Lecture 6

Master Particle Physics 1

Marcel Merk

September 21, 2011

Lectures PP1

L0 : Introduction

L1 : Particles and Fields (historical overview)
L2 : Wave Equations and Antiparticles
L3 : The Electromagnetic Field
L4 : Perturbation Theory and Fermi's Golden Rule
L5 : Electromagnetic Scattering of Spinless Particles

QED of Spinless Particles: Scattering Theory and Cross Sections

L6 : The Dirac Equation
L7 : Solutions of the Dirac Equation
L8 : Spin 1/2 Electrodynamics

QED for Fundamental Fermions

L9 : The Weak Interaction
(Fermi 4-point scattering: an analogy with QED)
L10: Local Gauge Invariance
(the role of symmetries in interactions)
L11: Electroweak Theory
L12: The Process: $e^+e^- \rightarrow \gamma, Z \rightarrow \mu^+\mu^-$

The Standard Model for massless particles $SU(2)_L \times U(1)_Y$

1) Free particle wave equations

K.G.: $\left(\partial_\mu \partial^\mu + m^2\right)\phi\left(x\right) = 0$

Dirac: $\left(i\gamma^\mu \partial_\mu - m\right)\psi\left(x\right) = 0$

Plane wave solutions:

$\phi\left(x\right) = Ne^{-ipx}$

$\psi\left(x\right) = u\left(p\right)e^{-ipx}$

Current: $\partial_\mu j^\mu = 0$

$j^\mu = i\left[\phi^*\left(\partial^\mu \phi\right) - \left(\partial^\mu \phi^*\right)\phi\right] \Rightarrow \rho = 2\left|N\right|^2 E$

$j^\mu = \bar{\psi}\gamma^\mu \psi \Rightarrow \rho = \psi^\dagger \psi \geq 0$

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2) Electromagnetic field

Maxwell $\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu$

or $\partial_\mu F^{\mu\nu} = j^\nu$

Gauge freedom:

$A_\mu \to A_\mu{}' = A_\mu + \partial_\mu \lambda$

with $\partial_\nu \partial^\nu \lambda = 0$

Lorentz condition:

$\partial_\mu A^\mu = 0$

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$A^\mu = N \varepsilon^\mu(p) e^{-ipx}$

(2 polarizations since m=0)

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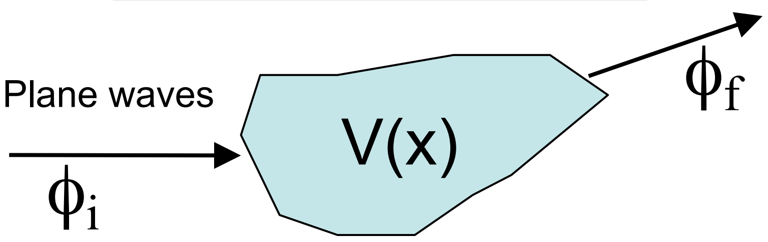
3) Scattering Perturbation Theory

A: non-relativistic derivation:



$$i\frac{\partial\psi}{\partial t} = \left(H_0 + V\left(x,t\right)\right)\psi$$

$$\psi = \sum_{n=0}^{\infty} a_n\left(t\right)\phi_n\left(t\right)e^{-iEt}$$

$$T_{fi} = a_f\left(t \rightarrow \infty\right) = -i\int d^4x\,\phi_f^*\left(x\right)V\left(x\right)\phi_i\left(x\right)$$

1-st order:

$$W_{fi} = \lim_{T\rightarrow\infty}\frac{\left|T_{fi}\right|^2}{T} \quad \text{with}$$

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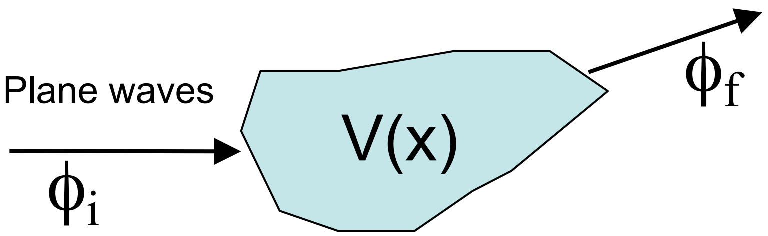

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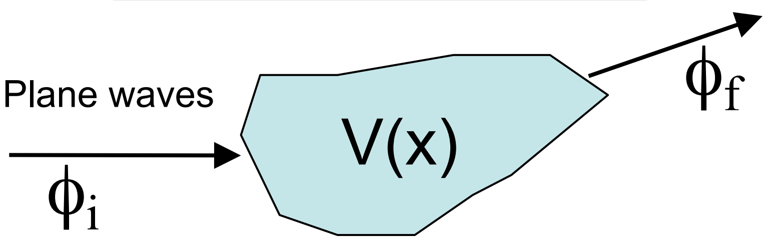

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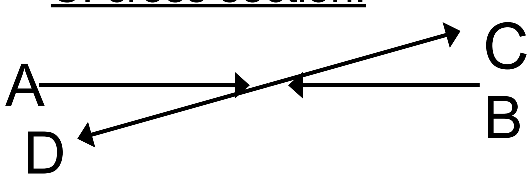
C: cross section:



$\frac{d\sigma}{d\Omega} = \frac{W_{fi}}{flux}d\Phi$

$flux = 4\sqrt{\left(p_A p_B\right)^2 - m_A^2 m_B^2}\,/V^2$

$d\Phi = \prod_i \frac{V}{(2\pi)^3}\frac{d^3p_i}{2E_i}$

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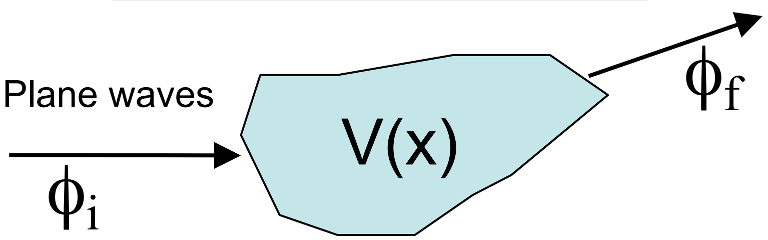

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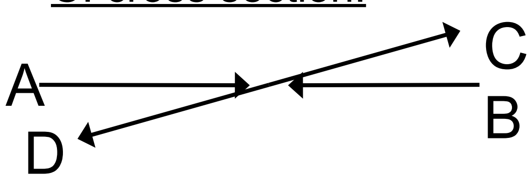

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$\partial^\mu \rightarrow \partial^\mu - ieA^\mu$

$$\left(\partial_\mu \partial^\mu + m^2\right)\phi(x) \rightarrow \left(\partial_\mu \partial^\mu + m^2 + V(x)\right)\phi(x) \; ; \; V(x) = -ie\partial_\mu A^\mu + A_\mu \partial^\mu \qquad T_{fi} = -i\int -ie\left[\phi_f^*\left(\partial_\mu \phi_i\right) - \left(\partial_\mu \phi_f^*\right)\phi_i\right] A^\mu\, d^4x$$

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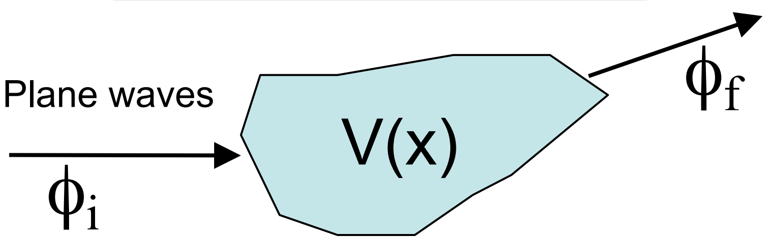

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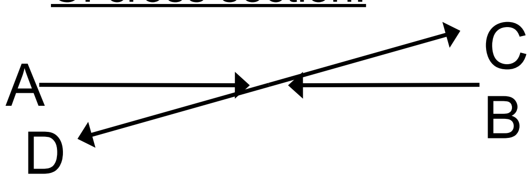

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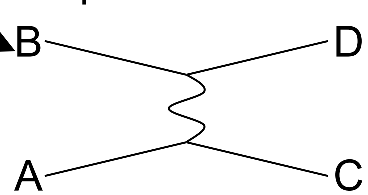
Spin 0 case:



$\phi_i = N_i e^{-ip_i x}$ $\phi_f^* = N_f^* e^{ip_f x}$

$\partial_\nu \partial^\nu A^\mu = -j_{BD}^\mu = -eN_B N_D^*\left(p_B^\mu + p_D^\mu\right) e^{i(p_D - p_B)}$

$A^\mu = -\frac{1}{q^2} j_{BD}^\mu$

$T_{fi} = -i\int j_\mu^{AC} A^\mu d^4x = -i\int j_{AC}^\mu \frac{-g_{\mu\nu}}{q^2} j_{BD}^\nu d^4x = -iN_A N_B N_C^* N_D^* (2\pi)^4 \delta^4\left(p_A + p_B - p_C - p_D\right)\mathcal{M}$

$-i\mathcal{M} = ie\left(p_A + p_C\right)^\mu \frac{-ig_{\mu\nu}}{q^2} ie\left(p_B + p_D\right)^\nu \Rightarrow$ “Feynman rules”