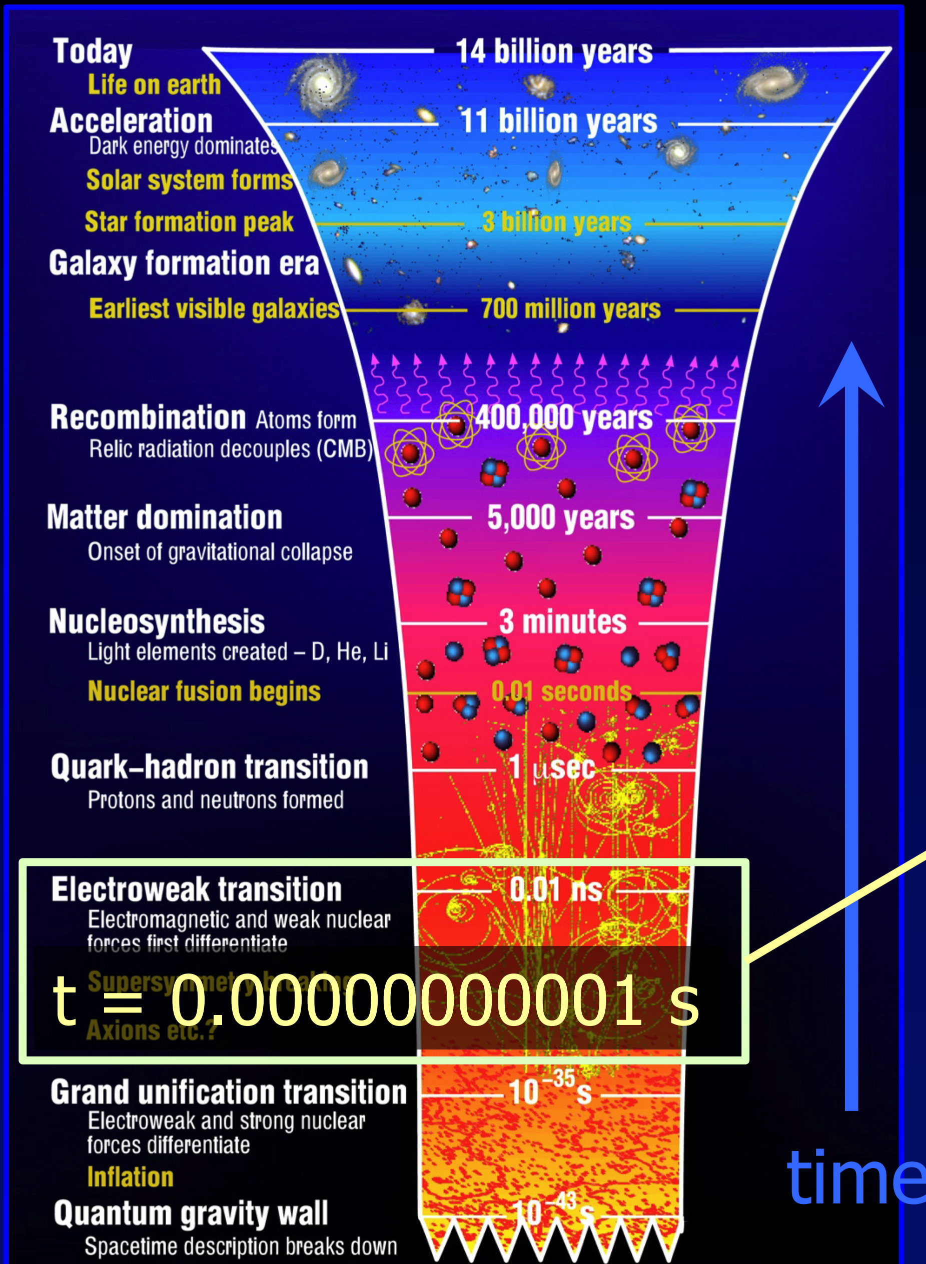


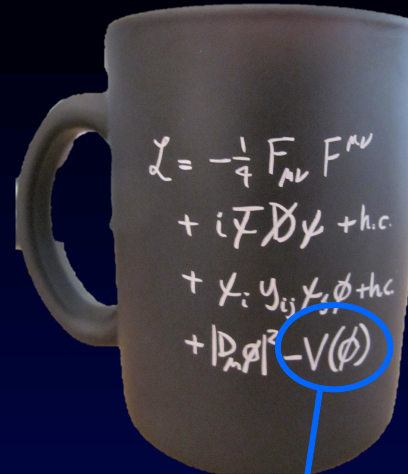
- Left and right not the same speed
- Happens only in decays with ***three quark generations*** present

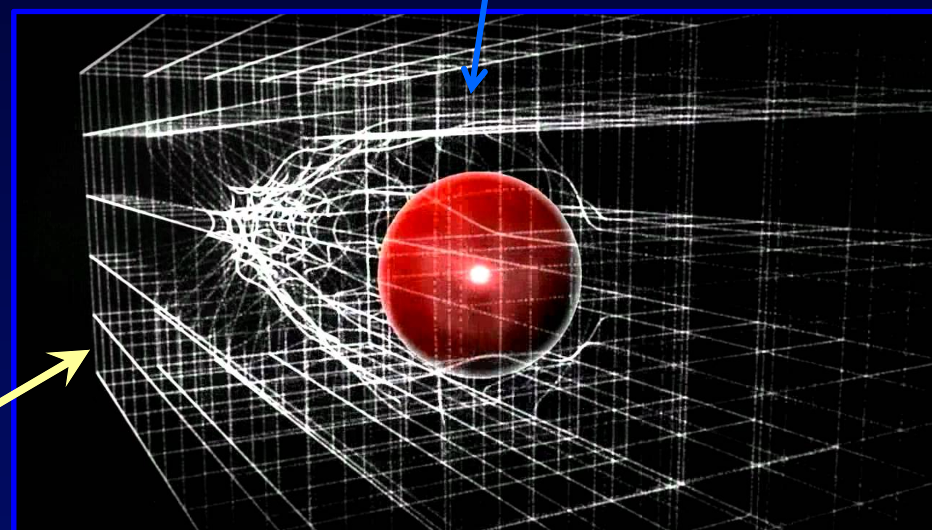
Big Bang


Big Bang
Inflation
Expansion
Present Day Acceleration
time

Higgs field emerged during Big Bang





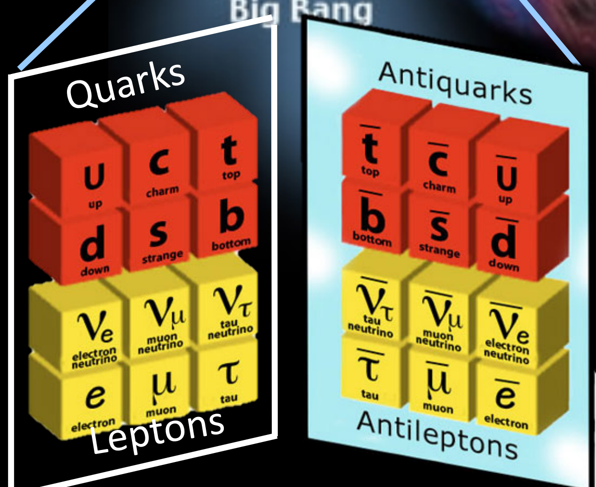


Higgs field:

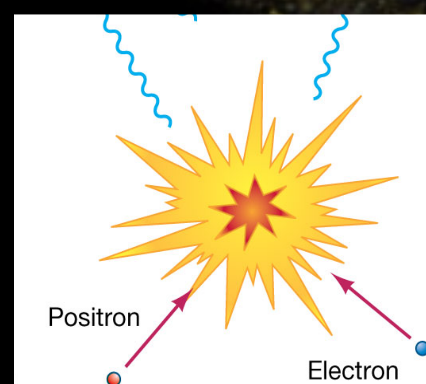
- Suddenly particles got mass
- Antimatter disappeared?

How did anti-matter disappear during Big Bang?

Standard model not enough

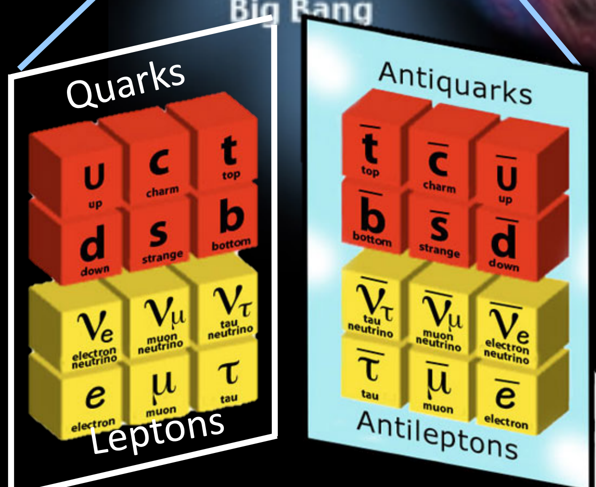


50.000001% 49.999999%



How did anti-matter disappear during Big Bang?

Why does nature provide 3 generations?



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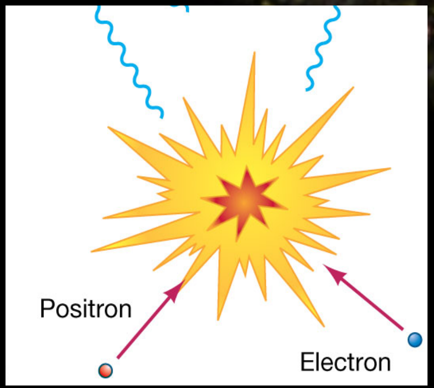

Lectures PP1

- 1: Wave Equations and Antiparticles
- 2: Perturbation Theory and Fermi's Golden Rule
- 3: The Electromagnetic Field
- 4: Scattering of Spinless Particles

QED for spin-0 particles
Scattering Theory & Cross sections

- 5: The Dirac Equation
- 6: Spin-1/2 Electrodynamics
- 7: The Weak Interaction

QED for fundamental Fermions
(spin- ½ particles)

- 8: Local Gauge Invariance
- 9: Electroweak Theory
- 10: The Process $e^+e^- \rightarrow \mu^+\mu^-$

Electroweak Standard model for
massless particles

- 11: Symmetry Breaking
- 12: The Higgs Mechanism
- 13: Fermion Masses, Higgs Decay and limits on m_h
- 14: Problems with the Higgs mechanism and beyond the SM

Standard model for massive
particles

Particle Physics 1
Introduction Lecture 1

Fall 2017

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Exercise classes: Laurent du Four, Maarten van Veghel

Standard Model of Particles

Fermions (Spin 1/2 particles) : Basic constituents of matter

Quarks :

$$\begin{pmatrix} u, u, u \\ d, d, d \end{pmatrix} \quad \begin{pmatrix} c, c, c \\ s, s, s \end{pmatrix} \quad \begin{pmatrix} t, t, t \\ b, b, b \end{pmatrix}$$

Leptons :

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

Quarks only occur in color “neutral” objects: “Hadrons”

Baryons: qqq

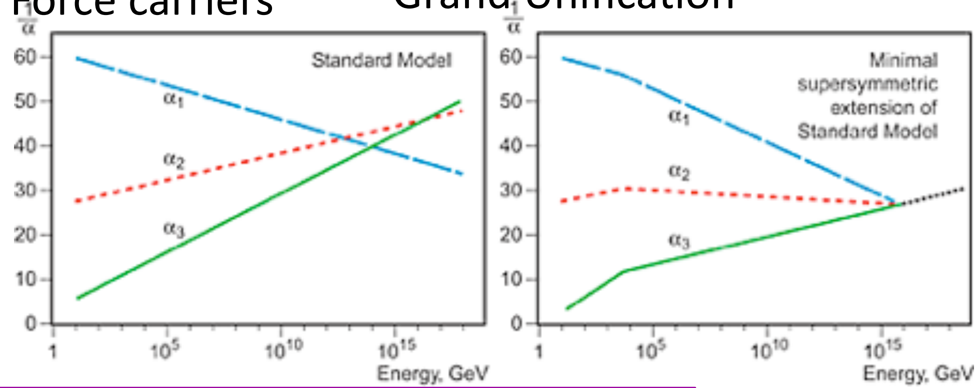
Mesons: qq

Three generations of fundamental particles exist!

Vector Bosons (Spin 1 particles) : Force carriers

Strong Interaction:	8 gluons
Weak Interaction:	$W^+ \, W^- \, Z^0$
Electromagnetism:	foton γ
Gravity:	graviton g ?

Grand Unification



Forces originate from principle of local gauge invariance: symmetry!

Scalar Boson (Spin 0 Higgs particle) : Masses via Spontaneous Symmetry Breaking

Matter Waves for Particles without Spin

	Non Relativistic	Relativistic
Kinematics:	$E = \frac{\vec{p}^2}{2m}$	$E^2 = \vec{p}^2 + m^2$
Quantum Mechanics:	$E \to i\frac{\partial}{\partial t}$ and $\vec{p} \to -i\vec{\nabla}$	$E \to i\frac{\partial}{\partial t}$ and $\vec{p} \to -i\vec{\nabla}$
Wave Equation:	$i\frac{\partial}{\partial t}\psi = -\frac{1}{2m}\nabla^2\psi$	$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\phi$

Continuity Equation: $\vec{\nabla}\cdot\vec{j} = -\frac{\partial\rho}{\partial t}$ or $\partial_\mu j^\mu = 0$

Probability density and current:

Non relativistic:

$$\rho = \psi^*\psi = |N|^2$$

$$\vec{j} = -\frac{i}{2m}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) = \frac{|N|^2}{m}\vec{p}$$

Relativistic:

$$\rho = i\left(\phi^*\frac{\partial\phi}{\partial t} - \phi\frac{\partial\phi^*}{\partial t}\right) = 2|N|^2 E$$

$$\vec{j} = -i\left(\phi^*\vec{\nabla}\phi - \phi\vec{\nabla}\phi^*\right) = 2|N|^2\vec{p}$$

Negative Energy solutions: $j^\mu(+e) = 2e|N|^2(E,\vec{p}) = -2e|N|^2(-E,-\vec{p})$

The negative energy solution of a particle travelling backwards in time =
the positive energy solution of an antiparticle travelling forwards in time.

Particle Physics 1
Introduction Lecture 3

Fall 2017

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Exercise classes: Laurent du Four, Maarten van Veghel

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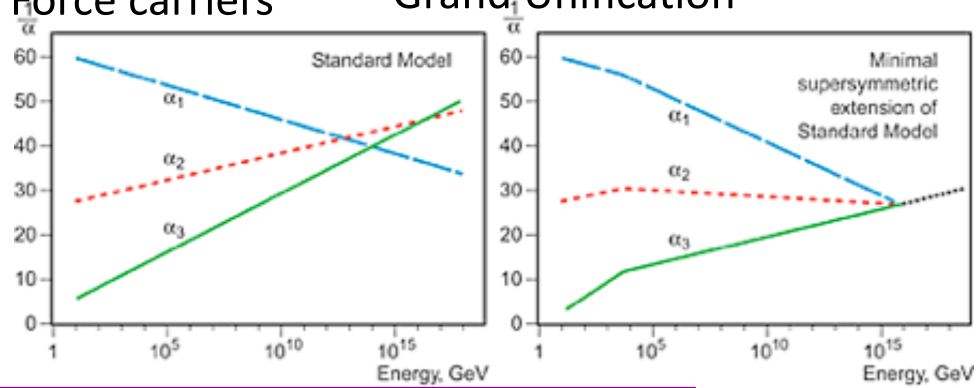

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Perturbation Theory (1)

$$i\frac{\partial \psi}{\partial t} = (H_0 + V(\vec{x},t))\ \psi \quad ; \quad \psi = \sum_{n=0}^{\infty} a_n(t)\ \phi_n(\vec{x})\ e^{-iE_nt}$$

$$\int \psi_f^* d^3x \times$$

$$\psi_f^* = \phi_f^* e^{iE_ft}$$

$$i\sum_{n=0}^{\infty} \frac{da_n(t)}{dt}\ \phi_n(\vec{x})\ e^{-iE_nt} + i\sum_{n=0}^{\infty}(-i)E_n\ a_n(t)\ \phi_n(\vec{x})\ e^{-iE_nt} =$$

$$\sum_{n=0}^{\infty} E_n\ a_n(t)\ \phi_n(\vec{x})\ e^{-iE_nt} + \sum_{n=0}^{\infty} V(\vec{x},t)\ a_n(t)\ \phi_n(\vec{x})\ e^{-iE_nt}$$

$$i\sum_{n=0}^{\infty} \frac{da_n(t)}{dt} \int d^3x\ \underbrace{\phi_f^*(\vec{x})\phi_n(\vec{x})}_{\delta_{fn}}\ e^{-i(E_n-E_f)t} =$$

$$\sum_{n=0}^{\infty} a_n(t) \int d^3x\ \phi_f^*(\vec{x})\ V(\vec{x},t)\ \phi_n(\vec{x}) e^{-i(E_n-E_f)t}$$

$$\frac{da_f(t)}{dt} = -i\sum_{n=0}^{\infty} a_n(t) \int d^3x\ \phi_f^*(\vec{x})\ V(\vec{x},t)\ \phi_n(\vec{x})\ e^{-i(E_n-E_f)t}$$

Perturbation Theory (2)

$$\frac{da_f(t)}{dt} = -i \sum_{n=0}^{\infty} a_n(t) \int d^3x \, \phi_f^*(\vec{x}) \, V(\vec{x}, t) \, \phi_n(\vec{x}) \, e^{-i(E_n - E_f)t}$$

$$\frac{da_f(t)}{dt} = -i \int d^3x \, \phi_f^*(\vec{x}) \, V(\vec{x}, t) \, \phi_i(\vec{x}) \, e^{-i(E_i - E_f)t}$$

$$a_f(t') = \int_{-T/2}^{t'} \frac{da_f(t)}{dt} \, dt$$

$$V_{fi} \equiv \int d^3x \, \phi_f^*(\vec{x}) \, V(\vec{x}) \, \phi_i(\vec{x})$$

$$= -i \int_{-T/2}^{t'} dt \int d^3x \, \left[\phi_f(\vec{x}) e^{-E_f t}\right]^* \, V(\vec{x}, t) \, \left[\phi_i(\vec{x}) e^{-E_i t}\right]$$

$$T_{fi} \equiv a_f(T/2) = -i \int_{-T/2}^{T/2} dt \int d^3x \, \phi_f^*(\vec{x}, t) \, V(\vec{x}, t) \, \phi_i(\vec{x}, t)$$

$$T_{fi} = a_f = -i \int d^4x \, \phi_f^*(x) \, V(x) \, \phi_i(x)$$

Relativistic Scattering

$$T_{fi} = -iN_A N_B N_C N_D \, (2\pi)^4 \, \delta^4(p_A + p_B - p_C - p_D) \, \mathcal{M}$$

$$\begin{aligned}
|T_{fi}|^2 &= |N_A N_B N_C N_D|^2 \; |\mathcal{M}|^2 \int d^4x \, e^{-i(p_A+p_B-p_C-p_D)x} \times \int d^4x' \, e^{-i(p_A+p_B-p_C-p_D)x'} \\
&= |N_A N_B N_C N_D|^2 \; |\mathcal{M}|^2 \; (2\pi)^2 \; \delta^4 \, (p_A + p_B - p_C - p_D) \times \lim_{T,V\to\infty} \int_{TV} d^4x \\
&= |N_A N_B N_C N_D|^2 \; |\mathcal{M}|^2 \; (2\pi)^2 \; \delta^4 \, (p_A + p_B - p_C - p_D) \times \lim_{T,V\to\infty} \; TV
\end{aligned}$$

$$\begin{aligned}
W_{fi} &= \lim_{T,V\to\infty} \frac{|T_{fi}|^2}{TV} \\
&= |N_a N_B N_C N_D|^2 \; |\mathcal{M}|^2 \; (2\pi)^4 \, \delta^4 \, (p_A + p_B - p_C - p_D)
\end{aligned}$$

Relativistic Scattering

$$d\sigma_{fi} = \frac{W_{fi}}{Flux}\, d\Phi$$

$$W_{fi} = \lim_{T,V\to\infty} \frac{|T_{fi}|^2}{TV} \quad ; \quad T_{fi} = -i \int d^4x\, \psi_f^*(x)\, V(x)\, \psi_i(x)$$

$$T_{fi} = -iN_A N_B N_C^* N_D^*\, (2\pi)^4\, \delta^4(p_A + p_B - p_C - p_D)\, \mathcal{M}$$

$$d\Phi = \sum_{i=1}^{N} \frac{V}{(2\pi)^3} \frac{d^3\vec{p}_i}{2E_i} \quad ; \quad Flux = 4\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2}\ /\ V^2$$

Cross Section and Decay Rate

Cross Section: A + B → C + D

$$d\sigma = \frac{(2\pi)^4\, \delta^4\, (p_A + p_B - p_C - p_D)}{4\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2}} \cdot |\mathcal{M}|^2 \cdot \frac{d^3 p_C}{(2\pi)^3\, 2E_C} \frac{d^3 p_D}{(2\pi)^3\, 2E_D}$$

Center of Mass:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2\, s} \frac{|\vec{p}_f|}{|\vec{p}_i|} |\mathcal{M}|^2$$

Decay rate: A → B + C

$$d\Gamma = \frac{(2\pi)^4\, \delta^4\, (p_A - p_B - p_C)}{2E_A} \cdot |\mathcal{M}|^2 \cdot \frac{d^3 p_B}{(2\pi)^3\, 2E_B} \frac{d^3 p_C}{(2\pi)^3\, 2E_C}$$

Center of Mass:

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2\, m_A^2} |\vec{p}_f|^2\, |\mathcal{M}|^2$$

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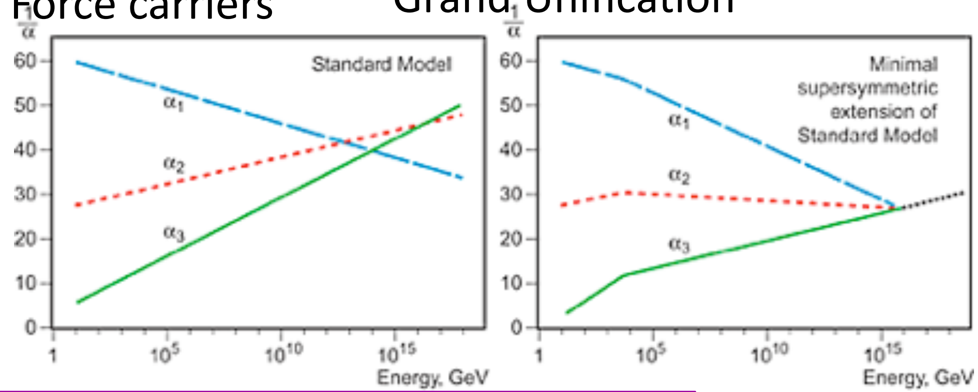
Gravity:	graviton g ?

Grand Unification



Forces originate from principle of local gauge invariance: symmetry!

<u>Scalar Boson</u> (Spin 0 Higgs particle) : Masses via Spontaneous Symmetry Breaking

Matter Waves and EM Field

Matter Waves

$$E \to i\frac{\partial}{\partial t} \quad ; \quad \vec{p} \to -i\vec{\nabla}$$

Non Relativistic

$$E = \frac{\vec{p}^2}{2m}$$

$$i\frac{\partial}{\partial t}\psi = \frac{-1}{2m}\nabla^2\psi$$

Relativistic

$$E^2 = \vec{p}^2 + m^2$$

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\phi$$

Continuity: $\partial_\mu j^\mu = 0$ with: $j^\mu = i\left(\phi^* \partial^\mu \phi - \phi\, \partial^\mu \phi^*\right)$

Scattering Theory: A + B → C + D

$$d\sigma_{fi} = \frac{W_{fi}}{Flux}\, d\Phi$$

$$W_{fi} = \lim_{T,V\to\infty} \frac{|T_{fi}|^2}{TV} \quad ; \quad T_{fi} = -i\int d^4x\, \psi_f^*(x)\, V(x)\, \psi_i(x)$$

$$T_{fi} = -iN_A N_B N_C^* N_D^*\, (2\pi)^4\, \delta^4(p_A + p_B - p_C - p_D)\, \mathcal{M}$$

$$d\Phi = \sum_{i=1}^{N} \frac{V}{(2\pi)^3}\frac{d^3\vec{p}_i}{2E_i} \quad ; \quad Flux = 4\sqrt{(p_A\cdot p_B)^2 - m_A^2 m_B^2}\;/\;V^2$$

Matter Waves and EM Field

EM Field

$$\vec{\nabla} \cdot \vec{E} = \rho$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$
$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j}$$

Maxwell Equations →

$$\vec{B} = \vec{\nabla} \times \vec{A}$$
$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla} V$$
$$A^\mu = \left(V, \vec{A}\right)$$
$$j^\nu = \partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu$$

Gauge Invariance: $A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu \lambda$

Lorentz Condition: $\partial_\mu A^\mu = 0$

Coulomb Condition: $A^0 = 0 \quad ; \quad \vec{\nabla} \cdot \vec{A} = 0$

Photon has 2 polarizations!

Aharanov – Bohm Experiment! Is the Vector field just a mathematical tool? Quantum particles ”feel” the A-field, even with E and B fields equal to zero.
➔ Even though it has gauge freedom the A-field is real!

Particle Physics 1
Introduction Lecture 5

Fall 2017

Lecturers: Wouter Hulsbergen, Marcel Merk, Ivo van Vulpen

Exercise classes: Laurent du Four, Maarten van Veghel

Quantum Electrodynamics

1) Free particle wave equations

K.G.: $\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0$

Dirac: $\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) = 0$

Plane wave solutions:

$\phi(x) = N e^{-ipx}$

$\psi(x) = u(p) e^{-ipx}$

Current: $\partial_\mu j^\mu = 0$

$j^\mu = i\left[\phi^*\left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right)\phi\right]$ ⇒ $\rho = 2|N|^2 E$

$j^\mu = \bar{\psi}\gamma^\mu \psi$ ⇒ $\rho = \psi^\dagger \psi \geq 0$

2) Electromagnetic field

Maxwell $\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu$

or $\partial_\mu F^{\mu\nu} = j^\nu$

Gauge freedom:

$A_\mu \rightarrow A_\mu' = A_\mu + \partial_\mu \lambda$

with $\partial_\nu \partial^\nu \lambda = 0$

Lorentz condition:

$\partial_\mu A^\mu = 0$

$\partial_\mu \partial^\mu A^\nu = j^\nu$

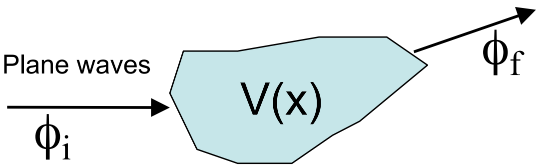
Plane wave solutions:

$A^\mu = N \varepsilon^\mu (p) e^{-ipx}$

(2 polarizations since m=0)

3) Scattering Perturbation Theory

A: non-relativistic derivation:



$i\frac{\partial \psi}{\partial t} = \left(H_0 + V(x,t)\right)\psi$

$\psi = \sum_{n=0}^{\infty} a_n(t)\phi_n(t) e^{-iEt}$

$T_{fi} = a_f(t \rightarrow \infty) = -i\int d^4x\, \phi_f^*(x) V(x) \phi_i(x)$

1-st order:

$W_{fi} = \lim_{T\rightarrow\infty} \frac{|T_{fi}|^2}{T}$ with

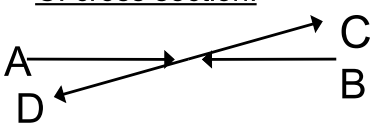
$T_{fi} = -2\pi i\, V_{fi}\, \delta\left(E_f - E_i\right)$ with

$V_{fi} = \int d^3x\, \phi_f^*(\vec{x}) V(\vec{x}) \phi_i(\vec{x})$

B: relativistic extension:

$W_{fi} = \lim_{T,V\rightarrow\infty} \frac{|T_{fi}|^2}{TV}$ and $T_{fi} = -iN_A N_B N_C^* N_D^* (2\pi)^4 \delta^4\left(p_A + p_B - p_C - p_D\right) \mathcal{M}$

C: cross section:



$\frac{d\sigma}{d\Omega} = \frac{W_{fi}}{flux} d\Phi$

$flux = 4\sqrt{\left(p_A p_B\right)^2 - m_A^2 m_B^2} / V^2$

$d\Phi = \prod_i \frac{V}{(2\pi)^3} \frac{d^3 p_i}{2E_i}$

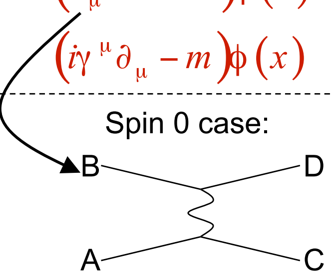
$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{s} \left|\frac{p_f}{p_i}\right| |\mathcal{M}|^2$

4) Electromagnetic Scattering $\partial^\mu \rightarrow \partial^\mu - ieA^\mu$

$\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) \rightarrow \left(\partial_\mu \partial^\mu + m^2 + V(x)\right)\phi(x)$; $V(x) = -ie\partial_\mu A^\mu + A_\mu \partial^\mu$ $\quad T_{fi} = -i\int -ie\left[\phi_f^*\left(\partial_\mu \phi_i\right) - \left(\partial_\mu \phi_f^*\right)\phi_i\right] A^\mu\, d^4x$

$\left(i\gamma^\mu \partial_\mu - m\right)\phi(x) \rightarrow \left(i\gamma^\mu \partial_\mu - m + V(x)\right)\phi(x)$; $V(x) = -e\gamma^0 \gamma_\mu A^\mu$ $\quad T_{fi} = -i\int -e\left[\bar{\psi}_f \gamma_\mu \psi_i\right] A^\mu\, d^4x$

Spin 0 case:



$\phi_i = N_i e^{-ip_i x}$ $\quad \phi_f^* = N_f^* e^{ip_f x}$ $\quad \partial_\nu \partial^\nu A^\mu = -j_{BD}^\mu = -eN_B N_D^*\left(p_B^\mu + p_D^\mu\right) e^{i(p_D - p_B)}$ $\quad A^\mu = -\frac{1}{q^2} j_{BD}^\mu$

$T_{fi} = -i\int j_\mu^{AC} A^\mu d^4x = -i\int j_{AC}^\mu \frac{-g_{\mu\nu}}{q^2} j_{BD}^\nu d^4x = -iN_A N_B N_C^* N_D^* (2\pi)^4 \delta^4\left(p_A + p_B - p_C - p_D\right)\mathcal{M}$

$-i\mathcal{M} = ie\left(p_A + p_C\right)^\mu \frac{-ig_{\mu\nu}}{q^2} ie\left(p_B + p_D\right)^\nu$ ⇒ “Feynman rules”

Particle Physics 1
Introduction Lecture 6

Fall 2017

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Particles with Spin=0

Particles with Spin=1/2

$$E^2 = \vec{p}^2 + m^2$$

$$E^2 = (\vec{\alpha}\cdot\vec{p} + \beta\, m)^2$$

Q.M.

$$E \to i\frac{\partial}{\partial t} \quad ; \quad \vec{p} \to -i\vec{\nabla}$$

Klein Gordon:

$$\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0$$

Dirac: $(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0$

$$\text{with} \ : \ \gamma^\mu = (\beta, \beta\vec{\alpha}) \quad ; \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$\longleftarrow$ Solutions $\longrightarrow$

$$\phi(x) = N e^{-ipx}$$

$$\left(\partial^\mu \partial_\mu + m^2\right)\phi(x) = 0 \quad \& \ \ \text{C.C.}$$

$$j^\mu = i\left[\phi^*(\partial^\mu \phi) - (\partial^\mu \phi^*)\phi\right]$$

$$\psi(x) = u(p)e^{-ipx} \qquad \overline{\psi} = \psi^\dagger \gamma^0$$

$$\left(i\partial_\mu \overline{\psi}\gamma^\mu + m\overline{\psi}\right) = 0 \quad (adjoint)$$

$$j^\mu = \overline{\psi}\gamma^\mu \psi$$

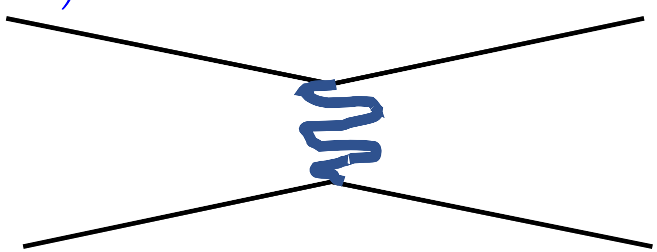
$$\partial_\mu j^\mu = 0$$

$$j^\mu_{fi} = \ i\ \left[\phi^*_f(\partial^\mu \phi_i) - (\partial^\mu \phi^*_f)\phi_i\right]$$

$$= \ -eN_i N^*_f \left(p^\mu_i + p^\mu_f\right) e^{i(p_f - p_i)x}$$

$$j^\mu_{fi} = \ \overline{\psi}_f \gamma^\mu \psi_i$$

$$= \ -e\,\overline{u}_f(p)\,\gamma^\mu\ u_i(p)\ e^{i(p_f - p_i)x}$$



Transition Currents

Feynman Rules

Solutions to the Dirac Equation

Dirac eq ~ 4 x K.G., so use ansatz:

$$(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0 \qquad \Rightarrow \qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} U_A(p) \\ U_B(p) \end{pmatrix} e^{-ipx}$$

Dirac becomes coupled linear equation:

$$\left[\begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} E - \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} p^i - \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} m \right] \begin{pmatrix} U_A \\ U_B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$(\vec{\sigma} \cdot \vec{p})\, U_B = (E - m)\, U_A$$
$$(\vec{\sigma} \cdot \vec{p})\, U_A = (E + m)\, U_B$$

$$U_A = \begin{pmatrix} * \\ * \end{pmatrix} \; ; \; U_B = \begin{pmatrix} * \\ * \end{pmatrix} \qquad \vec{\sigma} \cdot \vec{p} = \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -pz \end{pmatrix}$$

$\vec{p} = 0$

$$U^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \; ; \; U^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \; ; \; U^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \; ; \; U^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \cdot (e^{-ipx})$$

$p \neq 0$

1) Choose: $U_A^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow U_B^{(1)} = \begin{pmatrix} p_z/(E+m) \\ (p_x + ip_y)/(E+m) \end{pmatrix}$ $\quad U_A^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow U_B^{(2)} = \ldots$

2) Choose: $U_B^{(3)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow U_A^{(3)} = \ldots$ $\quad U_B^{(4)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow U_A^{(4)} = \ldots$

---

1) Solutions are orthogonal

2) Normalisation: $N = \sqrt{E + m}$

3) Adjoint: $(\not{p} - m)u = 0 \Rightarrow \overline{u}(\not{p} - m) = 0$

$(\not{p} + m)v = 0 \Rightarrow \overline{v}(\not{p} + m) = 0$

4) Completeness: $\sum_{S=1,2} u^{(s)}(p)\, \overline{u}^{(s)}(p) = \not{p} + m$

5) Helicity: $\lambda = \frac{1}{2} \vec{\Sigma} \cdot \vec{p} \; ; \; \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$

Particle Physics 1
Introduction Lecture 7

Fall 2017

Lecturers: Wouter Hulsbergen, Marcel Merk, Ivo van Vulpen

Exercise classes: Laurent du Four, Maarten van Veghel

QED

S=0 | **S=1/2**

Wave equation: $\left(\partial_\mu \partial^\mu + m^2\right)\ \phi(x) = 0$ | $(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0$

Solution: $\phi(x) = N\, e^{-ipx}$ | $\psi(x) = u(p)\, e^{-ipx}$

Conserved current: $j^\mu = i\left[\phi^* \left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right) \phi\right]$ | $j^\mu = \overline{\psi}\, \gamma^\mu\, \psi$

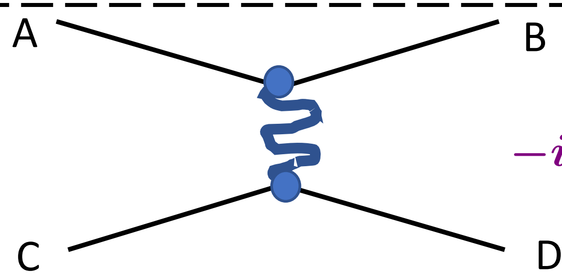
Perturbation Theory: $T_{fi} = -i \int d^4x\, \phi_f^*(x) V(x) \phi_i(x)$ | $T_{fi} = -i \int d^4x\, \psi_f^\dagger(x) V(x) \psi_i(x)$

Electromagnetic Field: $\partial_\mu F^{\mu\nu} = j^\nu \qquad \text{with :} \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

QED: $\partial^\mu \rightarrow \partial^\mu - ieA^\mu \qquad \Rightarrow \qquad T_{fi} = -i \int j_\mu^{fi}(x)\ A^\mu(x)\ d^4x$

Feynman Rules:

$$-i\mathcal{M} = \begin{array}{c} ie\left(p_A + p_C\right)^\mu \\ \cdot\ -ig_{\mu\nu}/q^2 \\ \cdot\ ie\left(p_B + p_D\right)^\nu \end{array}$$



$$-i\mathcal{M} = \begin{array}{c} ie\left(\overline{u}_C \gamma^\mu u_A\right) \\ \cdot\ -ig_{\mu\nu}/q^2 \\ \cdot\ ie\left(\overline{u}_D \gamma^\mu u_B\right) \end{array}$$

Cross Section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(\frac{3 + \cos\theta}{1 - \cos\theta}\right)^2 \quad \Leftarrow \quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}\overline{|\mathcal{M}|^2} \quad \Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\frac{4 + (1 + \cos\theta)^2}{(1 - \cos\theta)^2}$$

Mandelstam Variables & Crossing:

$$e^+ e^- \rightarrow \mu^+ \mu^- = \frac{\alpha^2}{4s}\left(1 + \cos^2\theta\right)$$

Particle Physics 1
Introduction Lecture 8

Fall 2017

Lecturers: Wouter Hulsbergen, Marcel Merk, Ivo van Vulpen

Exercise classes: Laurent du Four, Maarten van Veghel

QED

Electromagnetism/ Weak Interaction

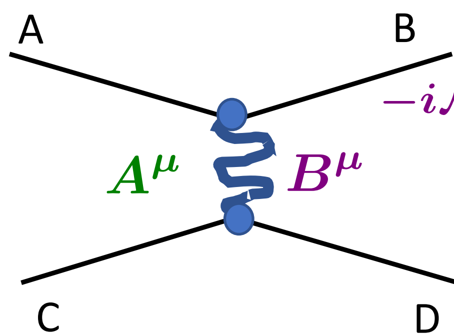
Weak

Matter Waves: $(i\gamma^\mu \partial_\mu - m)\,\psi(x) = 0 \quad ; \quad \psi(x) = u(p)\, e^{-ipx} \quad ; \quad (\not{p} - m)\, u(p) = 0$

	QED	Weak
Substitution:	$\partial^\mu \to \partial^\mu - ieA^\mu$	$\partial^\mu \to \partial^\mu + igB^\mu$
Field Equation:	$\partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu = j^\nu_{EM}$	$\partial_\mu \partial^\mu B^\nu - \partial_\mu \partial^\nu B^\mu + m^2 B^\nu = j^\nu_{\text{Weak}}$
Perturbation Theory	$T_{fi} = -i \int J^{fi}_{EM,\mu}\, A^\mu(x)\, d^4x$; $J^{fi}_{EM,\mu} = \overline{\psi}\gamma^\mu \psi$	$T_{fi} = -i \int J^{fi}_{\text{Weak},\mu}\, B^\mu(x)\, d^4x$; $J^{fi}_{\text{Weak},\mu} = \overline{\psi}\frac{1}{2}\gamma^\mu \left(1-\gamma^5\right)\psi$

Matrix Element:

$$-i\mathcal{M} = ie\left(\overline{u}_C \gamma^\mu u_A\right) \cdot -ig_{\mu\nu}/q^2 \cdot ie\left(\overline{u}_D \gamma^\mu u_B\right)$$



$$-i\mathcal{M} = i\frac{g}{\sqrt{2}}\left(\overline{u}_C \tfrac{1}{2}\gamma^\mu \left(1-\gamma^5\right) u_A\right) \cdot -ig_{\mu\nu}/\left(M^2 - q^2\right) \cdot i\frac{g}{\sqrt{2}}\left(\overline{u}_D \tfrac{1}{2}\gamma^\mu \left(1-\gamma^5\right) u_B\right)$$

$$e^+e^- \to \mu^+\mu^- \qquad \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right)$$

$$\mu^- \to e^- \nu_\mu \overline{\nu}_e \qquad \Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

General Matrix Element

$$\mathcal{M} = \sum_{i,j}^{S,V,T,P,A} C_{ij}\ \left(\overline{u}_C \mathcal{O}_i u_A\right) \cdot \text{prop} \cdot \left(\overline{u}_D \mathcal{O}_j u_C\right)$$

$$S = \overline{\psi}\psi\ ;\ V = \overline{\psi}\gamma^\mu\psi\ ;\ T = \overline{\psi}\sigma^{\mu\nu}\psi\ ;\ A = \overline{\psi}\gamma^\mu\gamma^5\psi\ ;\ P = \overline{\psi}\gamma^5\psi$$

Weak Interaction:

$$\textbf{leptons}:\ \begin{pmatrix}\nu_e\\ e\end{pmatrix};\ \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix};\ \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix} \Rightarrow \begin{pmatrix}\nu_e\\ \nu_\nu\\ \nu_\tau\end{pmatrix} = V_{PMNS}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$$\textbf{quarks}:\ \begin{pmatrix}u\\ d\end{pmatrix};\ \begin{pmatrix}c\\ s\end{pmatrix};\ \begin{pmatrix}t\\ b\end{pmatrix} \Rightarrow \begin{pmatrix}d'\\ s'\\ b'\end{pmatrix} = V_{CKM}\begin{pmatrix}d\\ s\\ b\end{pmatrix}$$

Particle Physics 1
Introduction Lecture 9

Fall 2017

Lecturers: Wouter Hulsbergen, Marcel Merk, Ivo van Vulpen

Exercise classes: Laurent du Four, Maarten van Veghel

Symmetries

Lagrangian density is the basic object for physics: $\mathcal{L}\left(\phi(x), \partial_\mu \phi(x)\right) = \overline{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi$

<u>Symmetry:</u> Require that the Lagrangian remains invariant under a symmetry operation

<u>4 symmetry groupes:</u>
- Permutation symmetries
- Continuous space-time symmetries
- Discrete symmetries $\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$
- *Unitary or Gauge symmetries*

Unitary Phase Symmetry U(1)

$\overline{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi$

Covariant Derivative:

$$\mathcal{D}_\mu = \partial_\mu + iqA_\mu(x)$$

Gauge Transformations:

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

$$A_\mu(x) \rightarrow A'_\mu(x) - \frac{1}{q}\partial_\mu \alpha(x)$$

(Maxwell Gauge Invariance)

Yang-Mills Symmetry U(1)

$$\psi(x) = \begin{pmatrix} p \\ n \end{pmatrix}$$

Covariant Derivative:

$$\mathcal{D}_\mu = \partial_\mu + igB_\mu(x) \quad \text{with} \quad B_\mu(x) = \frac{1}{2}\vec{\tau}\cdot\vec{b}_\mu$$

Gauge Transformations:

$$\psi(x) \rightarrow \psi'(x) = e^{i\frac{1}{2}\vec{\tau}\cdot\vec{\alpha}(x)}\psi(x)$$

$$B_\mu(x) \rightarrow B'_\mu(x) = GB_\mu(x)G^{-1} + \frac{i}{g}(\partial_\mu G)G^{-1}$$

$$\vec{b}_\mu(x) \rightarrow \vec{b}'_\mu(x) = \vec{b}_\mu - \vec{\alpha}\times\vec{b}_\mu - \frac{1}{g}\partial_\mu \alpha(x)$$

(“non Abelian”)

U(1) Lagrangian:

$$\begin{aligned} \overline{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi &= \overline{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi - q\overline{\psi}\gamma^\mu\psi A_\mu \\ \mathcal{L}_{U(1)} &= \mathcal{L}^{\text{free}}_{U(1)} + j^\mu A_\mu \end{aligned}$$

SU(2) Lagrangian:

$$\begin{aligned} \overline{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi &= \overline{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi - \frac{g}{2}\overline{\psi}\gamma^\mu\vec{\tau}\psi\vec{b}_\mu \\ \mathcal{L}_{SU(2)} &= \mathcal{L}^{\text{free}}_{SU(2)} + \vec{j}^\mu\vec{b}_\mu \end{aligned}$$

Particle Physics 1
Introduction Lecture 10

Fall 2017

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Standard Model of Electroweak Interactions

Origin of interactions is described via the principle of ***local gauge invariance***

Recipe: Take the Lagrangian of free Dirac particles $\mathcal{L}_{\text{Dirac}} = \overline{\psi}(x)\ (i\gamma^\mu \partial_\mu - m)\ \psi(x)$

and impose that it remains invariant under:

$$U(1)_Y \ :\ \psi(x) \to \psi'(x) = e^{iY\alpha(x)}\psi(x) \quad ; \quad Y = \text{hypercharge} \quad , \quad Q = T_3 + \frac{1}{2}Y$$

$$SU(2)_L \ :\ \psi_L(x) \to \psi'_L(x) = e^{i\vec{T}\cdot\vec{\alpha}(x)}\psi_L(x) \quad ; \quad \vec{T} = \frac{1}{2}\vec{\tau} = \text{weak isospin} \ , \quad \psi_L = \left(\frac{1-\gamma^5}{2}\right)\psi$$

To do that compensating gauge fields are introduced which transform with the Dirac fields:

$$\partial_\mu \ \to\ \mathcal{D}_\mu = \ \partial_\mu + g'\frac{Y}{2}a_\mu + g\vec{T}\cdot\vec{b}_\mu \qquad a_\mu = \text{hypercharge field}$$

$$\Rightarrow\ \mathcal{L} = \ \mathcal{L}_{\text{free}} - g'\frac{J_Y^\mu}{2}a_\mu - g\vec{J}_L^\mu\cdot\vec{b}_\mu \qquad b_\mu^1, b_\mu^2, b_\mu^3 = \text{weak isospin fields}$$

Physical currents: $C.C.:$

$$W_\mu^+ = \frac{b_\mu^1 - ib_\mu^2}{\sqrt{2}} \qquad W_\mu^- = \frac{b_\mu^1 + ib_\mu^2}{\sqrt{2}}$$

$N.C.:$

$$A_\mu = \ a_\mu \cos\theta_W + b_\mu^3 \sin\theta_W \qquad Z_\mu = -a_\mu \sin\theta_W + b_\mu^3 \cos\theta_W$$

The Lagrangian becomes:

$$\begin{aligned}\mathcal{L} \ = \ \mathcal{L}_{\text{free}} \ & -\frac{g}{\sqrt{2}}\,\overline{\psi}_u\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\psi_d\, W_\mu^+ \\ & -\frac{g}{\sqrt{2}}\,\overline{\psi}_d\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\psi_u\, W_\mu^- \\ & -e\,Q\,\overline{\psi}\,\gamma^\mu\,\psi\,A_\mu \\ & -g_z\,\overline{\psi}\,\gamma^\mu\frac{1}{2}\left(C_V^f - C_A^f\gamma^5\right)\psi\,Z_\mu\end{aligned}$$

$$e = g\sin\theta_W \qquad g'/g = \tan\theta_W$$

$$g_z = g/\cos\theta_W$$

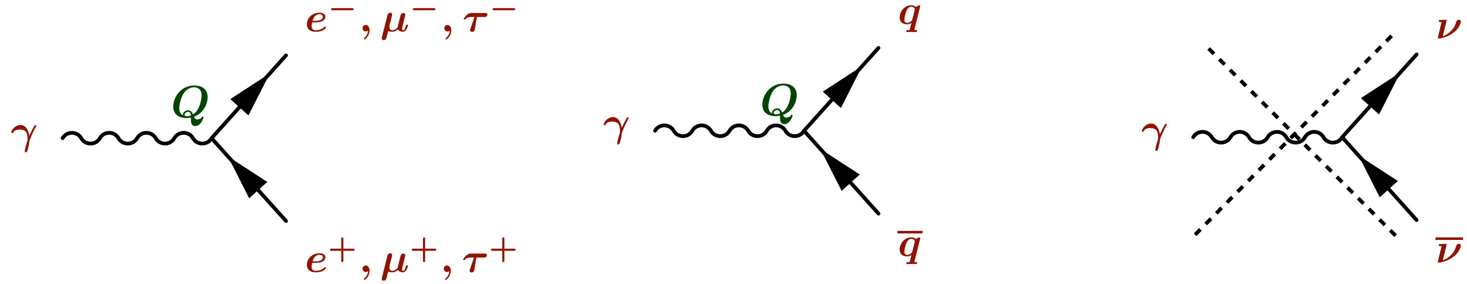
$$C_V^f = T_3^f - 2Q^f\sin^2\theta_W$$

$$C_A^f = T_3^f$$

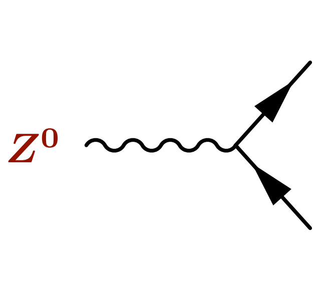
$$T_3 = \begin{matrix} 1/2 \\ -1/2 \end{matrix} \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} \nu \\ l \end{pmatrix}$$

Feynman Vertices

E.M.:



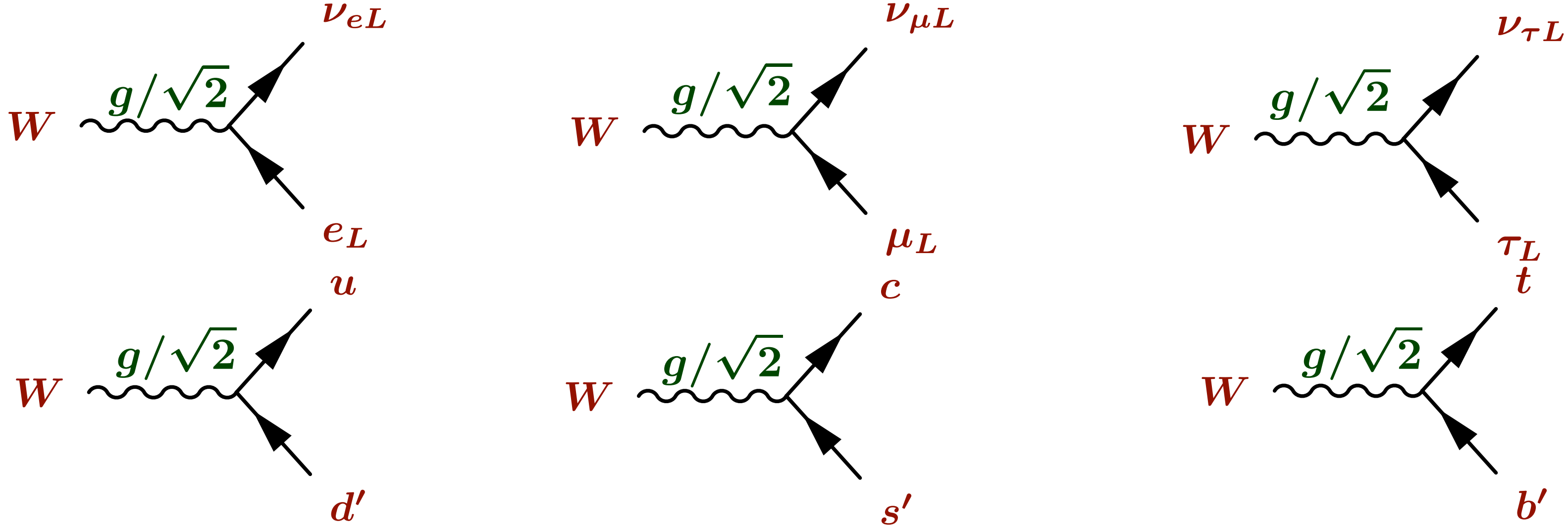
N.C.:



$$g_z = \frac{g}{\cos\theta_W}$$

$$C_V^f = T_3^f - 2Q^f \sin^2\theta_W$$
$$C_A^f = T_3^f$$

C.C.:



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} & V_{CKM} & \end{pmatrix} = \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} & V_{PMNS} & \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$