PHY3004: Nuclear and Particle Physics

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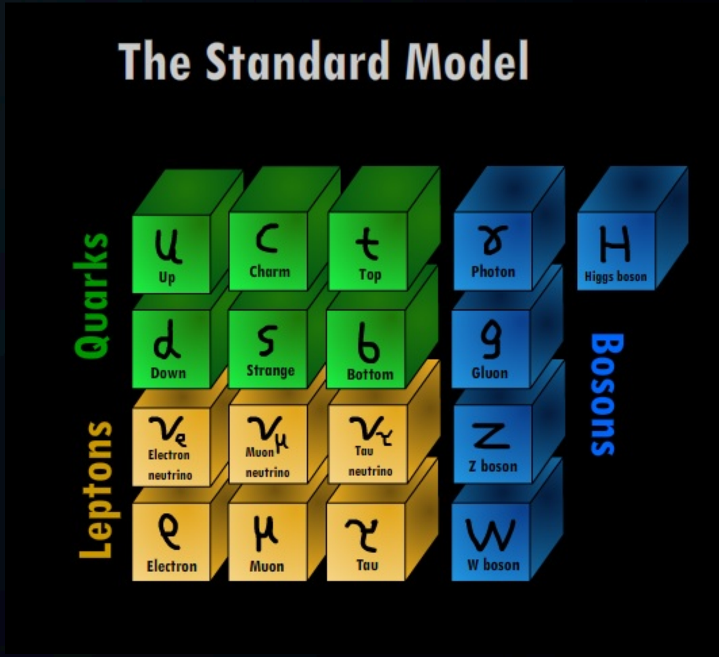
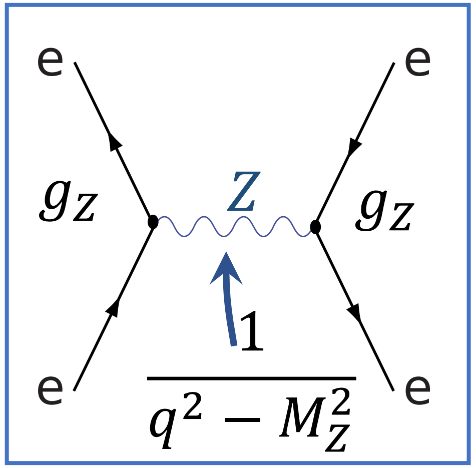
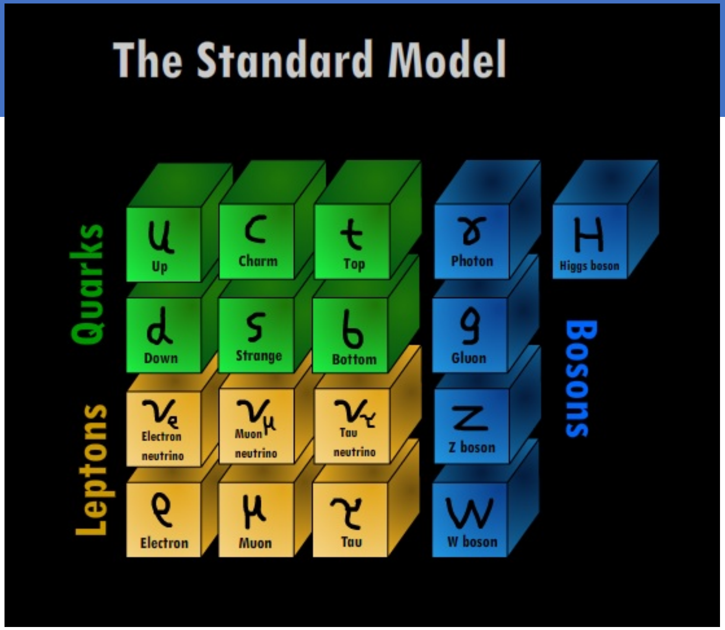
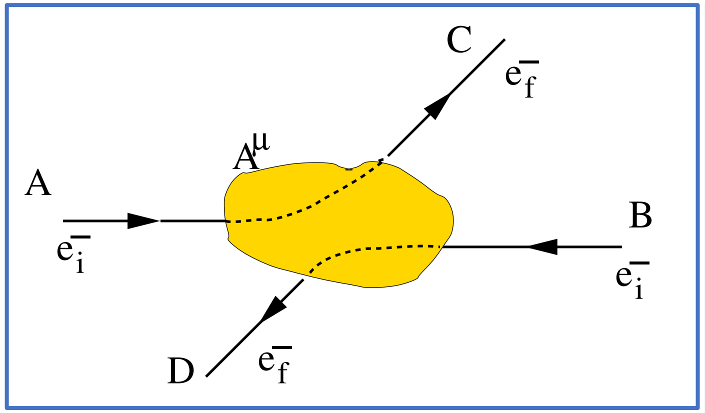


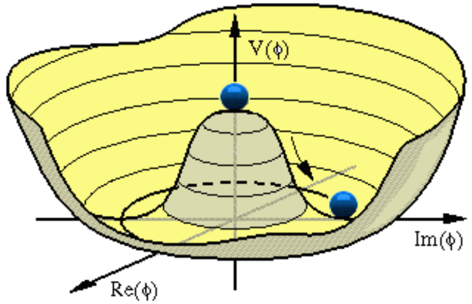
Recap: “Seeing the wood for the trees”

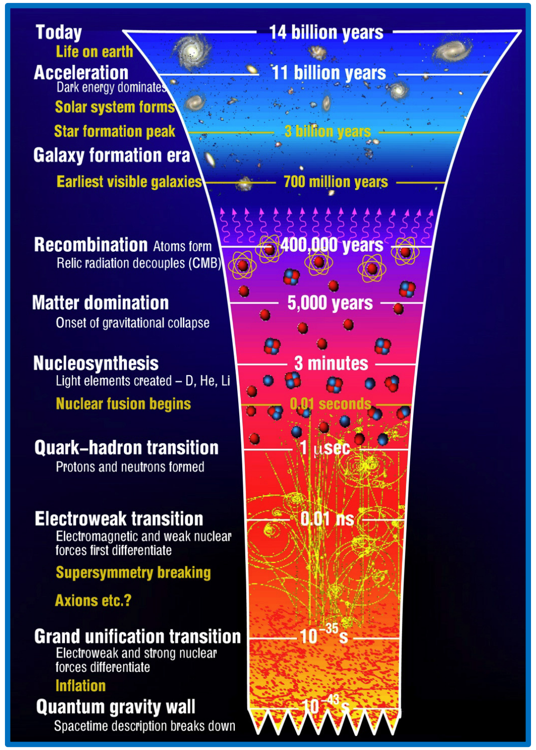
- Lecture 1: “Particles”
  - Zooming into constituents of matter
  - Skills: distinguish particle types, Spin
- Lecture 2: “Forces”
  - Exchange of quanta: EM, Weak, QCD
  - Skills: 4-vectors, Feynman diagrams
- Lecture 3: “Waves”
  - Quantum fields and gauge invariance
  - Dirac algebra, Lagrangian, co- & contra variant
- Lecture 4: “Symmetries”
  - Standard Model, Higgs, Discrete Symmetries
  - Skills: Lagrangians, Chirality & Helicity
- Lecture 5: “Scattering”
  - Cross section, decay, perturbation theory
  - Skills: Dirac-delta funtion Feynman Calculus
- Lecture 6: “Detectors”
  - Energy loss mechanisms, detection technologies



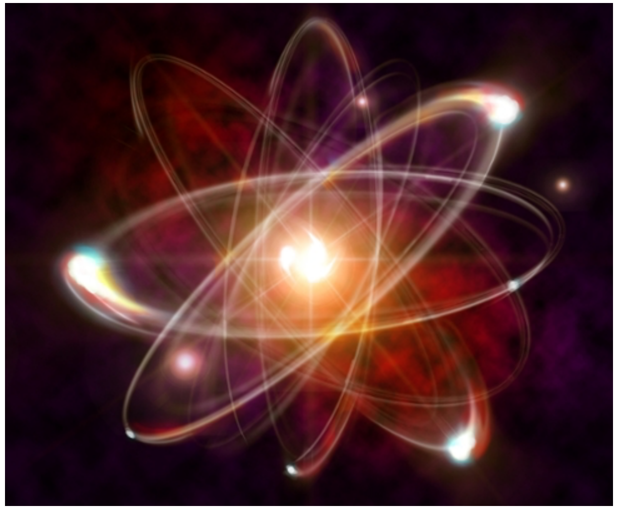


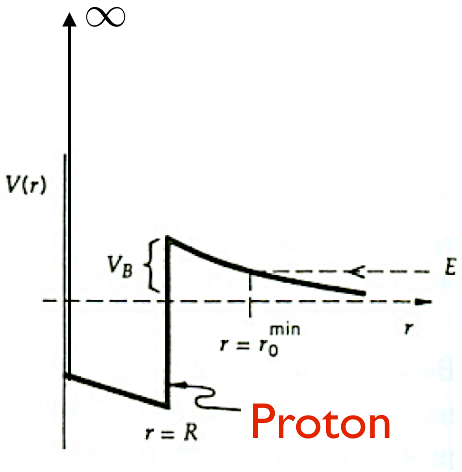


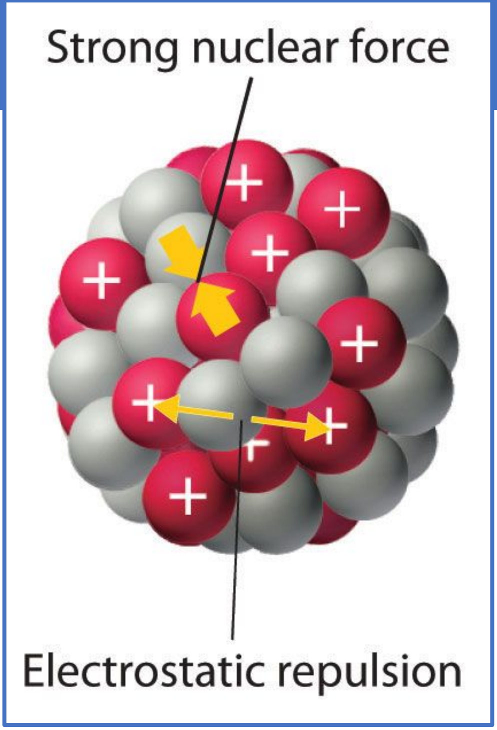


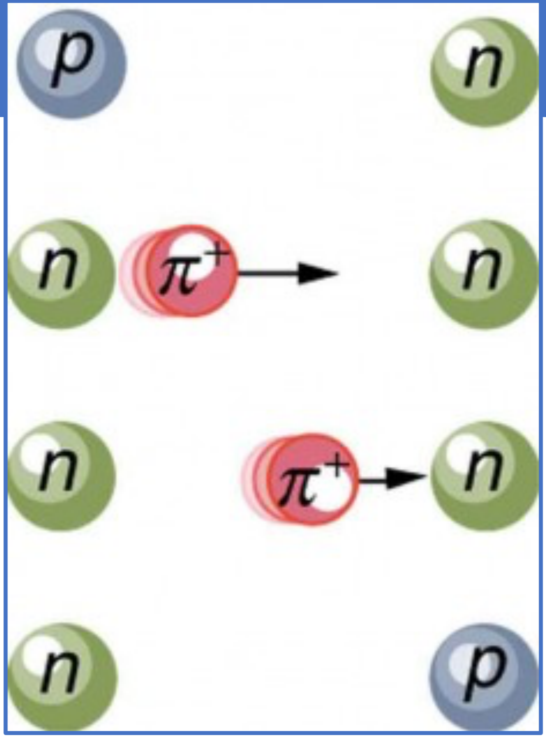


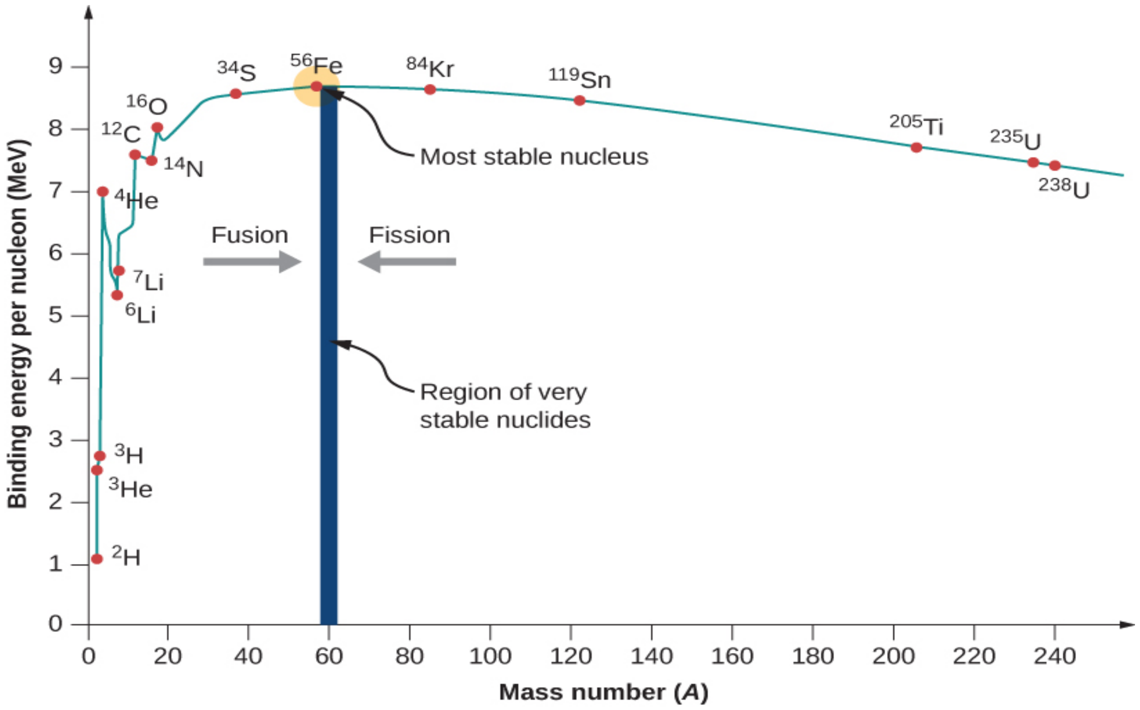
Lecture 1: “Particles” - Nuclear

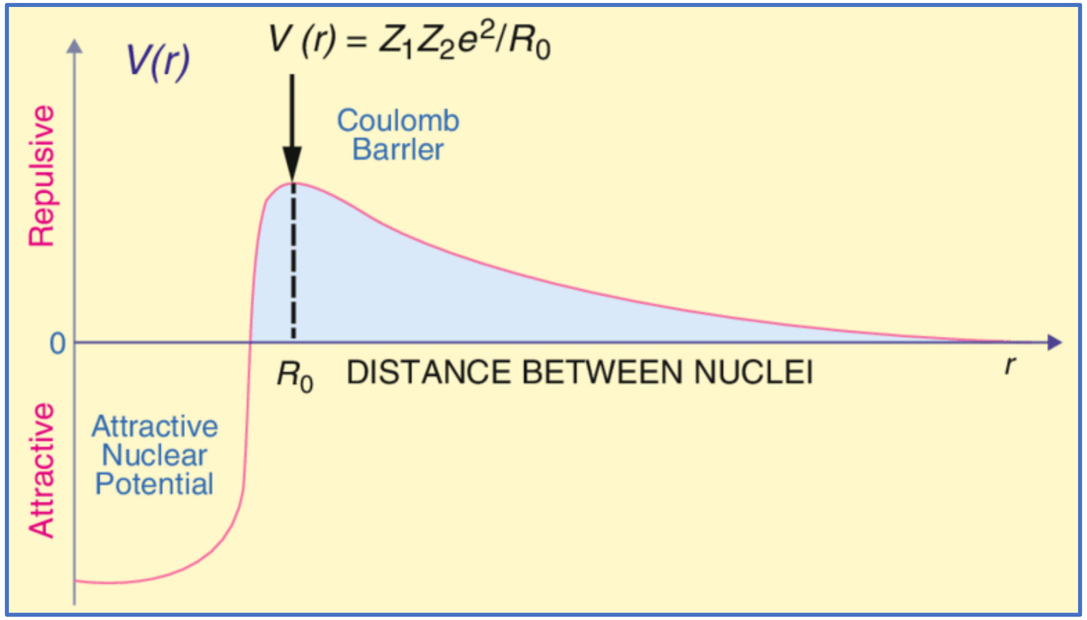




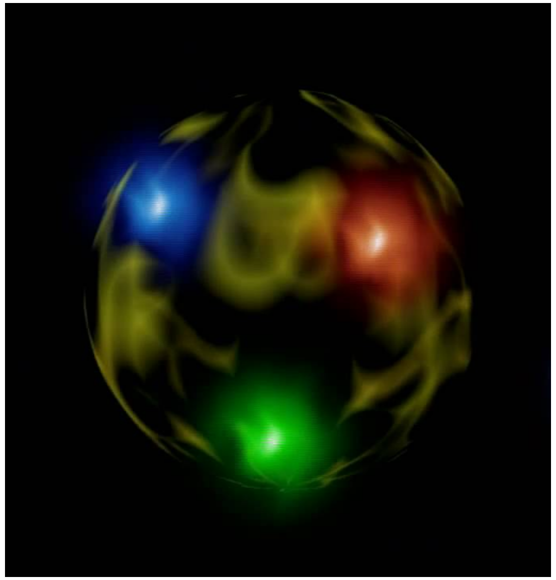


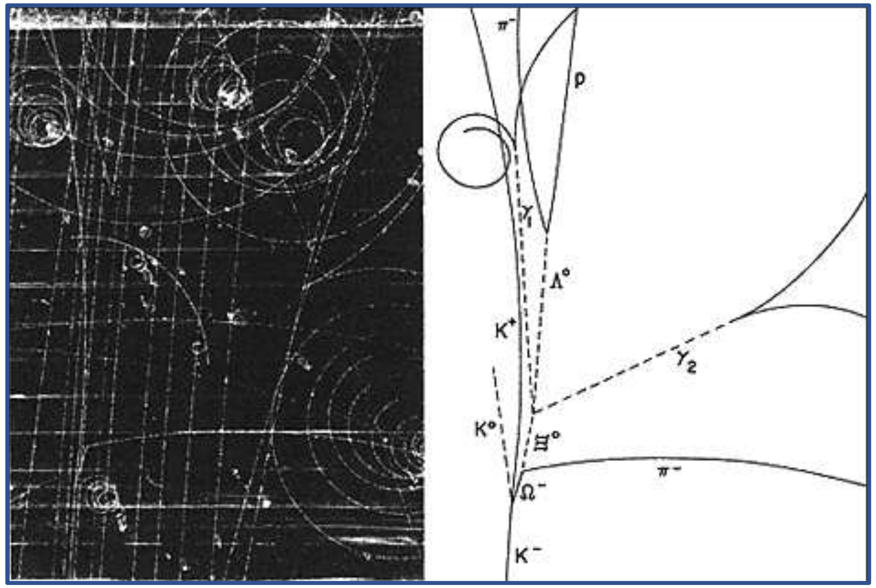


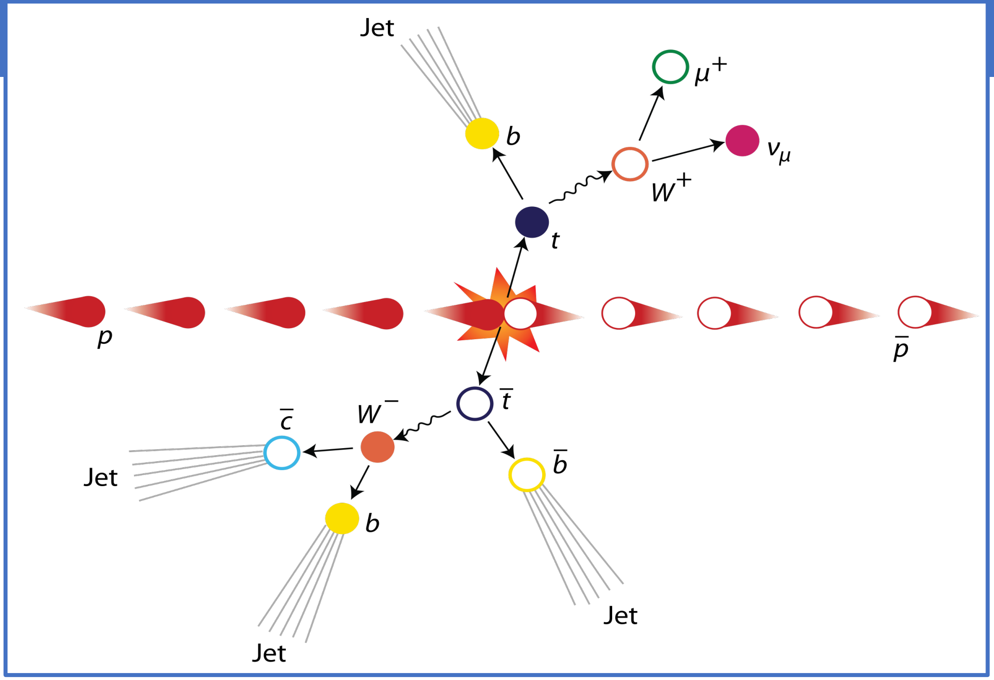


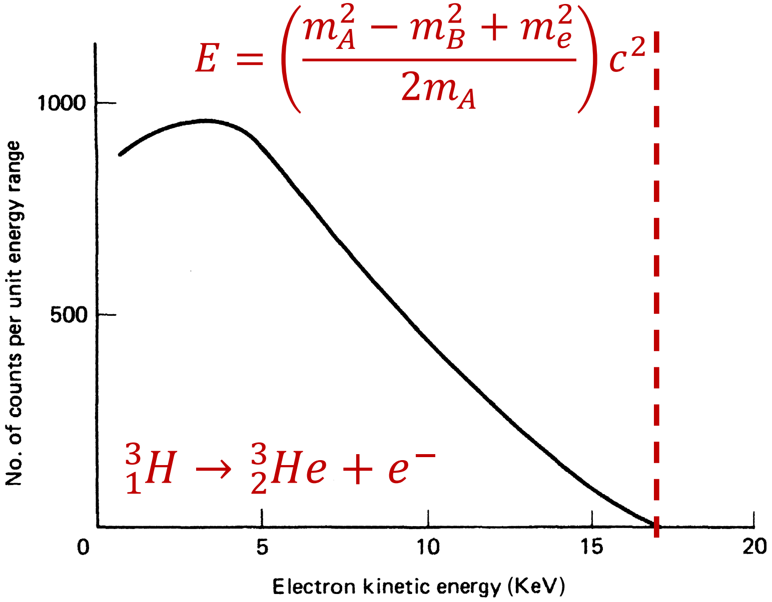


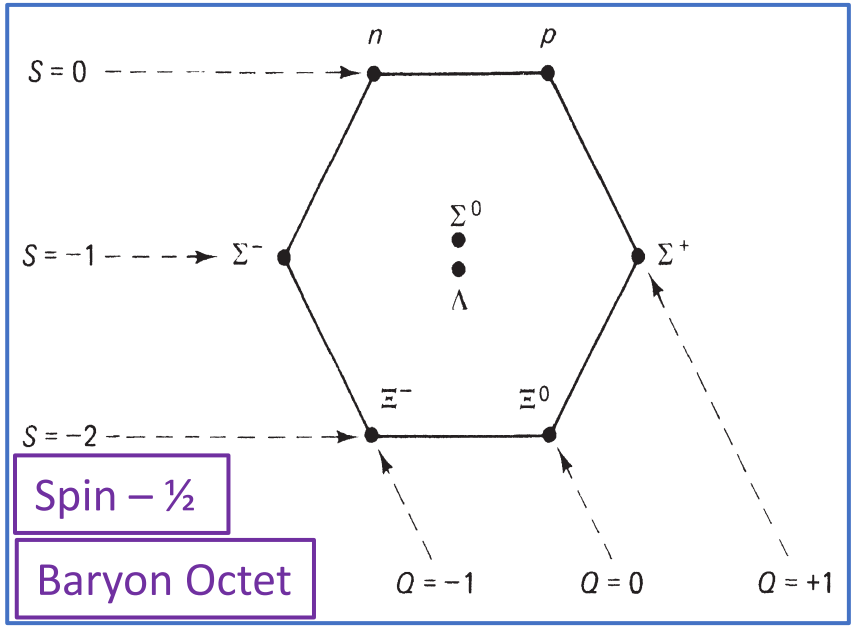
Lecture 1: “Particles” - subatomic

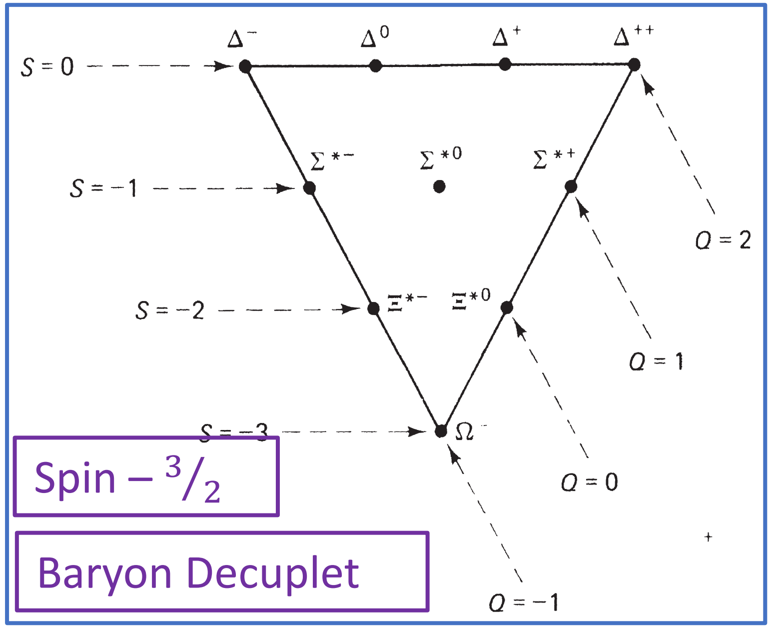




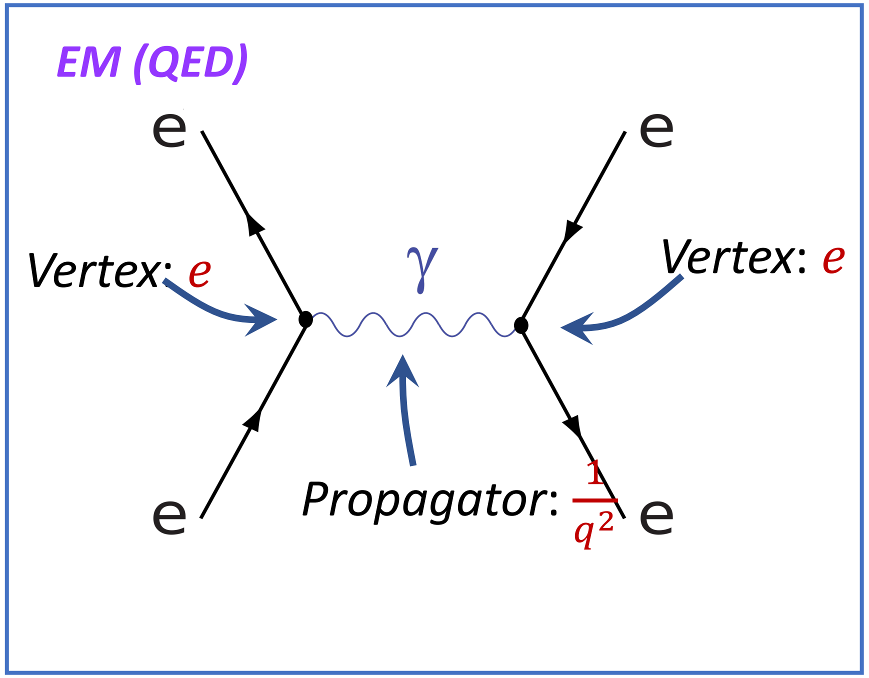


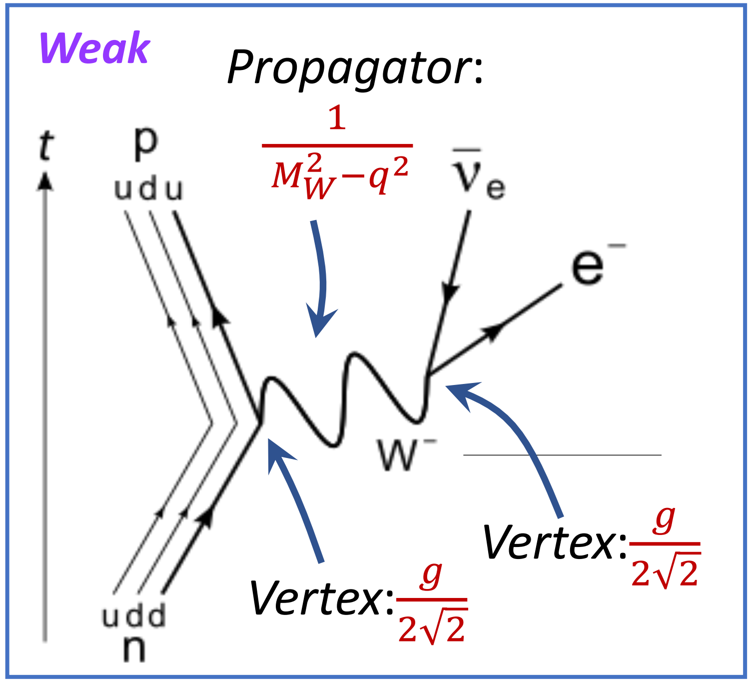


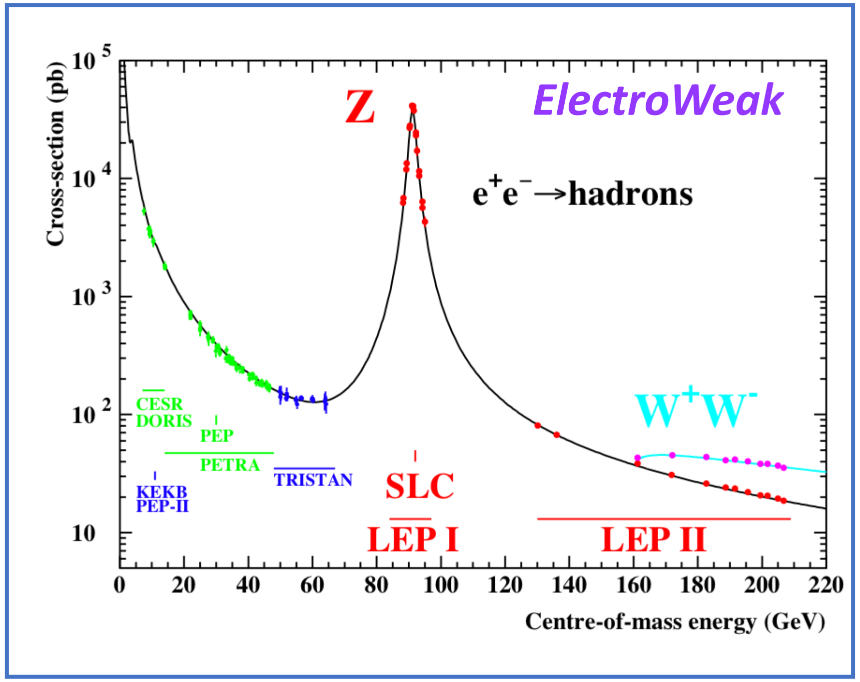


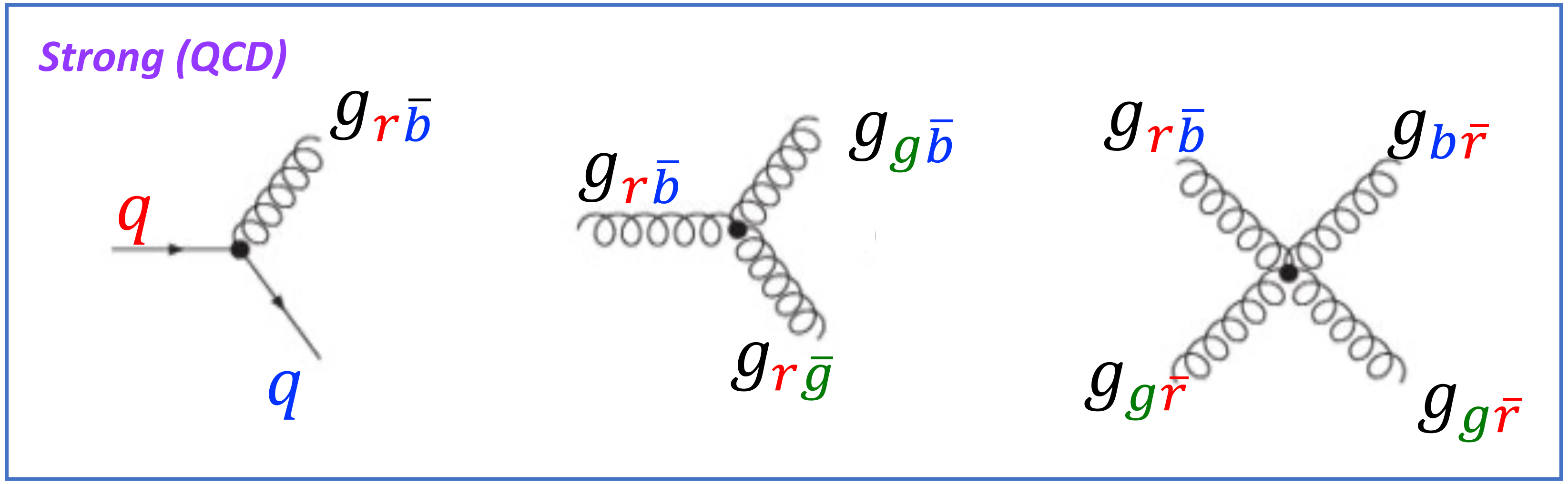


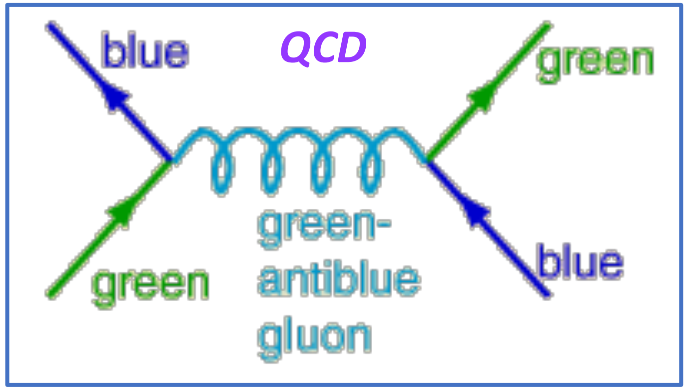
Lecture 2: "Forces"











Lecture 3: "Waves" – wave equations

Probability interpretation
(Continuity equation)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

Quantum Mechanics: $E \to \hat{E} = i\hbar \frac{\partial}{\partial t}$; $p \to \hat{p} = -i\hbar \vec{\nabla}$

Non-relativistic spin 0:

$$E = \frac{\vec{p}^2}{2m}$$

Schrödinger:

$$i\hbar \frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi \qquad \psi = Ne^{i(\vec{p}\vec{x}-Et)}$$

$$\rho \equiv \psi^*\psi = |N|^2$$

$$\vec{j} \equiv \frac{i\hbar}{2m}\left(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi\right) = \frac{|N|^2}{m}\vec{p}$$

Relativistic spin 0:

$$E^2 = p^2c^2 + m^2c^4$$

Klein-Gordon:

$$-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + \frac{m^2c^2}{\hbar^2}\phi$$

$$\partial_\mu\partial^\mu\phi + m^2\phi = 0 \qquad \phi = Ne^{i(\vec{p}\vec{x}-Et)}$$

$$\rho = 2|N|^2E$$

$$\vec{j} = 2|N|^2\vec{p}$$

$$j^\mu = 2|N|^2p^\mu$$

Relativistic spin- ½:

$$H = (\vec{\alpha} \cdot \vec{p} + \beta m)$$

Dirac:

$$i\frac{\partial}{\partial t}\psi = \left(-i\,\vec{\alpha} \cdot \vec{\nabla} + \beta m\right)\psi \qquad \psi = u(p)e^{i(\vec{p}\vec{x}-Et)}$$

$$\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0 \qquad u(p) = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$$j^0 = \bar{\psi}\gamma^0\psi = \psi^\dagger\psi = \sum_{i=1}^{4}|\psi_i|^2$$

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

Lecture 3: "Waves" – gauge invariance

Lagrangians

Spin 0 Scalar field: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - m^2\phi^2$

Spin ½ Dirac fermion $\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$

Spin 1 gauge boson (photon) : $\mathcal{L} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - j^\mu A_\mu$

Euler Lagrange lead to the wave equations:
$$\frac{\partial \mathcal{L}}{\partial \phi(x)} = \partial_\mu \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi(x)\right)}$$

Forces result from requiring a symmetry principle: Lagrangian should stay invariant

1) QED = U(1) symmetry

$$\psi(x) \rightarrow \psi'(x) = e^{iq\alpha(x)}\psi(x)$$
$$A^\mu(x) \rightarrow A'^\mu(x) = A^\mu(x) - \frac{1}{q}\partial^\mu\alpha(x)$$

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi \longrightarrow \mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu\psi - m\bar{\psi}\psi$$

Covariant derivative: $\partial^\mu \rightarrow D^\mu \equiv \partial^\mu + iqA^\mu$

$$\mathcal{L} = \underbrace{i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi}_{\text{"free"}} \underbrace{- q\bar{\psi}\gamma_\mu A^\mu\psi}_{\text{"interaction"}}$$

2) Weak = SU(2) symmetry

3) QCD = SU(3) symmetry

Lecture 4: "Symmetries" – Standard Model

- The Lagrangian of the Standard Model includes electromagnetic, weak and strong interactions according to the gauge field principle

- Construction of the Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{free}} - \mathcal{L}_{\text{interaction}} = \mathcal{L}_{\text{Dirac}} - gJ^{\mu}A_{\mu}$
  - With g a coupling constant, J^{μ} a current ($\bar{\psi}\mathrm{O}_i\psi$) and A_{μ} a force field

A. Local $U(1)$ gauge invariance: symmetry under complex phase rotations
   - Conserved quantum number: (hyper-) charge
   - Lagrangian: $\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi - q\,\underbrace{\bar{\psi}\gamma^{\mu}\psi}_{J^{\mu}_{EM}}\,A_{\mu}$

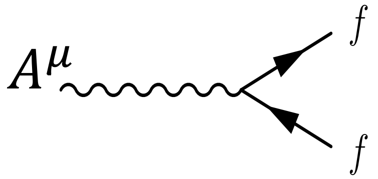
B. Local $SU(2)$ gauge invariance: symmetry under transformations in isospin doublet space.
   - Conserved quantum number: weak isospin
   - Lagrangian: $\mathcal{L} = \bar{\Psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\Psi = \bar{\Psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Psi - \frac{g}{2}\underbrace{\bar{\Psi}\gamma^{\mu}\vec{\tau}\Psi}_{J^{\mu}_{Weak}}\,\vec{b}_{\mu}$

C. Local $SU(3)$ gauge invariance: symmetry under transformations in colour triplet space
   - Conserved quantum number: color
   - Lagrangian: $\mathcal{L} = \bar{\Phi}\left(i\gamma^{\mu}D_{\mu} - m\right)\Phi = \bar{\Phi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Phi - \frac{g_s}{2}\underbrace{\bar{\Phi}\gamma^{\mu}\vec{\lambda}\Phi}_{\vec{J}^{\mu}_{QCD}}\,\vec{c}_{\mu}$

Lecture 4: "Symmetries" – Standard Model

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - qJ^{\mu}_{EM}A_{\mu} - \frac{g}{2}J^{\mu}_{\text{Weak}}\vec{b}_{\mu} - \frac{g_s}{2}J^{\mu}_{QCD}\,\vec{c}_{\mu}$$

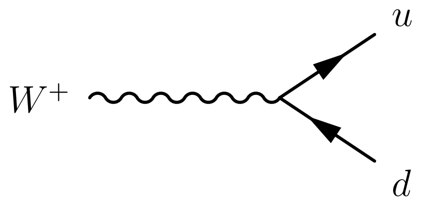
QED U(1) $\mathcal{L}_{\text{int}} = -J_{\mu}A^{\mu}$ with $J_{\mu} = q\bar{\psi}\gamma_{\mu}\psi$

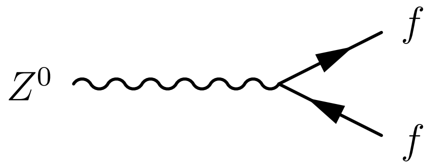


Weak SU(2) : $\mathcal{L}_{\text{int}} = -\vec{J}_{\mu}\vec{b}^{\mu}$ with $\vec{J}_{\mu} = \frac{g}{2}\,\bar{\Psi}\,\gamma_{\mu}\vec{\tau}\,\Psi$

$W^{\pm}_{\mu} \equiv \frac{1}{\sqrt{2}}(b^{1}_{\mu} \mp i b^{2}_{\mu})$ $\quad J^{\pm}_{\mu} = \frac{1}{\sqrt{2}}\bar{\Psi}\gamma_{\mu}\tau^{\pm}\Psi$ $\quad$ with $\tau^{\pm} = \frac{1}{2}(\tau_1 \pm i\tau_2)$

$Z_{\mu} = b^{3}_{\mu}$ $\quad J^{3}_{\mu} = \frac{1}{2}\bar{\Psi}\gamma_{\mu}\tau^{3}\Psi$ $\quad$ with $\tau^{\pm} = \frac{1}{2}(\tau_1 \pm i\tau_2)$





Electroweak SU(2)xU(1):

$$\gamma_{\mu} = A_{\mu}\cos\theta_W + b^{3}_{\mu}\sin\theta_W$$
$$Z_{\mu} = -A_{\mu}\sin\theta_W + b^{3}_{\mu}\cos\theta_W$$

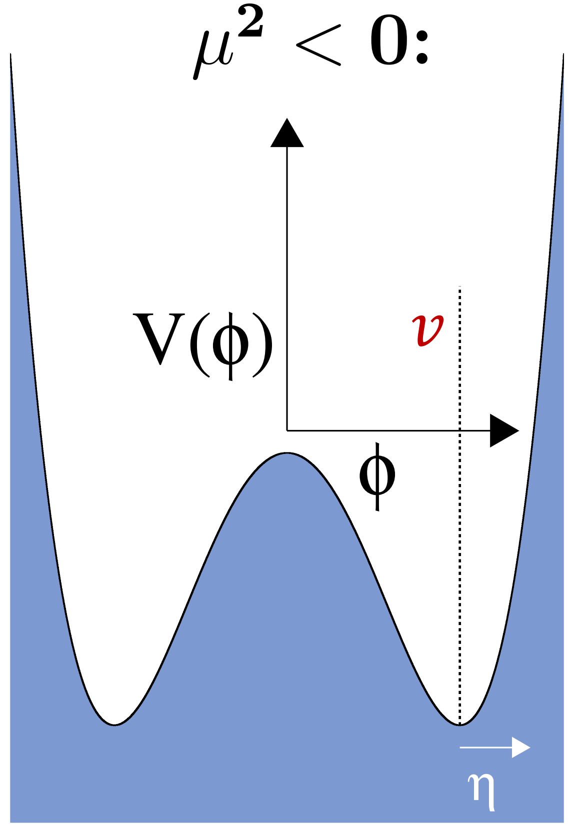
Standard Model: $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$

Lecture 4: "Symmetries" – Symmetry breaking

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

Massive Klein-Gordon term (Spin 0, mass $=\mu$)

Interaction term



The Lagrangian has a minimum for $\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} = v$ or $\mu^2 = -\lambda v^2$

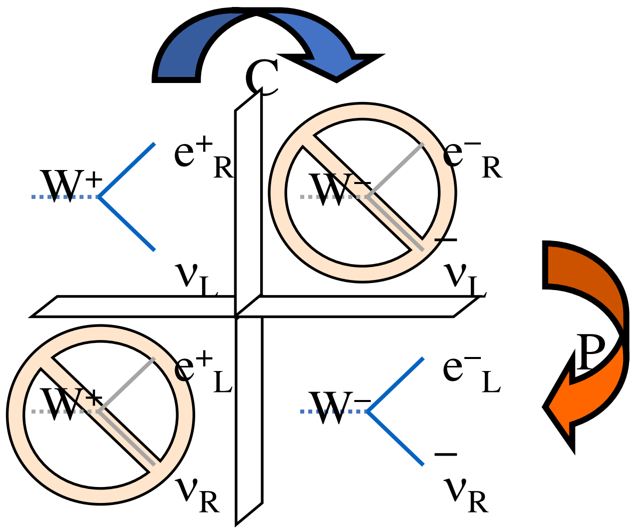
Conclusion:

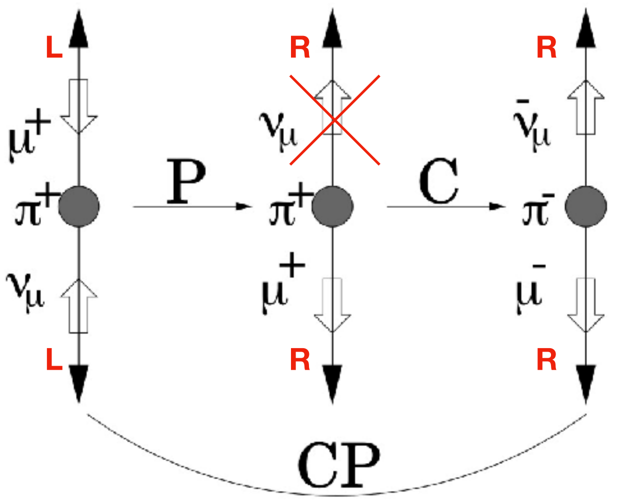
- The symmetry of the Lagrangian by adding a symmetric potential ϕ *has not been destroyed*
- The *vacuum is no longer* in a symmetric position

The real case includes a complex field ϕ

Lecture 4 : “Symmetries” – Violation

1) Weak interaction maximally violates parity “P” and also charge symmetry “C”





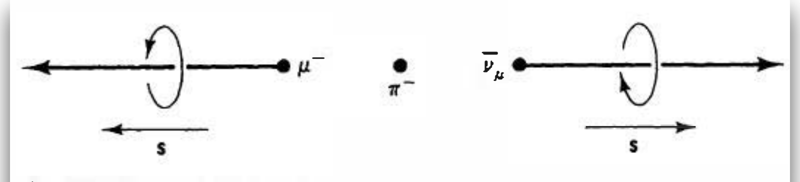
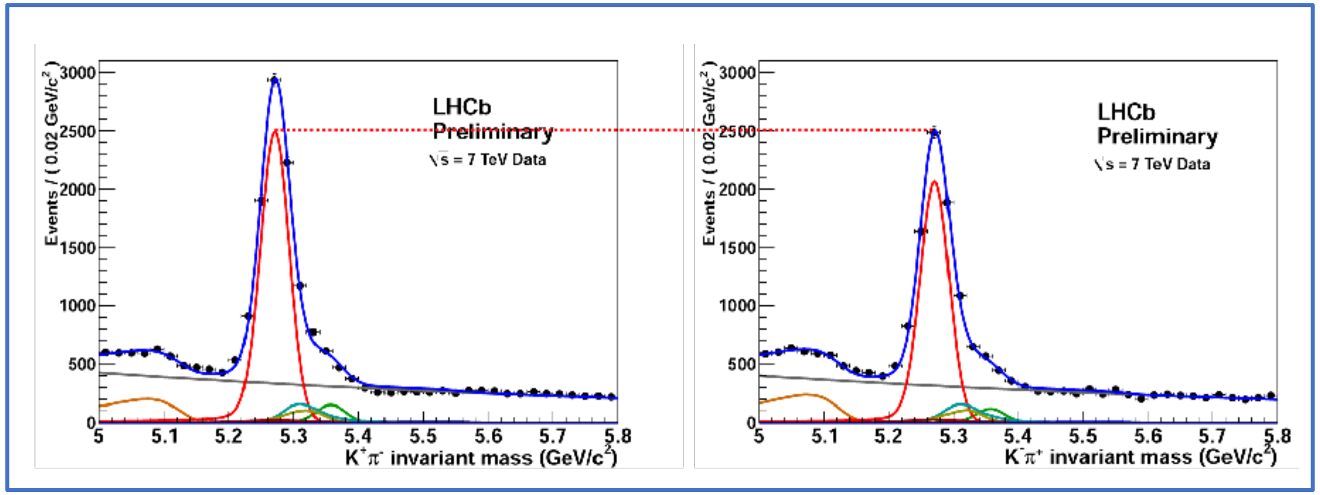


Fig. 4.10 Decay of π^- at rest.

2) Weak interaction subtly violates simultanous “CP”

- Requires Quantum Mechanical interference
- Requires existence of three particle generations (CKM)
- Not sufficient to explain absence antimatter in theuniverse



Lecture 5 : "Scattering" – non-Rel.

<u>Perturbation theory</u>

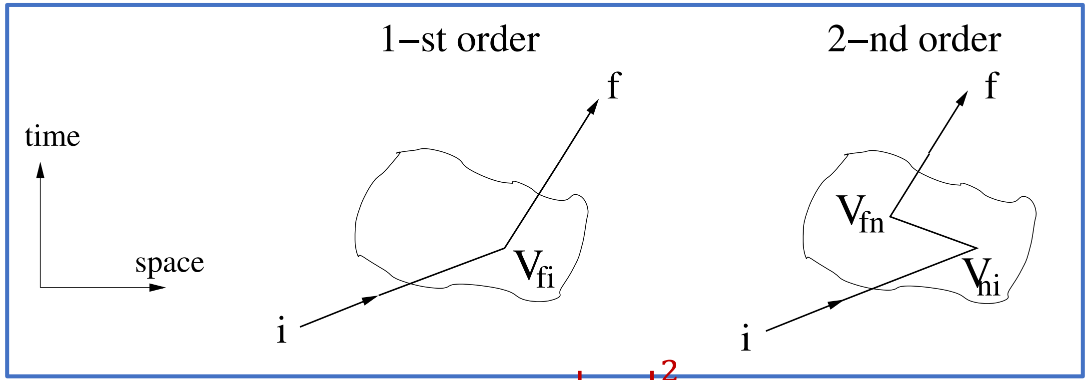
1) $V(x,t)$ is fixed

Solve wave equation Iteratively…

$$i\frac{\partial\psi}{\partial t} = \left(H_0 + V(\vec{x},t)\right)\psi$$

…use plane waves

$$\psi = \sum_{n=0}^{\infty} a_n(t)\phi_n(\vec{x})e^{-iE_n t}$$



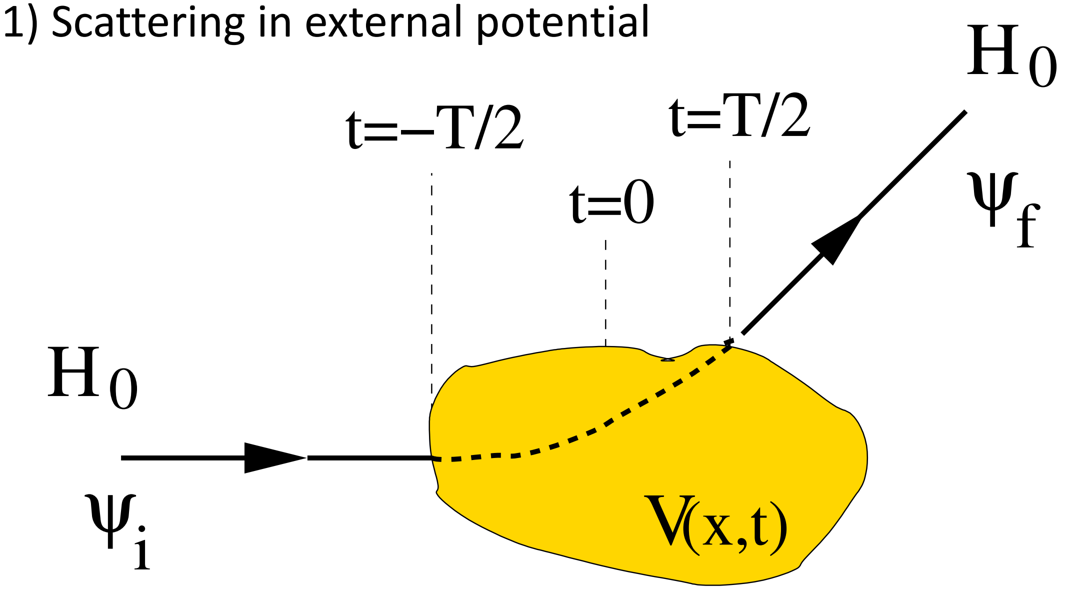
$$d\sigma = \frac{W_{fi}}{\text{flux}}d\Phi \qquad W_{fi} \equiv \lim_{T\to\infty}\frac{|T_{fi}|^2}{T}$$

$$T_{fi} = -i\int d^4x\, \psi_f^*(x)V(x)\psi_i(x) = -2\pi V_{fi}\delta(E_f - E_i)$$

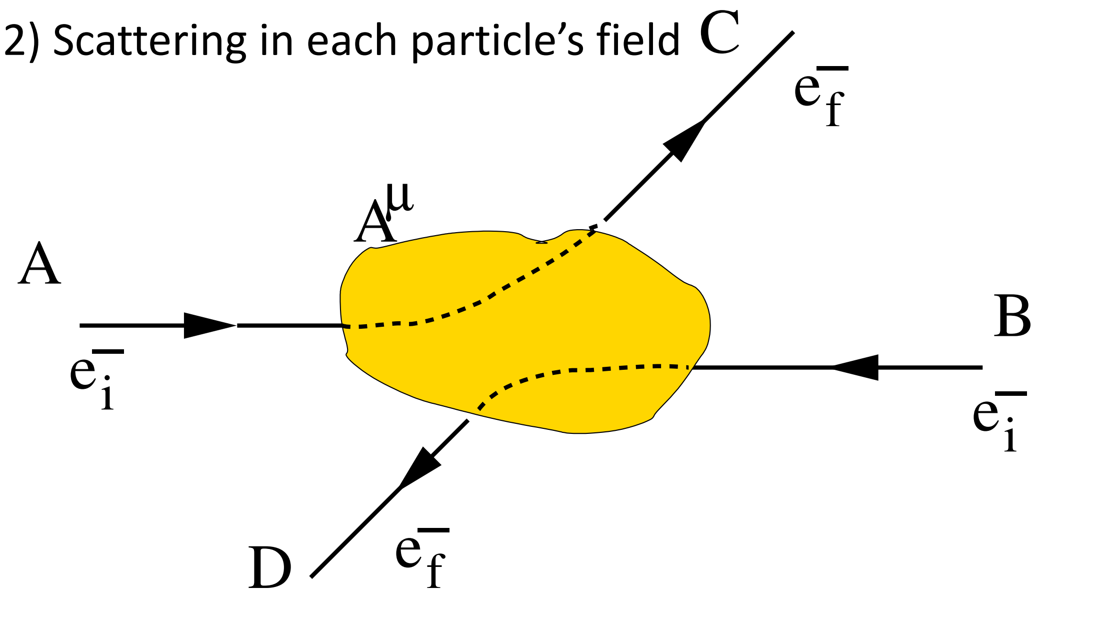
Relativistic: $V_{fi} \rightarrow \mathcal{M}$ "matrix element"

2) Determine V from A field scattering particles
(Solve Maxwell equation)

1) Scattering in external potential



2) Scattering in each particle's field

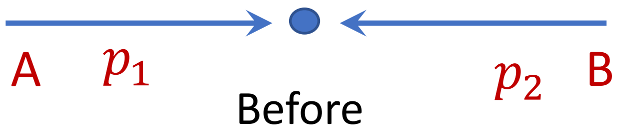


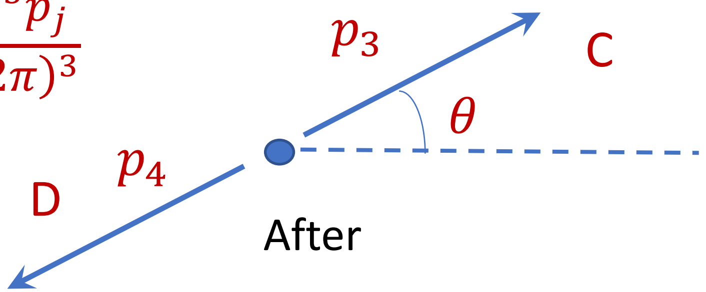
Lecture 5 : "Scattering" - Relativistic

Cross section:

$$\sigma = \frac{S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 \, (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \ldots - p_n) \times \prod_{j=3}^{n} \frac{1}{2E_j} \frac{d^3\vec{p}_j}{(2\pi)^3}$$

Eg: "2-to-2" scattering:





$$\sigma = \frac{S}{64\pi^2 (E_1 + E_2)|\vec{p}_1|} \int |\mathcal{M}|^2 \, (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \, \frac{d^3\vec{p}_3}{E_3} \frac{d^3\vec{p}_4}{E_4}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

How to determine $\mathcal{M}$? → Feynman rules (depend on actual theory/interaction):

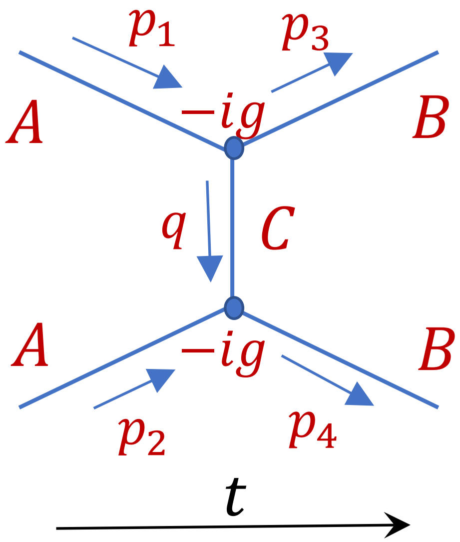
Feynman rules "ABC" theory:

1. Diagrams: see sketch
2. Labels: see sketch
3. One vertex: $-ig$
4. Propagators: no internal lines
5. Conservation of energy and momentum: $(2\pi)^4 \, \delta^4(p_1 - p_2 - p_3)$
6. Integrate: no internal momenta
7. Discard delta-function and multiply by i.

Example diagram:

$A + A \rightarrow B + B$

$$\mathcal{M} = \frac{g^2}{(p_4 - p_2)^2 - m_C^2}$$



Standard Model interactions require real Lagrangians and dealing with spinor objects → Master level education