Lecture 1

Master Particle Physics 1

Marcel Merk

September 5, 2011

Lectures PP1

L0 : Introduction

L1 : Particles and Fields (historical overview)
L2 : Wave Equations and Antiparticles
L3 : The Electromagnetic Field
L4 : Perturbation Theory and Fermi's Golden Rule
L5 : Electromagnetic Scattering of Spinless Particles

QED of Spinless Particles: Scattering Theory and Cross Sections

L6 : The Dirac Equation
L7 : Solutions of the Dirac Equation
L8 : Spin 1/2 Electrodynamics

QED for Fundamental Fermions

L9 : The Weak Interaction
(Fermi 4-point scattering: an analogy with QED)
L10: Local Gauge Invariance
(the role of symmetries in interactions)
L11: Electroweak Theory
L12: The Process: $e^+e^- \rightarrow \gamma, Z \rightarrow \mu^+\mu^-$

The Standard Model for massless particles $SU(2)_L \times U(1)_Y$

OCW

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Minister: Marja van Bijsterveld-Vliegenthart
Staatssecretaris: Halbe Zijlstra

→

NWO

Dutch organisation for Scientific Research
Voorzitter algemeen bestuur: Jos Engelen
General director: Hans de Groene

* Aard- en levenswetenschappen (ALW)
* Chemische wetenschappen (CW)
* Exacte wetenschappen (EW)
* Geesteswetenschappen (GW)
* Maatschappij- en gedragswetenschappen (MaGW)
* Medische wetenschappen (ZonMw)
* **Natuurkunde (N)**
* Technische wetenschappen (STW)

Nikhef

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FOM

Foundation for Fundamental Research of Matter
Director: Wim van Saarloos
Institutes:

- Rijnhuizen: Plasma physics
  director: Gerard Meijer
- Amolf: Atomic and Molecular Physics
  director: Albert Polman
- Nikhef: Dutch Institute for Particle Physics
  director: Frank Linde
- KVI: Nuclear Physics (closed)
  director: Klaus Jungmann

Universities

Nijmegen:	RU	Sijbrand de Jong
Utrecht:	UU	Thomas Peitzmann
Amsterdam:	VU	Jo van den Brand
	UvA	Stan Bentvelsen

+...Groningen, Twente, Leiden, Eindhoven...

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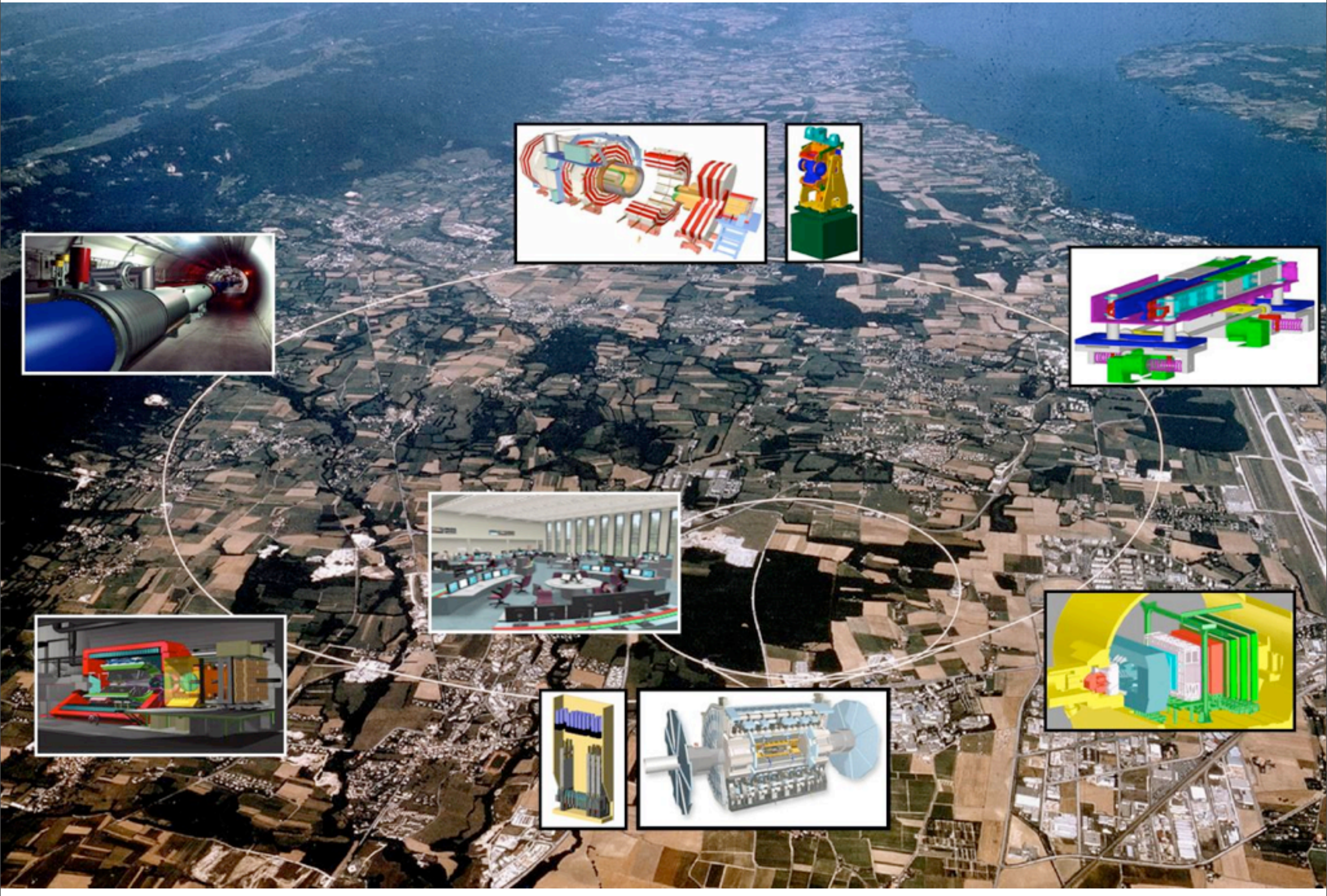
+...Groningen, Twente, Leiden, Eindhoven...

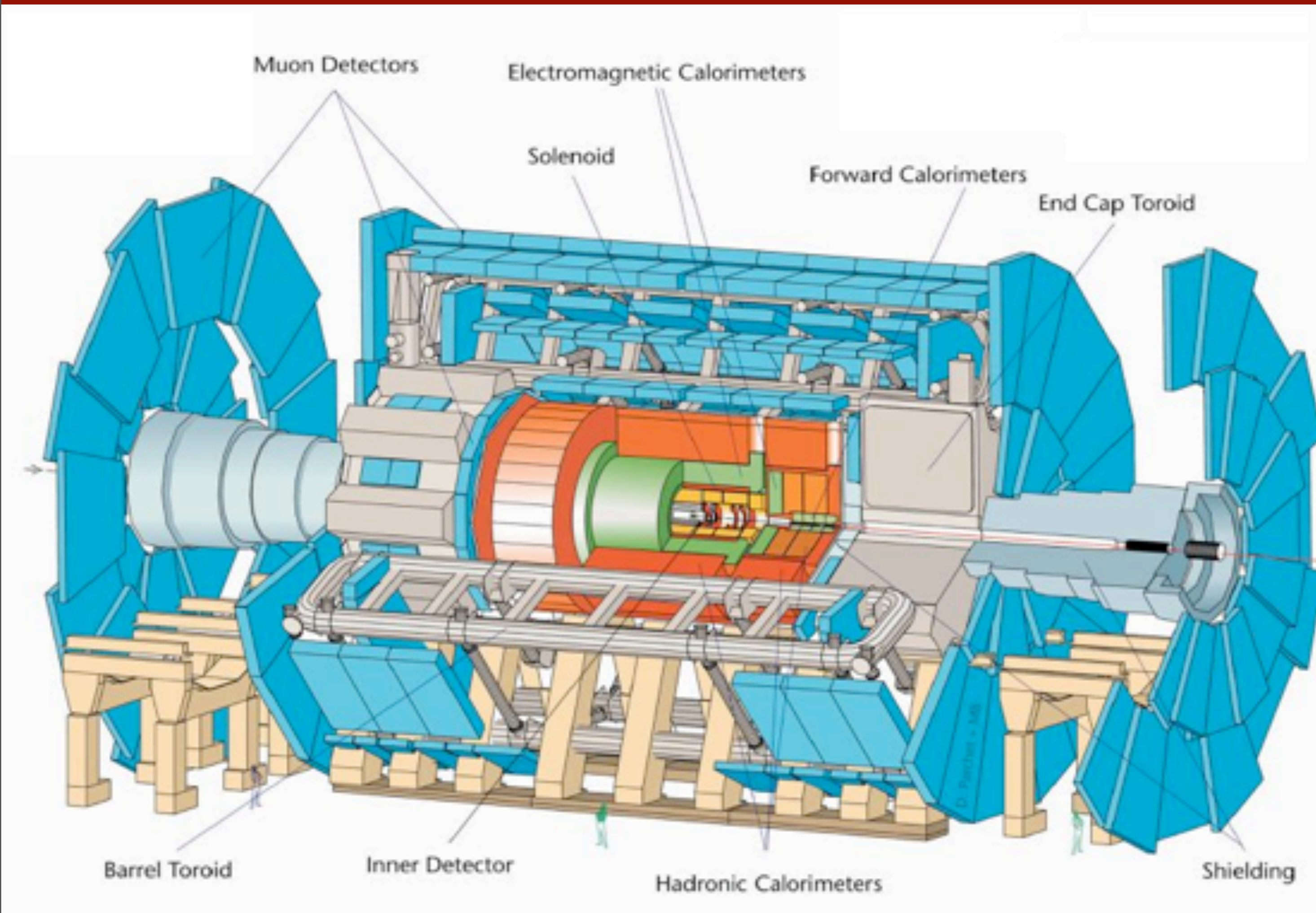
Nikhef Collaboration:
Nikhef institute + 4 Universities
Real basis for all particle physics in the Netherlands
Director: Frank Linde

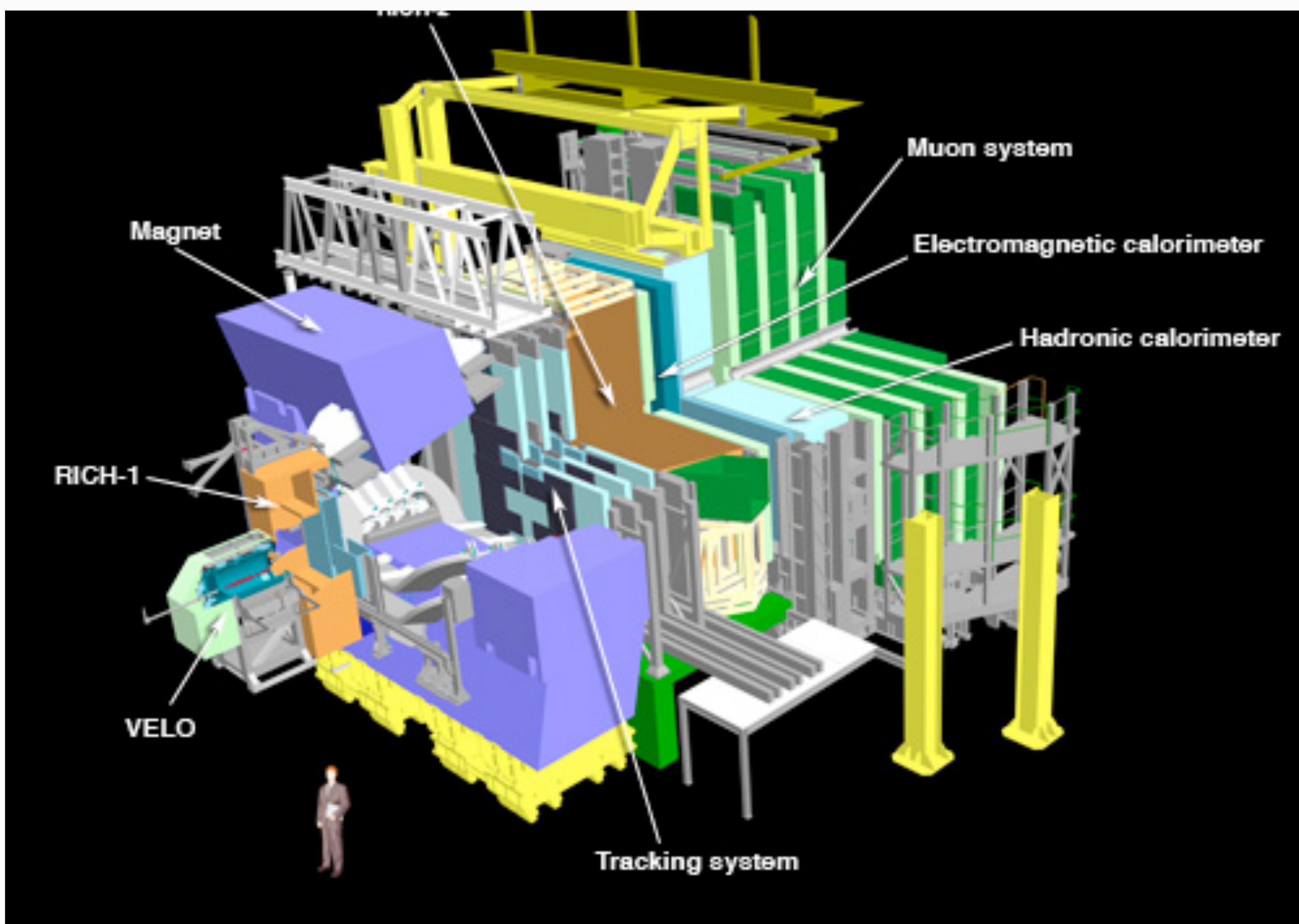
LHC:
Atlas: Stan Bentvelsen
LHCb: Marcel Merk
Alice: Thomas Peitzmann

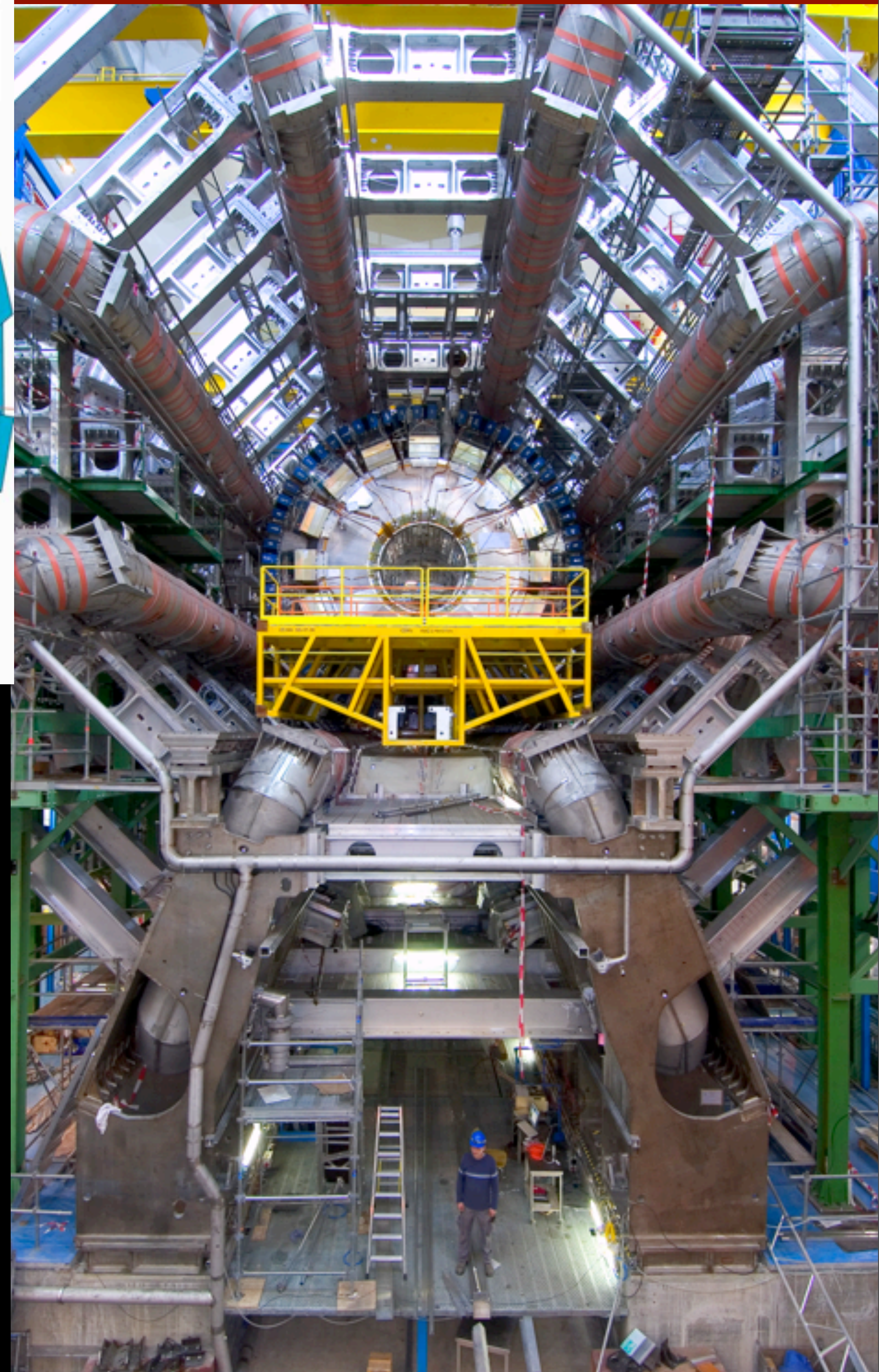
Astroparticle Physics:
Antares: Maarten de Jong
Auger: Charles Timmermans
Grav Waves: Jo van den Brand
Dark Matter: Patrick Decowski

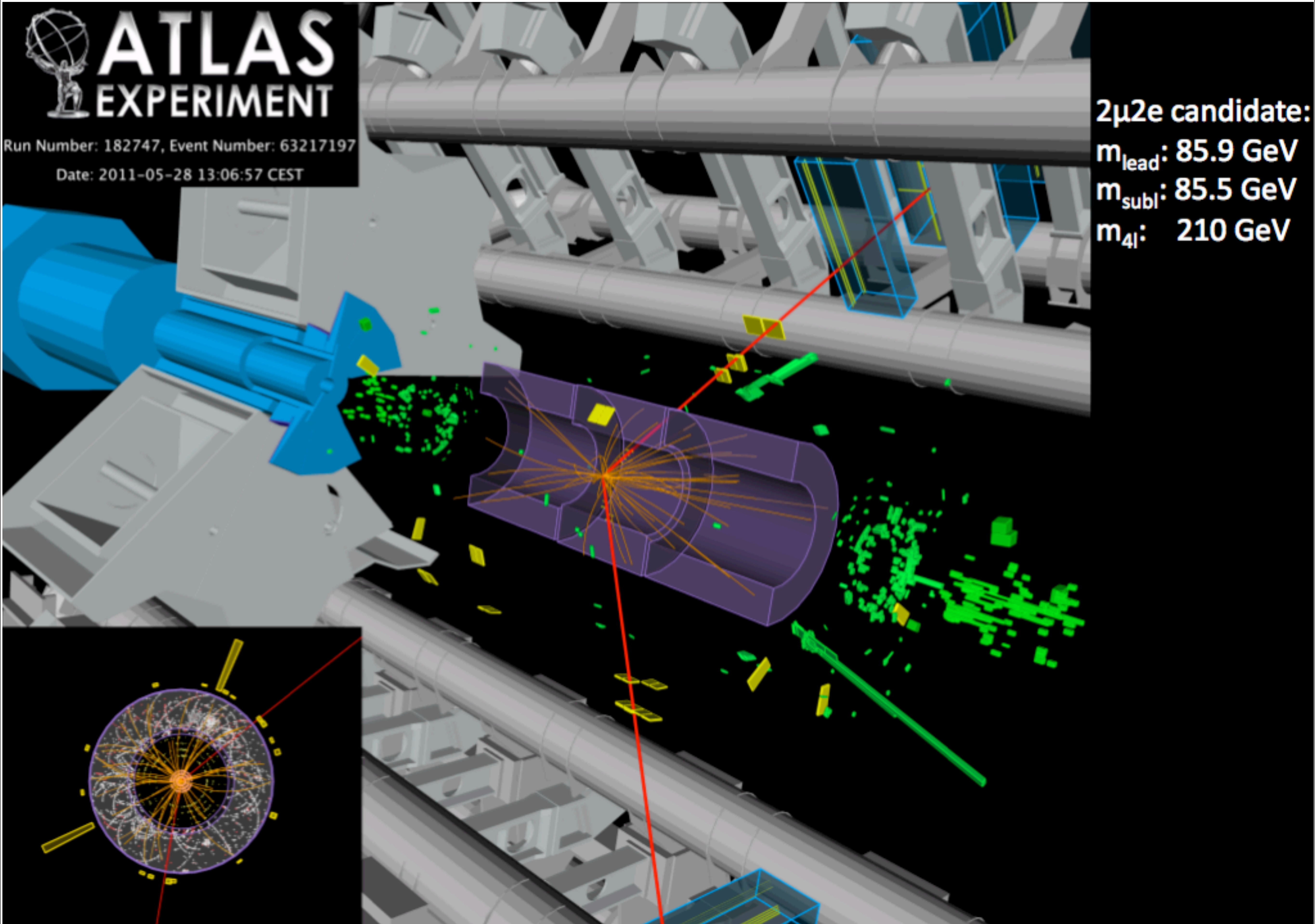
General:
Theory: Eric Laenen
Grid: Jeff Templon

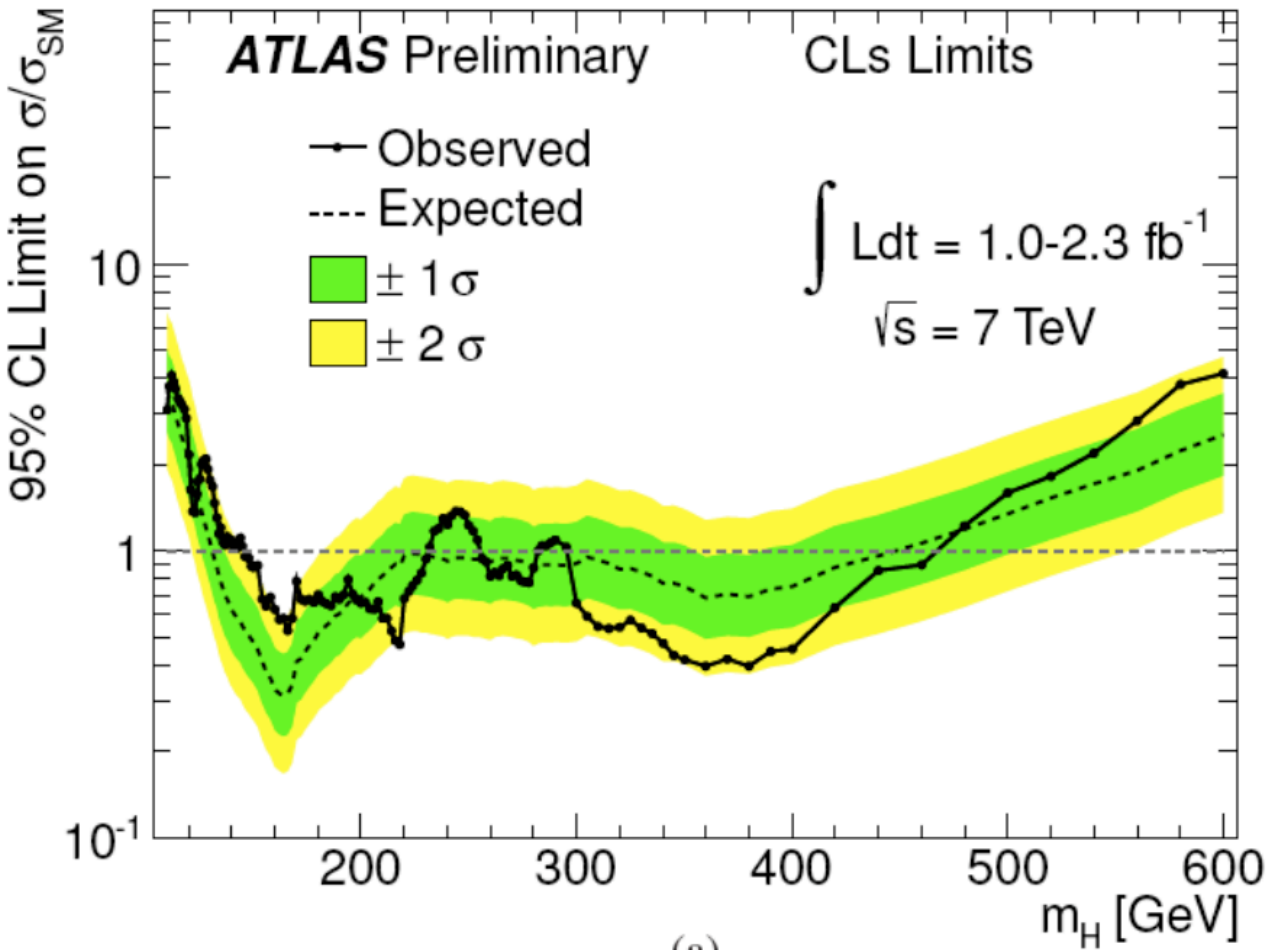



Muon Detectors
Electromagnetic Calorimeters
Solenoid
Forward Calorimeters
End Cap Toroid
Barrel Toroid
Inner Detector
Hadronic Calorimeters
Shielding

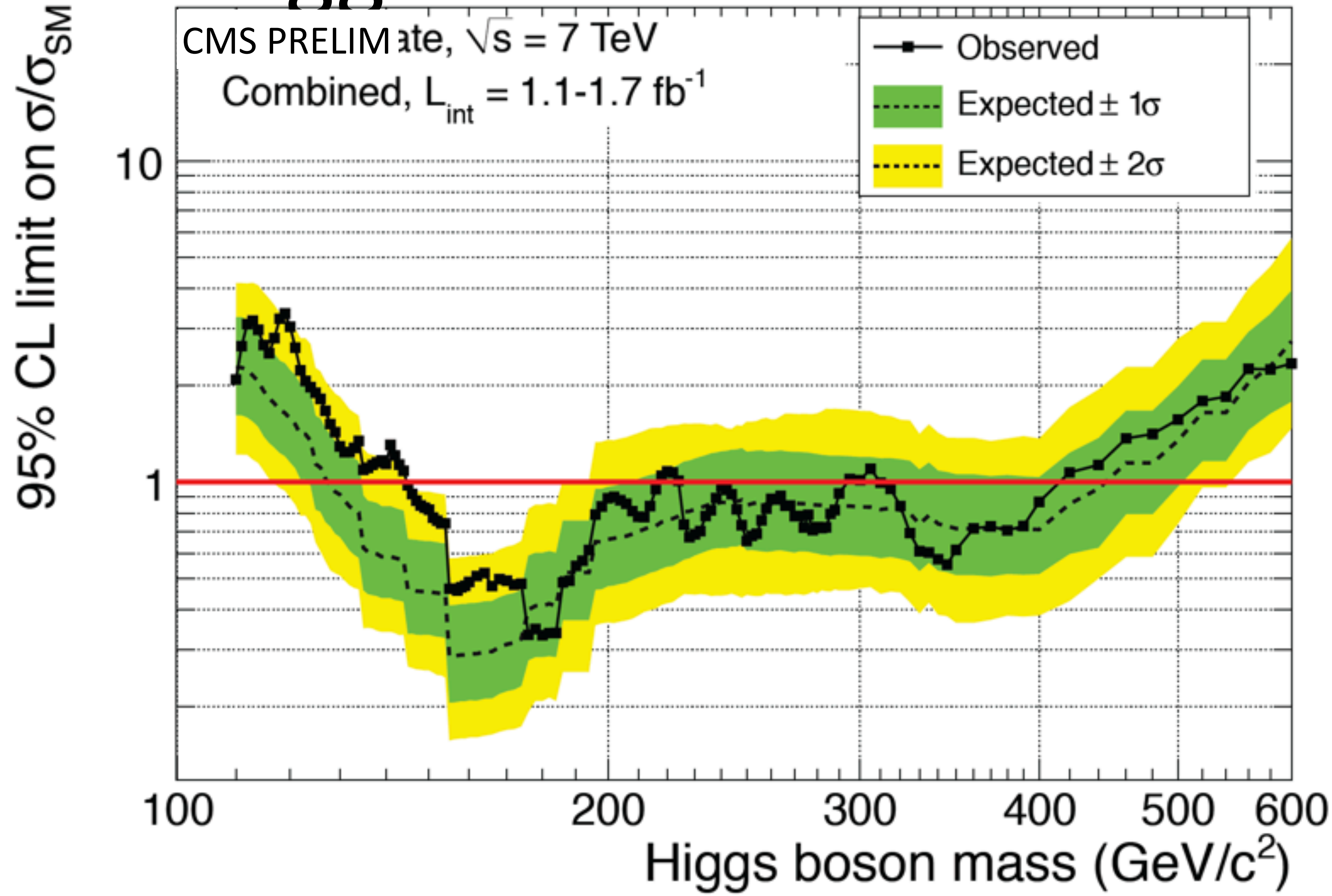

Muon system
Magnet
Electromagnetic calorimeter
Hadronic calorimeter
RICH-1
VELO
Tracking system




ATLAS EXPERIMENT
Run Number: 182747, Event Number: 63217197
Date: 2011-05-28 13:06:57 CEST
2μ2e candidate:
m_{lead}: 85.9 GeV
m_{subl}: 85.5 GeV
m_{4l}: 210 GeV

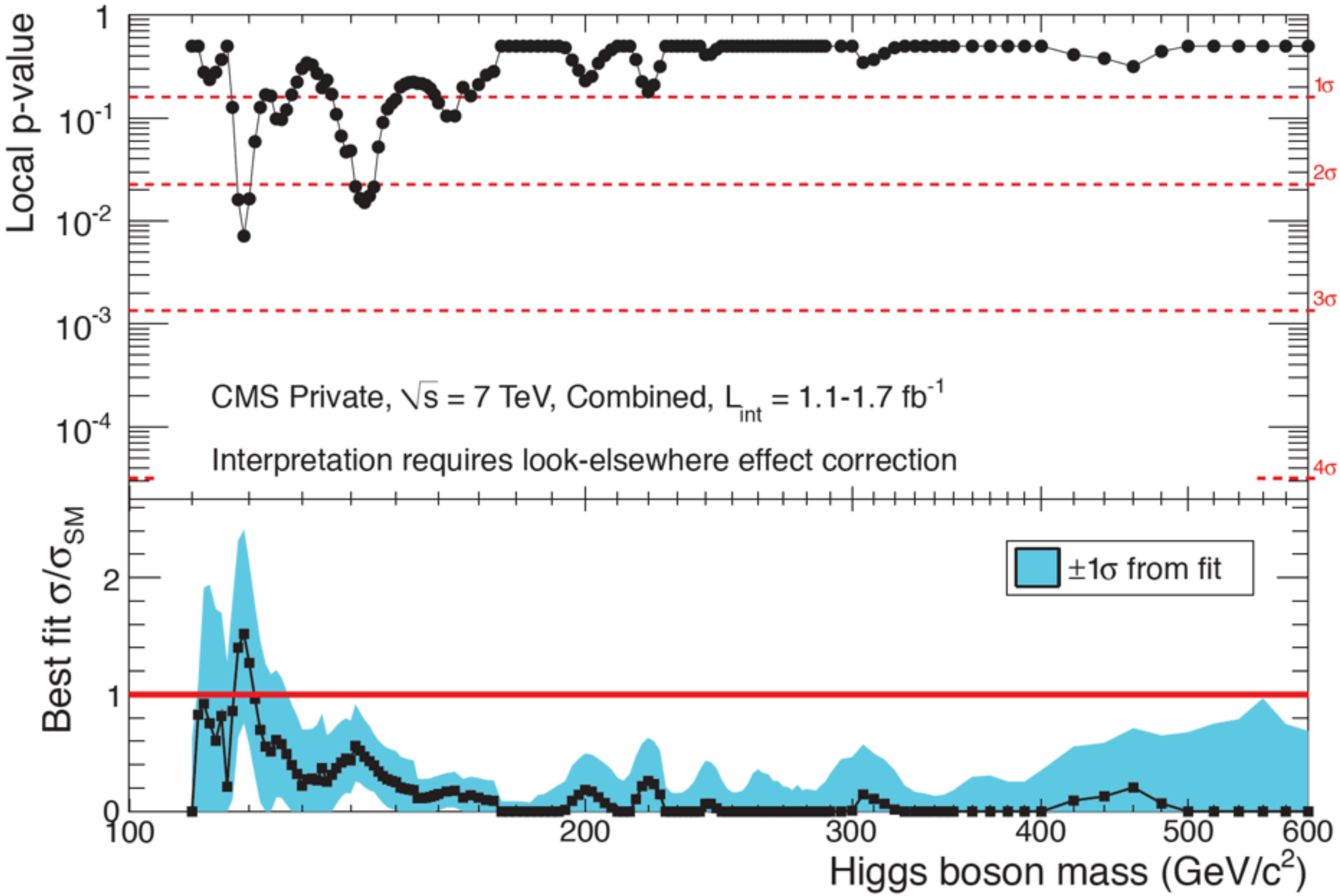

ATLAS Preliminary
CLs Limits
Observed
Expected
± 1 σ
± 2 σ
∫ Ldt = 1.0-2.3 fb-1
√s = 7 TeV
95% CL Limit on σ/σSM
10
1
10-1
200
300
400
500
600
mH [GeV]

SM Higgs Search Combination



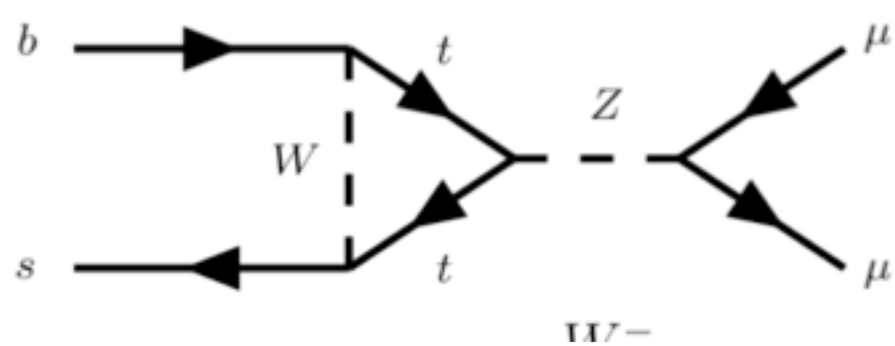
Expected exclusion mass range: 130 – 440 GeV
Observed exclusion mass range: 145-216, 226-288, 310-400 GeV

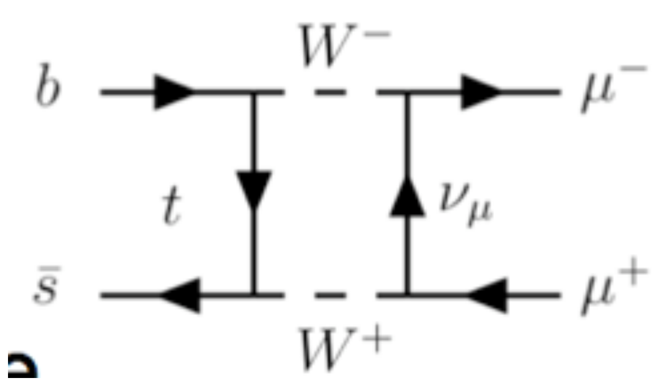
P-Values & Best Fit



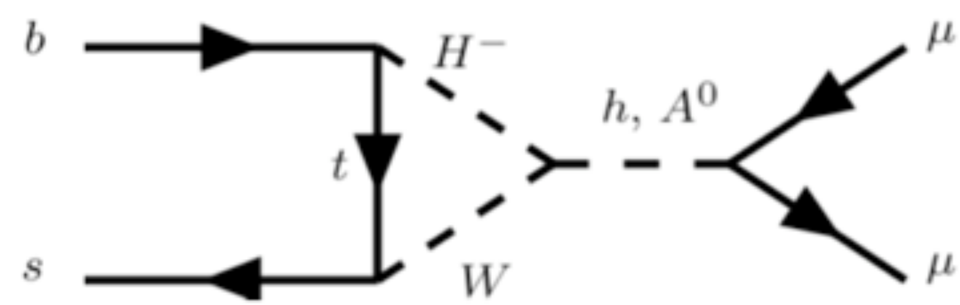
Search for New Physics via forbidden B-decays

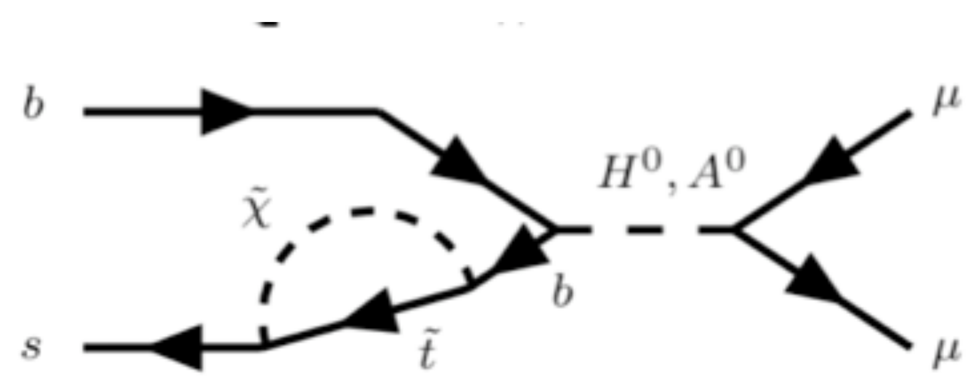
Standard Model





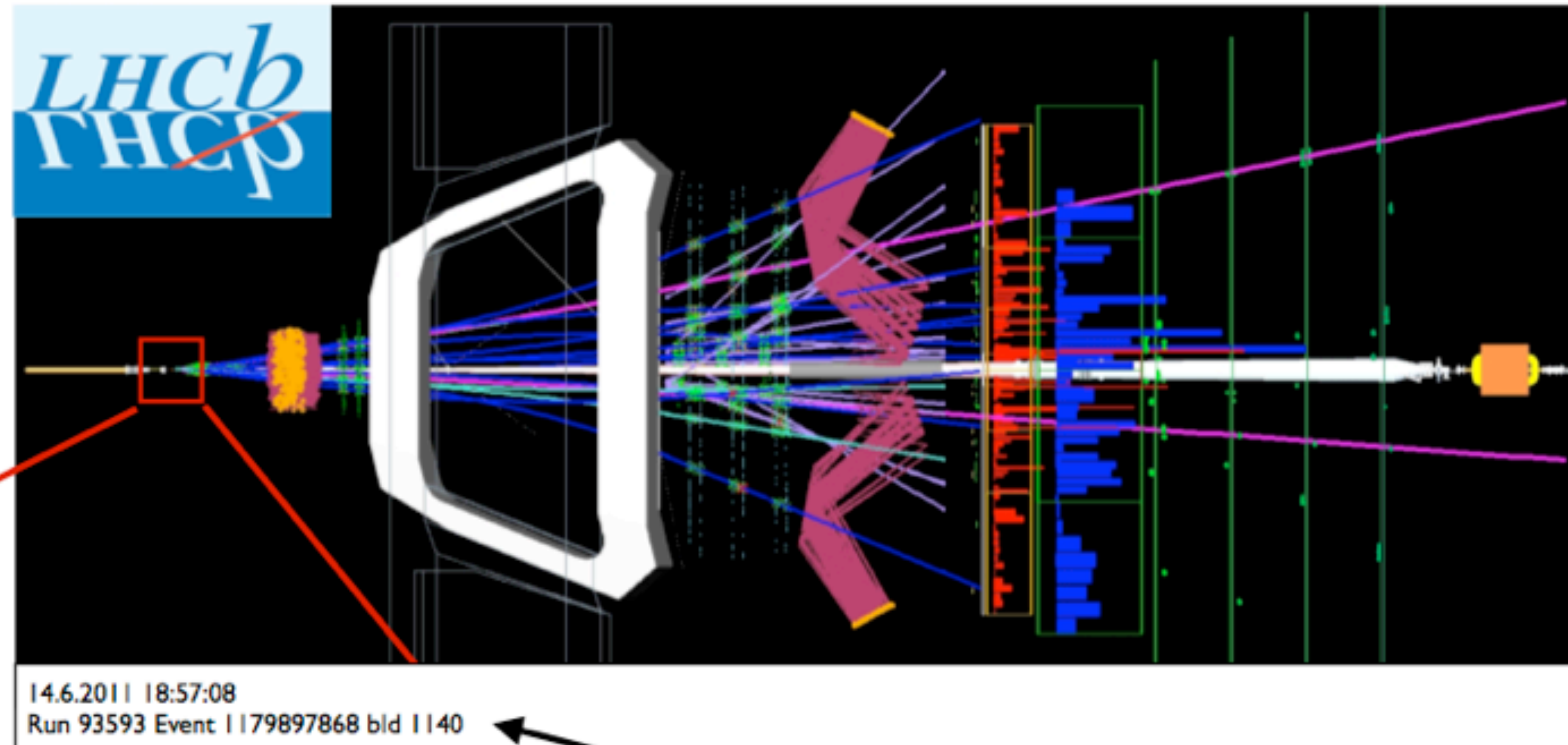
Supersymmetry

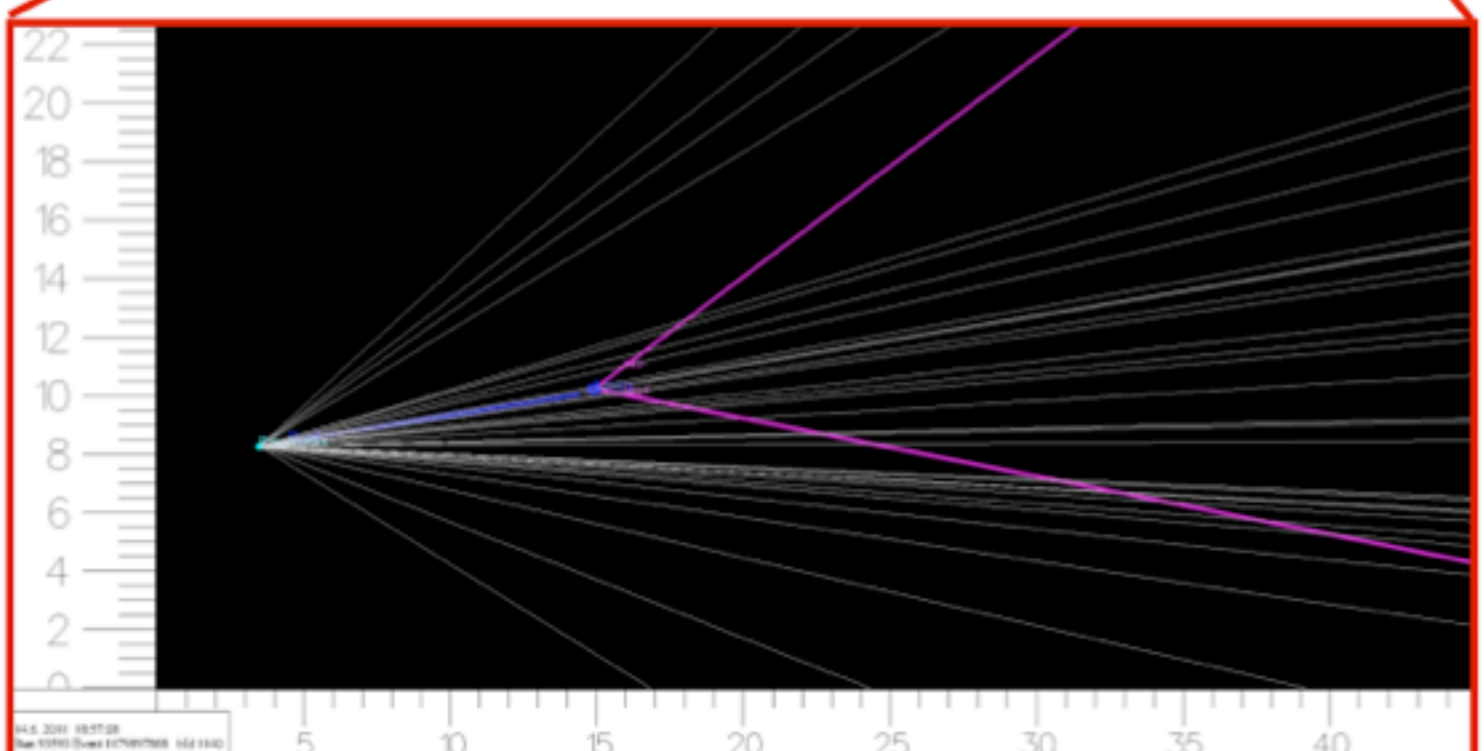


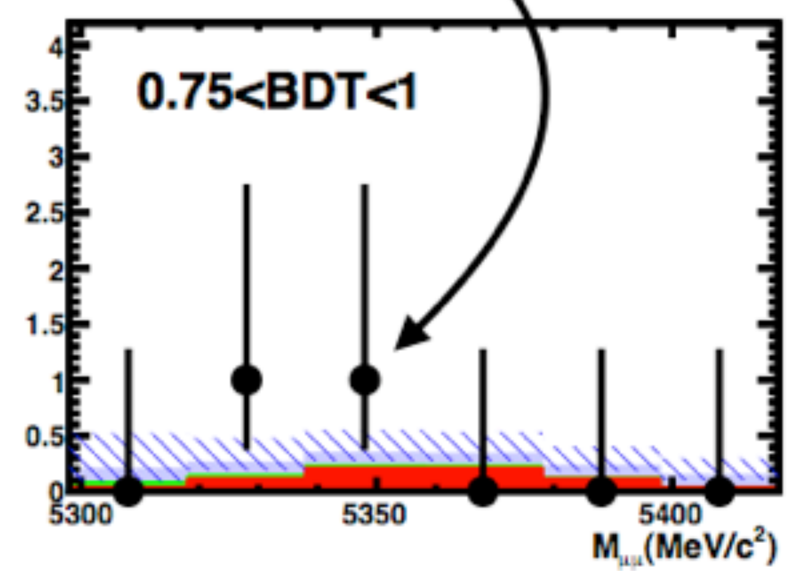


LHC*b*: $B_s \rightarrow \mu\mu$?

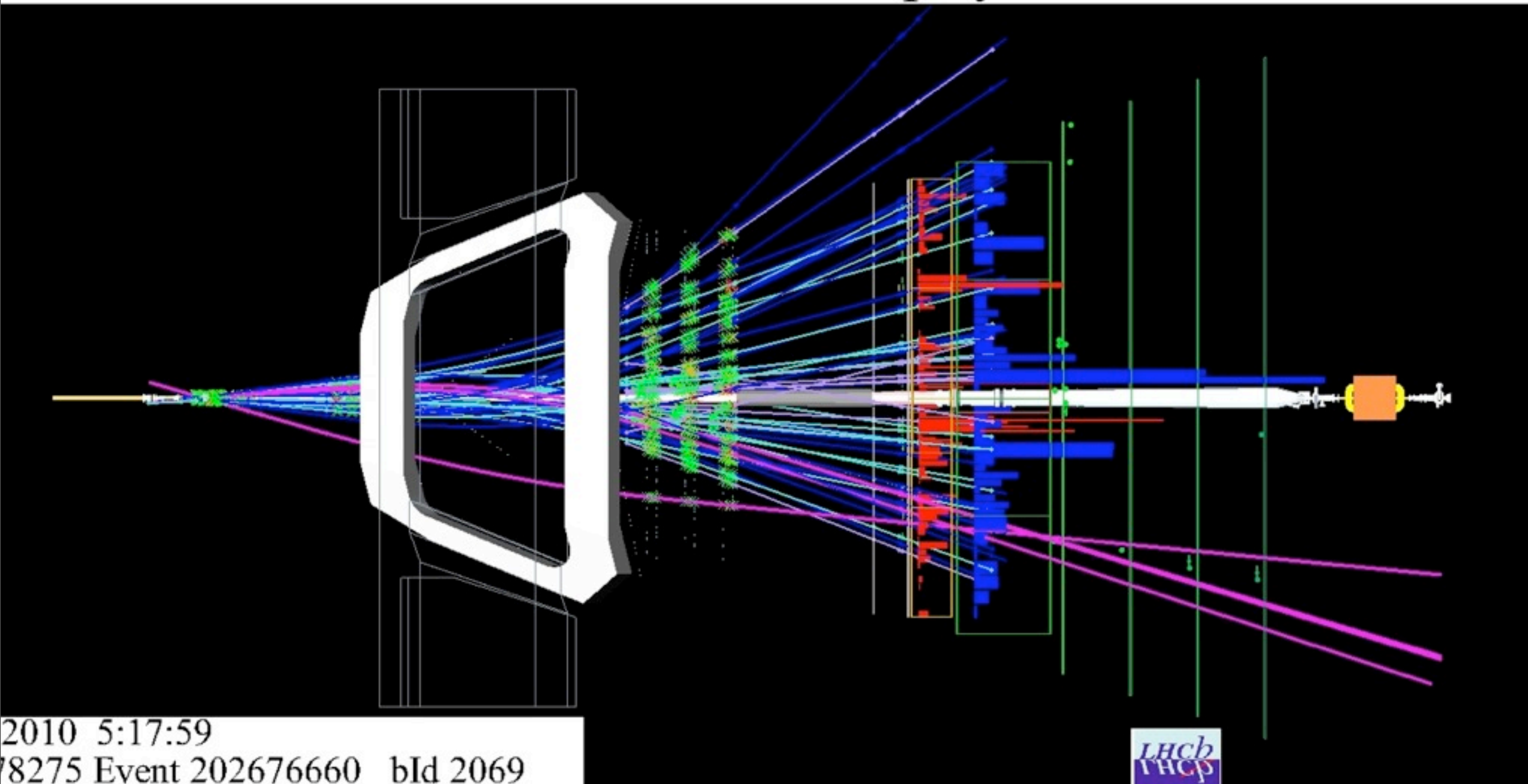
$m_{\mu\mu} = 5.357$ GeV
BDT = 0.90
Decay length = 11.5 mm
Tracks shown for $p_T > 0.5$ GeV



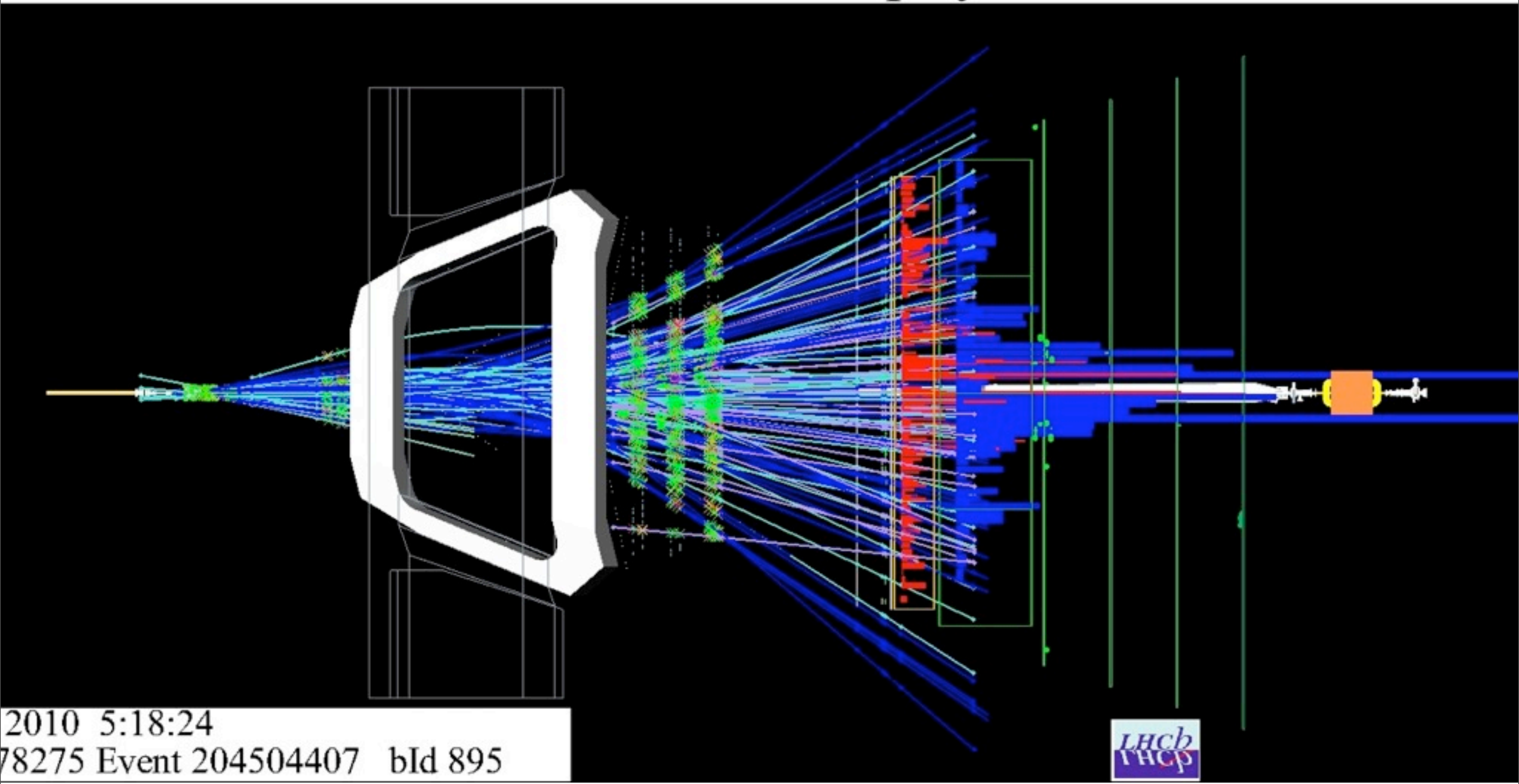




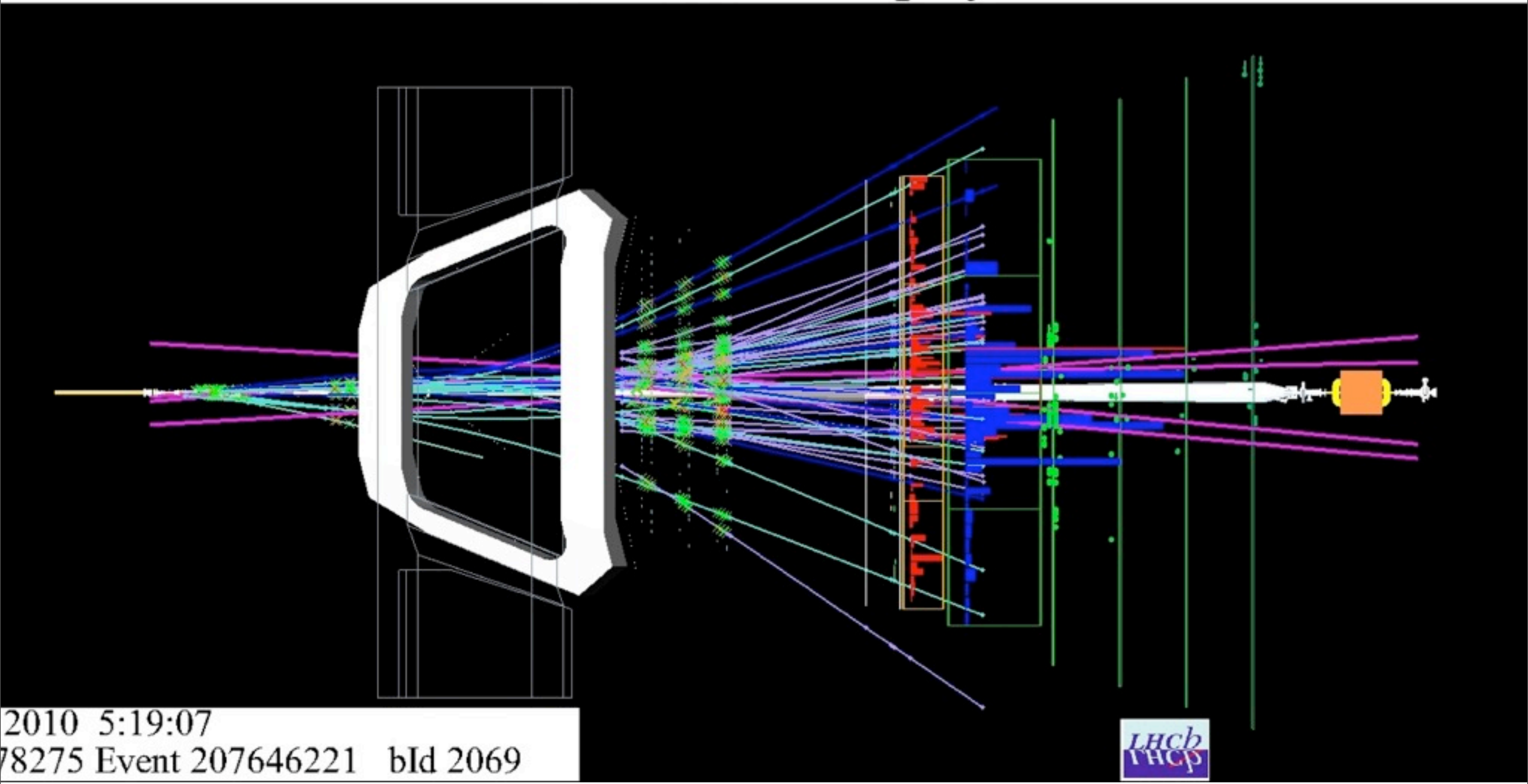
LHCb Event Display



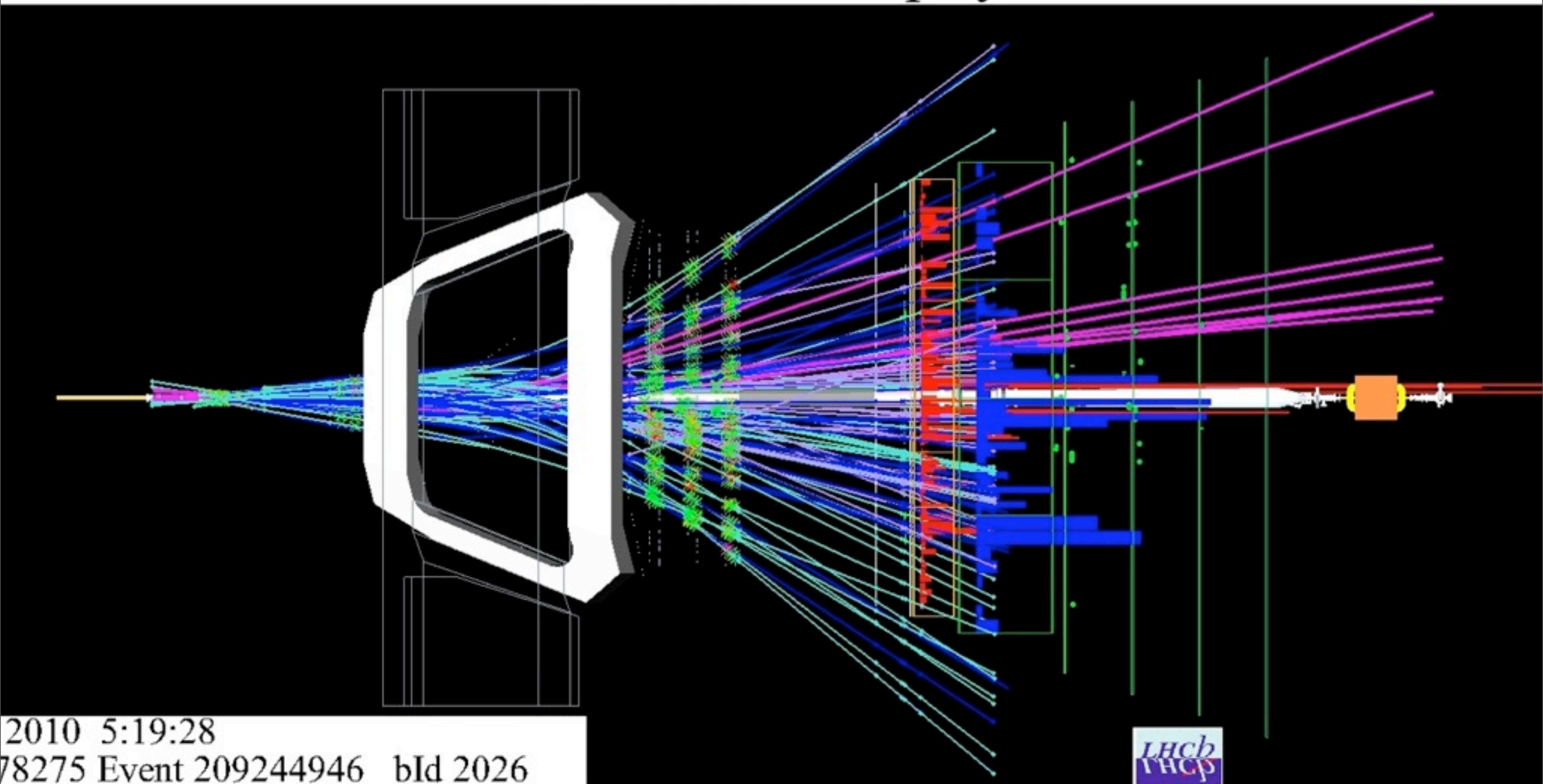
LHCb Event Display



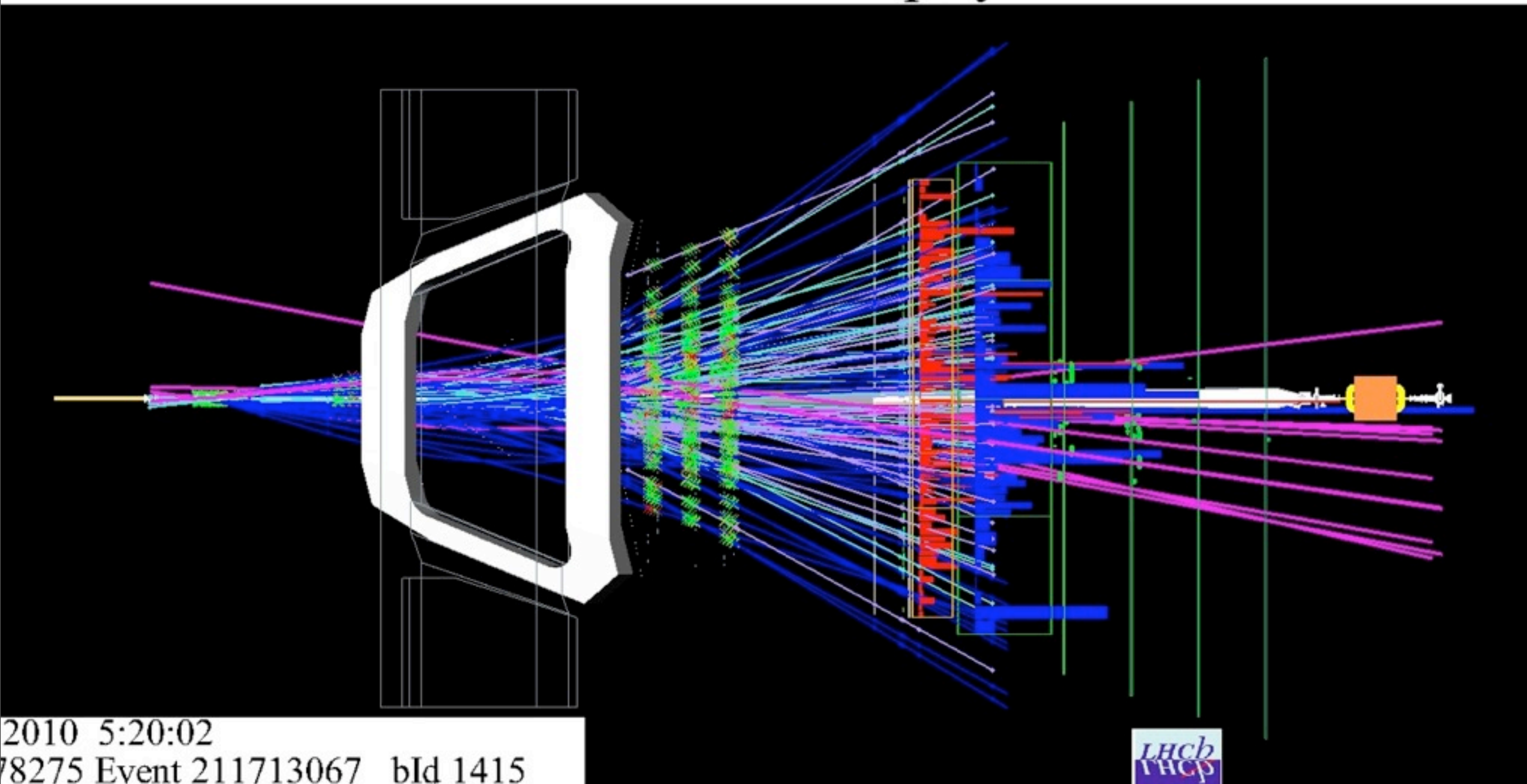
LHCb Event Display



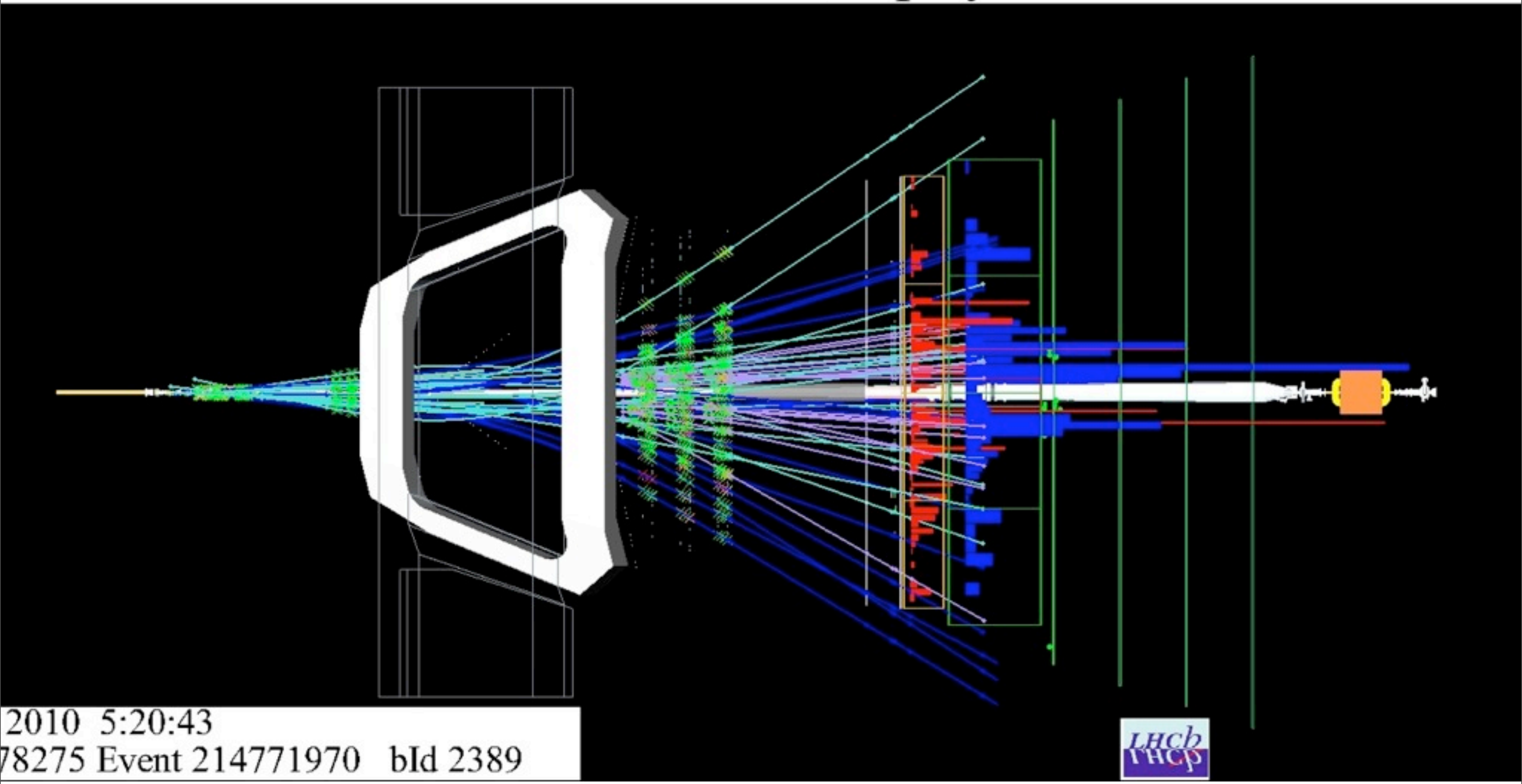
LHCb Event Display



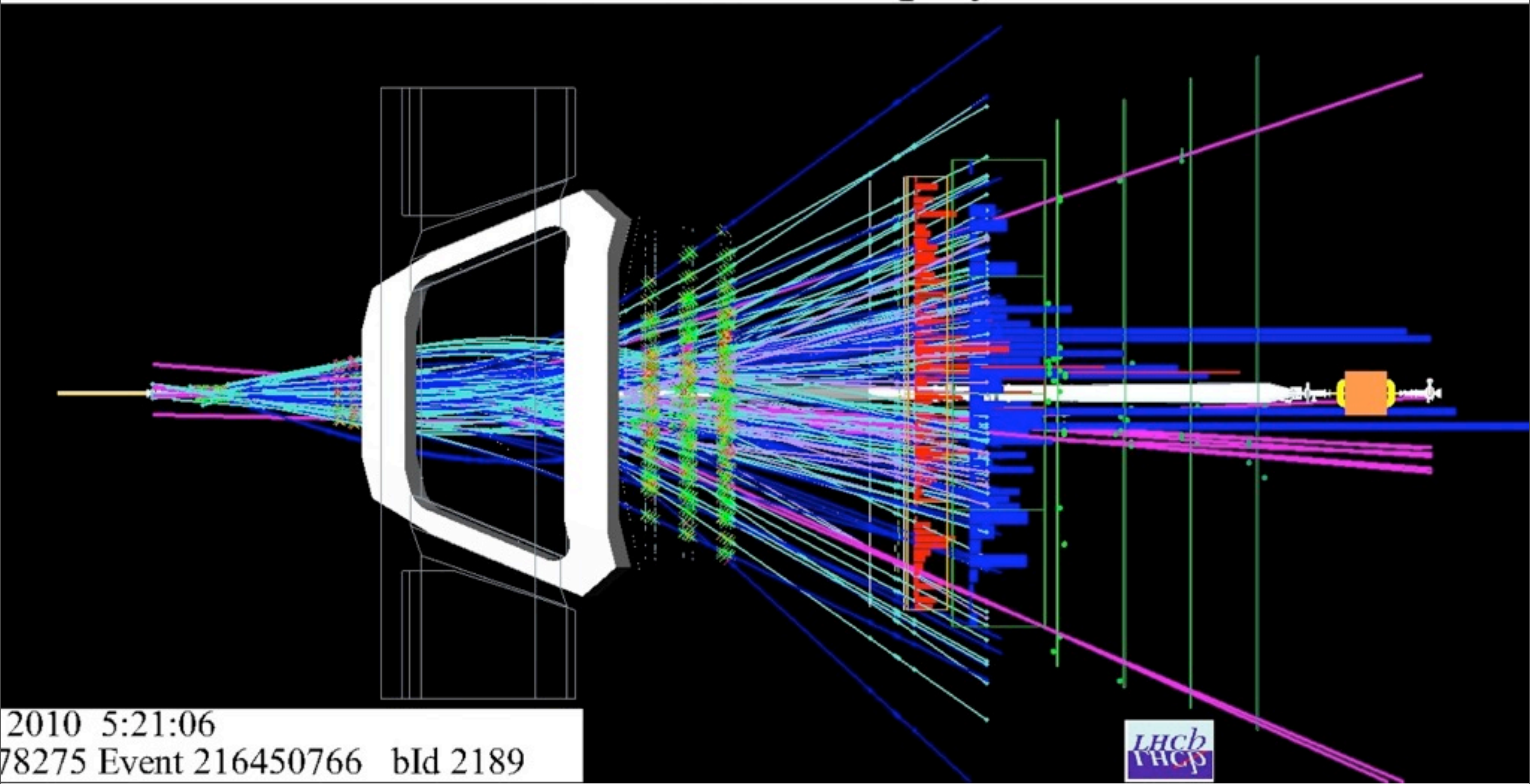
LHCb Event Display



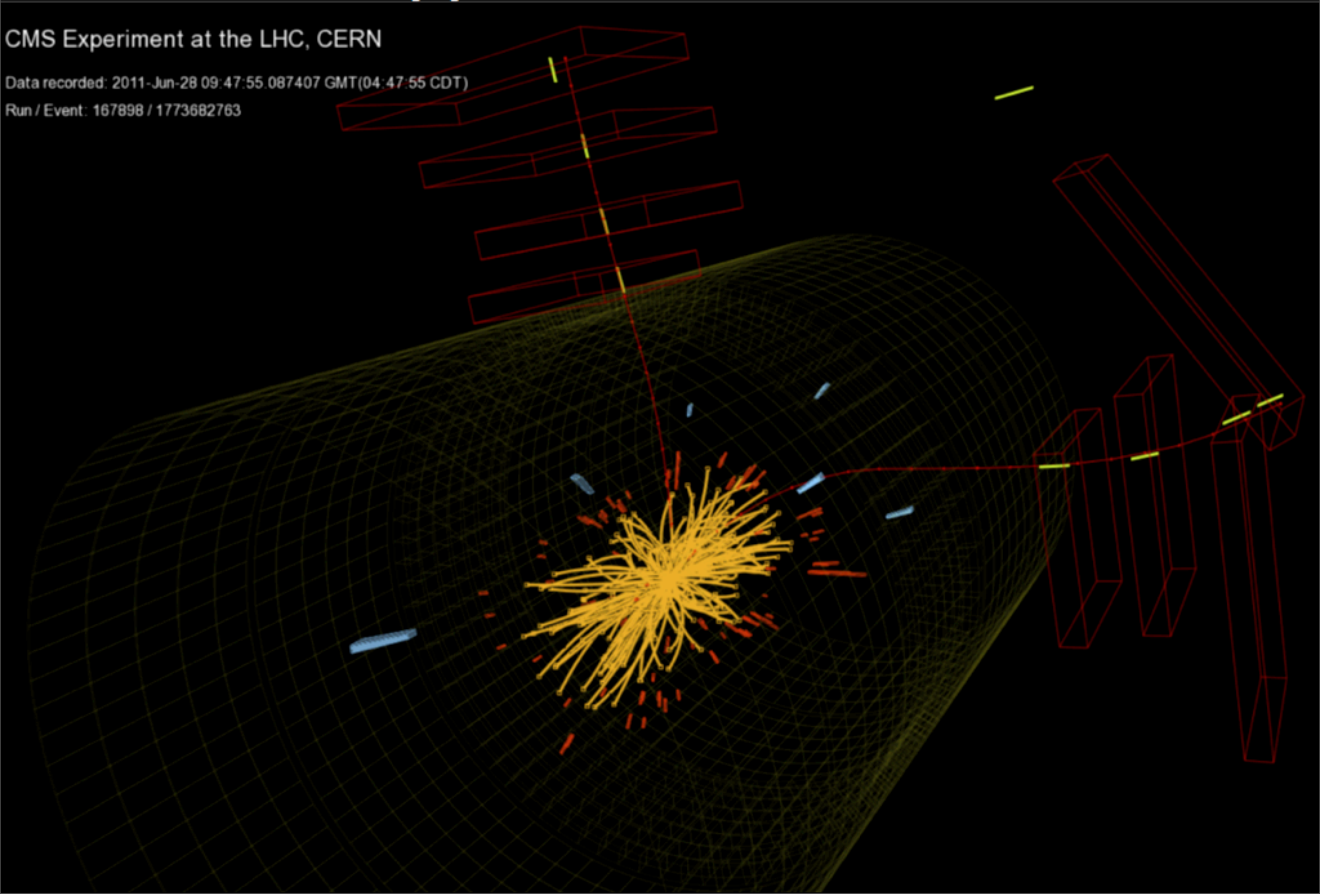
LHCb Event Display



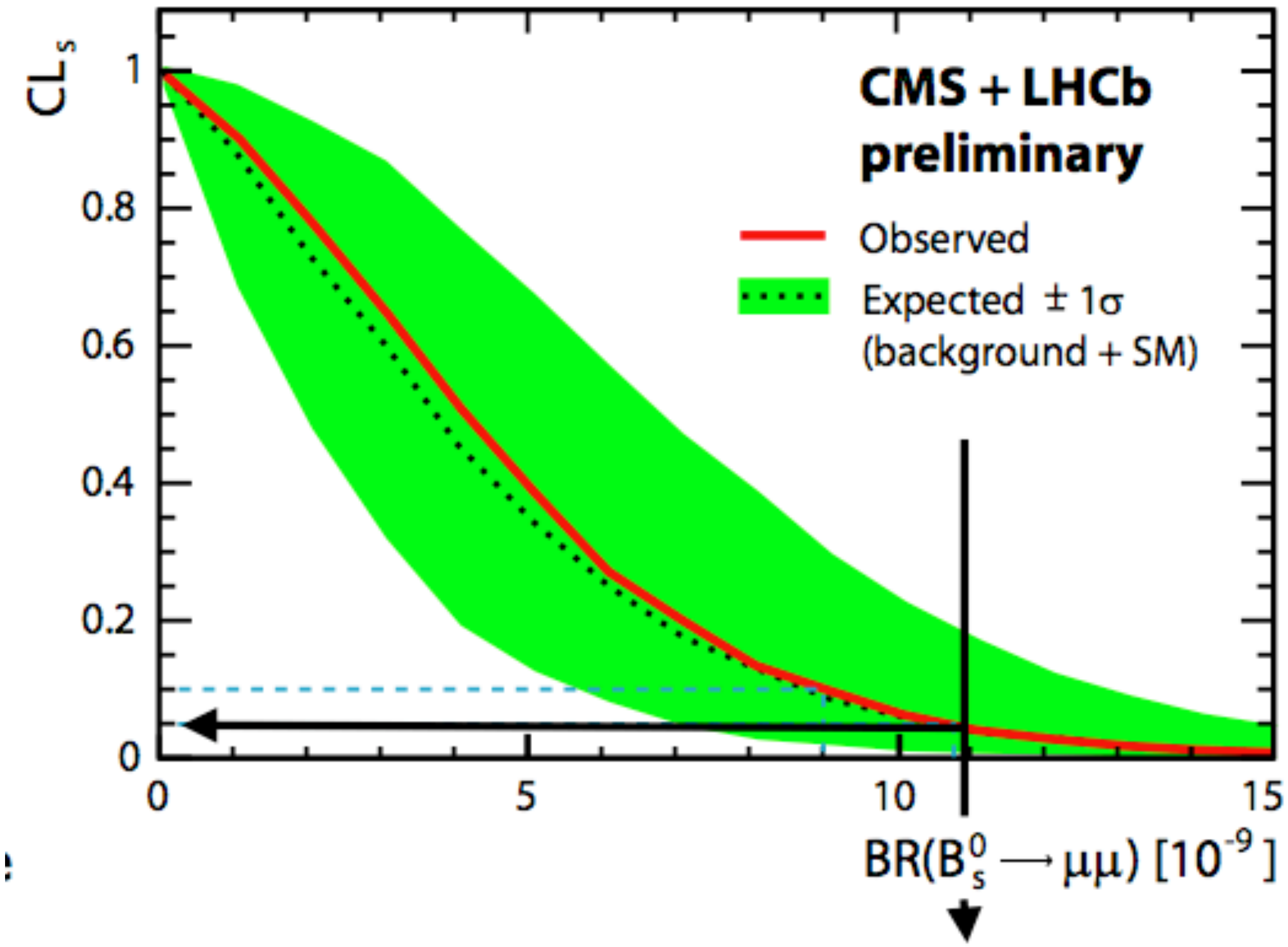
LHCb Event Display



CMS: $B_s \to \mu\mu$?



So far New Physics is not yet observed...

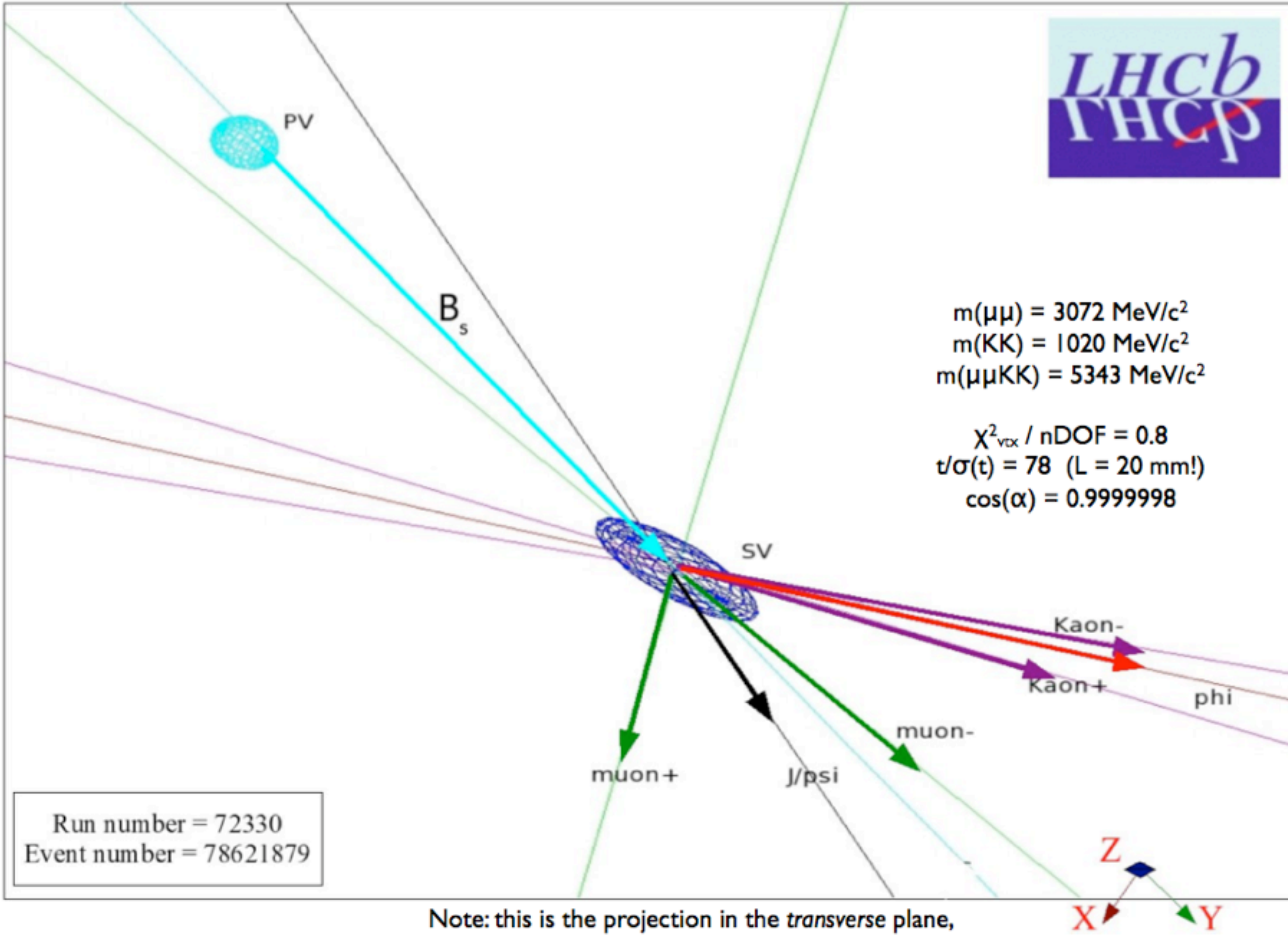


< 11×10^{-9} @ 95% CL

But we have just started... much more to come...

These are really exciting times!

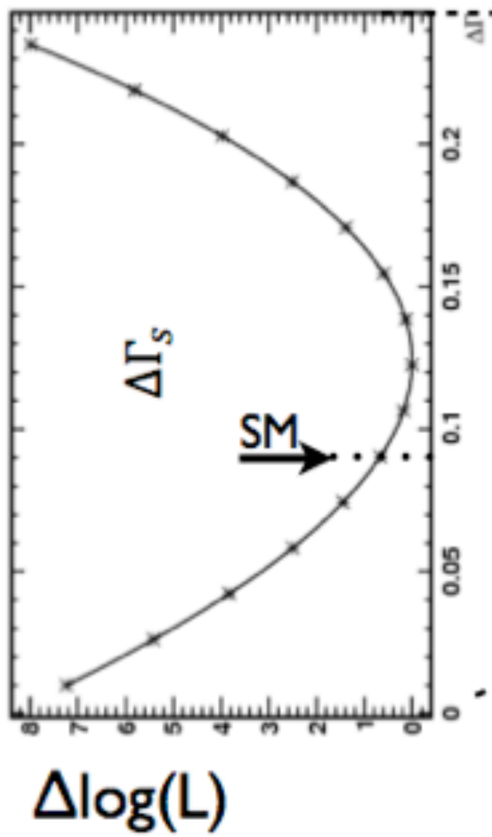
$B_s \rightarrow J/\psi\ \varphi$ The matter anti-matter asymmetry

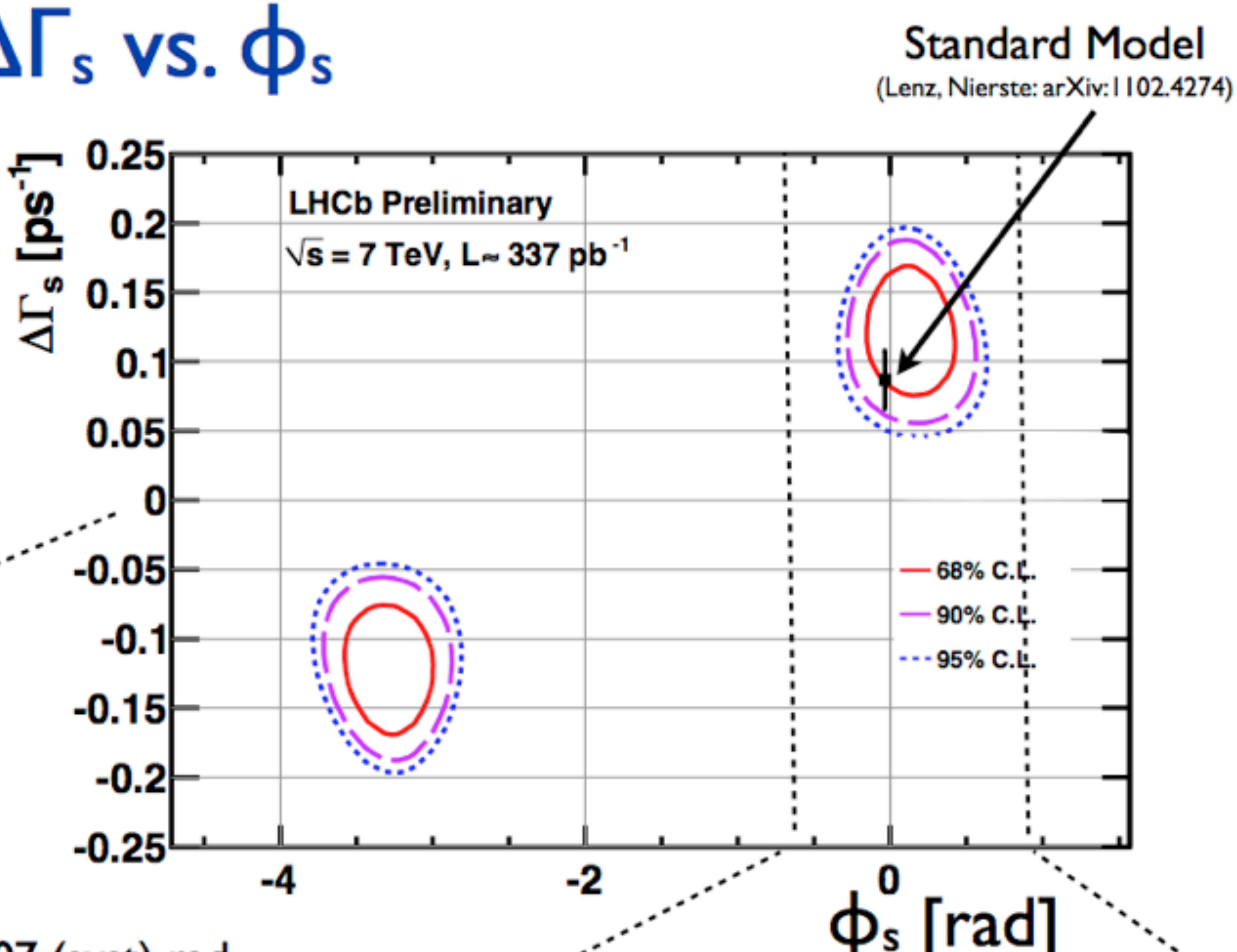


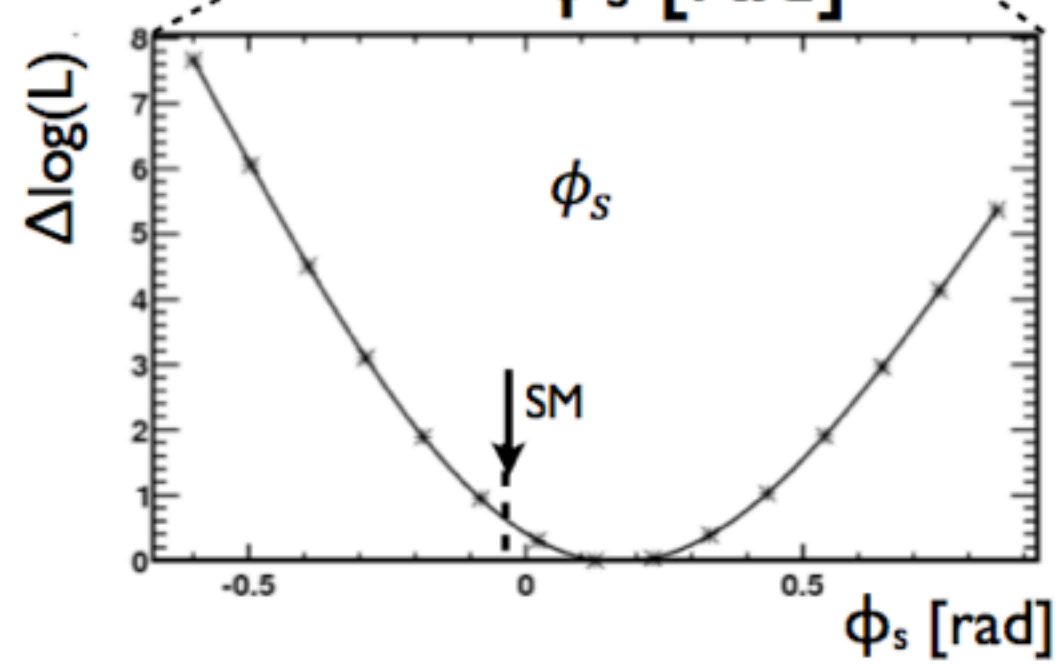
Note: this is the projection in the *transverse* plane,
Vertex separation is *much* larger in 3D.

$B_s \to J/\psi\varphi$: $\Delta\Gamma_s$ vs. ϕ_s

LHCb-CONF-2011-49







Most precise measurement of ϕ_s

- $\phi_s = 0.13 \pm 0.18$ (stat) ± 0.07 (syst) rad
- Consistent with SM

4 σ Evidence for $\Delta\Gamma_s \neq 0$:

- $\Delta\Gamma s = 0.123 \pm 0.029$ (stat) ± 0.008 (syst) ps^{-1}
- $\Gamma_s = 0.656 \pm 0.009$ (stat) ± 0.008 (syst) ps^{-1}

Lecture 2

Master Particle Physics 1
Marcel Merk
September 7, 2011

Standard Model for particles

Fermions (Spin 1/2 particles) :The basic constituents of matter

Quarks:

$$\begin{pmatrix} u, u, u \\ d, d, d \end{pmatrix} \begin{pmatrix} c, c, c \\ s, s, s \end{pmatrix} \begin{pmatrix} t, t, t \\ b, b, b \end{pmatrix}$$

Quarks only occur in colour neutral objects: “Hadrons”
Baryons: qqq
Mesons: $q\bar{q}$
⇒ Hadron particles occur in multiplets

Leptons:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

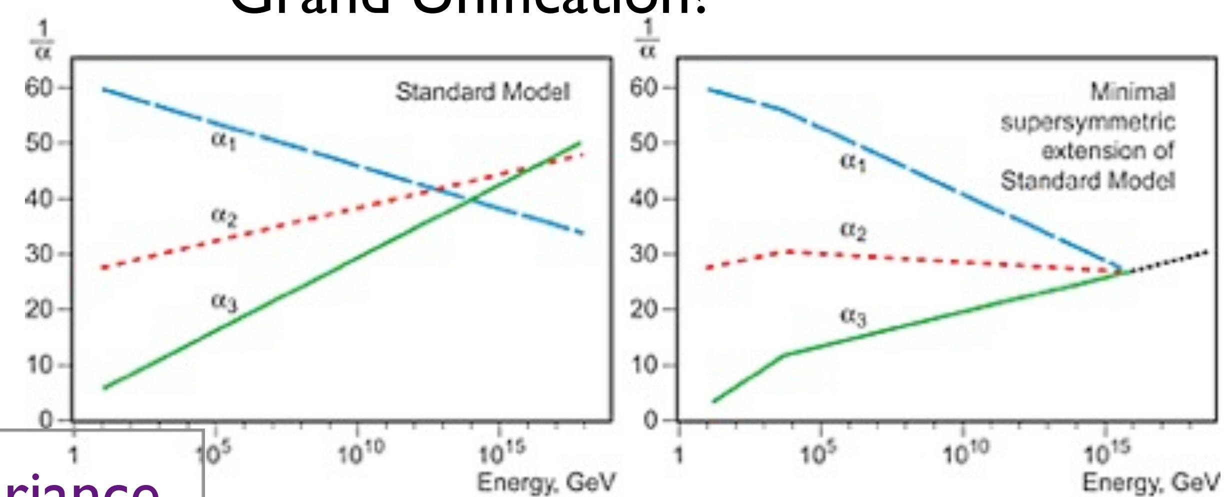
Only 3 generations of fundamental particles are known

Vector Bosons (Spin 1 particles). force carriers

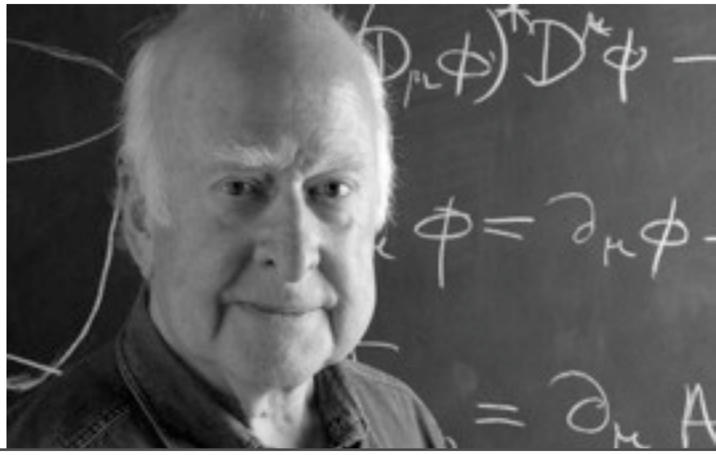
Strong interaction	8 gluons
Weak interaction	W^+ W^- Z^0
Electromagnetic interaction	γ
Gravity	g

⇒Forces originate from principle of local gauge invariance

Grand Unification?



Scalar Boson (Spin=0 particle).
Generates fermion masses via the Higgs mechanism



Lecture 3

Master Particle Physics 1

Marcel Merk

September 12, 2011

Matter Waves for Particles without Spin

	Non relativistic:	Relativistic:
Kinematics:	$E = \frac{\vec{p}^2}{2m}$	$E^2 = \vec{p}^2 + m^2$
Quantum Mechanics:	$E \to \frac{\partial}{\partial t}$ and $\vec{p} \to -i\vec{\nabla}$	$E \to i\frac{\partial}{\partial t}$ and $\vec{p} \to -i\vec{\nabla}$
Wave Equation:	$i\frac{\partial}{\partial t}\psi = \frac{-1}{2m}\nabla^2\psi$	$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\phi$

Continuity Equation: $\vec{\nabla} \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$ or $\partial_\mu j^\mu = 0$

Probability density and current:

$$\begin{aligned} \rho &= \psi^*\psi = |N|^2 \\ \vec{j} &= -\frac{i}{2m}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) \frac{|N|^2}{m}\vec{p} \end{aligned}$$

$$\begin{aligned} \rho &= i\left(\phi^*\frac{\partial \phi}{\partial t} - \phi\frac{\partial \phi^*}{\partial t}\right) = 2|N|^2 E \\ \vec{j} &= -i\left(\phi^*\vec{\nabla}\phi - \phi\vec{\nabla}\phi^*\right) = 2|N|^2\vec{p} \end{aligned}$$

Negative Energy solutions: $j^\mu(+e) = 2e|N|^2\ (E, \vec{p}) = -2e|N|^2\ (-E, -\vec{p})$

The negative energy solution of a particle traveling backwards in time =
the positive energy solution of the antiparticle traveling forwards in time

Lecture 4

Master Particle Physics 1

Marcel Merk

September 14, 2011

Matter Waves and EM Field

Matter Waves

$$E \rightarrow i\frac{\partial}{\partial t} \quad ; \quad \vec{p} \rightarrow -i\vec{\nabla}$$

Non-relativistic:

$$E = \frac{\vec{p}^2}{2m}$$

$$i\frac{\partial}{\partial t}\psi = \frac{-1}{2m}\nabla^2\psi$$

Relativistic:

$$E^2 = \vec{p}^2 + m^2$$

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\phi$$

Continuity: $\partial_\mu j^\mu = 0$ with: $j^\mu = i\left(\phi^* \, \partial^\mu \phi - \phi \, \partial^\mu \phi^*\right)$

EM Field

Maxwell Equations:

Maxwell 1+4
-> continuity

$$\begin{aligned} \vec{\nabla}\cdot\vec{E} &= \rho \\ \vec{\nabla}\cdot\vec{B} &= 0 \\ \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} &= 0 \\ \vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} &= \vec{j} \end{aligned} \quad \Bigg\} \longrightarrow \quad \boxed{\begin{aligned} \vec{B} &= \vec{\nabla}\times\vec{A} \\ \vec{E} &= -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}V \\ A^\mu &= \left(V, \vec{A}\right) \\ j^\nu &= \partial_\mu\partial^\mu A^\nu - \partial^\nu\partial_\mu A^\mu \end{aligned}}$$

Gauge Invariance: $A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu\lambda$

Lorentz Condition: $\partial_\mu A^\mu = 0$

Coulomb Condition: $A^0 = 0 \quad ; \quad \vec{\nabla}\cdot\vec{A} = 0$

Photon has 2-polarizations!

Bohm - Aharanov Experiment!

Cross Section: $A + B \to C + D$

$$d\sigma = \frac{(2\pi)^4 \ \delta^4 \left(p_A + p_B - p_C - p_D\right)}{4\sqrt{\left(p_A \cdot p_B\right)^2 - m_A^2 m_B^2}} \cdot \left|\mathcal{M}\right|^2 \cdot \frac{d^3 p_C}{(2\pi)^3 \ 2E_C} \frac{d^3 p_D}{(2\pi)^3 \ 2E_D}$$

Decay Rate: $A \to B + C$

$$d\Gamma = \frac{(2\pi)^4 \ \delta^4 \left(p_A - p_C - p_D\right)}{2E_A} \cdot \left|\mathcal{M}\right|^2 \cdot \frac{d^3 p_C}{(2\pi)^3 \ 2E_C} \frac{d^3 p_D}{(2\pi)^3 \ 2E_D}$$

Lecture 5

Master Particle Physics 1

Marcel Merk

September19, 2011

The Story Sofar...

1) Plane Waves

$$\text{Kinematics} \quad : \quad E^2 = \vec{p}^2 + m^2 \quad ; \quad E \to i\frac{\partial}{\partial t} \quad \vec{p} \to -i\vec{\nabla}$$

$$\text{Klein - Gordon} \quad : \quad \left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0 \quad ; \quad \phi(x) = N\, e^{-ip_\mu x^\mu}$$

$$\text{Current} \quad : \quad j^\mu = -ie\, \left[\phi^* \left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right)\phi\right] \quad ; \quad \partial_\mu j^\mu = 0$$

2) Electromagnetic Field

$$\text{Maxwell} \quad : \quad \partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu \quad \text{or} \quad \partial_\mu F^{\mu\nu} = j^\nu$$

$$\text{GaugeFreedom} \quad : \quad A_\mu \to A'_\mu = A_\mu + \partial_\mu \lambda$$

$$\text{LorentzCondition} \quad : \quad \partial_\mu A^\mu = 0 \quad \Rightarrow \text{Maxwell} : \quad \partial_\mu \partial^\mu A^\nu = j^\nu$$

$$\text{PlaneWaves} \quad : \quad A^\mu = N \mathcal{E}^\mu(\vec{p}) e^{-ip_\mu x^\mu} \quad \Rightarrow 2 \text{ polarizations}$$

3) Scattering: $A_i + B_i \to C_f + D_f + \ldots$

$$d\sigma_{fi} = \frac{W_{fi}}{Flux}\, d\Phi$$

$$W_{fi} = \lim_{T, V \to \infty} \frac{|T_{fi}|^2}{TV} \quad ; \quad T_{fi} = -i \int d^4x\, \psi_f^*(x)\, V(x)\, \psi_i(x)$$

$$T_{fi} = -iN_A N_B N_C^* N_D^* (2\pi)^4\, \delta^4(p_A + p_B - p_C - p_D)\, \mathcal{M}$$

$$d\Phi = \sum_{i=1}^{N} \frac{V}{(2\pi)^3} \frac{d^3\vec{p}_i}{2E_i} \quad ; \quad Flux = 4\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2} \,/\, V^2$$

The Story Sofar...

1) Plane Waves

$$\text{Kinematics} \quad : \quad E^2 = \vec{p}^2 + m^2 \quad ; \quad E \rightarrow i\frac{\partial}{\partial t} \quad \vec{p} \rightarrow -i\vec{\nabla}$$

$$\text{Klein} - \text{Gordon} \quad : \quad \left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0 \quad ; \quad \phi(x) = N\, e^{-ip_\mu x^\mu}$$

$$\text{Current} \quad : \quad j^\mu = -ie\,\left[\phi^*\left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right)\phi\right] \quad ; \quad \partial_\mu j^\mu = 0$$

2) Electromagnetic Field

$$\text{Maxwell} \quad : \quad \partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu \quad \text{or} \quad \partial_\mu F^{\mu\nu} = j^\nu$$

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$$\text{LorentzCondition} \quad : \quad \partial_\mu A^\mu = 0 \quad \Rightarrow \text{Maxwell} : \quad \partial_\mu \partial^\mu A^\nu = j^\nu$$

$$\text{PlaneWaves} \quad : \quad A^\mu = N\mathcal{E}^\mu(\vec{p})e^{-ip_\mu x^\mu} \quad \Rightarrow 2 \text{ polarizations}$$

3) Scattering: $A_i + B_i \rightarrow C_f + D_f + \ldots$

Today: Electromagnetic Scattering

- A particle in a potential
- Spinless π−K scattering

$$d\sigma_{fi} = \frac{W_{fi}}{Flux}\, d\Phi$$

$$W_{fi} = \lim_{T,V \rightarrow \infty} \frac{|T_{fi}|^2}{TV} \quad ; \quad T_{fi} = -i \int d^4x\, \psi_f^*(x)\, V(x)\, \psi_i(x)$$

$$T_{fi} = -iN_A N_B N_C^* N_D^* (2\pi)^4\, \delta^4(p_A + p_B - p_C - p_D)\, \mathcal{M}$$

$$d\Phi = \sum_{i=1}^{N} \frac{V}{(2\pi)^3} \frac{d^3\vec{p}_i}{2E_i} \quad ; \quad Flux = 4\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2}\ / \ V^2$$

Lecture 6

Master Particle Physics 1
Marcel Merk
September 21, 2011

1) Free particle wave equations

K.G.: $\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0$

Dirac: $\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) = 0$

Plane wave solutions:

$\phi(x) = N e^{-ipx}$

$\psi(x) = u(p) e^{-ipx}$

Current: $\partial_\mu j^\mu = 0$

$j^\mu = i\left[\phi^*\left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right)\phi\right]$ $\Rightarrow$ $\rho = 2|N|^2 E$

$j^\mu = \bar{\psi}\gamma^\mu \psi$ $\Rightarrow$ $\rho = \psi^\dagger \psi \geq 0$

1) Free particle wave equations

Plane wave solutions:

Current: $\partial_\mu j^\mu = 0$

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Dirac: $\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) = 0$ $\quad \psi(x) = u(p) e^{-ipx}$ $\quad j^\mu = \bar{\psi}\gamma^\mu \psi$ $\Rightarrow$ $\rho = \psi^\dagger \psi \geq 0$

2) Electromagnetic field

Maxwell $\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu$

or $\partial_\mu F^{\mu\nu} = j^\nu$

Gauge freedom:

$A_\mu \rightarrow A_\mu' = A_\mu + \partial_\mu \lambda$

with $\partial_\nu \partial^\nu \lambda = 0$

Lorentz condition:

$\partial_\mu A^\mu = 0$

$\partial_\mu \partial^\mu A^\nu = j^\nu$

Plane wave solutions:

$A^\mu = N \varepsilon^\mu(p) e^{-ipx}$

(2 polarizations since m=0)

1) Free particle wave equations

K.G.: $\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0$

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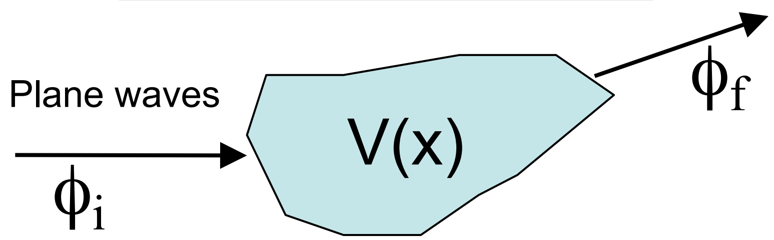
(2 polarizations since m=0)

3) Scattering Perturbation Theory

A: non-relativistic derivation:



$$i\frac{\partial \psi}{\partial t} = \left(H_0 + V(x,t)\right)\psi$$

$$\psi = \sum_{n=0}^{\infty} a_n(t)\phi_n(t)e^{-iEt}$$

$$T_{fi} = a_f(t \rightarrow \infty) = -i\int d^4x\, \phi_f^*(x)V(x)\phi_i(x)$$

1-st order:

$$W_{fi} = \lim_{T\rightarrow\infty} \frac{|T_{fi}|^2}{T} \quad \text{with}$$

$$T_{fi} = -2\pi i\, V_{fi}\, \delta\left(E_f - E_i\right) \quad \text{with}$$

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Quantum Electrodynamics

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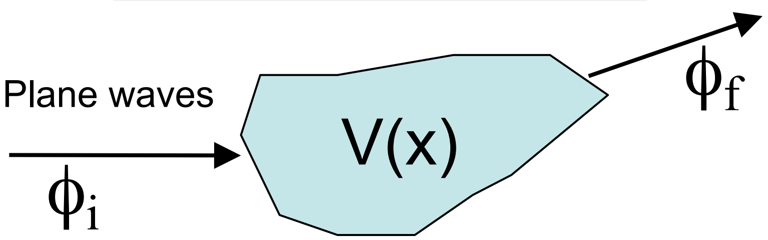

$$i\frac{\partial \psi}{\partial t} = \left(H_0 + V(x,t)\right)\psi$$

$$\psi = \sum_{n=0}^{\infty} a_n(t)\phi_n(t) e^{-iEt}$$

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B: relativistic extension:

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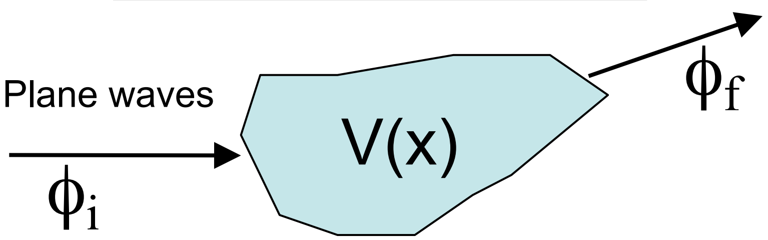

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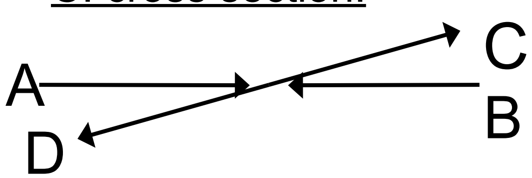
C: cross section:



$\frac{d\sigma}{d\Omega} = \frac{W_{fi}}{flux} d\Phi$

$flux = 4\sqrt{\left(p_A p_B\right)^2 - m_A^2 m_B^2} / V^2$

$d\Phi = \prod_i \frac{V}{(2\pi)^3} \frac{d^3 p_i}{2E_i}$

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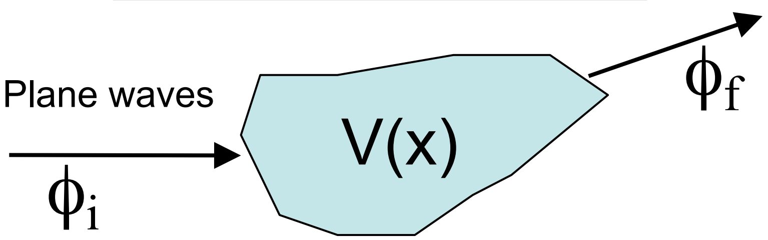

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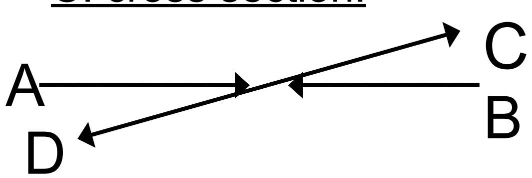

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4) Electromagnetic Scattering

$\boxed{\partial^\mu \rightarrow \partial^\mu - ieA^\mu}$

$\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) \rightarrow \left(\partial_\mu \partial^\mu + m^2 + V(x)\right)\phi(x)$; $V(x) = -ie\partial_\mu A^\mu + A_\mu \partial^\mu$

$T_{fi} = -i\int -ie\left[\phi_f^*\left(\partial_\mu \phi_i\right) - \left(\partial_\mu \phi_f^*\right)\phi_i\right] A^\mu\, d^4x$

$\left(i\gamma^\mu \partial_\mu - m\right)\phi(x) \rightarrow \left(i\gamma^\mu \partial_\mu - m + V(x)\right)\phi(x)$; $V(x) = -e\gamma^0 \gamma_\mu A^\mu$

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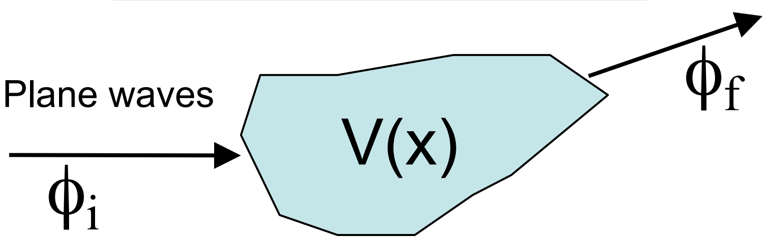

$i\frac{\partial \psi}{\partial t} = \left(H_0 + V(x,t)\right)\psi$

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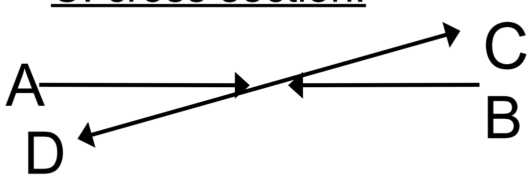

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$T_{fi} = -i\int -ie\left[\phi_f^*\left(\partial_\mu \phi_i\right) - \left(\partial_\mu \phi_f^*\right)\phi_i\right] A^\mu\, d^4x$

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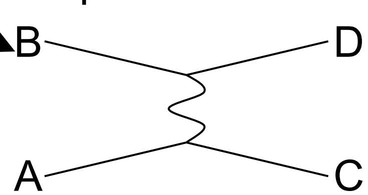
Spin 0 case:



$\phi_i = N_i e^{-ip_i x}$ $\phi_f^* = N_f^* e^{ip_f x}$

$\partial_\nu \partial^\nu A^\mu = -j^\mu_{BD} = -eN_B N_D^*\left(p_B^\mu + p_D^\mu\right)e^{i(p_D - p_B)}$

$A^\mu = -\frac{1}{q^2} j^\mu_{BD}$

$T_{fi} = -i\int j_\mu^{AC} A^\mu d^4x = -i\int j^\mu_{AC} \frac{-g_{\mu\nu}}{q^2} j^\nu_{BD} d^4x = -iN_A N_B N_C^* N_D^* (2\pi)^4 \delta^4(p_A + p_B - p_C - p_D)\mathcal{M}$

$-i\mathcal{M} = ie\left(p_A + p_C\right)^\mu \frac{-ig_{\mu\nu}}{q^2} ie\left(p_B + p_D\right)^\nu \Rightarrow$ "Feynman rules"

Lecture 7

Master Particle Physics 1
Marcel Merk
September 26, 2011

Particles with Spin = 0 | **Particles with Spin = 1/2**

$$E^2 = \vec{p}^2 + m^2$$

Q.M.:

$$E \to i\frac{\partial}{\partial t} \quad ; \quad \vec{p} \to -i\vec{\nabla}$$

$$E^2 = (\vec{\alpha}\cdot\vec{p} + \beta m)^2$$

Klein Gordon:

$$\left(\partial_\mu \partial^\mu + m^2\right)\ \phi(x) = 0$$

Dirac: $\left(i\gamma^\mu \partial_\mu - m\right)\ \psi(x) = 0$

$$\text{with}:\ \gamma^\mu = (\beta, \beta\vec{\alpha}) \quad ; \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

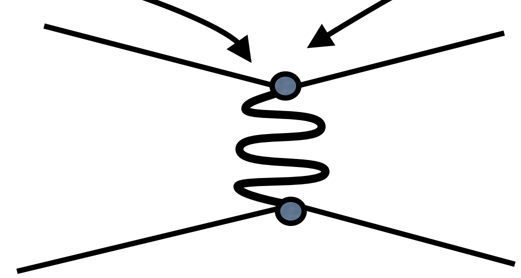
$\phi(x) = Ne^{-ipx}$ ⟵ Solutions ⟶ $\psi(x) = u(p)e^{-ipx} \qquad \overline{\psi} = \psi^\dagger\gamma^0$

$\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0 \quad$: C.C. $\qquad$ Adjoint : $\left(i\partial_\mu \overline{\psi}\gamma^\mu + m\overline{\psi}\right) = 0$;

$j^\mu = i\left[\phi^*\left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right)\phi\right] \qquad \partial_\mu j^\mu = 0 \qquad j^\mu = \overline{\psi}\gamma^\mu\psi$

$$\begin{aligned} j^\mu_{fi} &= i\left[\phi^*_f\left(\partial^\mu \phi^*_i\right) - \left(\partial^\mu \phi^*_f\right)\phi_i\right] \\ &= -eN_iN^*_f\left(p^\mu_i + p^\mu_f\right)e^{i(p_f - p_i)x} \end{aligned}$$

$$\begin{aligned} j^\mu_{fi} &= \overline{\psi}_f\gamma^\mu\psi_i \\ &= -e\,\overline{u}_f(p)\,\gamma^\mu\,u_i(p)\,e^{i(p_f - p_i)x} \end{aligned}$$



Transition currents $\qquad$ Feynman rules

Lecture 8

Master Particle Physics 1

Marcel Merk

September 28, 2011

Solutions to the Dirac Equation

$$(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0 \qquad \Rightarrow \qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} U_A(p) \\ U_B(p) \end{pmatrix} e^{-ipx}$$

Solutions to the Dirac Equation

$$(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0 \qquad \Rightarrow \qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} U_A(p) \\ U_B(p) \end{pmatrix} e^{-ipx}$$

$$\left[\begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix} E - \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} p^i - \begin{pmatrix} 1\!\!1 & 0 \\ 0 & 1\!\!1 \end{pmatrix} m \right] \begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} (\vec{\sigma} \cdot \vec{p})\, U_B &= (E - m)\, U_A \\ (\vec{\sigma} \cdot \vec{p})\, U_A &= (E + m)\, U_B \end{aligned} \qquad U_A = \begin{pmatrix} * \\ * \end{pmatrix} \quad ; \quad U_B = \begin{pmatrix} * \\ * \end{pmatrix} \qquad \vec{\sigma} \cdot \vec{p} = \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix}$$

Solutions to the Dirac Equation

$$(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0 \qquad \Rightarrow \qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} U_A(p) \\ U_B(p) \end{pmatrix} e^{-ipx}$$

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$$\begin{aligned} (\vec{\sigma} \cdot \vec{p})\, U_B &= (E - m)\, U_A \\ (\vec{\sigma} \cdot \vec{p})\, U_A &= (E + m)\, U_B \end{aligned} \qquad U_A = \begin{pmatrix} * \\ * \end{pmatrix} \quad ; \quad U_B = \begin{pmatrix} * \\ * \end{pmatrix} \qquad \vec{\sigma} \cdot \vec{p} = \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix}$$

$\boxed{\vec{p} = 0}$

$$U^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad ; \quad U^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad ; \quad U^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad ; \quad U^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \cdot\ e^{-ipx}$$

Solutions to the Dirac Equation

$$(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0 \qquad \Rightarrow \qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} U_A(p) \\ U_B(p) \end{pmatrix} e^{-ipx}$$

$$\left[\begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix} E - \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} p^i - \begin{pmatrix} 1\!\!1 & 0 \\ 0 & 1\!\!1 \end{pmatrix} m \right] \begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} (\vec{\sigma} \cdot \vec{p})\, U_B &= (E - m)\, U_A \\ (\vec{\sigma} \cdot \vec{p})\, U_A &= (E + m)\, U_B \end{aligned} \qquad U_A = \begin{pmatrix} * \\ * \end{pmatrix} \quad ; \quad U_B = \begin{pmatrix} * \\ * \end{pmatrix} \qquad \vec{\sigma} \cdot \vec{p} = \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix}$$

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$\boxed{\vec{p} \neq 0}$

1)choose : $U_A^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; ; \; U_A^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad U_B^{(1)} = \begin{pmatrix} p_z/(E+m) \\ (p_x + ip_y)/(E+m) \end{pmatrix} \; ; \; U_B^{(2)} = \ldots \quad \text{etc}$

2)choose : $U_B^{(3)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; ; \; U_B^{(4)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad U_A^{(3)} = \ldots. \quad ; \quad U_A^{(4)} = \ldots$

Solutions to the Dirac Equation

$$(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0 \qquad \Rightarrow \qquad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} U_A(p) \\ U_B(p) \end{pmatrix} e^{-ipx}$$

$$\left[\begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix} E - \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} p^i - \begin{pmatrix} 1\!\!1 & 0 \\ 0 & 1\!\!1 \end{pmatrix} m \right] \begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} (\vec{\sigma} \cdot \vec{p})\, U_B &= (E - m)\, U_A \\ (\vec{\sigma} \cdot \vec{p})\, U_A &= (E + m)\, U_B \end{aligned} \qquad U_A = \begin{pmatrix} * \\ * \end{pmatrix} \quad ; \quad U_B = \begin{pmatrix} * \\ * \end{pmatrix} \qquad \vec{\sigma} \cdot \vec{p} = \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix}$$

$\vec{p} = 0$

$$U^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \; ; \; U^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \; ; \; U^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \; ; \; U^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-ipx}$$

$\vec{p} \neq 0$

1)choose : $U_A^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; ; \; U_A^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad U_B^{(1)} = \begin{pmatrix} p_z/(E+m) \\ (p_x + ip_y)/(E+m) \end{pmatrix} \; ; \; U_B^{(2)} = \ldots$ etc

2)choose : $U_B^{(3)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; ; \; U_B^{(4)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad U_A^{(3)} = \ldots. \; ; \; U_A^{(4)} = \ldots$

---

1) Solutions are orthogonal

2) Normalisation: $N = \sqrt{E+m}$

3) Adjoint:

$$(\not{p} - m)\, u = 0 \qquad \Rightarrow \qquad \overline{u}\, (\not{p} - m) = 0$$

$$(\not{p} + m)\, v = 0 \qquad \Rightarrow \qquad \overline{v}\, (\not{p} + m) = 0$$

4) Completeness: $\displaystyle\sum_{s=1,2} u^{(s)}(p)\, \overline{u}^{(s)}(p) = \not{p} + m$

5) Helicity: $\lambda = \frac{1}{2} \vec{\Sigma} \cdot \vec{p} \; ; \; \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$

Lecture 9

Master Particle Physics 1

Marcel Merk

October 3, 2011

QED

S=0 | **S=1/2**

Wave equation: $(\partial_\mu \partial^\mu + m^2)\ \phi(x) = 0$ | $(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0$

Solution: $\phi(x) = N\, e^{-ipx}$ | $\psi(x) = u(p) e^{-ipx}$

Conserved current: $j^\mu = i\left[\phi^* \left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right) \phi\right]$ | $j^\mu = \overline{\psi}\gamma^\mu \psi$

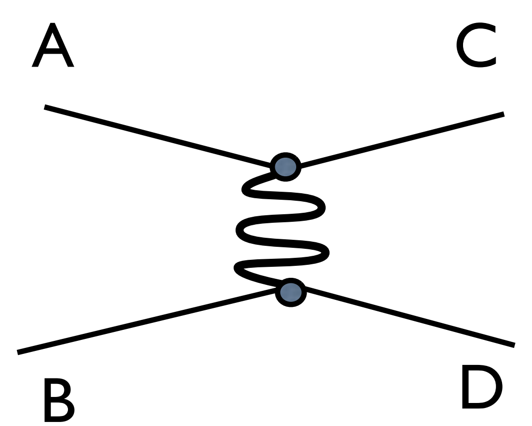
Perturbation Theory $T_{fi} = -i \int d^4x\ \phi_f^*(x) V(x) \phi_i(x)$ | $T_{fi} = -i \int d^4x\ \psi^\dagger(x) V(x) \psi_i(x)$

Electromagnetic Field: $\partial_\mu F^{\mu\nu} = j^\nu$ with : $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

QED: $\partial^\mu \rightarrow \partial^\mu - ieA^\mu \quad \Rightarrow \quad T_{fi} = -i \int j_\mu^{fi}(x)\ A^\mu(x)\ d^4x$

Feynman Rules:

$$-i\mathcal{M} = \begin{array}{c} ie(p_A + p_C)^\mu \\ \cdot\ -ig_{\mu\nu}/q^2 \\ \cdot\ ie(p_B + p_D)^\nu \end{array}$$



$$-i\mathcal{M} = \begin{array}{c} ie(\overline{u}_C \gamma^\mu u_A) \\ \cdot\ -ig_{\mu\nu}/q^2 \\ \cdot\ ie(\overline{u}_D \gamma^\nu u_B) \end{array}$$

Cross Section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2 \quad \Leftarrow \quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}\overline{|\mathcal{M}|}^2 \quad \Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{\alpha}{2s}\frac{4+(1+\cos\theta)^2}{(1-\cos\theta)^2}$$

Mandelstam Variables & Crossing:

$$\boxed{e^+e^- \rightarrow \mu^+\mu^- = \frac{\alpha^2}{4s}(1+\cos^2\theta)}$$

Lecture 10

Master Particle Physics 1

Marcel Merk

October 5, 2011

QED Electromagnetism / Weak Interaction Weak

Matter Waves: $i\left(\gamma^\mu \partial_\mu - m\right)\psi(x) = 0 \quad ; \quad \psi(x) = U(p)e^{-ip_\mu x^\mu} \quad ; \quad (\not p - m)\, u(p) = 0$

Substitution: $\partial^\mu \rightarrow \partial^\mu - ieA^\mu$ $\qquad$ $\partial^\mu \rightarrow \partial^\mu + igB^\mu$

Field Equation: $\partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu = j^\nu_{EM}$ $\qquad$ $\partial_\mu \partial^\mu B^\nu - \partial_\mu \partial^\nu B^\mu + m^2 B^\nu = j^\nu_{\text{Weak}}$

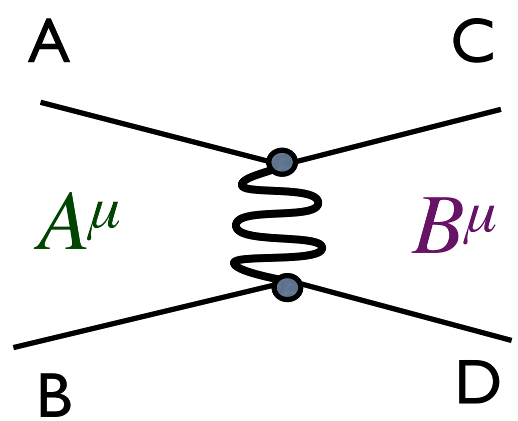
Perturbation Theory

$$T_{fi} = -i \int j^{fi}_{EM,\mu}\, A^\mu(x)\, d^4x \qquad T_{fi} = -i \int j^{fi}_{\text{Weak},\mu}\, B^\mu(x)\, d^4x$$

$$j^{fi}_{EM,\mu} = \overline{\psi}_f \gamma^\mu \psi_i \qquad j^{fi}_{\text{Weak},\mu} = \overline{\psi}_f\, \frac{1}{2}\gamma^\mu (1-\gamma^5)\, \psi_f$$

Matrix Element:

$$-i\mathcal{M} = \begin{array}{c} ie(\overline{u}_C \gamma^\mu u_A) \\ \cdot \; -ig_{\mu\nu}/q^2 \\ \cdot \; ie(\overline{u}_D \gamma^\nu u_B) \end{array}$$



$$-i\mathcal{M} = \begin{array}{c} i\frac{g}{\sqrt{2}}(\overline{u}_C \frac{1}{2}\gamma^\mu (1-\gamma^5) u_A) \\ \cdot \; -ig_{\mu\nu}/(M^2 - q^2) \\ \cdot \; i\frac{g}{\sqrt{2}}(\overline{u}_D \frac{1}{2}\gamma^\nu (1-\gamma^5) u_B) \end{array}$$

$$\boxed{\begin{array}{rcl} e^+e^- & \rightarrow & \mu^+\mu^- \\ \dfrac{d\sigma}{d\Omega} & = & \dfrac{\alpha^2}{4s}\left(1+\cos^2\theta\right) \end{array}} \qquad \boxed{\begin{array}{rcl} \mu^- & \rightarrow & e^-\nu_\mu \overline{\nu}_e \\ \Gamma & = & \dfrac{G_F^2 m_\mu^5}{192\pi^3} \end{array}}$$

General Matrix Element: $\mathcal{M} = \sum_{i,j}^{S,V,T,P,A} C_{ij}\; (\overline{u}_C \mathcal{O}_i u_A) \cdot \text{prop} \cdot (\overline{u}_D \mathcal{O}_j u_B)$

$S = \overline{\psi}\psi \; ; \; V = \overline{\psi}\gamma^\mu\psi \; ; \; T = \overline{\psi}\sigma^{\mu\nu}\psi \; ; \; A = \overline{\psi}\gamma^5\gamma^\mu\psi \; ; P = \overline{\psi}\gamma^5\psi$

Weak Interaction:

Leptons : $\begin{pmatrix} \nu_e \\ e \end{pmatrix} \; ; \; \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \; ; \; \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$

Quarks : $\begin{pmatrix} u \\ d \end{pmatrix} \; ; \; \begin{pmatrix} c \\ s \end{pmatrix} \; ; \; \begin{pmatrix} t \\ b \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

Lecture 11

Master Particle Physics 1

Marcel Merk

Ocober 10, 2011

Symmetries

Lagrangian density is the basic object for physics: $\mathcal{L}\left(\phi(x), \partial_\mu \phi(x)\right) = \overline{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi$

<u>Symmetry</u>: Require that the Lagrangian remains invariant under a symmetry operation.

<u>4 Basic symmetry groups:</u>
- Permutation symmetries
- Continuous space-time symmetries ("external" symmetries)
- Discrete symmetries: *C, P, T*
- *<u>Unitary or Gauge symmetries ("internal" symmetries)</u>*

Unitary Phase Symmetry U(1) $\quad \overline{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi$

Covariant Derivative:

$$\mathcal{D}_\mu = \partial_\mu + iqA_\mu(x)$$

Gauge Transformations:

$$\psi(x) \to \psi'(x) = e^{iq\alpha(x)}\psi(x)$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x)$$

(Maxwell gauge invariance)

U(1) Lagrangian:

$$\begin{aligned} \overline{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi &= \overline{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi - q\overline{\psi}\gamma^\mu\psi A_\mu \\ \mathcal{L}_{U(1)} &= \mathcal{L}^{\text{free}}_{U(1)} + j^\mu A_\mu \end{aligned}$$

Yang Mills Symmetry SU(2) $\quad \psi(x) = \begin{pmatrix} p \\ n \end{pmatrix}$

Covariant Derivative:

$$\mathcal{D}_\mu = \partial_\mu + igB_\mu(x) \quad \text{with} \quad B_\mu(x) = \frac{1}{2}\vec{\tau}\cdot\vec{b}_\mu$$

Gauge Transformations:

$$\psi(x) \to \psi'(x) = e^{i\frac{1}{2}\vec{\tau}\cdot\vec{\alpha}(x)}\psi(x)$$

$$B_\mu(x) \to B'_\mu(x) = G B_\mu(x) G^{-1} + \frac{i}{g}\left(\partial_\mu G\right)G^{-1}$$

$$\vec{b}_\mu(x) \to \vec{b}'_\mu(x) = \vec{b}_\mu - \vec{\alpha}\times\vec{b}_\mu - \frac{1}{g}\partial_\mu \alpha(x)$$

"non-Abelian"

SU(2) Lagrangian:

$$\begin{aligned} \overline{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi &= \overline{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi - \frac{g}{2}\overline{\psi}\gamma^\mu\vec{\tau}\psi\,\vec{b}_\mu \\ \mathcal{L}_{SU(2)} &= \mathcal{L}^{\text{free}}_{SU(2)} + \vec{j}^\mu\,\vec{b}_\mu \end{aligned}$$

Lecture 12

Master Particle Physics 1

Marcel Merk

Ocober 12, 2011

Standard Model of Electroweak Interactions

Origin of interactions is described via the principle of *local gauge invariance*
Recipe: Take the Lagrangian of free Dirac particles:

$$\mathcal{L}_{\text{Dirac}} = \overline{\psi}(x)\ (i\gamma^\mu \partial_\mu - m)\ \psi(x)$$

and impose that it should remain invariant under:

$$U(1)_Y \ :\ \psi(x) \to \psi'(x) = e^{iY\alpha(x)}\psi(x) \quad ; \quad Y = \text{hypercharge}\ ,\ Q = T_3 + \frac{1}{2}Y$$

$$SU(2) \ :\ \psi_L(x) \to \psi'_L(x) = e^{i\vec{T}\cdot\vec{\alpha}(x)}\psi_L(x) \quad ; \quad \vec{T} = \frac{1}{2}\vec{\tau} = \text{weak isospin}\ ,\ \psi_L = \left(\frac{1-\gamma^5}{2}\right)\psi$$

To keep the Lagrangian invariant compensating gauge fields must be introduced which transform simultaneously with the Dirac fields:

$$\partial_\mu \ \to\ \mathcal{D}_\mu \ =\ \partial_\mu + g'\,\frac{Y}{2}\,a_\mu \ +\ g\,\vec{T}\cdot\vec{b}_\mu \qquad\qquad a_\mu = \text{hypercharge field}$$

$$\mathcal{L} \ =\ \mathcal{L}_{\text{free}} - g'\,\frac{J_Y^\mu}{2}\,a_\mu \ -\ g\,\vec{J}_L^\mu\cdot\vec{b}_\mu \qquad\qquad b_\mu^1, b_\mu^2, b_\mu^3 = \text{weak isospin field}$$

The physical currents are: $\text{C.C.}:\ W_\mu^+ = \dfrac{b_\mu^1 - i b_\mu^2}{\sqrt{2}}$ $\qquad$ $\text{N.C.}:\ Z_\mu = -a_\mu \sin\theta_W + b_\mu^3 \cos\theta_W$

$$W_\mu^- = \frac{b_\mu^1 + i b_\mu^2}{\sqrt{2}} \qquad\qquad A_\mu = a_\mu \cos\theta_W + b_\mu^3 \sin\theta_W$$

The interaction Lagrangian is:

$$\boxed{\begin{aligned}\mathcal{L} \ =\ \mathcal{L}_{\text{free}} &- \frac{g}{\sqrt{2}}\,\overline{\psi}_u\,\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\ \psi_d\ W_\mu^+ \\ &- \frac{g}{\sqrt{2}}\,\overline{\psi}_d\,\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\ \psi_u\ W_\mu^- \\ &- e\,Q\,\overline{\psi}\,\gamma^\mu\,\psi\ A_\mu \\ &- g_z\,\overline{\psi}\,\gamma^\mu\frac{1}{2}\left(C_V^f - C_A^f\gamma^5\right)\psi\ Z_\mu\end{aligned}}$$

$$e = g\sin\theta_W \qquad g'/g = \tan\theta_W$$

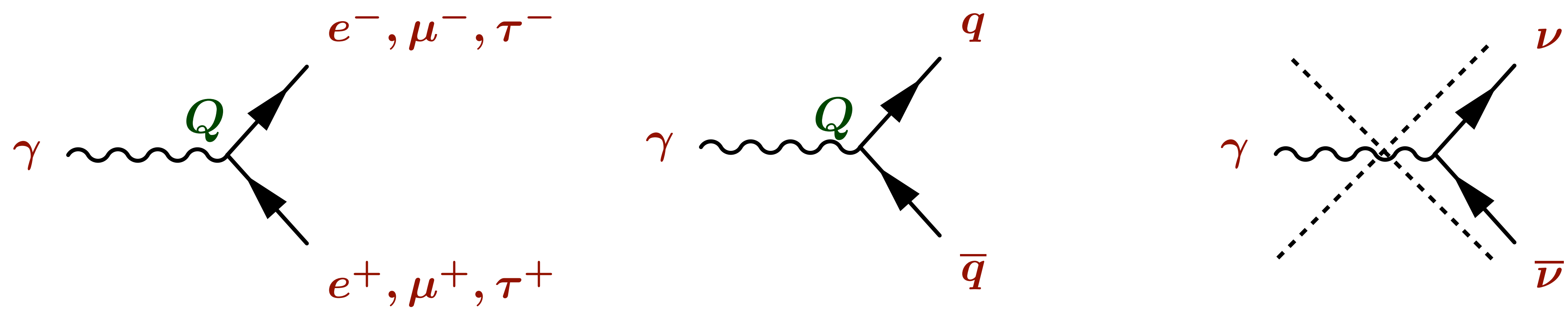
$$g_z = g\ /\ \cos\theta_W$$

$$\begin{aligned} C_V^f &= T_3^f - 2Q^f\sin^2\theta_W \\ C_A^f &= T_3^f \end{aligned}$$

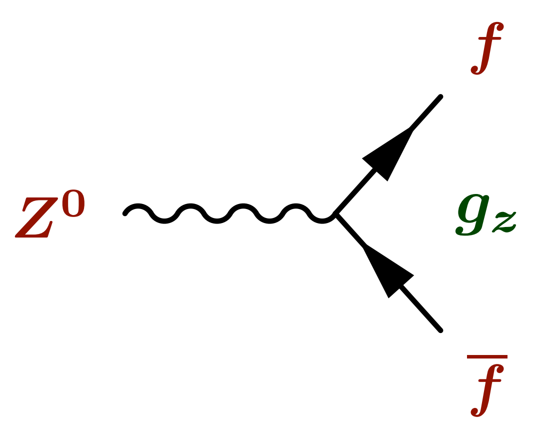
$$T_3 = \begin{matrix} 1/2 \\ -1/2 \end{matrix}\ \begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} \nu \\ l \end{pmatrix}$$

Feynman Vertices

E.M.:



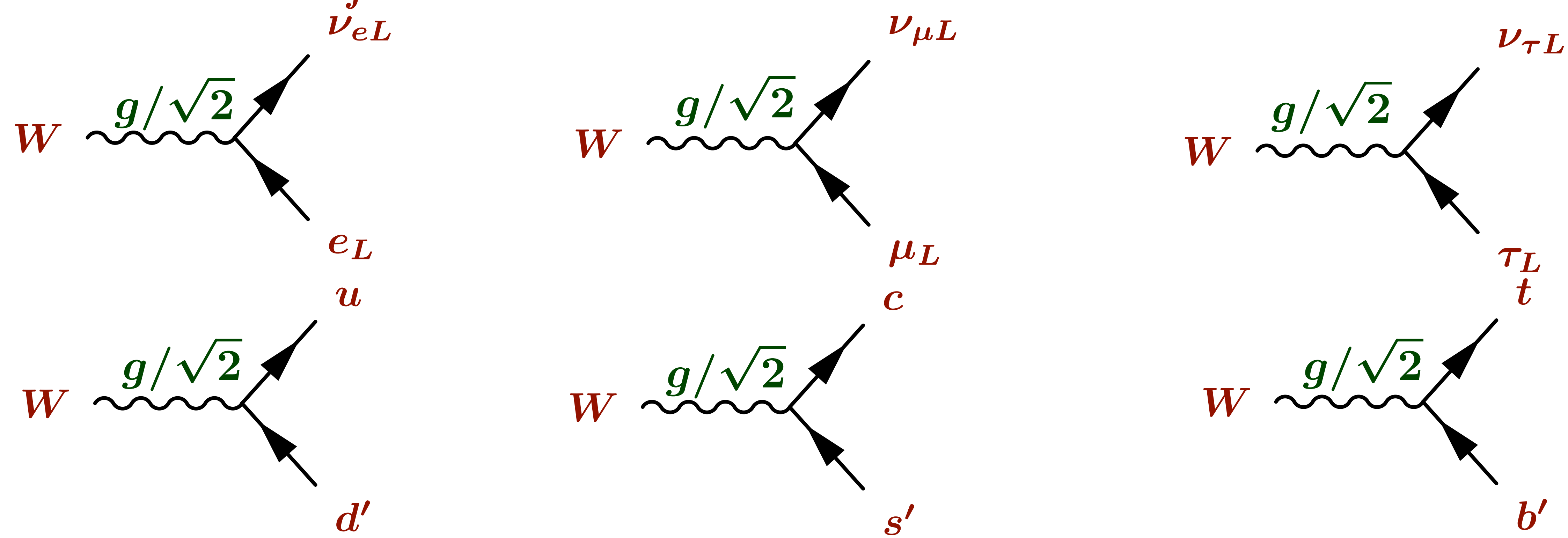
N.C.:



$$g_z = \frac{g}{\cos\theta_W}$$

$$C_V^f = T_3^f - 2Q^f \sin^2\theta_W$$
$$C_A^f = T_3^f$$

C.C.:



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \Big(\quad V_{CKM} \quad \Big) = \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \Big(\quad V_{PMNS} \quad \Big) = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$