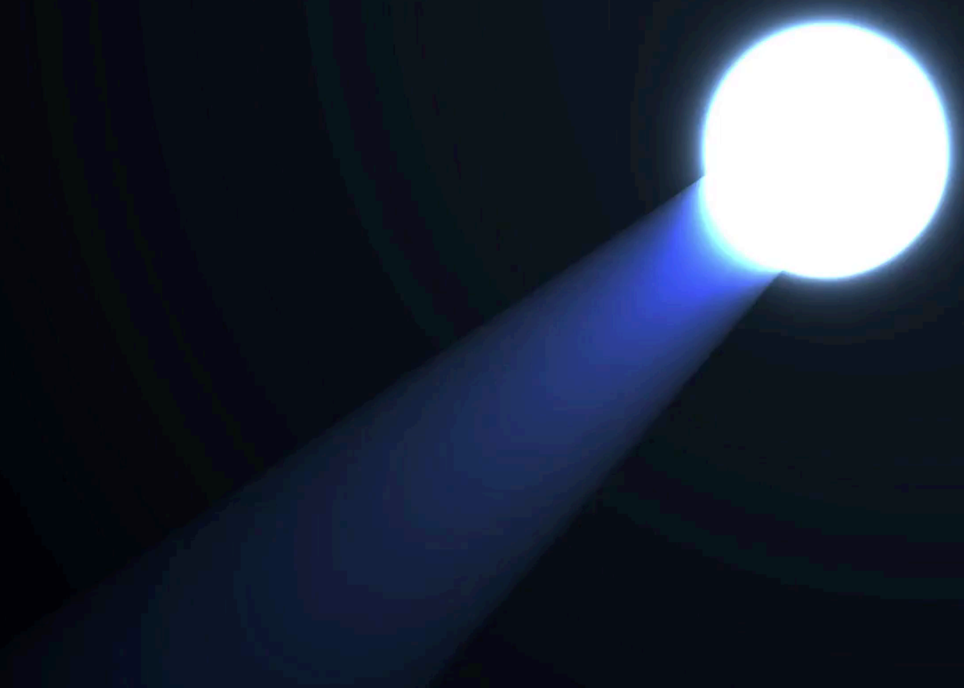
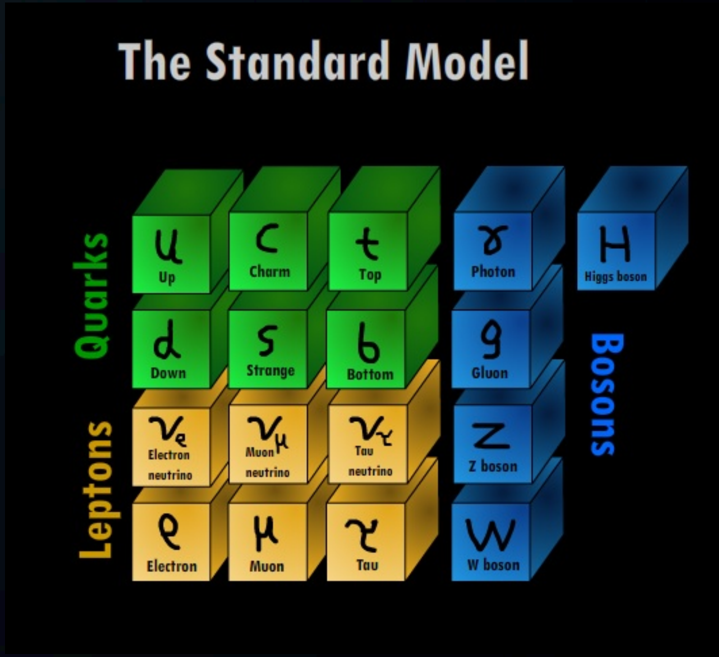
PHY3004: Nuclear and Particle Physics

Marcel Merk, Jacco de Vries, …

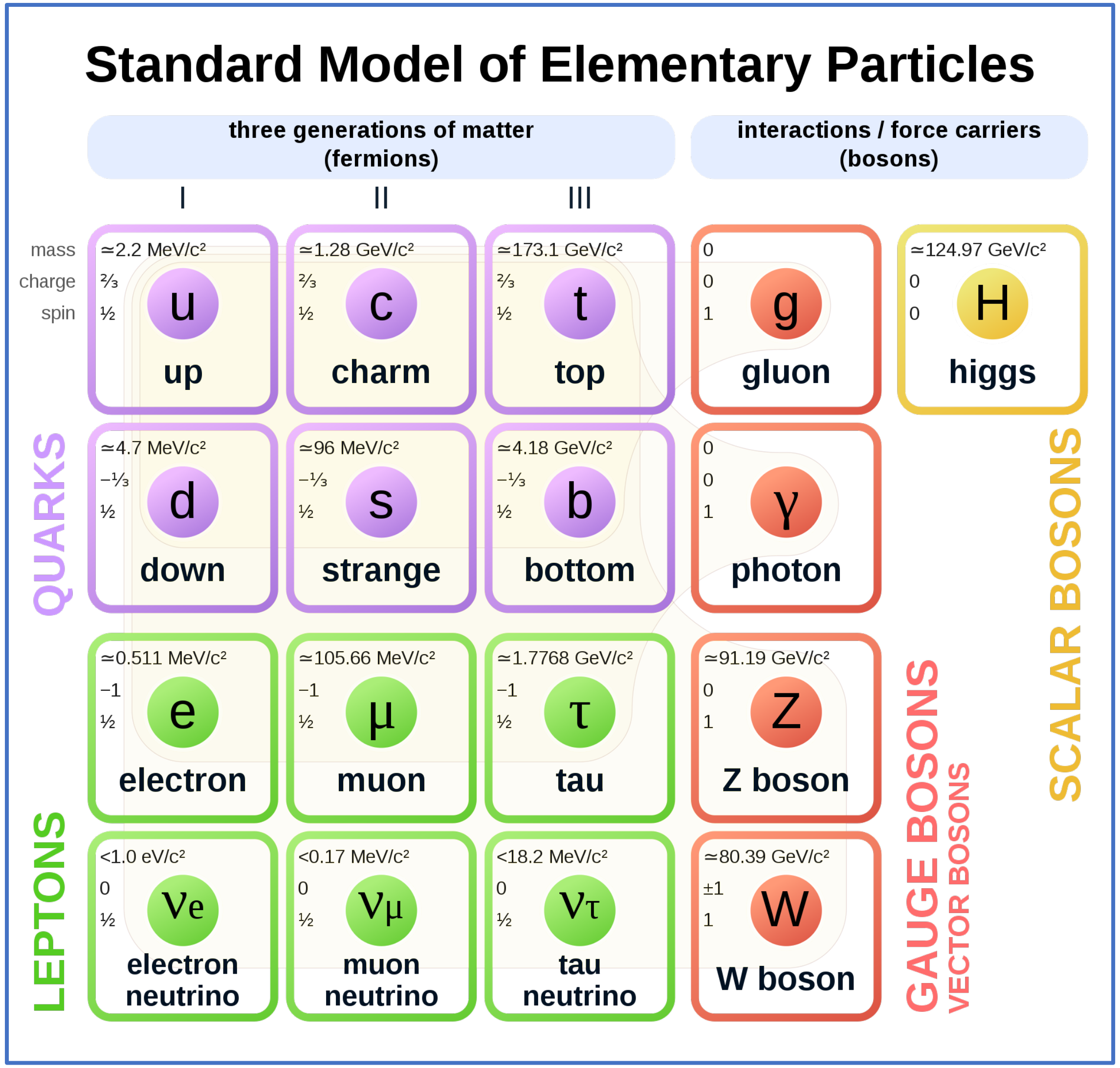




Elementary particles

Classification of particles

- Lepton: fundamental particle
- Hadron: consist of quarks
- ***Fermion***: particle with half-integer spin.
  - All fundamental quarks and leptons are spin-½
- ***Boson***: particle with integer spin
  - Higgs (S=0)
  - Force carriers: γ, W, Z, g (S=1); graviton(S=2)
- Wave equations:
  - Spin-0: Klein-Gordon
  - Spin-½: Dirac
  - Spin-1: Maxwell
- Gauge Invariance:
  - EM: U(1) , Weak SU(2), Strong SU(3)



Lecture 4: “Symmetries”

- Gauge Symmetries: Standard Model
- Symmetry Breaking: Higgs Mechanism
- Discrete Symmetries

Griffiths 9.7, PP1 Lect 9

Griffiths 10.7-9, PP1 Lect 11

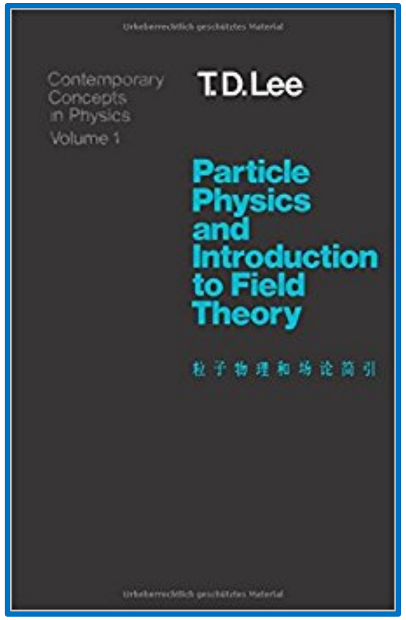
Griffiths chapter 4

Symmetry and non-observables

T.D.Lee: "The root to all *symmetry* principles lies in the assumption that it is impossible to observe certain basic quantities; the *non-observables*"

There are four main types of symmetry:

- Permutation symmetry:
  Bose-Einstein and Fermi-Dirac Statistics
- Continuous space-time symmetries:
  translation, rotation, velocity, acceleration,…
- Discrete symmetries:
  space inversion, time reversal, charge conjugation,…
- Unitary symmetries: gauge invariances:
  U_1(charge), SU_2(isospin), SU_3(color),…



$\Rightarrow$ If a quantity is fundamentally non-observable it is related to an *exact* symmetry

$\Rightarrow$ If it could in principle be observed by an improved measurement; the symmetry is said to be *broken*

Noether Theorem: symmetry ⇔ conservation law

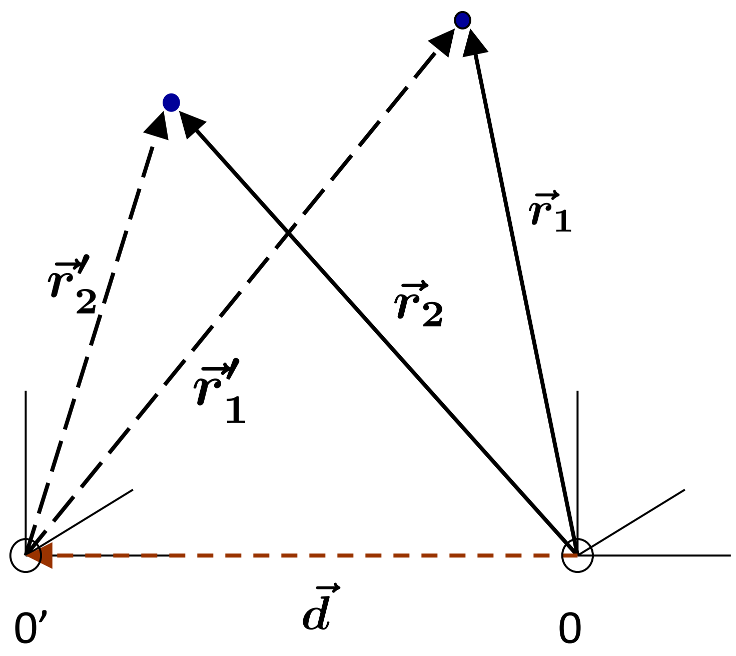
Symmetry and non-observables

Non-observables	Symmetry Transformations	Conservation Laws or Selection Rule
Difference between identical particles	Permutation	B.-E. or F.-D. statistics
Absolute spatial position	Space translation: $\vec{r} \to \vec{r} + \vec{\Delta}$	momentum
Absolute time	Time translation: $t \to t + \tau$	energy
Absolute spatial direction	Rotation: $\vec{r} \to \vec{r}'$	angular momentum
Absolute velocity	Lorentz transformation	generators of the Lorentz group
Absolute right (or left)	$\vec{r} \to -\vec{r}$	parity
Absolute sign of electric charge	$e \to -e$	charge conjugation
Relative phase between states of different charge Q	$\psi \to e^{i\theta Q}\psi$	charge
Relative phase between states of different baryon number B	$\psi \to e^{i\theta N}\psi$	baryon number
Relative phase between states of different lepton number L	$\psi \to e^{i\theta L}\psi$	lepton number
Difference between different coherent mixture of p and n states	$\binom{p}{n} \to U\binom{p}{n}$	isospin

Symmetry and non-observables: example

- Simple example: potential energy V between two charged particles:

Absolute position is a non-observable:
The interaction is independent on the choice of the origin 0.



Symmetry:
V is invariant under arbitrary space translations:

$$\vec{r_1} \to \vec{r}_1 + \vec{d} \qquad \vec{r_2} \to \vec{r}_2 + \vec{d}$$

Consequently:

$$V = V\left(\vec{r}_1 - \vec{r}_2\right)$$

Total momentum is conserved:

$$\frac{d}{dt}\underbrace{\left(\vec{p}_1 + \vec{p}_2\right)}_{\vec{p}_{\mathrm{tot}}} = \vec{F}_1 + \vec{F}_2 = -\left(\vec{\nabla}_1 + \vec{\nabla}_2\right)V = 0$$

Lecture 4: “Symmetries”

Part 1

Gauge Symmetries in

The Standard Model

Standard Model

- The Lagrangian of the Standard Model includes electromagnetic, weak and strong interactions according to the gauge field principle

- Construction of the Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{free}} - \mathcal{L}_{\text{interaction}} = \mathcal{L}_{\text{Dirac}} - gJ^{\mu}A_{\mu}$
  - With g a coupling constant, J^{μ} a current ($\bar{\psi}\mathrm{O}_i\psi$) and A_{μ} a force field

A. Local $U(1)$ gauge invariance: symmetry under complex phase rotations
   - Conserved quantum number: (hyper-) charge
   - Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - q\underbrace{\bar{\psi}\gamma^{\mu}\psi}_{J^{\mu}_{EM}}A_{\mu}$

B. Local $SU(2)$ gauge invariance: symmetry under transformations in isospin doublet space.
   - Conserved quantum number: weak isospin
   - Lagrangian: $\mathcal{L} = \bar{\Psi}(i\gamma^{\mu}D_{\mu} - m)\Psi = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - \frac{g}{2}\underbrace{\bar{\Psi}\gamma^{\mu}\vec{\tau}\Psi}_{J^{\mu}_{Weak}}\vec{b}_{\mu}$

C. Local $SU(3)$ gauge invariance: symmetry under transformations in colour triplet space
   - Conserved quantum number: color
   - Lagrangian: $\mathcal{L} = \bar{\Phi}(i\gamma^{\mu}D_{\mu} - m)\Phi = \bar{\Phi}(i\gamma^{\mu}\partial_{\mu} - m)\Phi - \frac{g_s}{2}\underbrace{\bar{\Phi}\gamma^{\mu}\vec{\lambda}\Phi}_{\vec{J}^{\mu}_{QCD}}\vec{c}_{\mu}$

Standard Model

- The Lagrangian of the Standard Model includes electromagnetic, weak and strong interactions according to the gauge field principle
- Construction of the Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{free}} - \mathcal{L}_{\text{interaction}} = \mathcal{L}_{\text{Dirac}} - gJ^{\mu}A_{\mu}$
  - With g a coupling constant, J^{μ} a current ($\bar{\psi}\mathrm{O}_i\psi$) and A_{μ} a force field

Standard Model Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - qJ^{\mu}_{EM}A_{\mu} - \frac{g}{2}J^{\mu}_{\text{Weak}}\vec{b}_{\mu} - \frac{g_s}{2}J^{\mu}_{QCD}\,\vec{c}_{\mu}$$

Implements U(1), SU(2) and SU(3) symmetries simultaneous

Requiring the Lagrangian to be invariant (symmetry) implies that the EM, Weak and Strong force fields must exist and the interactions respectively conserve charge weak isospin and color.

Electromagnetism and Weak force

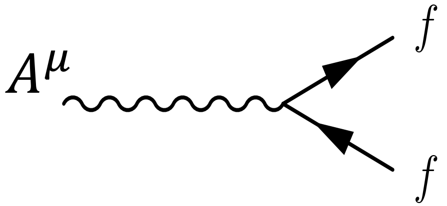
- U(1) gauge transformations require that the laws of physics (i.e. the Lagrangian) is invariant under:

$$\psi(x) \rightarrow \psi'(x) = e^{iq\alpha(x)}\psi(x)$$

$$A^{\mu}(x) \rightarrow A'^{\mu}(x) = A^{\mu}(x) - \frac{1}{q}\partial^{\mu}\alpha(x)$$

Electromagnetic field gauge transformation

- This leads to the interaction: $\mathcal{L}_{\text{int}} = -J_{\mu}A^{\mu}$ with $J_{\mu} = q\bar{\psi}\gamma_{\mu}\psi$



- SU(2) gauge transformations require that the laws of physics (i.e. the Lagrangian) is invariant under:

$$\Psi(x) \rightarrow \Psi'(x) = e^{ig\frac{1}{2}\vec{\tau}\cdot\vec{\alpha}(x)}\Psi(x)$$

With doublets $\Psi = \begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix}$ and $\overline{\Psi} = (\psi_u, \psi_d)$

- This leads to the interaction: $\mathcal{L}_{\text{int}} = -\vec{J}_{\mu}\vec{b}^{\mu}$ with $\vec{J}_{\mu} = \frac{g}{2}\,\overline{\Psi}\,\gamma_{\mu}\vec{\tau}\,\Psi$

Weak Isospin: $T_i = \frac{1}{2}\tau_i$

$\vec{\tau} = \tau_1, \tau_2, \tau_3$ are the Pauli matrices: $\tau_1 = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}, \tau_2 = \begin{pmatrix}0 & -i \\ i & 0\end{pmatrix}, \tau_3 = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}$

The weak force

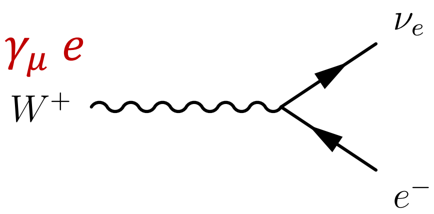
- The weak interaction includes charged (J_μ^1 and J_μ^2) and neutral (J_μ^3) currents

- It turns out the following charge current fields are realized in Nature:
  - $W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(b_\mu^1 \mp i b_\mu^2\right)$ and $Z_\mu = b_\mu^3$ (see exercise)

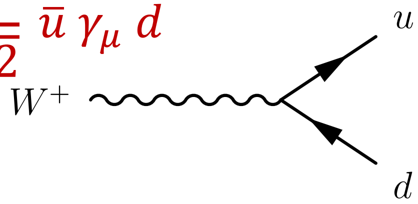
- The charged current becomes
  - $J_\mu^\pm = \frac{1}{\sqrt{2}}\overline{\Psi}\gamma_\mu\tau^\pm\Psi$ with $\tau^\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$

$$\tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \tau^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

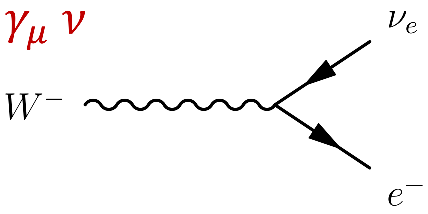
Charge raising interaction: $J_\mu^+ = \frac{1}{2\sqrt{2}}\,\bar{\nu}\,\gamma_\mu\,e$



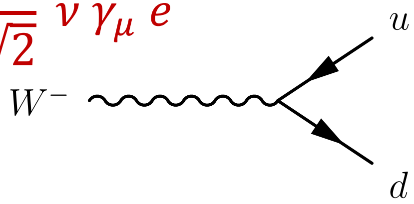
$J_\mu^+ = \frac{1}{2\sqrt{2}}\,\bar{u}\,\gamma_\mu\,d$



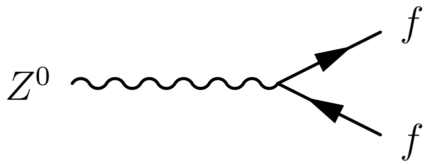
Charge lowering interaction: $J_\mu^- = \frac{1}{2\sqrt{2}}\,\bar{e}\,\gamma_\mu\,\nu$



$J_\mu^- = \frac{1}{2\sqrt{2}}\,\overline{\nu}\,\gamma_\mu\,e$



- The neutral current is:
  - $J_\mu^3 = \frac{1}{2}\overline{\Psi}\gamma_\mu\tau^3\Psi$ with $\tau^\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$



Exercise – 20 : Charge Current

- Show that the definition $W_\mu^\pm = \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}$ leads to the charged current:

  $\mathcal{L} = -W_\mu^+ J^{\mu^+} - W_\mu^- J^{\mu^-}$ with $J^{\mu^+} = \frac{g}{\sqrt{2}} \overline{\Psi} \gamma_\mu \tau^+ \Psi$ and $J^{\mu^-} = \frac{g}{\sqrt{2}} \overline{\Psi} \gamma_\mu \tau^- \Psi$

Electroweak unification

- A strange phenomenon for the neutral current
  - The $SU(2)$ gauge field b_μ^3 and and the $U(1)$ gauge field A_μ are not physical
  - The physical fields are: $\gamma_\mu = A_\mu \cos\theta_W + b_\mu^3 \sin\theta_W$ (“*mixing*”)
    $Z_\mu = -\mathrm{A}_\mu \sin\theta_W + b_\mu^3 \cos\theta_W$
  - The electromagnetic and weak interaction are linear combinations of the $U(1)$ and $SU(2)$ symmetries
    - We speak of a *unified electroweak force*

- The $U(1)$ symmetry is related to the quantity ”hypercharge” Y
  - The charge of a particle is given by the relation: $Q = T_3 + \frac{1}{2}Y$

- The Standard Model of interactions implements the symmetry:

$$SU(3)_{\mathrm{color}} \times SU(2)_L \times U(1)_Y$$

  - Mystery 1: How do gauge bosons and fermions acquire a mass
  - Mystery 2: The weak interaction is only *left-handed*

Lecture 4: ”Symmetries”

Part 2

Electroweak Symmetry Breaking

The Higgs Mechanism

Symmetry breaking

- Massive particles are forbidden in the SM Lagrangian
  - A hypothetical mass term in the Lagrangian for the photon is not gauge invariant:

$$m^2 A_\mu A^\mu \rightarrow m^2 \left(A_\mu + \frac{1}{e}\partial_\mu \alpha\right)\left(A^\mu + \frac{1}{e}\partial^\mu \alpha\right) \neq m^2 A_\mu A^\mu$$

- The same holds (harder to show) for the weak mediators W, Z
  - However they are massive
  - → SU(2) symmetry is *broken*

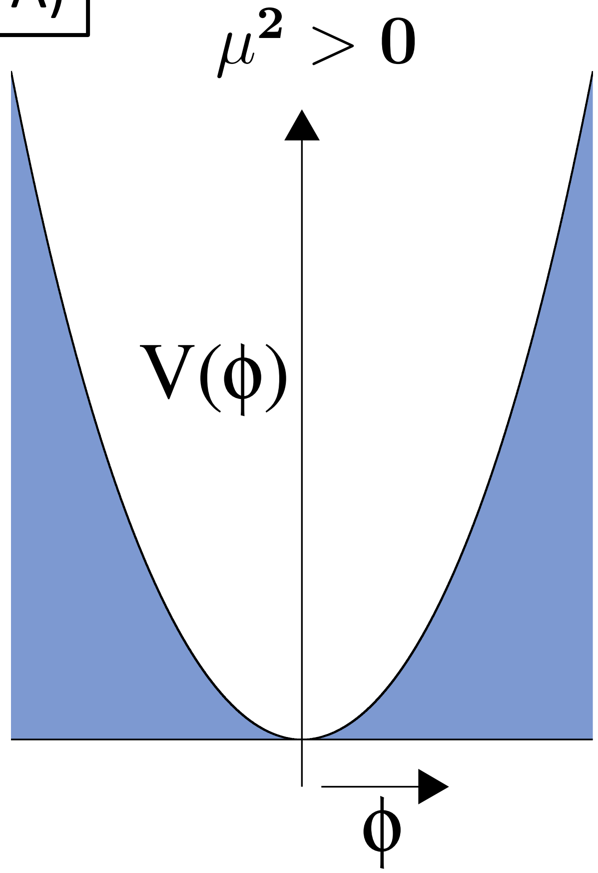
- We will give an example how mass terms can be generated without destroying the symmetry of the Lagrangian

The idea of symmetry breaking with a new field ϕ

- Add a new field ϕ to the Lagrangian
  - Chose a scalar field ($S = 0$)
  - Include a potential $V(\phi)$:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2}_{\text{Massive Klein-Gordon term (Spin 0, mass }=\mu)} \underbrace{- \frac{1}{4}\lambda\phi^4}_{\text{Interaction term}}$$

Case A)



- The vacuum (lowest energy state) has $\phi = 0$
  - This means no-field in the vacuum.
- The Lagrangian describes a new particle with $S = 0$ and $m = \mu$

The idea of symmetry breaking with a new field ϕ

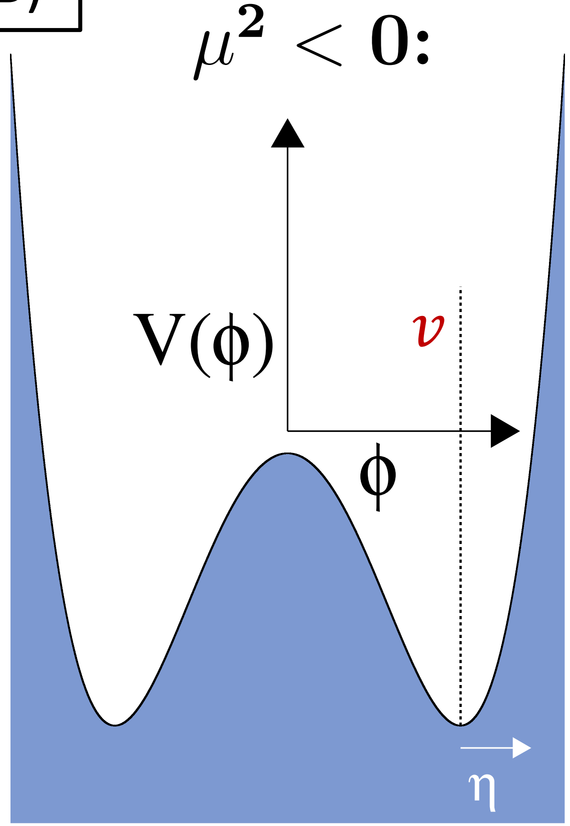
- Add a new field ϕ to the Lagrangian
  - Chose a scalar field ($S = 0$)
  - Include a potential $V(\phi)$:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

Massive Klein-Gordon term (Spin 0, mass $=\mu$)

Interaction term

Case B)

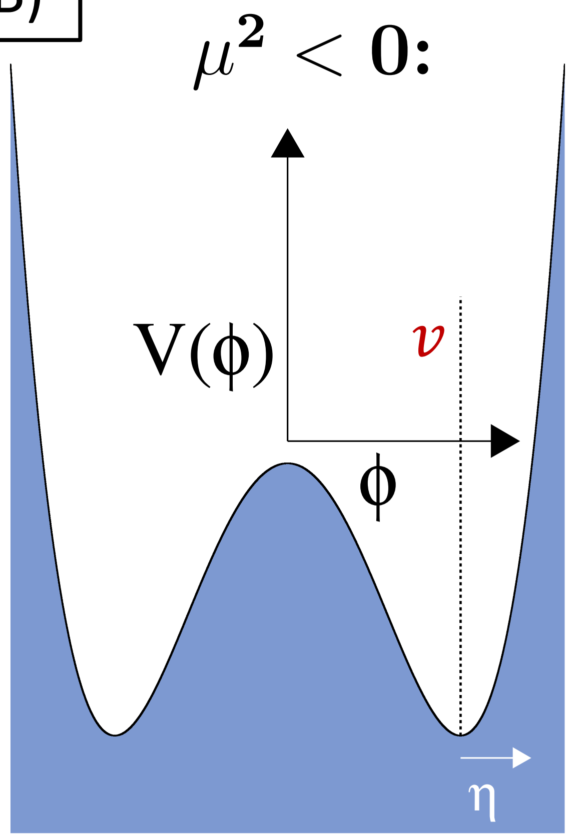


- The particle has imaginary mass? $\mu^2 < 0$
- The Lagrangian has a minimum for $\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} = v$ or $\mu^2 = -\lambda v$
  - The lowest energy (vacuum) includes a field with value v

Exercise – 21 : Symmetry breaking

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

Case B)



- Redefine coordinates: $\eta \equiv \phi - v$
- Exercise: re-write the Lagrangian in ϕ and v to show:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4$$

  - Ignore the constant term $\frac{1}{4}\lambda v^4$ and neglect higher order η^3:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2$$

  - This describes a new scalar field η with a mass $\frac{1}{2}m_\eta^2 = \lambda v^2 \;\Rightarrow\; m_\eta = \sqrt{2\lambda v^2}\;\; (= \sqrt{-2\mu^2})$
  - Price to pay: Lagrangian is no longer symmetric under $\eta \to -\eta$ in the new field.

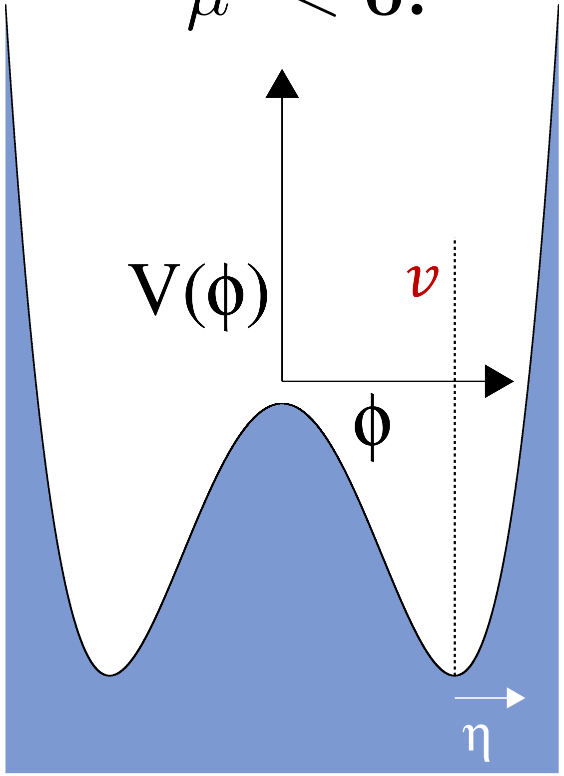
The idea of symmetry breaking with a new field ϕ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

- Redefine coordinates: $\eta \equiv \phi - v$

Case B)

$\mu^2 < 0$:



<u>Conclusion:</u>

- The symmetry of the Lagrangian by adding a symmetric potential ϕ *has not been destroyed*
- The *vacuum is no longer* in a symmetric position

The real case includes a complex field ϕ

- This describes a new scalar field η with a mass $\frac{1}{2}m_\eta^2 = \lambda v^2 \;\Rightarrow\; m_\eta = \sqrt{2\lambda v^2}\;\; (= \sqrt{-2\mu^2})$
- Price to pay: Lagrangian is no longer symmetric under $\eta \rightarrow -\eta$ in the new field.

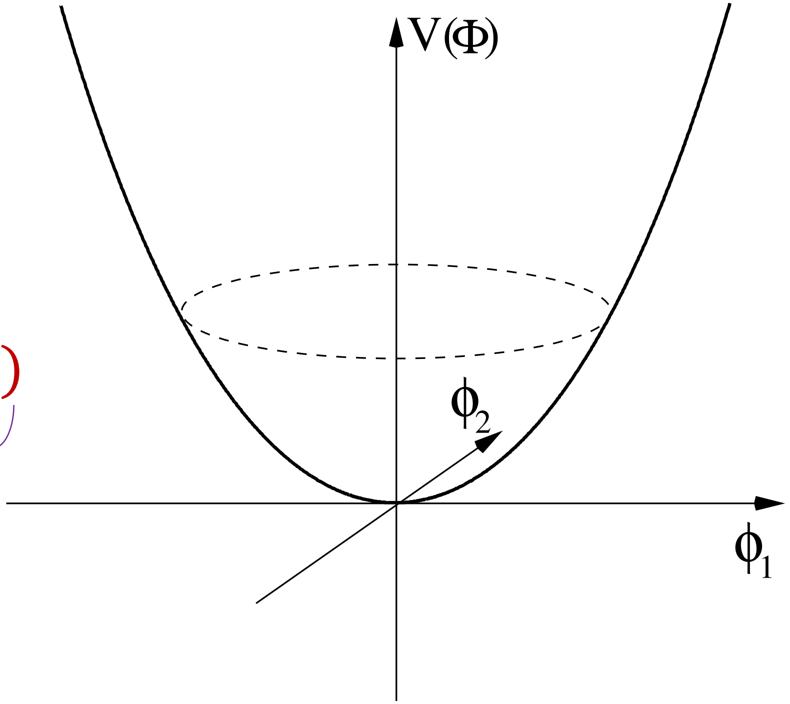
Symmetry breaking with a *complex* field ϕ

- Introduce a complex scalar field: $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$
- The Lagrangian term is: $\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - V(\phi)$, with $V(\phi) = \mu^2(\phi^*\phi) + \lambda\,(\phi^*\phi)^2$
- Lagrangian:

$$\mathcal{L}(\phi_1, \phi_2) = \frac{1}{2}\,(\partial_\mu \phi_1)^2 + \frac{1}{2}\,(\partial_\mu \phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

Case A)

$\boldsymbol{\mu^2 > 0}$



- Lagrangian:

$$\mathcal{L}(\phi_1, \phi_2) = \underbrace{\frac{1}{2}\,(\partial_\mu \phi_1)^2 - \frac{1}{2}\mu^2(\phi_1^2)}_{\text{Particle } \phi_1 \text{, mass } \mu} + \underbrace{\frac{1}{2}\,(\partial_\mu \phi_2)^2 - \frac{1}{2}\mu^2(\phi_2^2)}_{\text{Particle } \phi_2 \text{, mass } \mu}$$

Particle ϕ_1, mass μ

Particle ϕ_2, mass μ

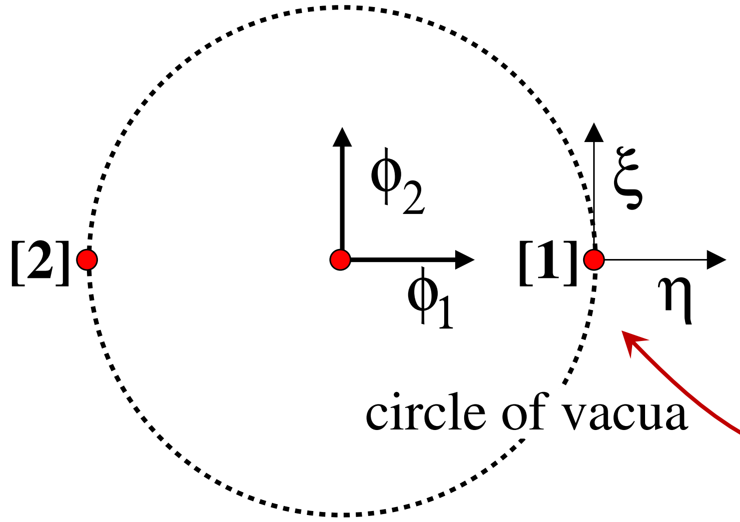
+ interaction terms

Symmetry breaking with a *complex* field ϕ

- Introduce a complex scalar field: $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$
- The Lagrangian term is: $\mathcal{L} = \left(\partial_\mu \phi\right)^* (\partial^\mu \phi) - V(\phi)$, with $V(\phi) = \mu^2(\phi^*\phi) + \lambda\,(\phi^*\phi)^2$
- Lagrangian:

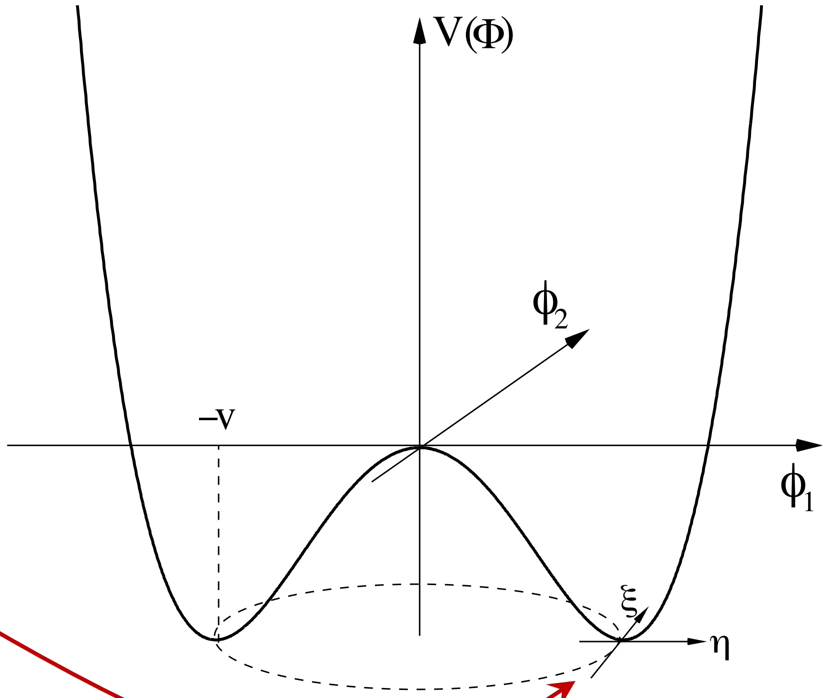
$$\mathcal{L}(\phi_1, \phi_2) = \frac{1}{2}\left(\partial_\mu \phi_1\right)^2 + \frac{1}{2}\left(\partial_\mu \phi_2\right)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

Case B)



- We now have a whole circle of vacua to chose from:

$$\sqrt{\phi_1^2 + \phi_2^2} = \sqrt{\frac{-\mu^2}{\lambda}} = v$$



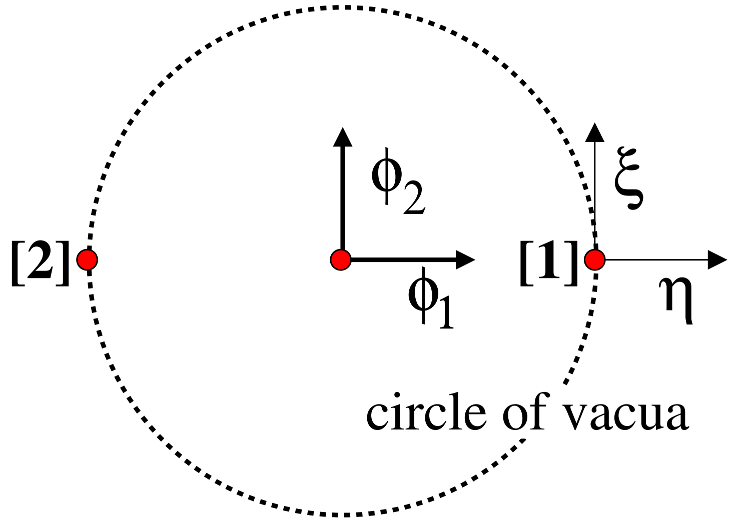
- Symmetry breaking: chose [1]: $\phi_0 = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$

Symmetry breaking with a *complex* field ϕ

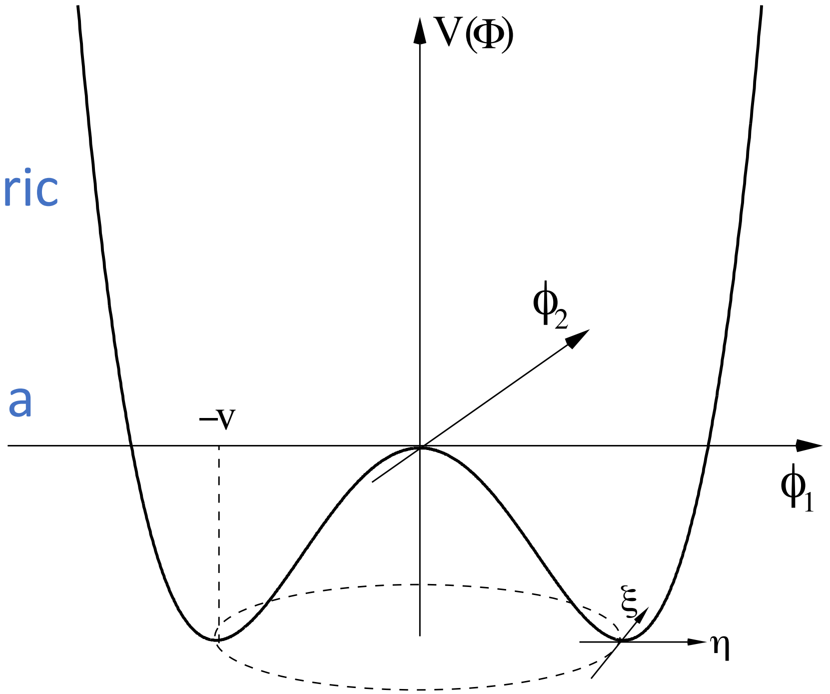
- Redefine coordinates: $\eta = \phi_1 - v \quad , \quad \xi = \phi_2 \quad , \quad \phi_0 = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$
- Exercise: rewrite the Lagrangian ignoring constant terms and higher order terms:

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \eta)^2 - (\lambda v^2)\eta^2}_{\textit{massive}\text{ scalar particle } \mu} + \underbrace{\frac{1}{2}(\partial_\mu \xi)^2 - 0 \cdot \eta^2}_{\textit{massless}\text{ scalar particle } \xi} + \text{higher order terms}$$

Case B)



- The Lagrangian is still symmetric
- The vacuum is no longer symmetric
- We have a *massive* scalar and a *massless* scalar
  - The latter is called a Goldstone boson.



- Symmetry breaking: chose [1]: $\phi_0 = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$

Higgs Mechanism

- The Higgs mechanism breaks the symmetry of the (electro-)weak interaction
  - Works along the lines as described in previous slides; introduce a complex SU(2) doublet
  - Details beyond the scope these lectures, idea as follows:
- Electroweak Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi + (D_{\mu}\phi)^{\dagger}(D^{\mu}\phi) - V(\phi)$
  - Where the covariant derivatives:

U(1): $\psi(x) \rightarrow \psi'(x) = e^{iq\alpha(x)}\psi(x)$

$$A^{\mu}(x) \rightarrow A'^{\mu}(x) = A^{\mu}(x) - \frac{1}{q}\partial^{\mu}\alpha(x)$$

$$\Rightarrow \quad D^{\mu} = \partial^{\mu} + iqA^{\mu}$$

and SU(2): $\psi(x) \rightarrow \psi'(x) = G(x)\psi(x)$

with $G(x) = \exp\left(\frac{i}{2}\vec{\tau}\cdot\vec{\alpha}(x)\right)$

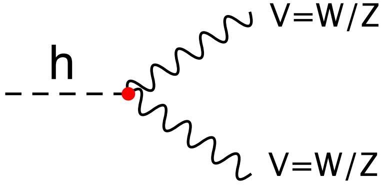
$$B'_{\mu} = GB_{\mu}G^{-1} + \frac{i}{g}(\partial_{\mu}G)G^{-1}$$

$$\Rightarrow \quad D_{\mu} = I\partial_{\mu} + igB_{\mu}$$

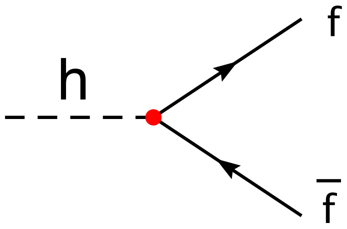
- Higgs field is weak isospin doublet: $\phi = \begin{pmatrix}\phi^{+}\\ \phi^{0}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2\\ \phi_3 + i\phi_4\end{pmatrix}$; $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$
  - With the potential: $V(\phi) = \mu^2(\phi^{\dagger}\phi) + \lambda(\phi^{\dagger}\phi)^2$ where: $\mu^2 < 0$

Higgs Mechanism : Resulting phenomenology

- The Higgs mechanism breaks the symmetry of the (electro-)weak interaction
  - The Higgs choses a preferred direction in weak isospin space
  - One massive Higgs scalar field remains – due to field excitations around v; the earlier η term
  - Three massless Goldstone bosons appear, but they are re-written as massterms for the gauge fields of the broken symmetry.
    - The W^+, W^-, Z^0 bosons acquire mass.
  - The photon remains massless



- Higgs and fermions:
  - The SM allows to couple the Higgs field to fermions isospin doublets:
    - The vacuum expectation value of the Higgs gives rise the fermion masses
    - Mass term: $m_f = Y_f \cdot \frac{1}{\sqrt{2}} v$ where Y_f is a particle constant.
      - For the top quark: $Y_f = 1$?!



- Mass eigenstates and interaction eigenstates:
  - The Higgs and the W boson do not agree on the ”generation” eigenstates, see lecture 2.
  - The Higgs couplings give rise to the CKM elements

Exercise – 22 : Mass of the proton

Besides giving mass to the weak vector bosons, it was briefly flashed that the same Higgs mechanism is responsible for giving mass to the fermion masses in the Standard Model, through ad-hoc Yukawa couplings. The mass of a 'naked' quark can be estimated through models of soft QCD, where it enters as a parameter for e.g. the binding energy of a meson. For up and down, they are found to be roughly 2 resp. 5 MeV/c .

a) What fraction of the proton mass is due to the Higgs mass of the constituent quarks?

b) Can you find out where the other part of the proton mass comes from?

- See also:
- https://en.wikipedia.org/wiki/Mathematical_formulation_of_the_Standard_Model

Lecture 4: ”Symmetries”

Part 3

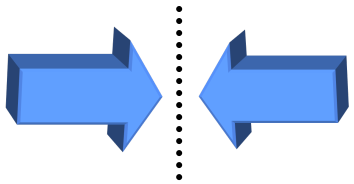
Discrete Symmetries

Discrete Symmetries

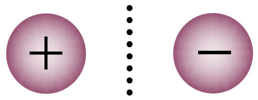
- Is nature invariant if we look at it through a mirror?

Discrete C, P, T Symmetries

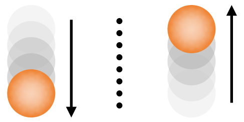
- Parity, P: *unobservable: (absolute handedness)*
  - Reflects a system through the origin.
    Converts right-handed to left-handed.
    - $\vec{\boldsymbol{x}} \to -\vec{\boldsymbol{x}}$, $\vec{\boldsymbol{p}} \to -\vec{\boldsymbol{p}}$ (vectors) but $\vec{\boldsymbol{L}} = \vec{\boldsymbol{x}} \times \vec{\boldsymbol{p}}$ (axial vectors)



- Charge Conjugation, C: *unobservable: (absolute charge)*
  - Turns internal charges to opposite sign.
    - $\boldsymbol{e}^+ \to \boldsymbol{e}^-$, $\boldsymbol{K}^- \to \boldsymbol{K}^+$



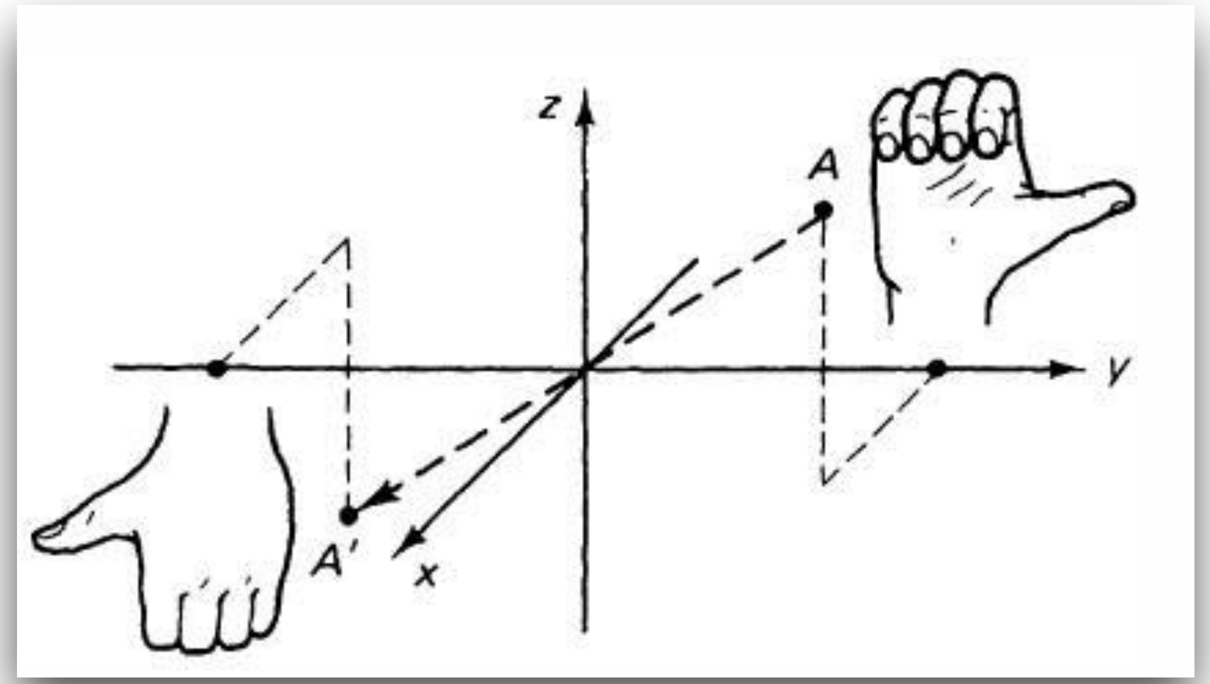
- Time Reversal, T: *unobservable: (direction of time)*
  - Changes direction of motion of particles
    - $\boldsymbol{t} \to -\boldsymbol{t}$



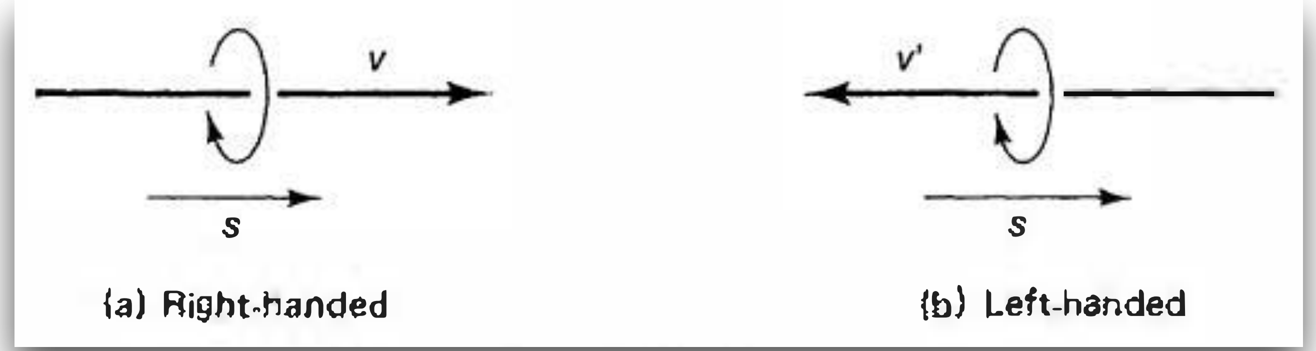
- CPT Theorem:
  - All interactions are invariant under combined C, P and T operation
  - A particle ***is*** an antiparticle travelling backward in time
  - Implies e.g. particle and anti-particle have equal masses and lifetimes

Parity: Helicity and Chirality

- Parity image
  - $\vec{L} = \vec{r} \times \vec{p} \quad \rightarrow \quad -\vec{r} \times -\vec{p} = \vec{L}$
  - Same for spin $\vec{S}$
- Helicity: spin projection on momentum
  - $\lambda = \vec{\sigma} \cdot \vec{p} \quad \rightarrow \quad \vec{\sigma} \cdot -\vec{p} = -\lambda$
  - The mirror of left-handed = right-handed



- Chirality:
  - If you, as observer overtake the electron, it changes from left handed to right-handed
  - How is it for a neutrino – zero mass?
    - You cannot overtake it.
    - Chirality is the helicity in the relativistic limit: $m \rightarrow 0 \; ; v \rightarrow c$



Exercise – 23 : Parity

a) Find the eigenvalue of the parity operator P, for the function $y(x) = 10x^5 + 3x^3$

b) (Optional for die-hards only – not required)
Find an expression for the parity of the Y_{lm} functions. Show that the parity is $(-1)^l$. This means that if a state has orbital angular momentum, there is an additional factor of $(-1)^l$ to the Parity eigenvalue!

c) [Griffiths 4.37 a)] Explain why the decay $\eta \rightarrow \pi\pi$ is forbidden for both strong and electromagnetic interactions.

Exercise – 24 : Helicity vs Chirality

a) Write out the chirality operator γ^5 in the Dirac-Pauli representation.

b) The helicity operator is defined as $\lambda = \vec{\sigma} \cdot \hat{p}$. Show that helicity operator and the chirality operator have the same effect on a spinor solution, i.e.

$$\gamma^5 \psi = \gamma^5 \begin{pmatrix} \chi^{(s)} \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi^{(s)} \end{pmatrix} \approx \lambda \begin{pmatrix} \chi^{(s)} \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi^{(s)} \end{pmatrix} = \lambda \psi$$

with: $\chi^{(1)} = \binom{1}{0}; \chi^{(2)} = \binom{0}{1}$

in the relativistic limit where $E \gg m$

c) Show explicitly that for a Dirac spinor:

$\bar{\psi} \gamma^\mu \psi = \overline{\psi_L} \gamma^\mu \psi_L + \overline{\psi_R} \gamma^\mu \psi_R$ making use of $\psi = \psi_L + \psi_R$ and the projection operators: $\psi_L = \frac{1}{2}(1 - \gamma^5)$ and $\psi_R = \frac{1}{2}(1 + \gamma^5)$

d) Explain why the weak interaction is called left-handed.

Wolfgang Pauli

"I cannot believe God is a weak left-hander."

The weak interaction for particles is "left-handed"

- Look at the weak pion decay: $\pi^- \to \mu^- \bar{\nu}_\mu$ in the pion:

Spin = ½ Spin = 0 Spin = ½

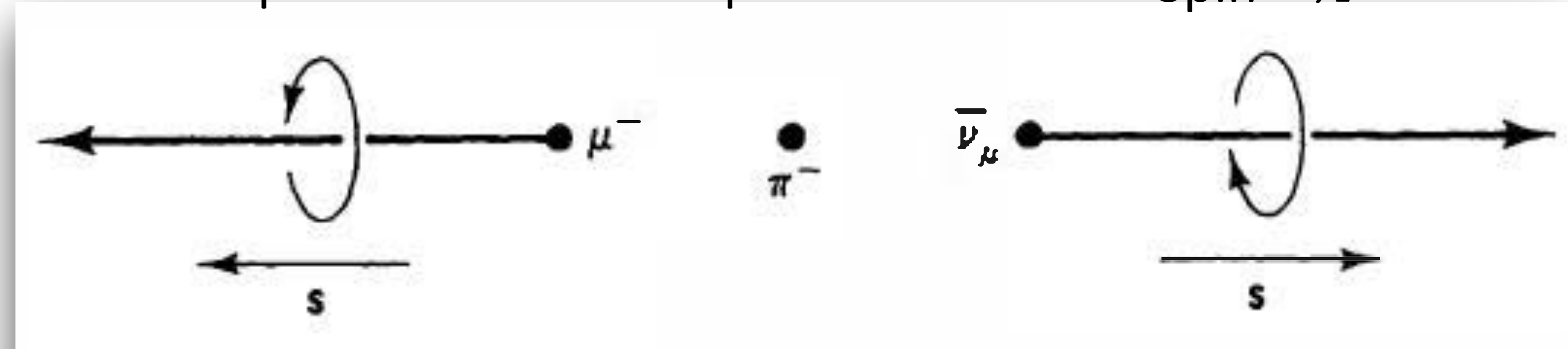


Fig. 4.10 Decay of π^- at rest.

- Compare to the decay: $\pi^+ \to \mu^+ \nu$ and measure the spin of the muon:
  - $\pi^+ \to \mu^+ \nu$: anti-muon spin was found left-handed: neutrino is also left handed
  - $\pi^- \to \mu^- \bar{\nu}$: muon spin was found right: handed: anti-neutrino is also right handed
- Since neutrino's are ultra-relativistic ($m \approx 0$): *neutrino's are always left-handed*

anti-neutrino's are always right handed

➔ The weak interaction maximally violates parity symmetry!

Classical Mirror Worlds → Invariant!

- Parity P: $\vec{x} \to -\vec{x}$, $\vec{p} \to -\vec{p}$
  - Mass m — $P\, m = m$: scalar
  - Force $\vec{F}$ $(\vec{F} = d\vec{p}/dt)$ — $P\, \vec{F} = P\, d\vec{p}/dt = -\, d\vec{p}/dt = -\vec{F}$: vector
  - Acceleration $\vec{a}$ $(\vec{a} = d^2\vec{x}/dt^2)$ — $P\, \vec{a} = -\, d^2x/dt^2 = -\vec{a}$: vector
  - Angular momentum $\vec{L}, \vec{S}, \vec{J}$ $(\vec{L} = \vec{x} \times \vec{p})$ — $P\ \vec{L} = -\vec{x} \times -\vec{p} = \vec{L}$: axial vector

- Parity: Newton's law is *invariant* under P-operation (i.e. the same in the mirror world):

$$\vec{F} = m\,\vec{a} \xrightarrow{P} -\vec{F} = -m\vec{a} \quad \Leftrightarrow \quad \vec{F} = m\vec{a}$$

- Charge: Lorentz Force in the C-mirror world is *invariant*:

$$\vec{F} = q\left[\vec{E} + \vec{v} \times \vec{B}\right] \xrightarrow{C} \vec{F} = -q\left[-\vec{E} + \vec{v} \times -\vec{B}\right]$$

- Time: laws of physics are also *invariant* unchanged under T-reversal, since:

$$\vec{F} = m\,\vec{a} = m\,\frac{d^2\vec{x}}{dt^2} \xrightarrow{T} \vec{F} = m\,\frac{d^2\vec{x}}{d(-t)^2} \quad \Leftrightarrow \quad \vec{F} = m\vec{a}$$

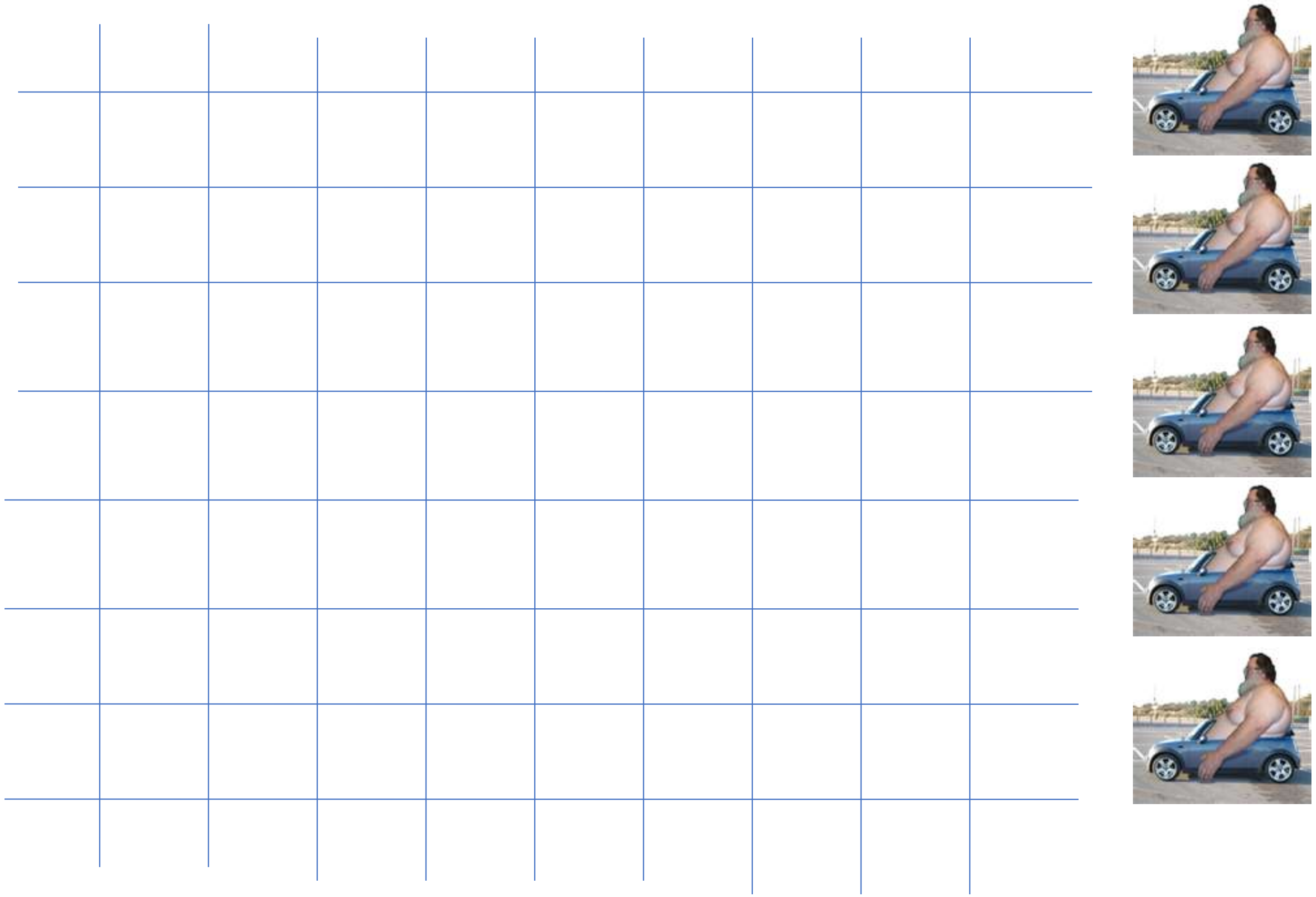
  - QM: Consider Schrodinger's equation $(t \to -t)$: $i\hbar\dfrac{\partial\psi}{\partial t} = -\dfrac{\hbar^2}{2m}\vec{\nabla}^2\psi$

    Complex conjugation is required to stay invariant: $\psi \xrightarrow{T} \psi^*$

C-, P-, T- Symmetry

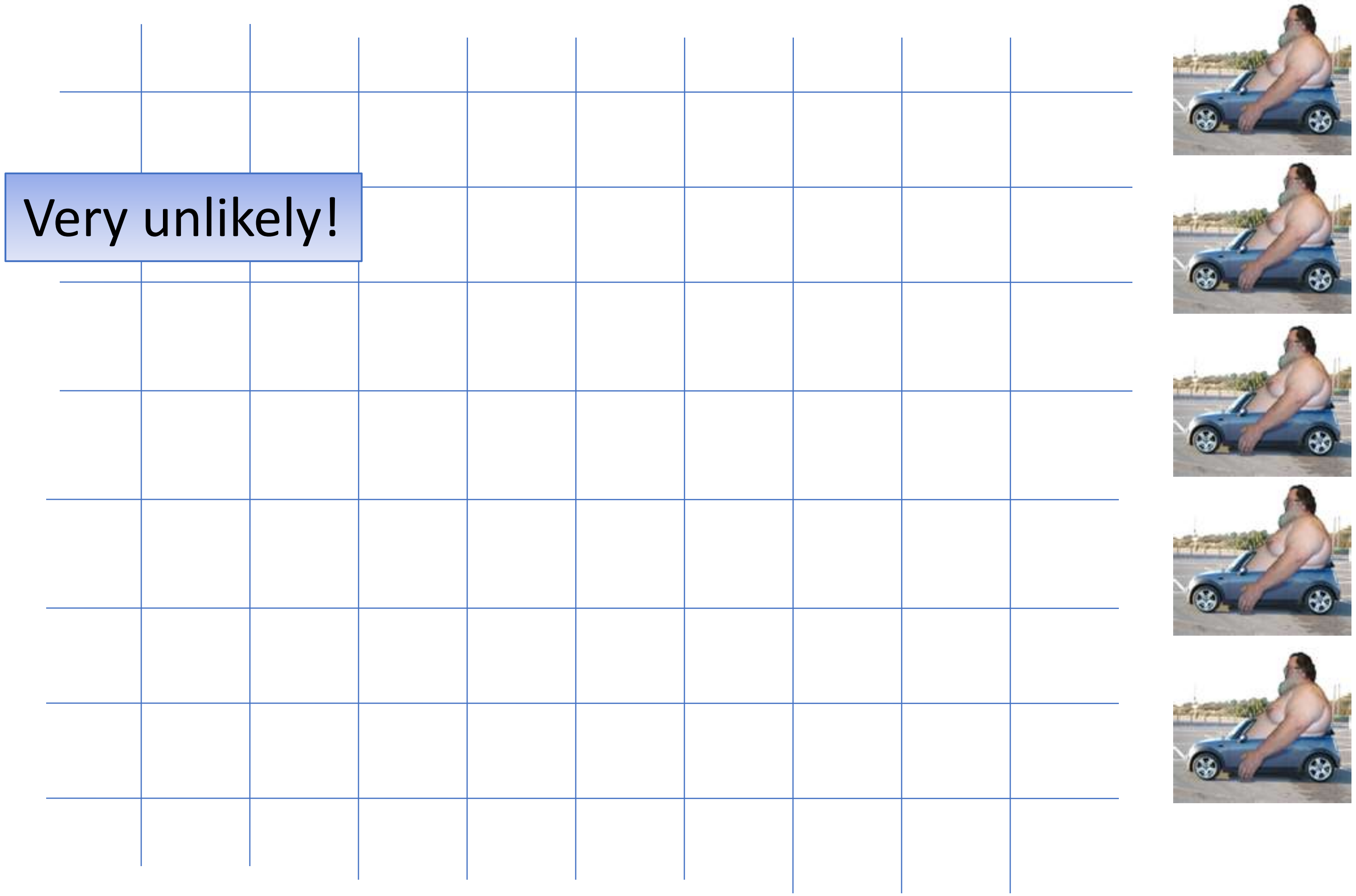
- Classical Theory is invariant under C, P, T operations; i.e. they conserve C, P, T symmetry
  - Newton mechanics, Maxwell electrodynamics.
- Suppose we watch some physical event. Can we determine unambiguously whether:
  - We are watching the event where all ***charges are reversed*** or not?
  - We are watching the event ***in a mirror*** or not?
    - Macroscopic biological asymmetries are considered *accidents of evolution* rather than fundamental asymmetry in the laws of physics.
  - We are watching the event in a ***film running backwards*** or not?
    - The arrow of time is due to thermodynamics: i.e. the realization of a macroscopic final state is *statistically more probable* than the initial state

Macroscopic time reversal (T.D. Lee)



- At each crossing: 50% - 50% choice to go left or right
- After many decisions: reverse the velocity of the final state and return
- Do we end up with the initial state?

Macroscopic time reversal (T.D. Lee)



- At each crossing: 50% - 50% choice to go left or right
- After many decisions: reverse the velocity of the final state and return
- Do we end up with the initial state?

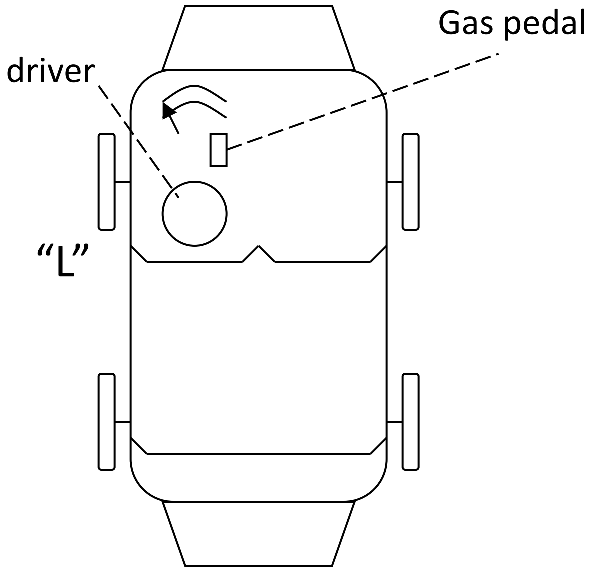
Macroscopic time reversal

This is why we don't teach our children about entropy until much later...

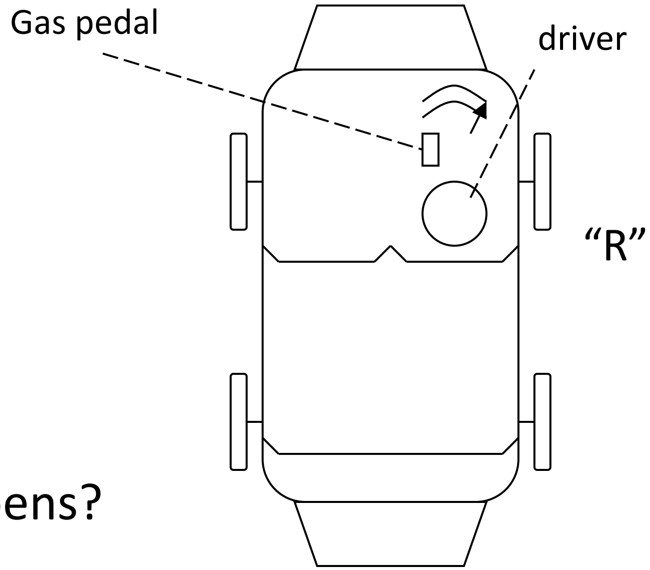
Parity Violation

Before 1956 physicists were **convinced** that the laws of nature were left-right symmetric. Strange?

A "gedanken" experiment: consider two perfectly mirror symmetric cars:



"L" and "R" are fully symmetric,
Each nut, bolt, molecule etc.
However the engine is a black box



Person "L" gets in, starts, ….. 60 km/h

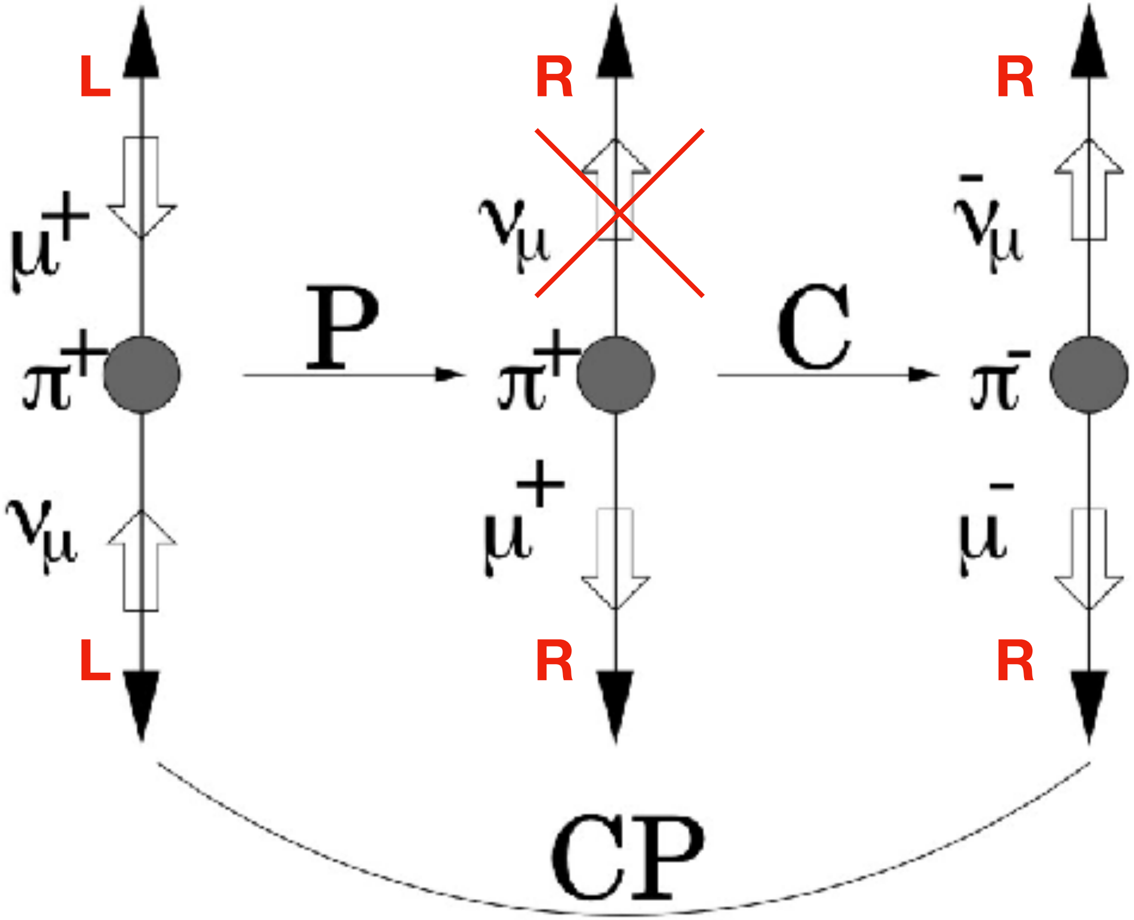
Person "R" gets in, starts, ….. What happens?

What happens in case the ignition mechanism uses, say, Co^{60} β decay?

The weak interaction is left handed

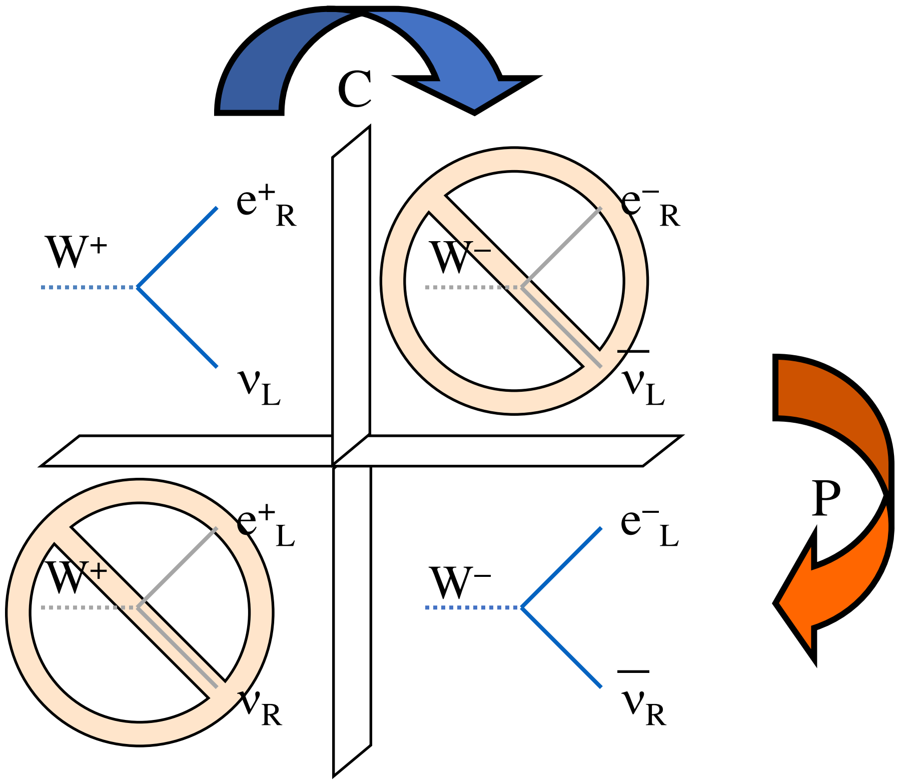
- Look again at pion decay

Fig. 4.10 Decay of π^- at rest.



- Both Parity P as well as charge conjugation C symmetry are violated
  - But happens if we do both: CP?

Weak Force breaks C and P, is CP really OK?

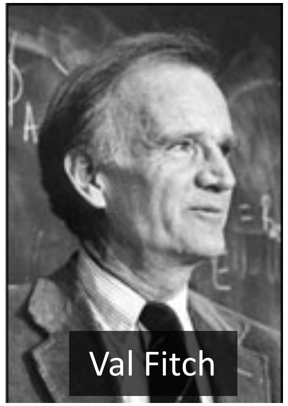


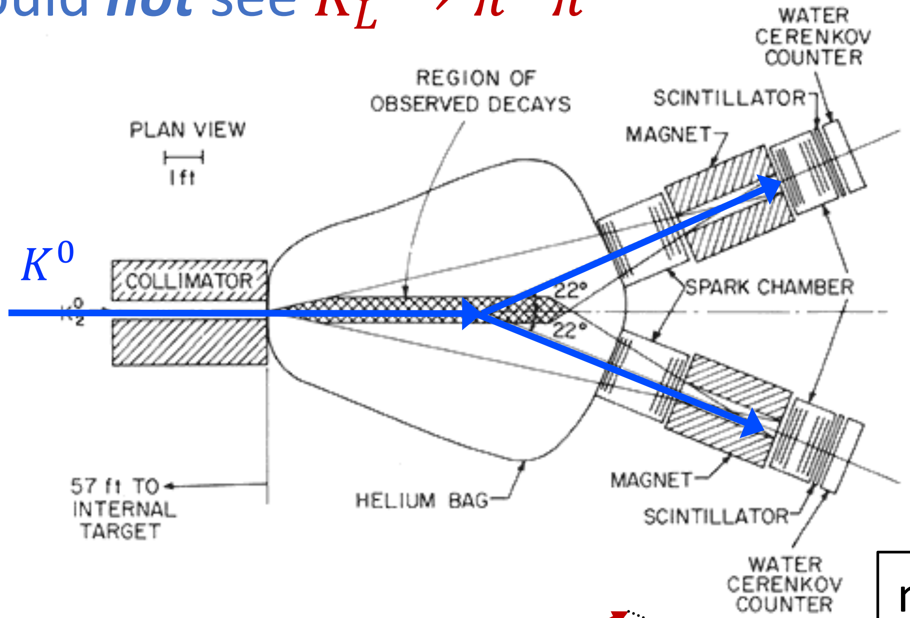
- Weak interaction breaks ***C*** and ***P*** symmetry maximally!
  - Nature is left-handed for matter and right-handed for antimatter.
- Despite *maximal* violation of ***C*** and ***P***, combined ***CP*** seemed *conserved*…
- But in 1964, Christenson, Cronin, Fitch and Turlay observed ***CP*** *violation* in decays of neutral kaons!

Discovery of CP-Violation with Kaons

K_S: Short-lived is CP even:
$K_1^0 \to \pi^+\pi^-$ (fast)
K_L: Long-lived is CP odd:
$K_2^0 \to \pi^+\pi^-\pi^0$ (slow)

- Create a pure K_L beam ("wait" for K_S to decay)
- If CP is conserved, should ***not*** see $K_L \to \pi^+\pi^-$

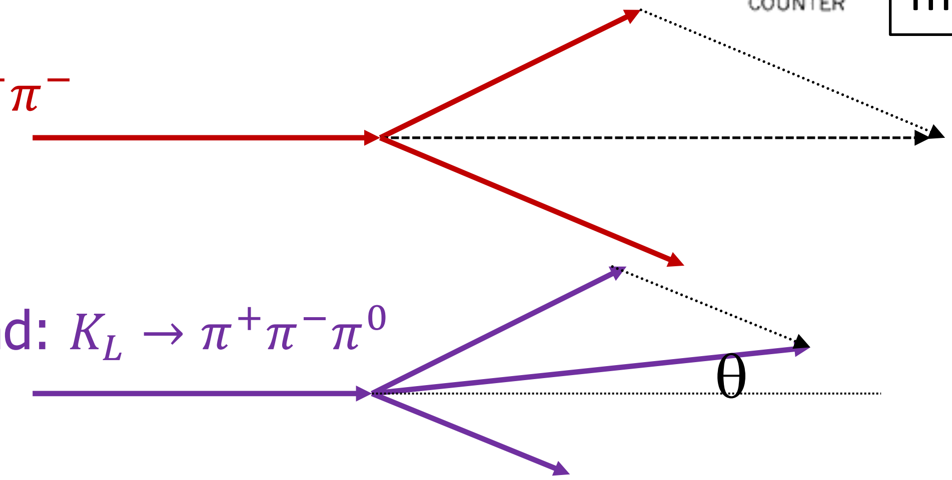


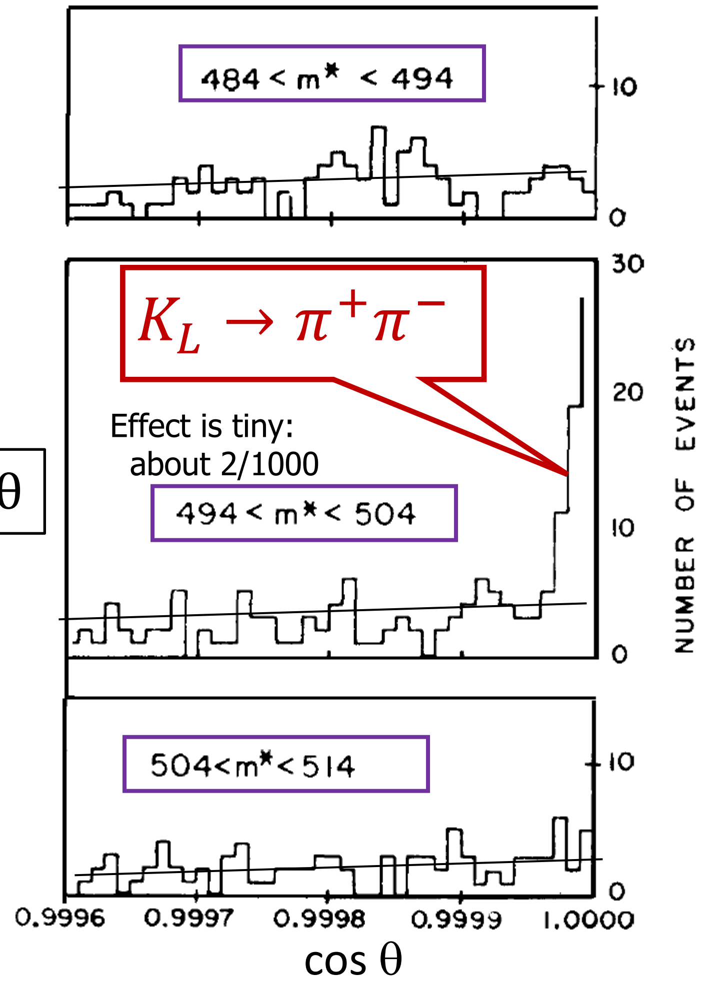


mass, θ

CPV Signal: $K_L \to \pi^+\pi^-$

Expected Background: $K_L \to \pi^+\pi^-\pi^0$



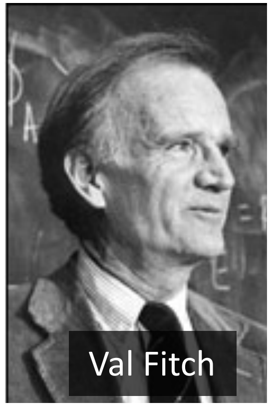


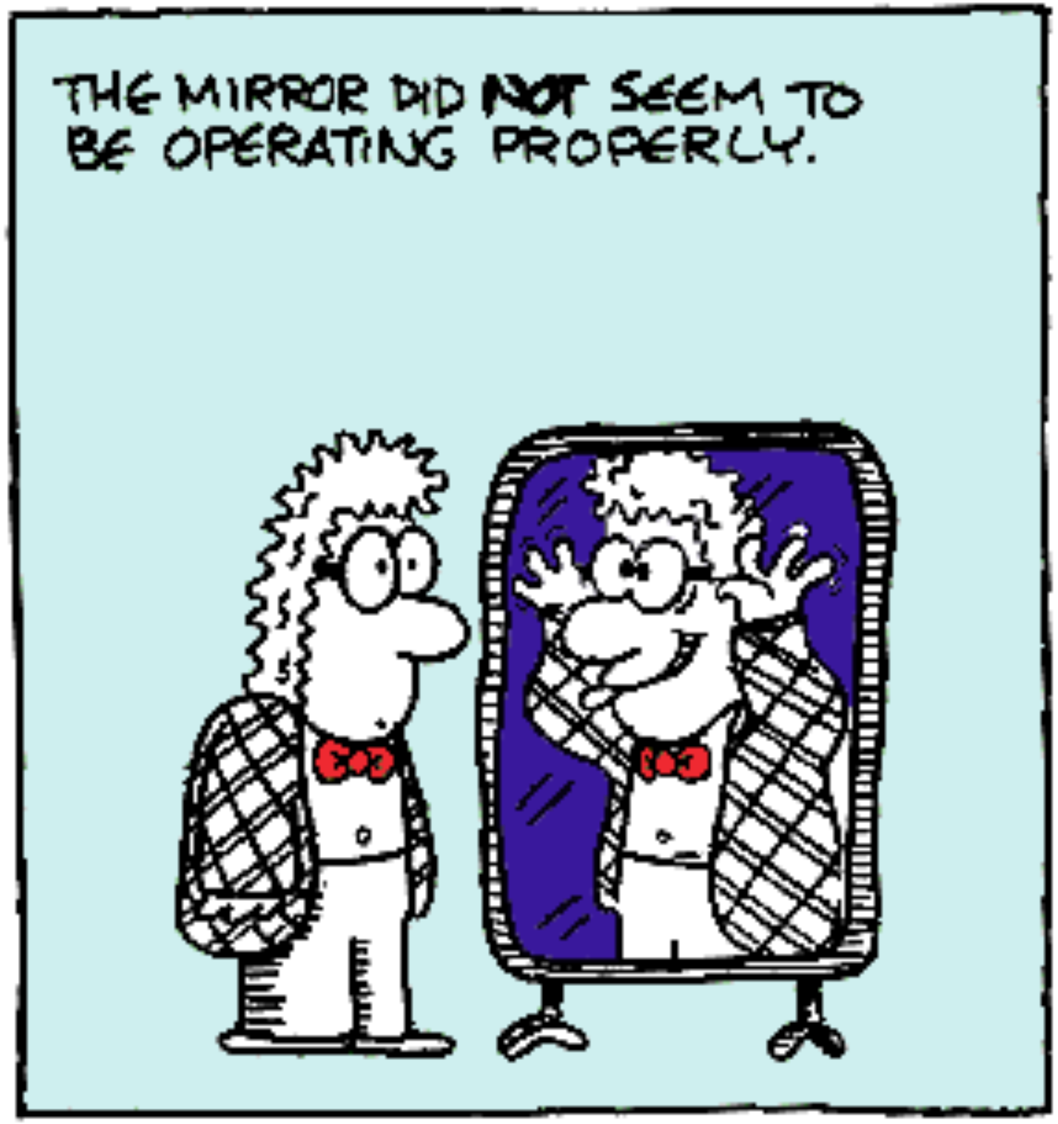
Discovery of CP-Violation with Kaons

K_S: Short-lived is CP even:
$K_1^0 \to \pi^+\pi^-$ (fast)
K_L: Long-lived is CP odd:
$K_2^0 \to \pi^+\pi^-\pi^0$ (slow)

- Create a pure K_L be
- If CP is conserved,

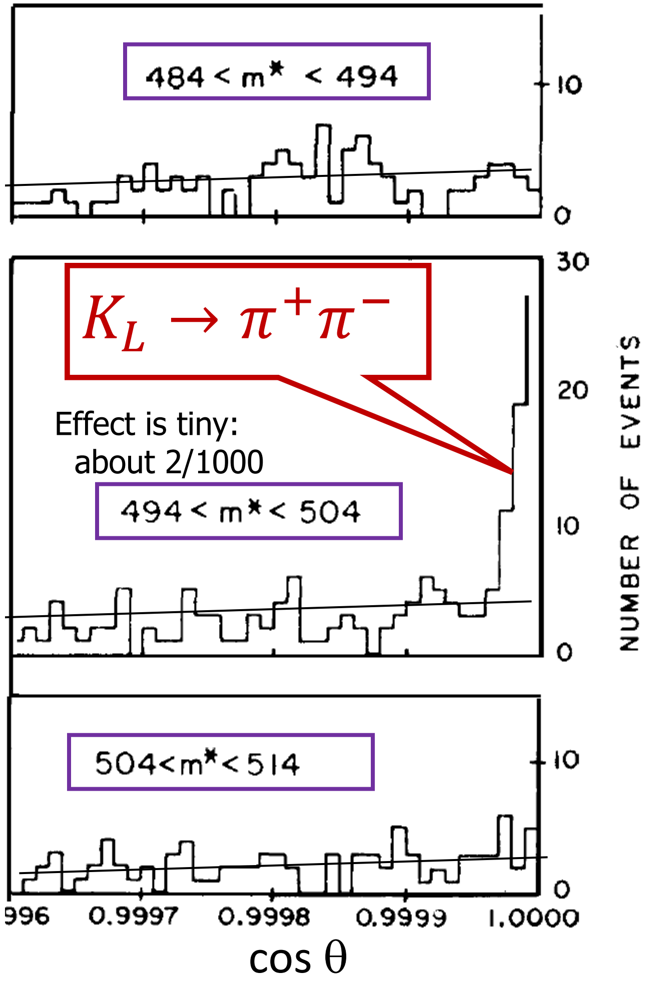

James Cronin


Val Fitch



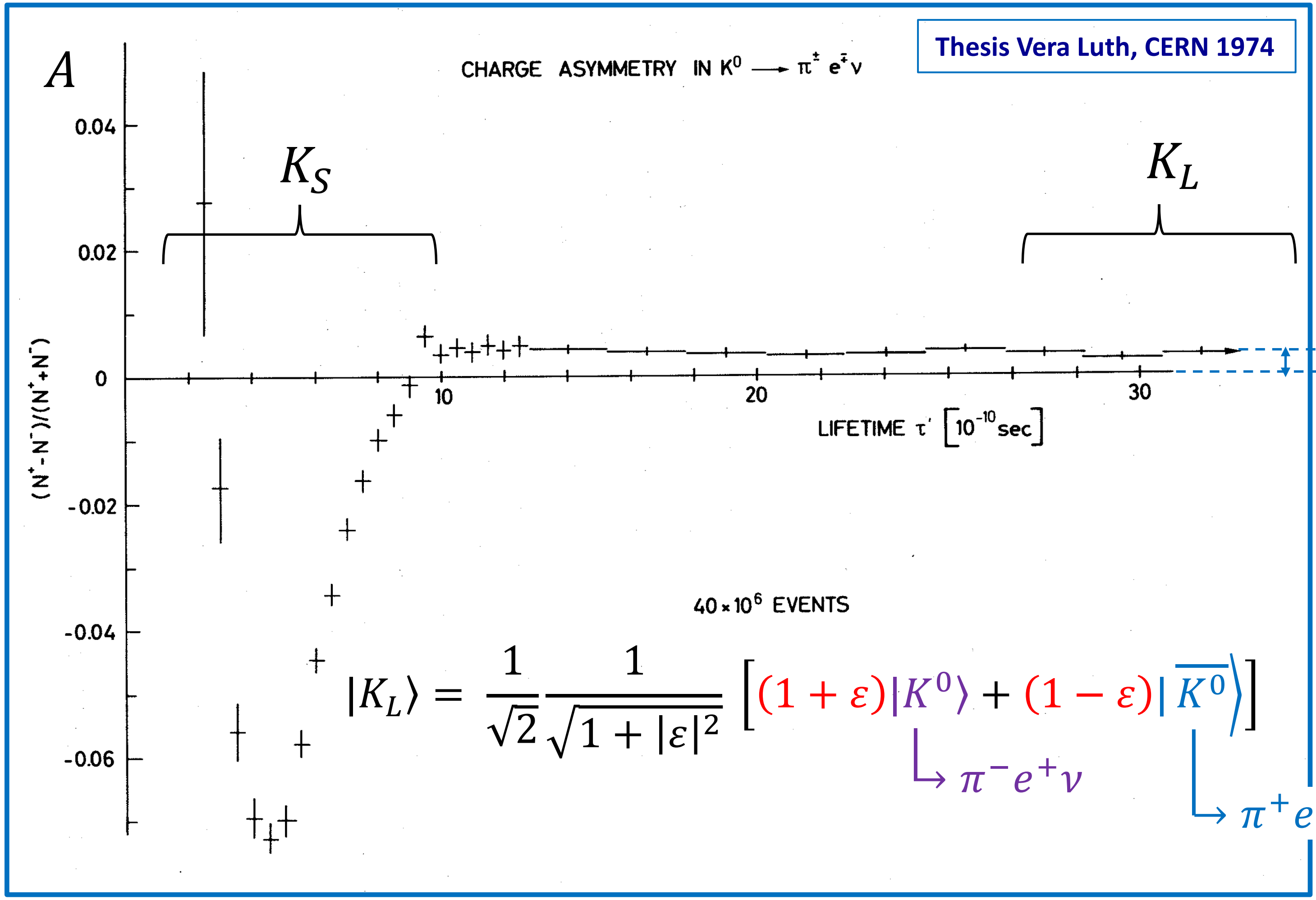
CPV Signal: $K_L \to$

Expected Backgro



Alternative: Charge Asymmetry in K^0 decays

Measure $A = \frac{N^+ - N^-}{N^+ + N^-}$ with $N^+ = K^0 \to \pi^- e^+ \nu$, $N^- = \overline{K^0} \to \pi^+ e^- \bar{\nu}$ vs the K^0 decay time



$$|K_L\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\overline{K^0}\rangle \right]$$

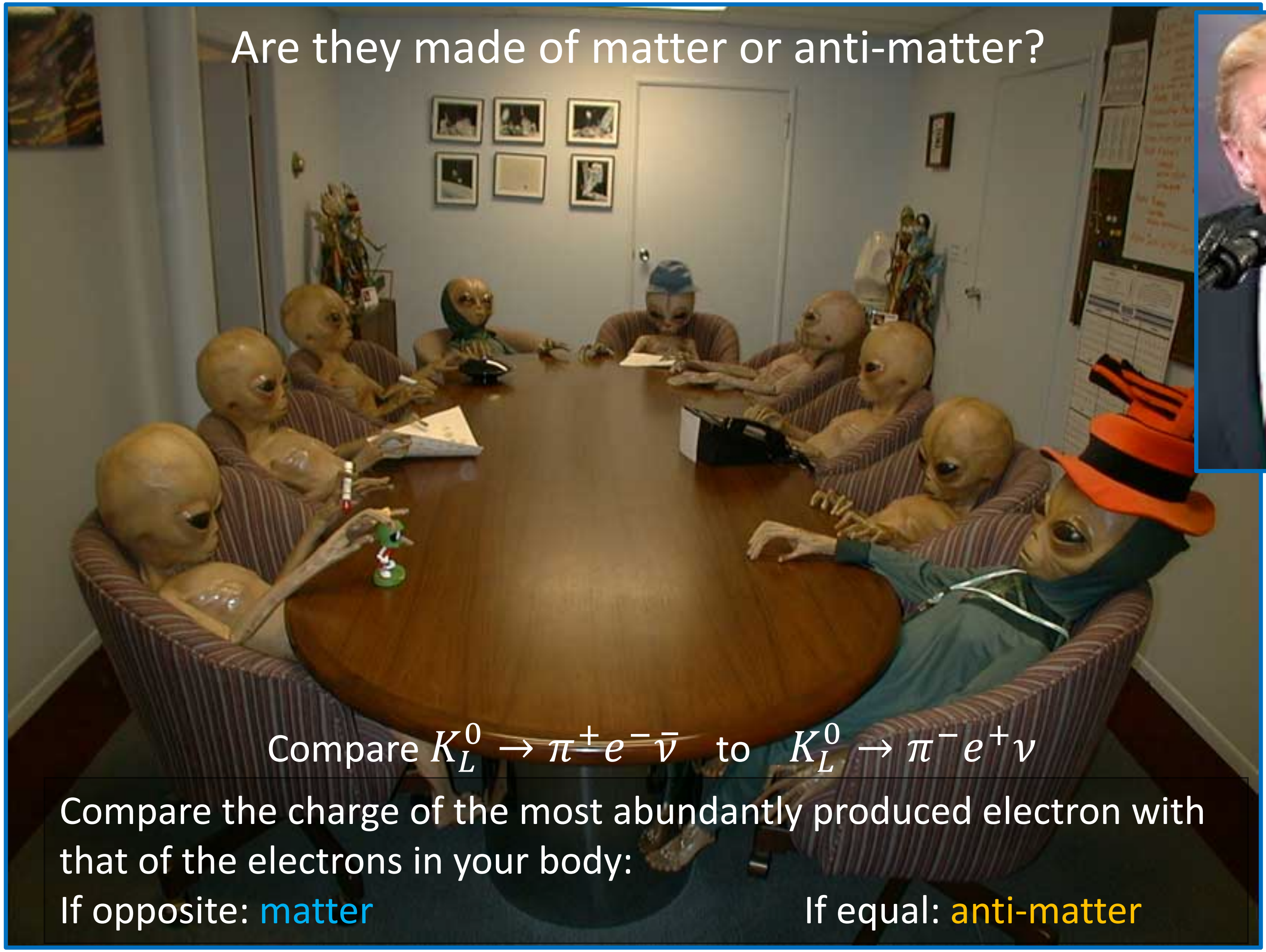
$(1+\varepsilon)|K^0\rangle \to \pi^- e^+ \nu$; $(1-\varepsilon)|\overline{K^0}\rangle \to \pi^+ e^- \bar{\nu}$

$$4\Re(\varepsilon) = \left| \frac{(1+\varepsilon)}{(1-\varepsilon)} \right|^2$$

$K^0 \to \pi^- e^+ \nu$ happens a bit (ε) more than $\overline{K^0} \to \pi^+ e^- \bar{\nu}$

CP violation in meson mixing.

Contact with Aliens !



Are they made of matter or anti-matter?

Compare $K_L^0 \to \pi^+ e^- \bar{\nu}$ to $K_L^0 \to \pi^- e^+ \nu$

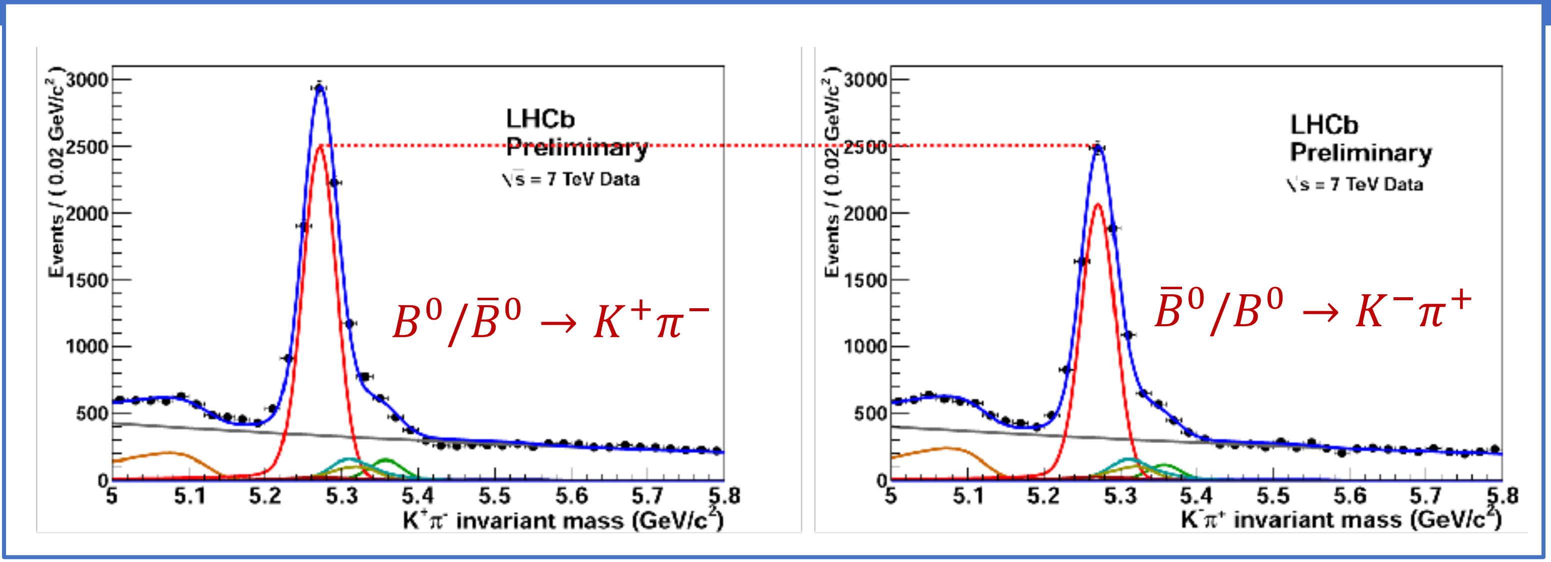
Compare the charge of the most abundantly produced electron with that of the electrons in your body:

If opposite: matter

If equal: anti-matter

CP Violation & B-mesons

- Example: $B^0 \rightarrow K^+\pi^-$
  - Two quantum amplitudes (Feynman diagrams)
  - Interference gives rise to CP violation
    - Requires “strong” and “weak” phases



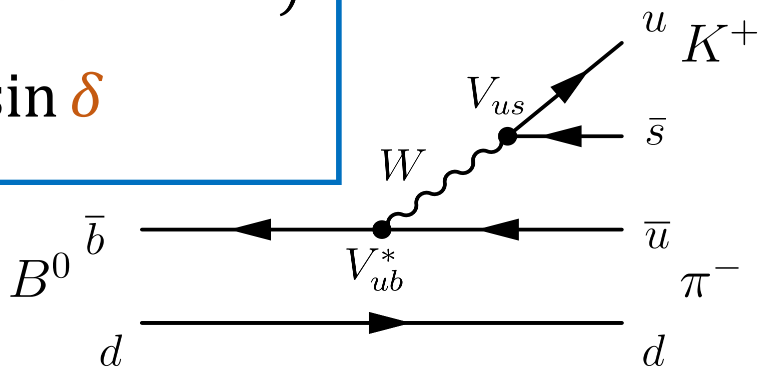
$$A = A_1 + A_2 e^{i\phi} e^{i\delta} \qquad \bar{A} = A_1 + A_2 e^{-i\phi} e^{i\delta}$$

$$|A|^2 = |A_1|^2 + |A_2|^2 + A_1A_2\left(e^{i\phi}e^{i\delta} + e^{-i\phi}e^{-i\delta}\right)$$

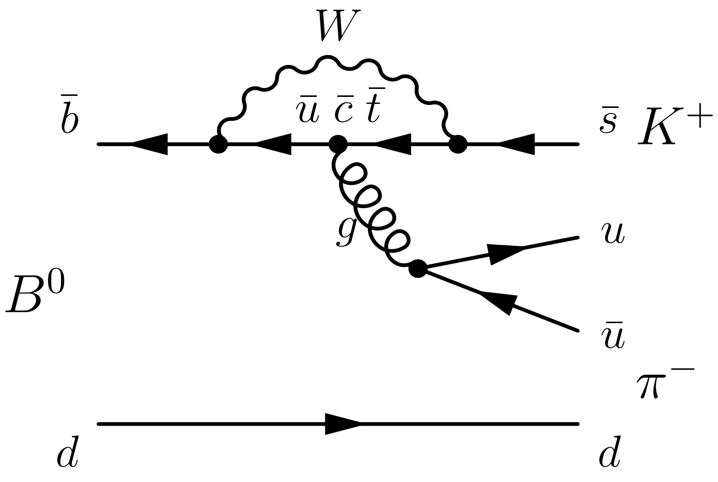
$$|\bar{A}|^2 = |A_1|^2 + |A_2|^2 + A_1A_2\left(e^{-i\phi}e^{i\delta} + e^{i\phi}e^{\ i\delta}\right)$$

$$|A - \bar{A}|^2 = 4\, A_1A_2 \sin\phi \sin\delta$$

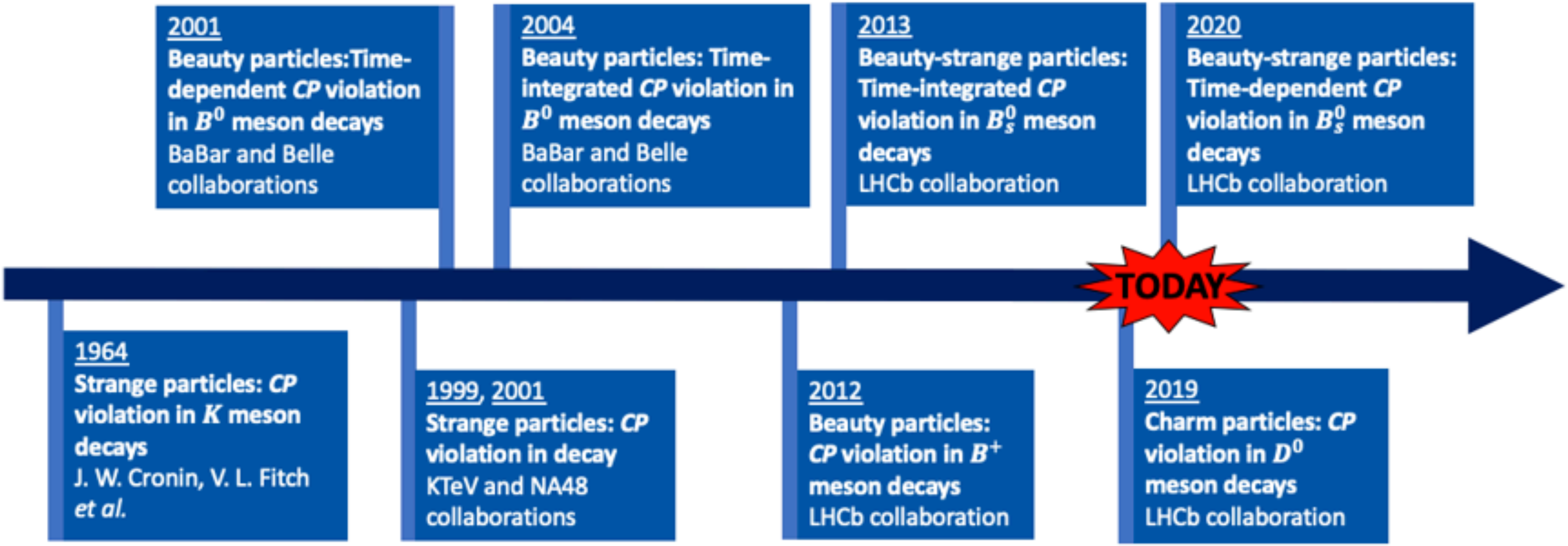
“Tree”= A_1



“Penguin”= A_2



CP Violation is a hot topic at the LHCb experiment



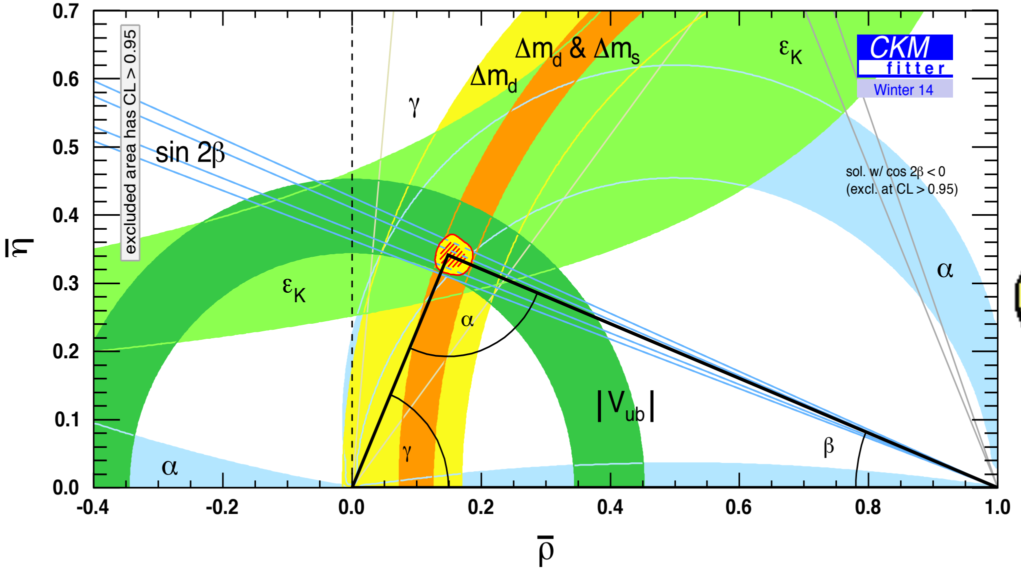
CPT Violation…

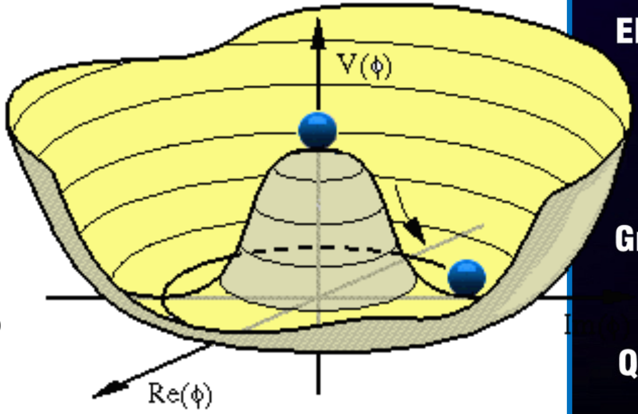
CPT symmetry implies that an antiparticle is ***identical*** to a particle travelling backwards in time.

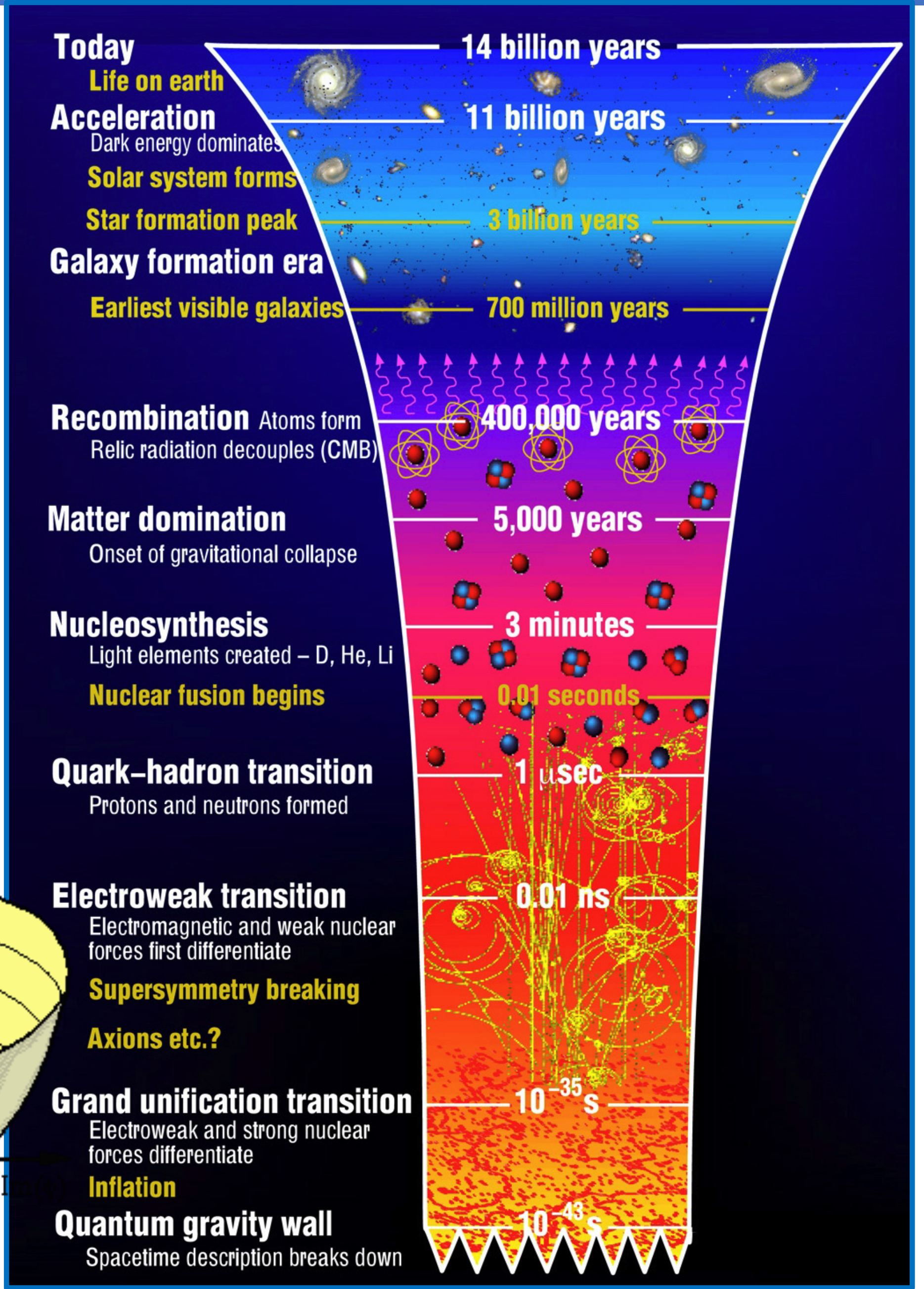
Symmetry breaking in the early universe

- Higgs mechanism generates mass
  - For the weak bosons
  - For the fermions
- Higgs couplings lead to CKM couplings
- 3 generations allow for CP violation

- Can it explain the matter anti-matter asymmetry?
  - So far: no!







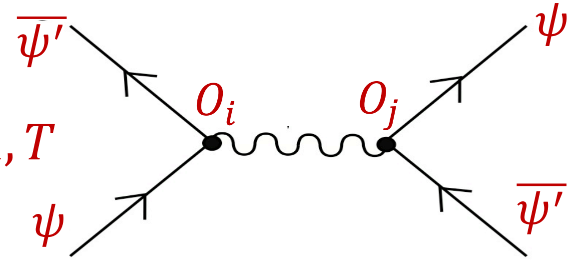
Exercise – 25 : Bilinear Covariants

- What is the reason that gamma matrices *cannot be* Lorentz 4-vectors?
- The space-time dependence is included by combining the wave function $\psi(t,x)$ with the gamma matrices γ^{μ}. Show explicitly that each of the following so-called *bilinear covariants* no longer carry Dirac spinor indices:

  $S\colon \bar{\psi}\psi,\ \ V\colon \bar{\psi}\gamma^{\mu}\psi,\ \ T\colon \bar{\psi}\sigma^{\mu\nu}\psi,\ \ A\colon \bar{\psi}\gamma^{5}\gamma^{\mu},\ \ P\colon \bar{\psi}\gamma^{5}\psi$ where $\sigma^{\mu\nu} \equiv \frac{i}{2}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu})$
- How many Lorentz indices does each of these combinations have?
- Can you think of additional independent forms that contract two Dirac spinors into an object without Dirac indices? These bilinear combinations are generally called currents as they are the most general form in which currents can occur.
- Explain the names of these currents:

  S = Scalar, V = Vector, T = Tensor, A = Axial vector, P = Pseudo-scalar
- The most general type of interaction will be of the form represents :

$$\mathcal{M} = G \sum_{i,j}^{S,P,V,A,T} C_{ij}(\bar{\psi}O_i\psi')(\bar{\psi}O_j\psi')$$ where Operator O_i represents S, P, V, A, T



Exercise – 26 : Symmetries

a) What do you think is the difference between an exact and a broken symmetry?

b) Can you explain the name *spontaneous* symmetry breaking means?

c) Which symmetry is involved in the gauge theories below? Which of these gauge symmetries are exact? Why/Why not?

   i. U1(Q) symmetry
   ii. SU2(u-d-flavour) symmetry
   iii. SU3(u-d-s-flavour) symmetry
   iv. SU6(u-d-s-c-b-t) symmetry
   v. SU3(colour) symmetry
   vi. SU5(Grand unified) symmetry
   vii. SuperSymmetry