Lecture 7

Master Particle Physics 1

Marcel Merk

September 26, 2011

Lectures PP1

L0 : Introduction

L1 : Particles and Fields (historical overview)
L2 : Wave Equations and Antiparticles
L3 : The Electromagnetic Field
L4 : Perturbation Theory and Fermi's Golden Rule
L5 : Electromagnetic Scattering of Spinless Particles

QED of Spinless Particles: Scattering Theory and Cross Sections

L6 : The Dirac Equation
L7 : Solutions of the Dirac Equation
L8 : Spin 1/2 Electrodynamics

QED for Fundamental Fermions

L9 : The Weak Interaction
(Fermi 4-point scattering: an analogy with QED)
L10: Local Gauge Invariance
(the role of symmetries in interactions)
L11: Electroweak Theory
L12: The Process: $e^+e^- \rightarrow \gamma, Z \rightarrow \mu^+\mu^-$

The Standard Model for massless particles $SU(2)_L \times U(1)_Y$

Particles with Spin = 0

Particles with Spin = 1/2

$$E^2 = \vec{p}^2 + m^2$$

Q.M.:

$$E \to i\frac{\partial}{\partial t} \quad ; \quad \vec{p} \to -i\vec{\nabla}$$

$$E^2 = (\vec{\alpha}\cdot\vec{p} + \beta m)^2$$

Klein Gordon:

$$\left(\partial_\mu \partial^\mu + m^2\right)\ \phi(x) = 0$$

Dirac: $(i\gamma^\mu\partial_\mu - m)\ \psi(x) = 0$

$$\text{with}:\ \gamma^\mu = (\beta, \beta\vec{\alpha}) \quad ; \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$\phi(x) = Ne^{-ipx}$$

$\longleftarrow$ Solutions $\longrightarrow$

$$\psi(x) = u(p)e^{-ipx} \qquad \overline{\psi} = \psi^\dagger\gamma^0$$

$$\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0 \quad : \text{C.C.}$$

Adjoint : $\left(i\partial_\mu\overline{\psi}\gamma^\mu + m\overline{\psi}\right) = 0\ ;$

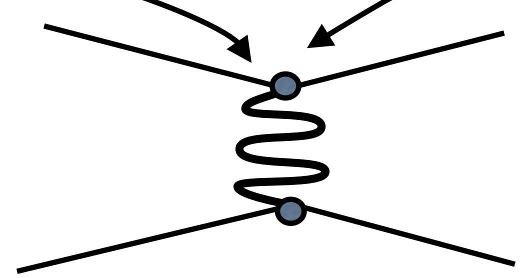
$$j^\mu = i\left[\phi^*\left(\partial^\mu\phi\right) - \left(\partial^\mu\phi^*\right)\phi\right] \qquad \partial_\mu j^\mu = 0 \qquad j^\mu = \overline{\psi}\gamma^\mu\psi$$

$$j^\mu_{fi} = i\left[\phi^*_f\left(\partial^\mu\phi^*_i\right) - \left(\partial^\mu\phi^*_f\right)\phi_i\right]$$

$$= -eN_iN^*_f\left(p^\mu_i + p^\mu_f\right)e^{i(p_f - p_i)x}$$

$$j^\mu_{fi} = \overline{\psi}_f\gamma^\mu\psi_i$$

$$= -e\,\overline{u}_f(p)\,\gamma^\mu\,u_i(p)\,e^{i(p_f - p_i)x}$$



Transition currents

Feynman rules

1) Free particle wave equations

Plane wave solutions:

Current: $\partial_\mu j^\mu = 0$

K.G.: $\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0$ $\qquad \phi(x) = N e^{-ipx}$ $\qquad j^\mu = i\left[\phi^*\left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right)\phi\right]$ $\Rightarrow$ $\rho = 2|N|^2 E$

Dirac: $\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) = 0$ $\qquad \psi(x) = u(p) e^{-ipx}$ $\qquad j^\mu = \bar{\psi}\gamma^\mu \psi$ $\qquad \rho = \psi^\dagger \psi \geq 0$

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2) Electromagnetic field

Maxwell $\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu$

or $\partial_\mu F^{\mu\nu} = j^\nu$

Gauge freedom:

$A_\mu \to A_\mu' = A_\mu + \partial_\mu \lambda$

with $\partial_\nu \partial^\nu \lambda = 0$

Lorentz condition:

$\partial_\mu A^\mu = 0$

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Plane wave solutions:

$A^\mu = N \varepsilon^\mu(p) e^{-ipx}$

(2 polarizations since m=0)

Quantum Electrodynamics

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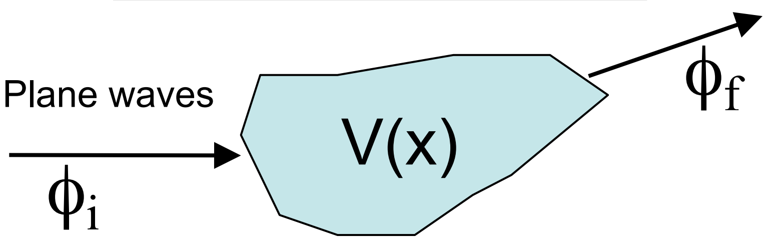
3) Scattering Perturbation Theory

A: non-relativistic derivation:



$$i\frac{\partial\psi}{\partial t} = \left(H_0 + V(x,t)\right)\psi$$

$$\psi = \sum_{n=0}^{\infty} a_n(t)\phi_n(t) e^{-iEt}$$

$$T_{fi} = a_f(t \rightarrow \infty) = -i\int d^4x\, \phi_f^*(x) V(x) \phi_i(x)$$

1-st order:

$$W_{fi} = \lim_{T\to\infty} \frac{|T_{fi}|^2}{T} \quad \text{with}$$

$$T_{fi} = -2\pi i\, V_{fi}\, \delta\left(E_f - E_i\right) \quad \text{with}$$

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zondag 25 september 2011

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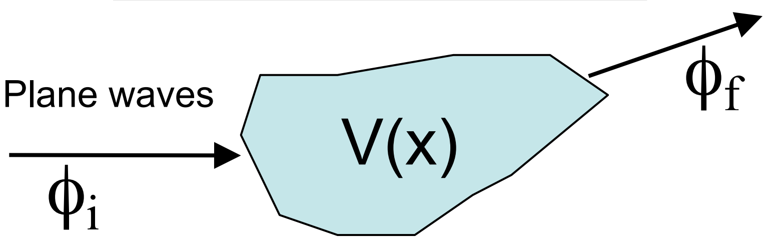

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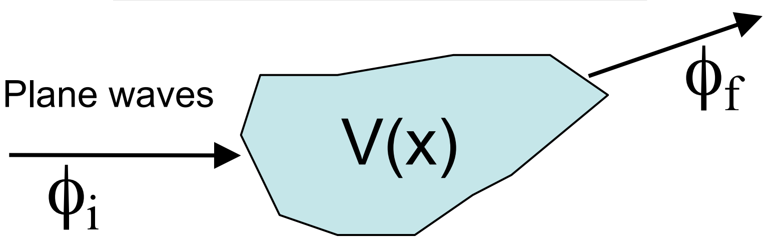

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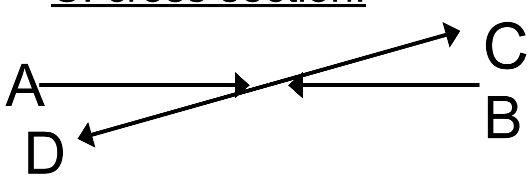
C: cross section:



$\frac{d\sigma}{d\Omega} = \frac{W_{fi}}{flux} d\Phi$

$flux = 4\sqrt{\left(p_A p_B\right)^2 - m_A^2 m_B^2} / V^2$

$d\Phi = \prod_i \frac{V}{(2\pi)^3} \frac{d^3 p_i}{2E_i}$

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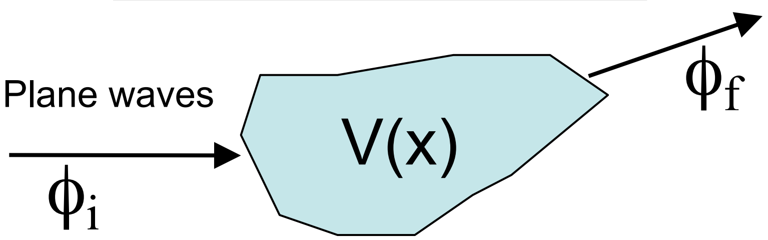

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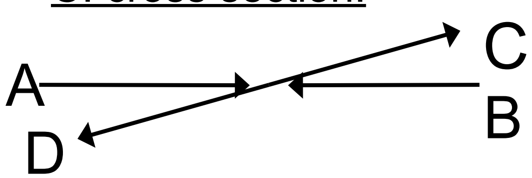

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$\boxed{\partial^\mu \to \partial^\mu - ieA^\mu}$

$\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) \to \left(\partial_\mu \partial^\mu + m^2 + V(x)\right)\phi(x) \quad ; \quad V(x) = -ie\partial_\mu A^\mu + A_\mu \partial^\mu$

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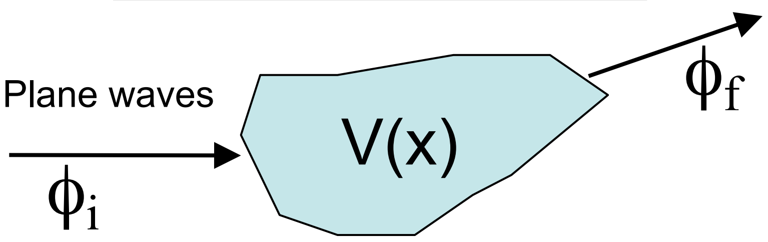

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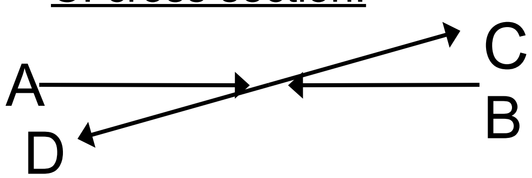

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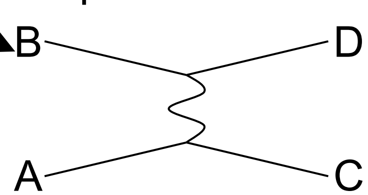
Spin 0 case:



$\phi_i = N_i e^{-ip_i x}$ $\phi_f^* = N_f^* e^{ip_f x}$

$\partial_\nu \partial^\nu A^\mu = -j_{BD}^\mu = -eN_B N_D^*\left(p_B^\mu + p_D^\mu\right) e^{i(p_D - p_B)}$

$A^\mu = -\frac{1}{q^2} j_{BD}^\mu$

$T_{fi} = -i\int j_\mu^{AC} A^\mu d^4x = -i\int j_{AC}^\mu \frac{-g_{\mu\nu}}{q^2} j_{BD}^\nu d^4x = -iN_A N_B N_C^* N_D^* (2\pi)^4 \delta^4(p_A + p_B - p_C - p_D)\mathcal{M}$

$-i\mathcal{M} = ie\left(p_A + p_C\right)^\mu \frac{-ig_{\mu\nu}}{q^2} ie\left(p_B + p_D\right)^\nu \Rightarrow$ “Feynman rules”