Lecture 9

Master Particle Physics 1

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Lectures PP1

L0 : Introduction

L1 : Particles and Fields (historical overview)
L2 : Wave Equations and Antiparticles
L3 : The Electromagnetic Field
L4 : Perturbation Theory and Fermi's Golden Rule
L5 : Electromagnetic Scattering of Spinless Particles

QED of Spinless Particles: Scattering Theory and Cross Sections

L6 : The Dirac Equation
L7 : Solutions of the Dirac Equation
L8 : Spin 1/2 Electrodynamics

QED for Fundamental Fermions

L9 : The Weak Interaction
(Fermi 4-point scattering: an analogy with QED)
L10: Local Gauge Invariance
(the role of symmetries in interactions)
L11: Electroweak Theory
L12: The Process: $e^+e^- \to \gamma, Z \to \mu^+\mu^-$

The Standard Model for massless particles $SU(2)_L \times U(1)_Y$

QED

S=0 | **S=1/2**

Wave equation: $(\partial_\mu \partial^\mu + m^2)\ \phi(x) = 0$ | $(i\gamma^\mu \partial_\mu - m)\ \psi(x) = 0$

Solution: $\phi(x) = N\, e^{-ipx}$ | $\psi(x) = u(p) e^{-ipx}$

Conserved current: $j^\mu = i\,[\phi^* \,(\partial^\mu \phi) - (\partial^\mu \phi^*)\,\phi]$ | $j^\mu = \overline{\psi}\gamma^\mu \psi$

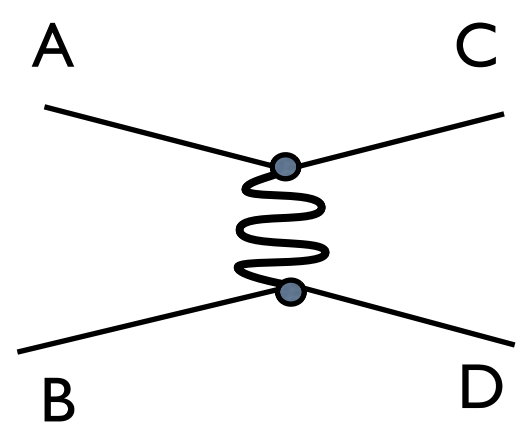
Perturbation Theory $T_{fi} = -i \int d^4x\ \phi_f^*(x) V(x) \phi_i(x)$ | $T_{fi} = -i \int d^4x\ \psi^\dagger(x) V(x) \psi_i(x)$

Electromagnetic Field: $\partial_\mu F^{\mu\nu} = j^\nu \quad \text{with :} \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

QED: $\partial^\mu \to \partial^\mu - ieA^\mu \quad \Rightarrow \quad T_{fi} = -i \int j_\mu^{fi}(x)\ A^\mu(x)\ d^4x$

Feynman Rules:

$$-i\mathcal{M} = \begin{matrix} ie(p_A + p_C)^\mu \\ \cdot\ -ig_{\mu\nu}/q^2 \\ \cdot\ ie(p_B + p_D)^\nu \end{matrix}$$



$$-i\mathcal{M} = \begin{matrix} ie(\overline{u}_C \gamma^\mu u_A) \\ \cdot\ -ig_{\mu\nu}/q^2 \\ \cdot\ ie(\overline{u}_D \gamma^\nu u_B) \end{matrix}$$

Cross Section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2 \quad \Leftarrow \quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\,\frac{1}{s}\,\overline{|\mathcal{M}|}^2 \quad \Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{\alpha}{2s}\,\frac{4+(1+\cos\theta)^2}{(1-\cos\theta)^2}$$

Mandelstam Variables & Crossing:

$$\boxed{e^+e^- \to \mu^+\mu^- = \frac{\alpha^2}{4s}\,(1+\cos^2\theta)}$$

Quantum Electrodynamics

1) Free particle wave equations

K.G.: $\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0$

Dirac: $\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) = 0$

Plane wave solutions:

$\phi(x) = N e^{-ipx}$

$\psi(x) = u(p) e^{-ipx}$

Current: $\partial_\mu j^\mu = 0$

$j^\mu = i\left[\phi^*\left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right)\phi\right] \Rightarrow \rho = 2|N|^2 E$

$j^\mu = \overline{\psi}\gamma^\mu \psi \Rightarrow \rho = \psi^\dagger \psi \geq 0$

2) Electromagnetic field

Maxwell $\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu$

or $\partial_\mu F^{\mu\nu} = j^\nu$

Gauge freedom:

$A_\mu \rightarrow A_\mu' = A_\mu + \partial_\mu \lambda$

with $\partial_\nu \partial^\nu \lambda = 0$

Lorentz condition:

$\partial_\mu A^\mu = 0$

$\partial_\mu \partial^\mu A^\nu = j^\nu$

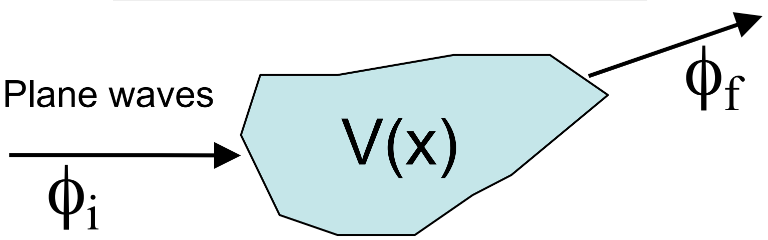
Plane wave solutions:

$A^\mu = N \varepsilon^\mu(p) e^{-ipx}$

(2 polarizations since m=0)

3) Scattering Perturbation Theory

A: non-relativistic derivation:



$i\dfrac{\partial \psi}{\partial t} = \left(H_0 + V(x,t)\right)\psi$

$\psi = \sum_{n=0}^{\infty} a_n(t)\phi_n(t) e^{-iEt}$

$T_{fi} = a_f(t \rightarrow \infty) = -i\int d^4x\, \phi_f^*(x) V(x) \phi_i(x)$

1-st order:

$W_{fi} = \lim_{T\rightarrow\infty} \dfrac{|T_{fi}|^2}{T}$ with

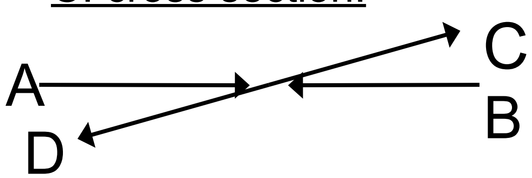
$T_{fi} = -2\pi i V_{fi}\, \delta\left(E_f - E_i\right)$ with

$V_{fi} = \int d^3x\, \phi_f^*(\vec{x}) V(\vec{x}) \phi_i(\vec{x})$

B: relativistic extension:

$W_{fi} = \lim_{T,V\rightarrow\infty} \dfrac{|T_{fi}|^2}{TV}$ and $T_{fi} = -iN_A N_B N_C^* N_D^* (2\pi)^4 \delta^4\left(p_A + p_B - p_C - p_D\right)\mathcal{M}$

C: cross section:



$\dfrac{d\sigma}{d\Omega} = \dfrac{W_{fi}}{flux} d\Phi$

$flux = 4\sqrt{\left(p_A p_B\right)^2 - m_A^2 m_B^2}\,/V^2$

$d\Phi = \prod_i \dfrac{V}{(2\pi)^3}\dfrac{d^3 p_i}{2E_i}$

$\dfrac{d\sigma}{d\Omega} = \dfrac{1}{64\pi^2}\dfrac{1}{s}\left|\dfrac{p_f}{p_i}\right| |\mathcal{M}|^2$

4) Electromagnetic Scattering

$\partial^\mu \rightarrow \partial^\mu - ieA^\mu$

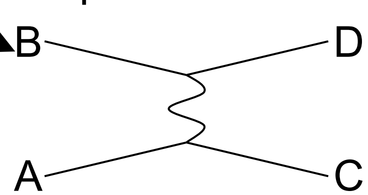
$\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) \rightarrow \left(\partial_\mu \partial^\mu + m^2 + V(x)\right)\phi(x)$; $V(x) = -ie\partial_\mu A^\mu + A_\mu \partial^\mu$

$T_{fi} = -i\int -ie\left[\phi_f^*\left(\partial_\mu \phi_i\right) - \left(\partial_\mu \phi_f^*\right)\phi_i\right] A^\mu\, d^4x$

$\left(i\gamma^\mu \partial_\mu - m\right)\phi(x) \rightarrow \left(i\gamma^\mu \partial_\mu - m + V(x)\right)\phi(x)$; $V(x) = -e\gamma^0 \gamma_\mu A^\mu$

$T_{fi} = -i\int -e\left[\overline{\psi}_f \gamma_\mu \psi_i\right] A^\mu\, d^4x$

Spin 0 case:



$\phi_i = N_i e^{-ip_i x}$ $\phi_f^* = N_f^* e^{ip_f x}$

$\partial_\nu \partial^\nu A^\mu = -j_{BD}^\mu = -eN_B N_D^*\left(p_B^\mu + p_D^\mu\right) e^{i(p_D - p_B)}$

$A^\mu = -\dfrac{1}{q^2} j_{BD}^\mu$

$T_{fi} = -i\int j_\mu^{AC} A^\mu d^4x = -i\int j_{AC}^\mu \dfrac{-g_{\mu\nu}}{q^2} j_{BD}^\nu d^4x = -iN_A N_B N_C^* N_D^* (2\pi)^4 \delta^4\left(p_A + p_B - p_C - p_D\right)\mathcal{M}$

$-i\mathcal{M} = ie\left(p_A + p_C\right)^\mu \dfrac{-ig_{\mu\nu}}{q^2} ie\left(p_B + p_D\right)^\nu \Rightarrow$ "Feynman rules"