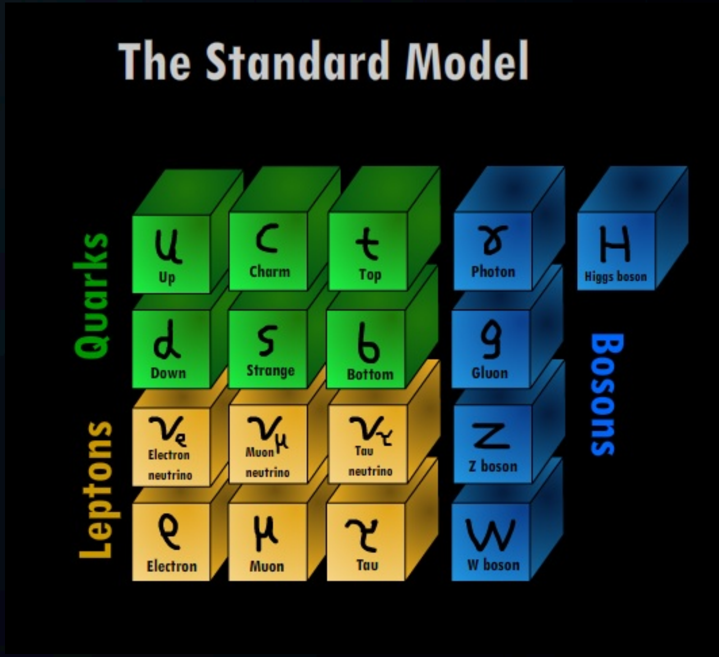
PHY3004: Nuclear and Particle Physics

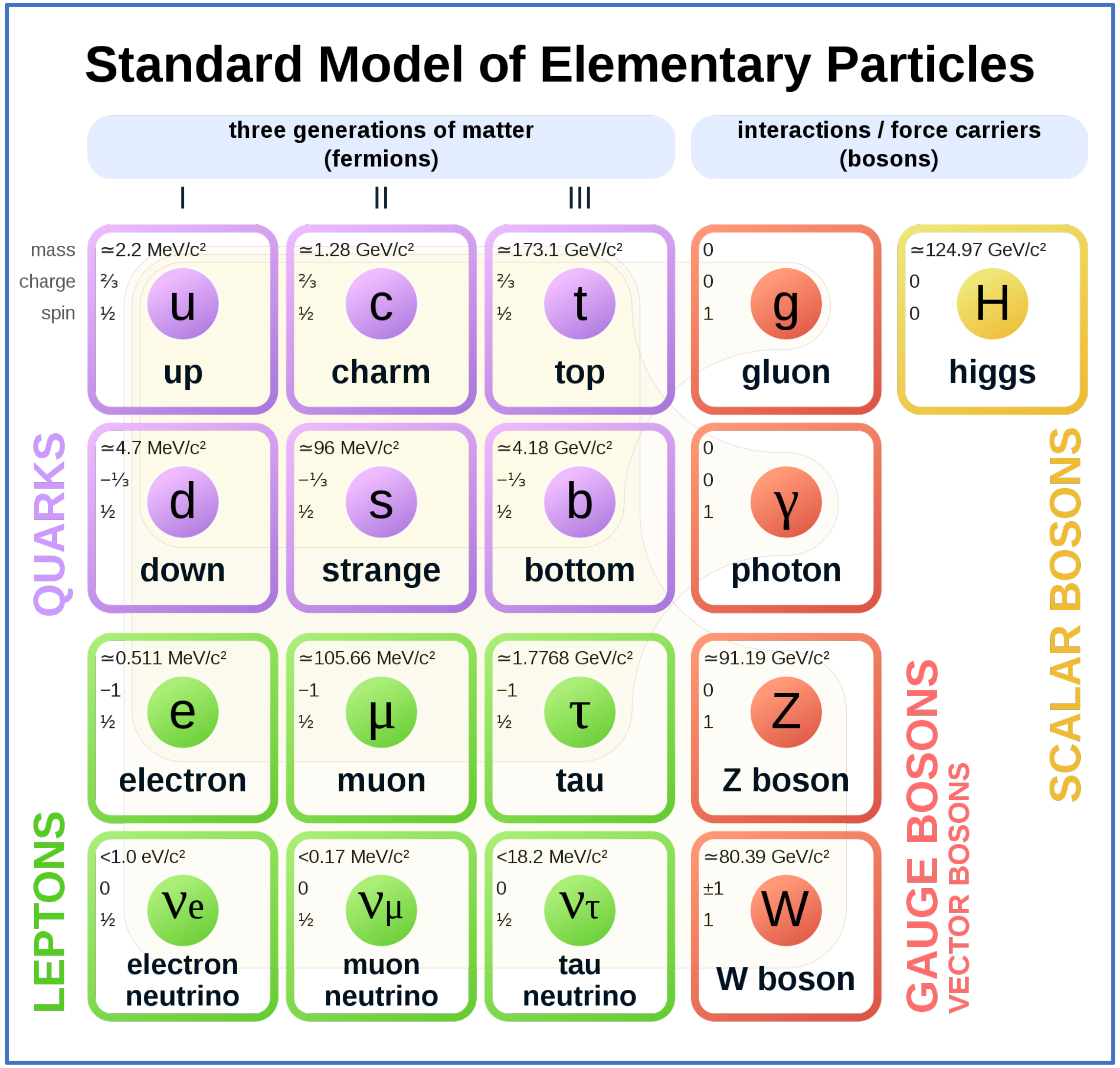
Marcel Merk, Jacco de Vries, …



Elementary particles

Classification of particles

- Lepton: fundamental particle
- Hadron: consist of quarks
  - Meson: 1 quark + 1 antiquark (π^+,B_s^0, …)
  - Baryon: 3 quarks (p ,n , Λ, …)
    - Anti-baryon: 3 anti-quarks
- ***Fermion***: particle with half-integer spin.
  - Antisymmetric wave function: obeys Pauli-exclusion principle and Pauli-Dirac statistics
  - All fundamental quarks and leptons are spin-½
  - Baryons (S=$^1/_2$, $^3/_2$)
- ***Boson***: particle with integer spin
  - Symmetric wave function: Bose-Einstein statistics
  - Mesons: (S=0, 1), Higgs (S=0)
  - Force carriers: γ, W , Z, g (S=1); graviton(S=2)



Lecture 2: “Forces”

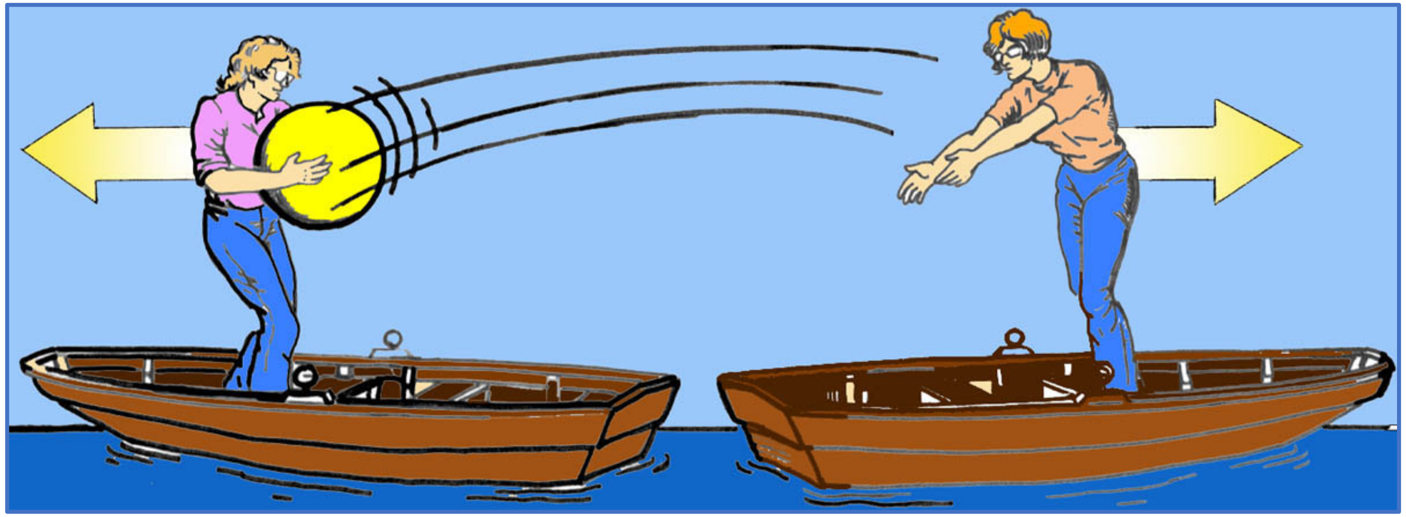
Griffiths chapter 2

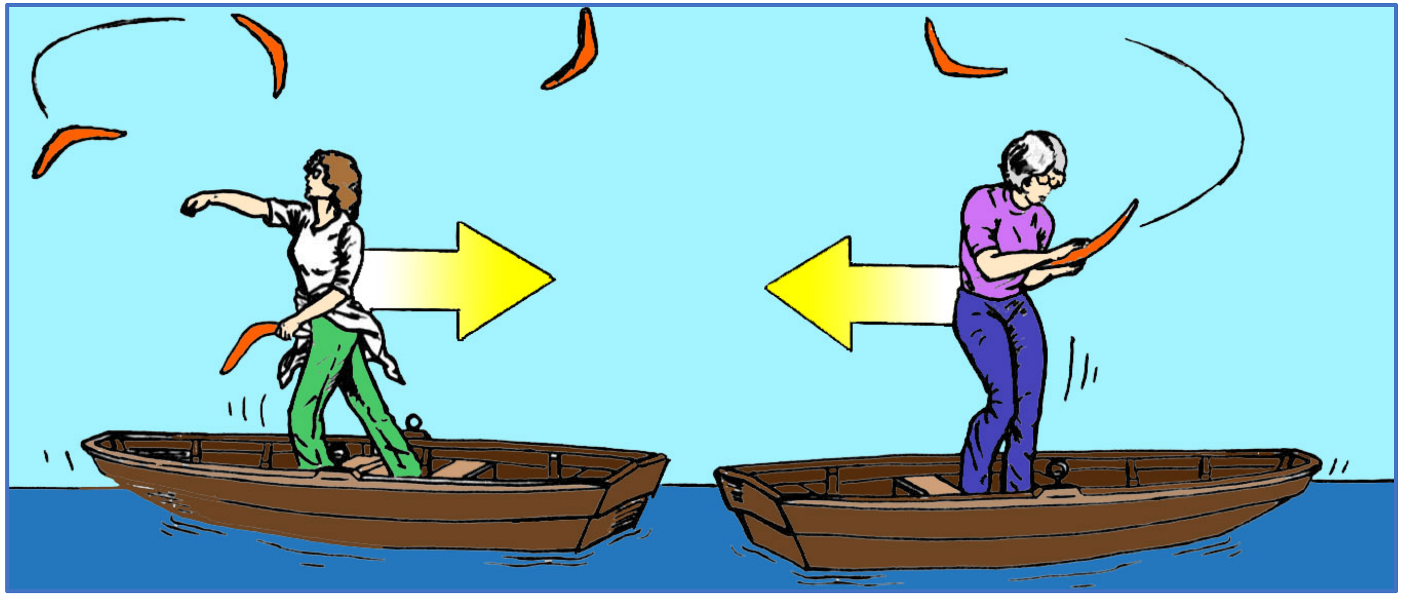
- Electromagnetism
- Weak Interaction
- Strong Interaction
- No Gravitation

Lecture 2 : Forces

	Gravity	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong
Carried By	Graviton (not yet observed)	W^+ W^- Z^0	Photon	Gluon
Acts on	All	Quarks and Leptons	Quarks and Charged Leptons and W^+ W^-	Quarks and Gluons
Strength	0.000000000000000 0000000000000000 0000000001	0.0001	1	60

Attractive and Repulsive forces and the quantum exchange

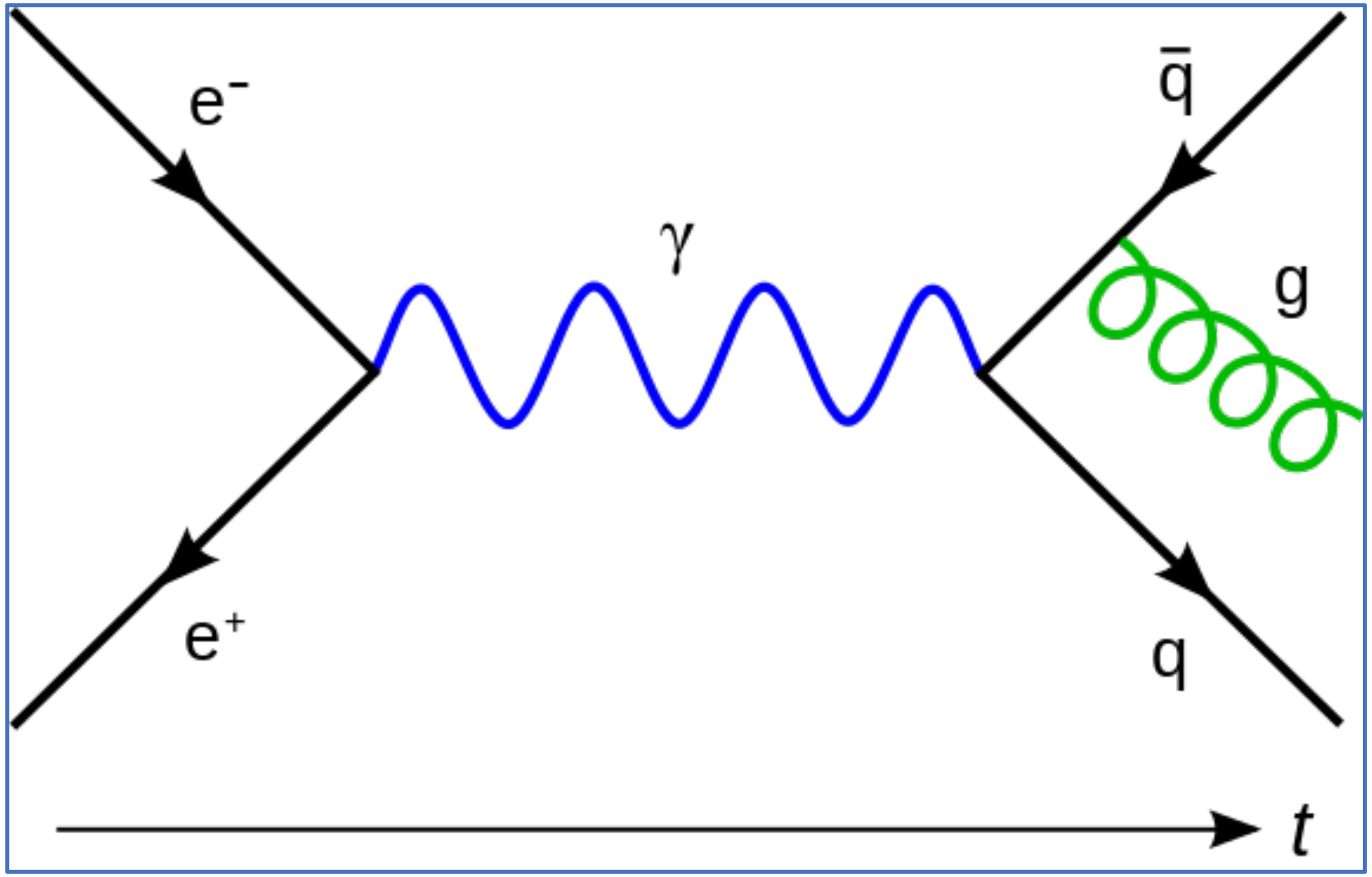


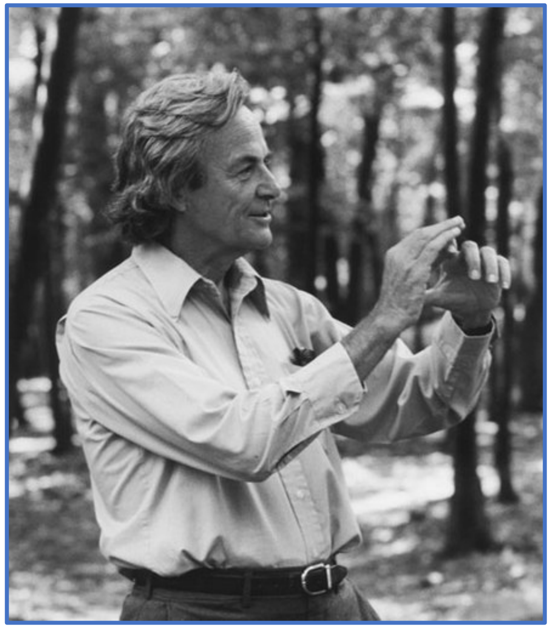


There is no
"action at a distance"

EM: photon
Weak: W, Z bosons
Strong: gluons
Gravitation: graviton(?)

Example of a quantum process





Feynman diagram

- Electron-positron annihilation
- Produce a "virtual" photon
- Photon produces a quark + antiquark
- Quark can radiate a gluon

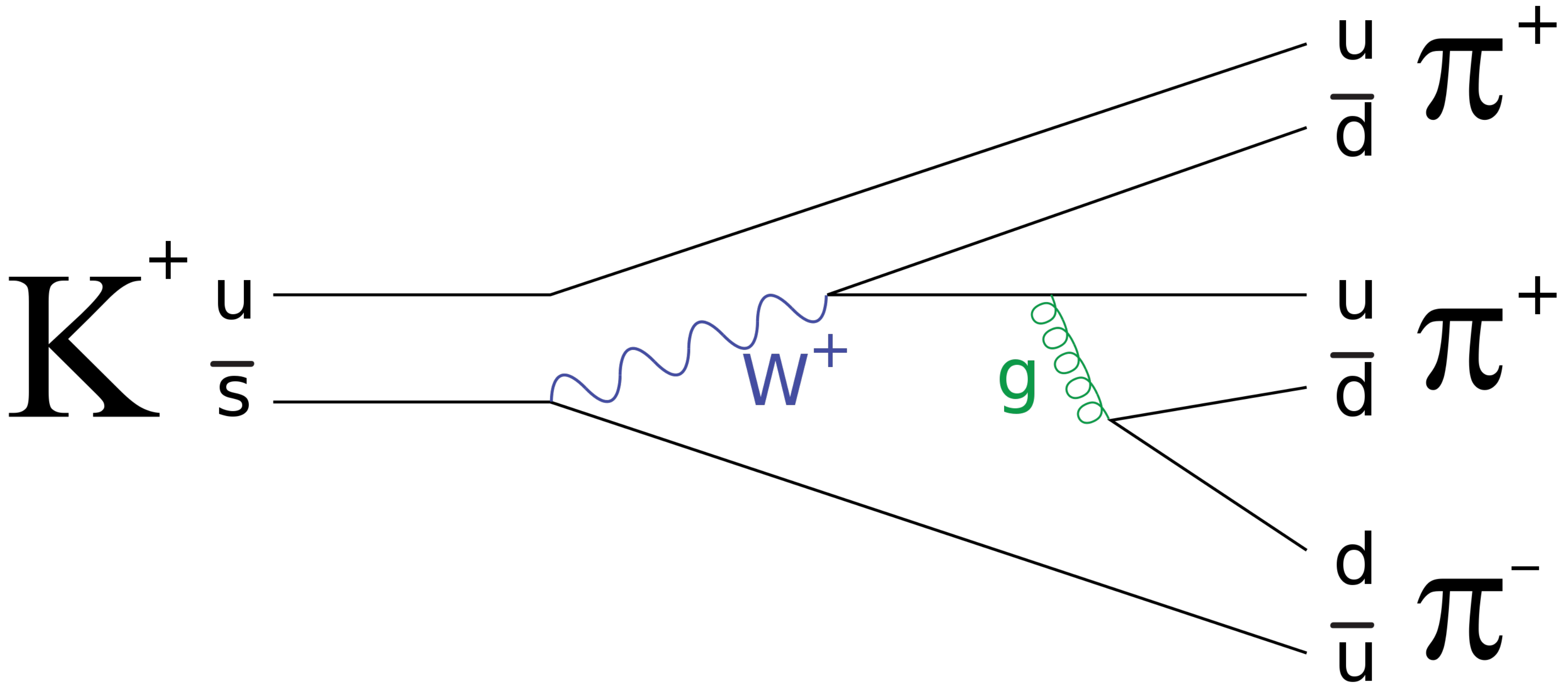
Note:

This is a ***graphic*** representation of a ***calculation*** that represents a quantum event. We use it to ***talk*** about it.

Do ***not*** take it ***literally*** as what happens

Another example

- Kaon decay with *weak* interaction mediated by a W-boson
- Quark anti-quark produced by the *strong* interaction mediated by a gluon



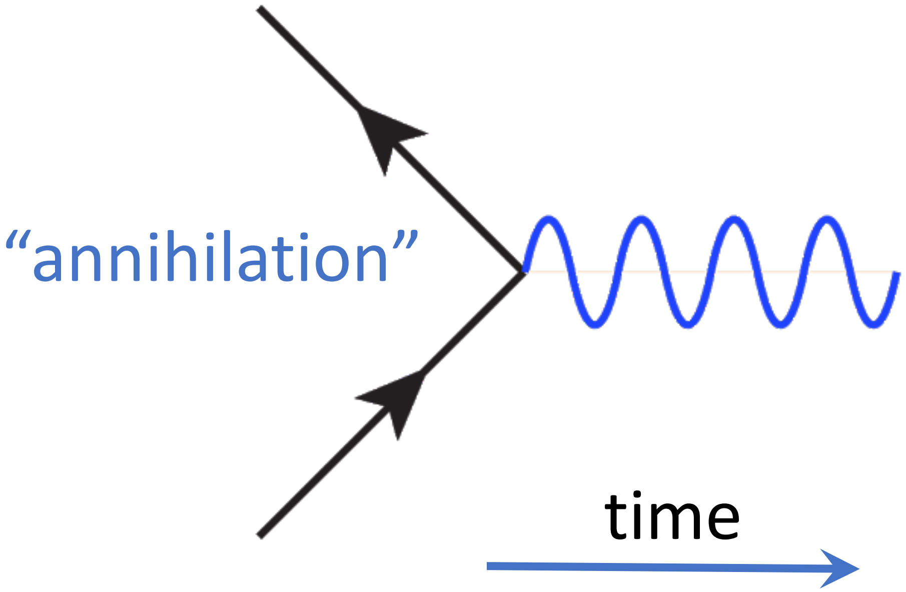
Part 1

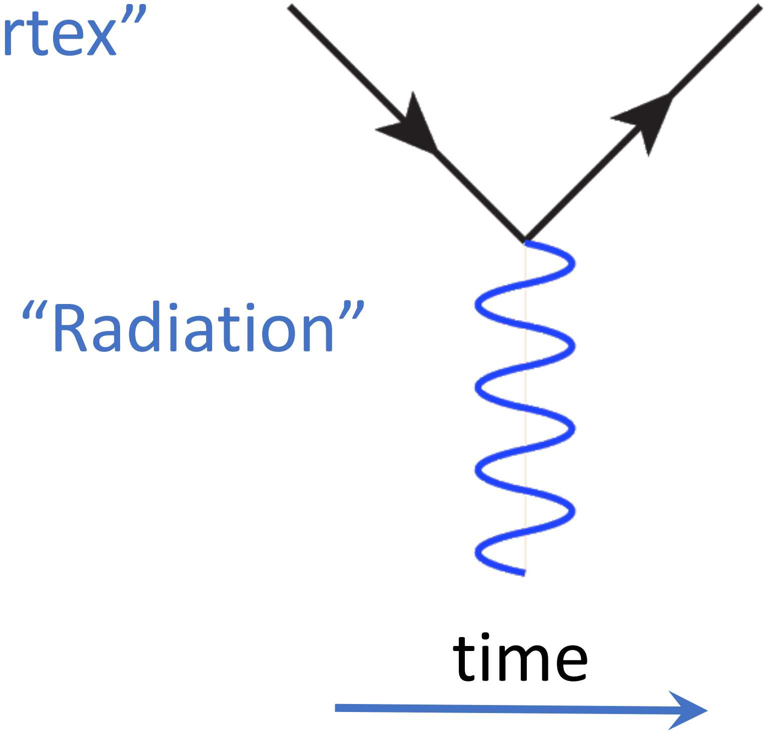
The Electromagnetic Interaction

Quantum Electrodynamics (QED)

Quantum Electrodynamics (QED)

- There is only one fundamental quantum interaction process: ie one "vertex"



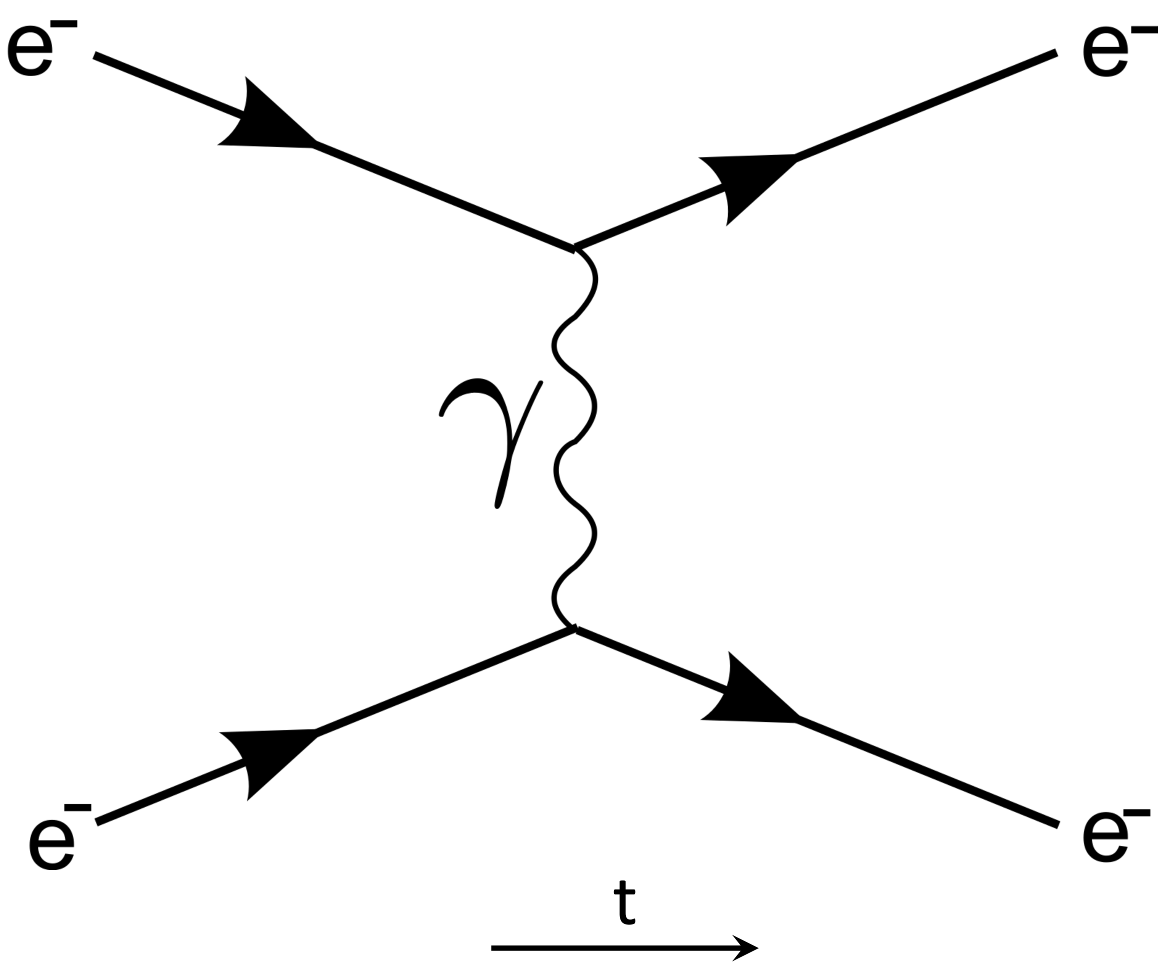


- "strength" of the "vertex coupling" is equal to the elementary charge e
- Note: this vertex can only be part of a process since energy and momentum cannot be conserved at the same time in a "2-to-1" process
  - This is an off-shell or virtual photon

A Real QED Process: Möller scattering

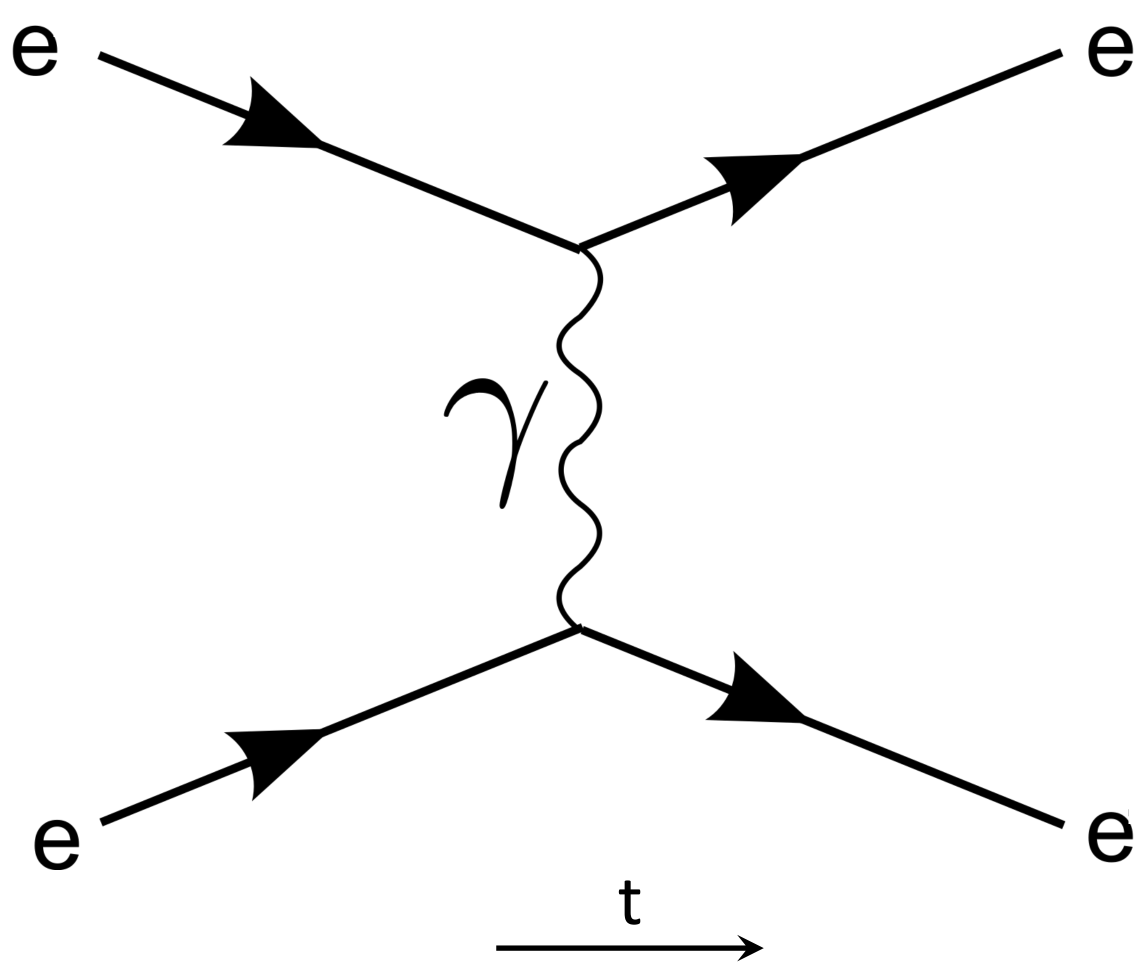
- "Coulomb" repulsion of colliding electrons

- Convention: the direction of an arrow w.r.t. time determines whether it is a colliding particle or antiparticle
  - Should leave out the charge to be unambiguous



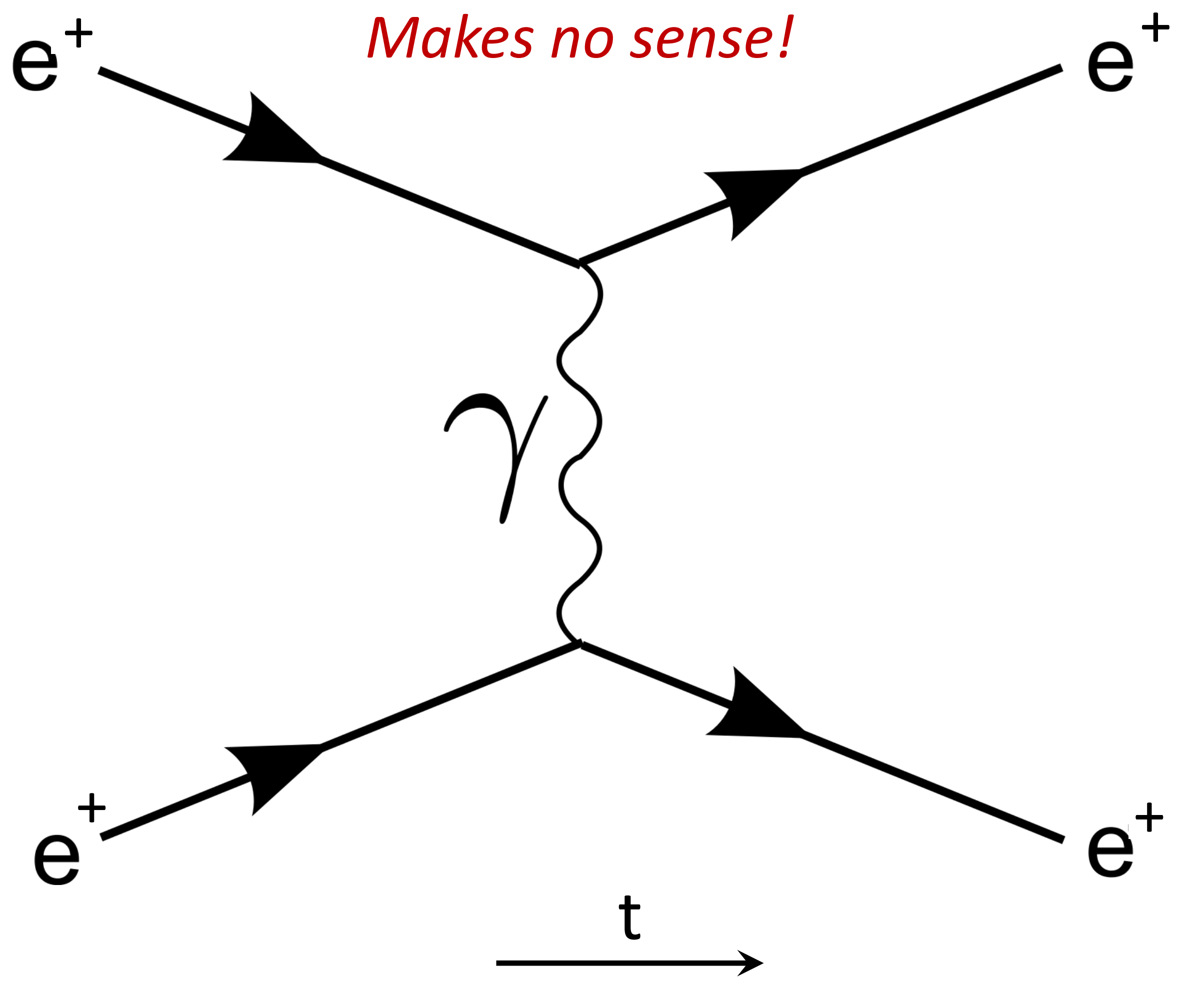
A Real QED Process: Möller scattering

- “Coulomb” repulsion of colliding electrons
- Convention: the direction of an arrow w.r.t. time determines whether it is a colliding particle or antiparticle
  - Should leave out the charge to be unambiguous



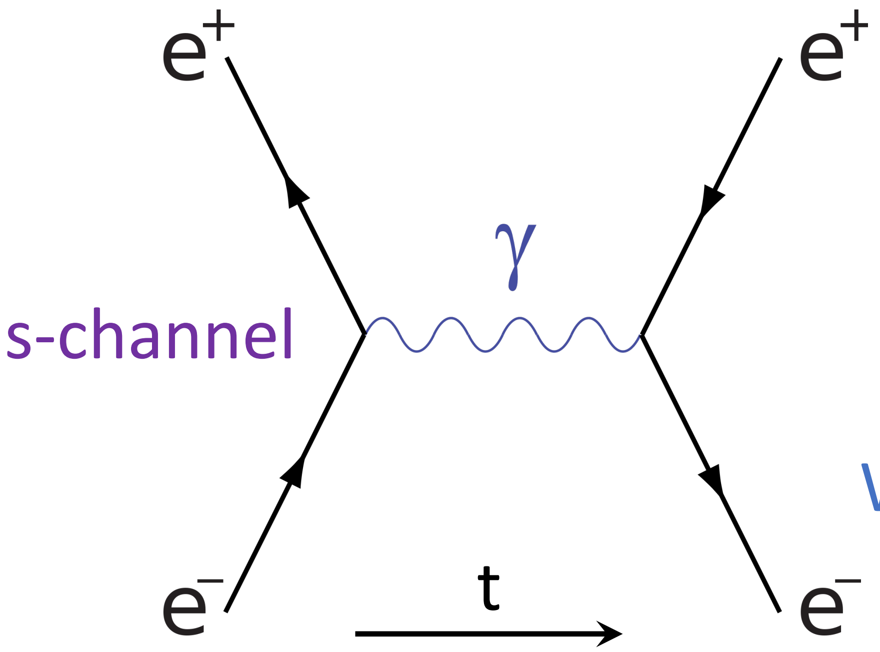
A Real QED Process: Möller scattering

- “Coulomb” repulsion of colliding electrons
- Convention: the direction of an arrow w.r.t. time determines whether it is a colliding particle or antiparticle
  - Should leave out the charge to be unambiguous
- Feynman diagrams are always drawn with *particles*
  - *Anti-particles* are represented with an arrow pointing against the direction of time.

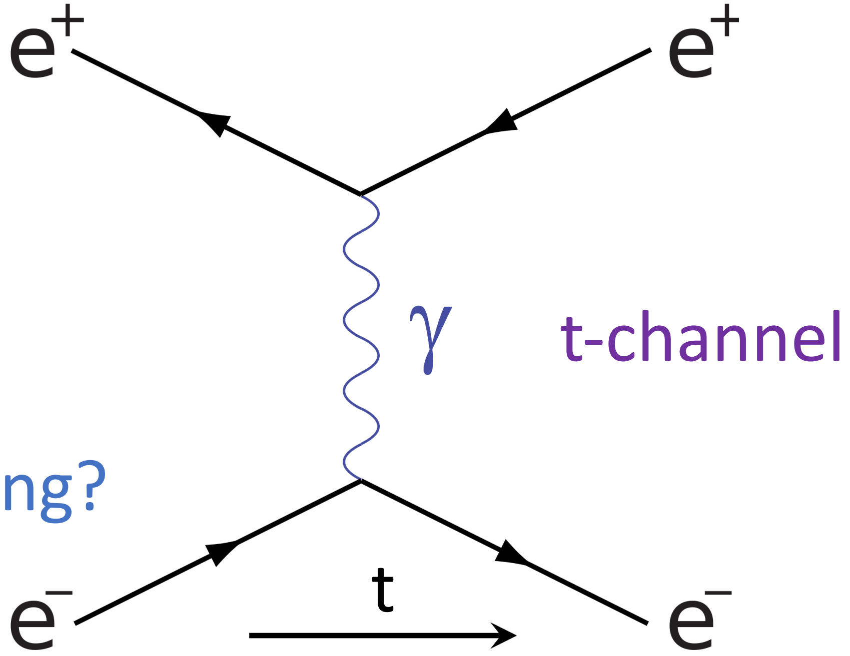


A Real Process: Bhabha Scattering

- Scattering of electron and positron, two quantum processes ("Feynman diagrams") in one real process:
  - Spacelike "s-channel" exchange
    - e^- and e^+ annihilate into a photon, which converts back to a e^- and e^+ pair
  - Timelike "t-channel" exchange
    - e^- and e^+ scatter in each others EM field



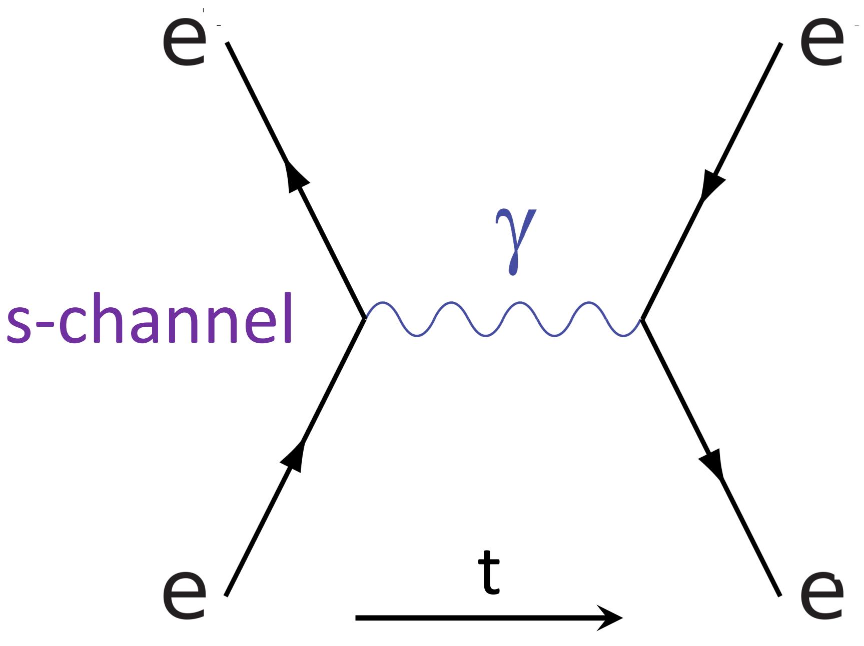
+



What is wrong?

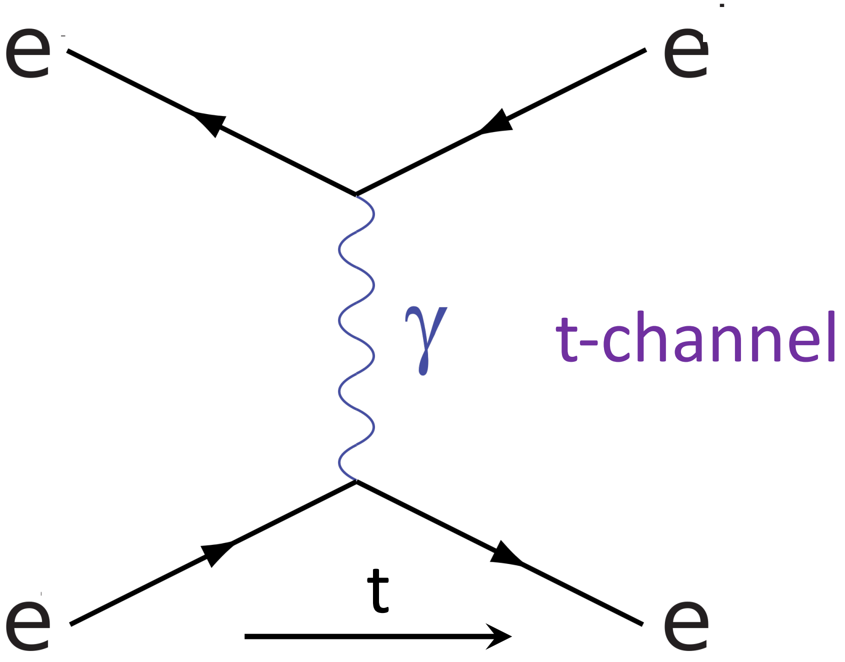
A Real Process: Bhabha Scattering

- Scattering of electron and positron, two quantum processes ("Feynman diagrams") in one real process:
  - Spacelike "s-channel" exchange
    - e^- and e^+ annihilate into a photon, which converts back to a e^- and e^+ pair
  - Timelike "t-channel" exchange
    - e^- and e^+ scatter in each others EM field



+

Now its OK!



Feynman Diagram

- Calculation using Feynman rules:

$$\mathcal{M} = \frac{e^2}{q^2}$$

- Fine structure constant:

$$\alpha = \frac{e^2}{4\pi}$$

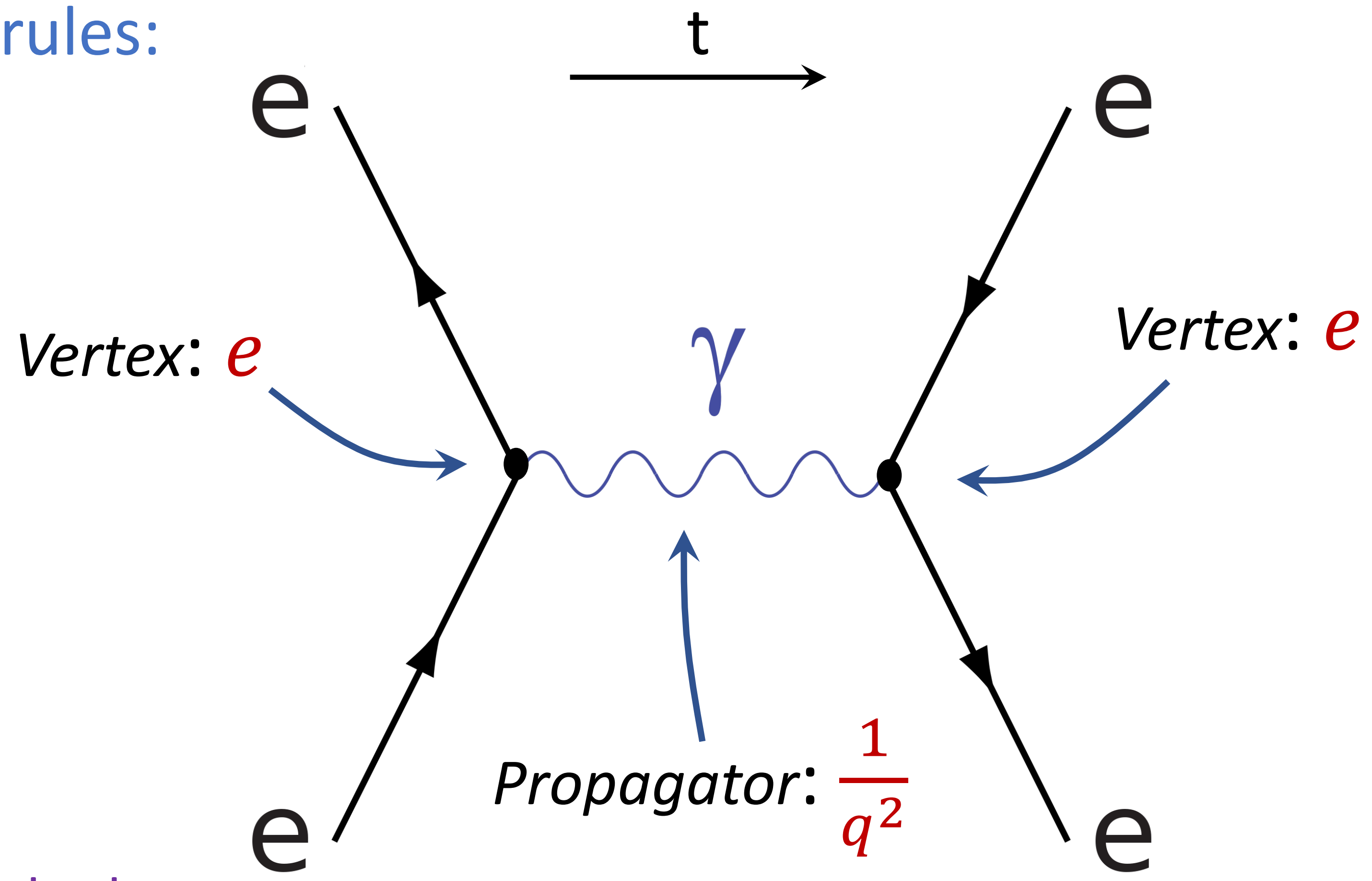
Hence: $\mathcal{M} \propto \frac{\alpha}{q^2}$

q^2 is “virtuality” of the photon

- Strength of the interaction
  - Determined by the coupling constant e:

$$\alpha = \frac{e^2}{4\pi} = 1/137$$

why???

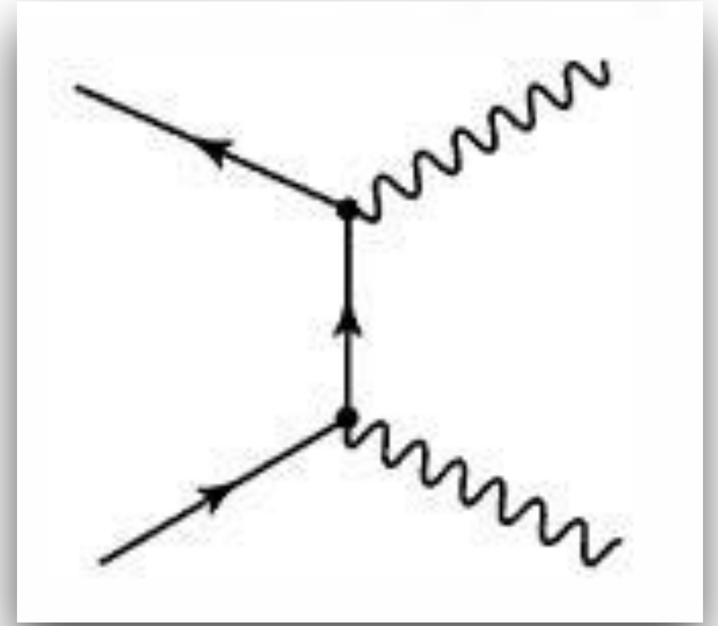


Note: a photon should have $q^2 = 0$!

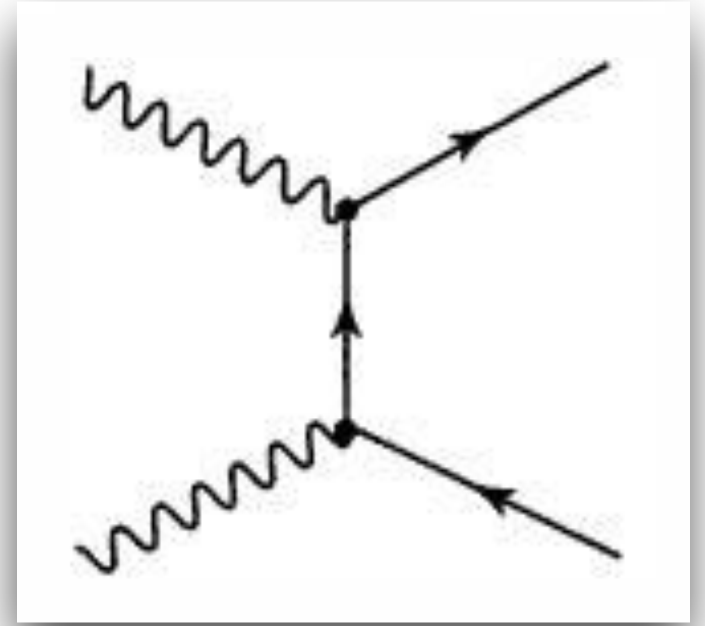
Scattering with photons

Real processes with external photon lines

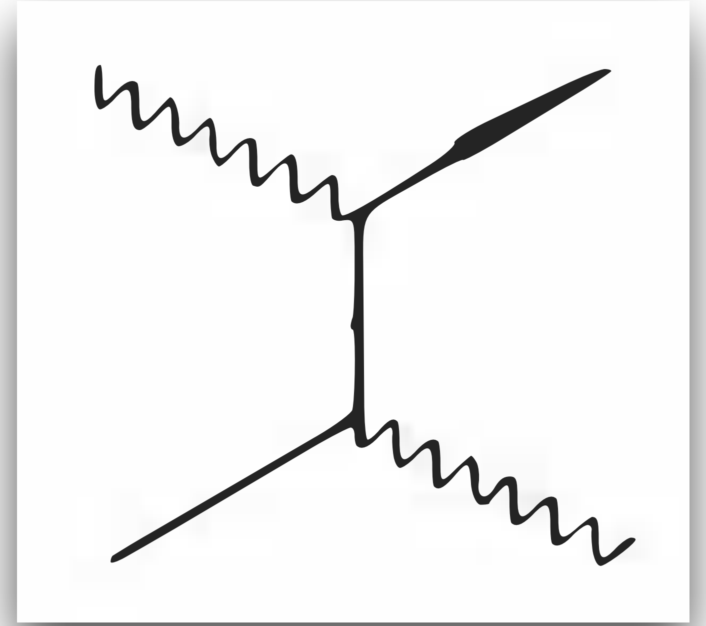
t



Pair annihilation



Pair production



Compton scattering

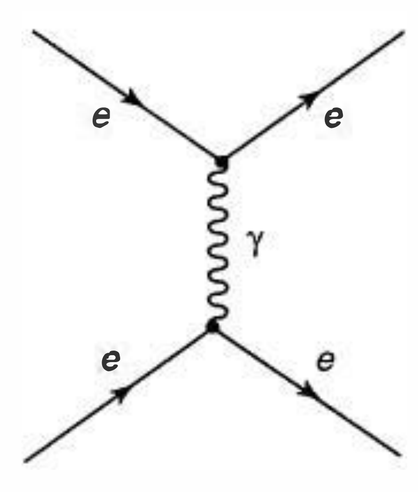
Perturbation Theory

See lecture 4

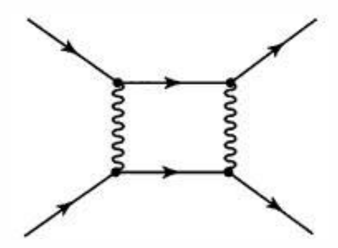
Feynman diagrams we use describe the lowest order "Perturbation Theory".
Calculations with Feynman diagrams are never exact, always an approximation.

- We simply do not know how to do exact calculations in particle physics
- QED is extremely precise because many complicated diagrams have been calculated

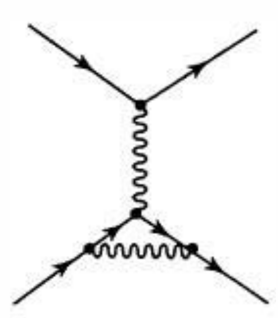
Coulomb scattering: $e^- e^- \rightarrow e^- e^-$



+

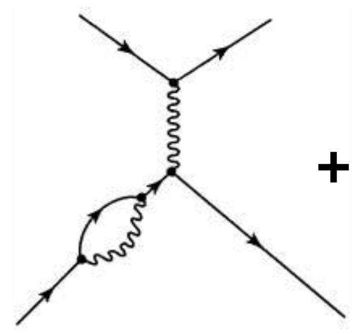


+



+

+



+ $O(\alpha^6)$

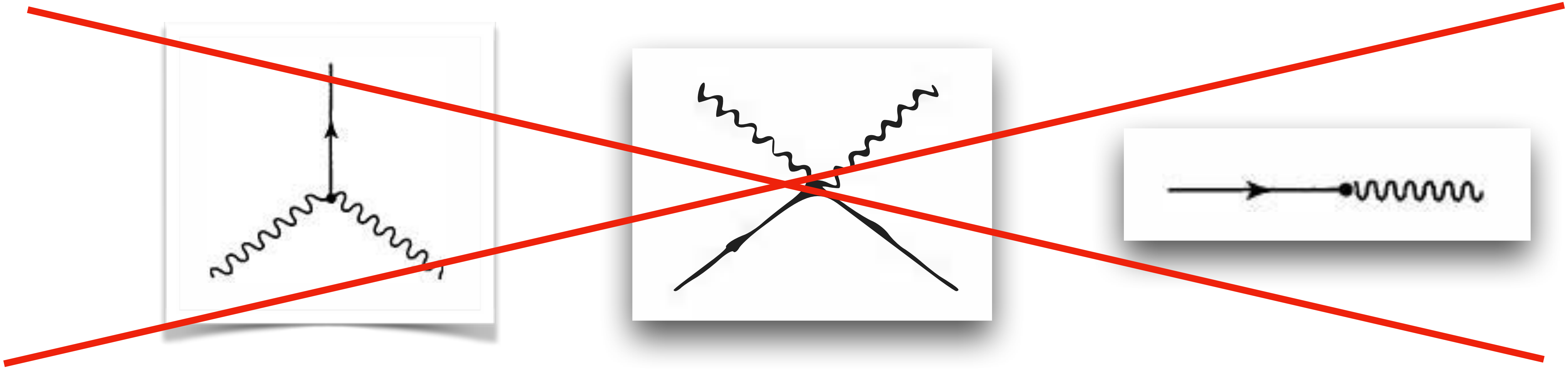
~ α^2 ~ α^4 ~ α^4 ~ α^4 ~ α^4

'Leading Order': LO

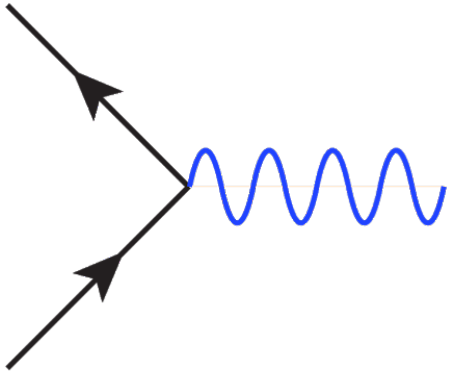
'Next-to Leading Order': NLO

NNLO

Not all diagrams exist



- You can only use combinations of the fundamental QED vertex:



The background of Feynman Diagrams

- More about scattering and Feynman diagrams in Lecture 4

- For an intuitive approach to Feynman diagrams, see wikipedia: https://en.wikipedia.org/wiki/Quantum_electrodynamics and references therein, or the booklet of Feynman:
  - “The strange theory of light and matter”

Griffiths section 2.4

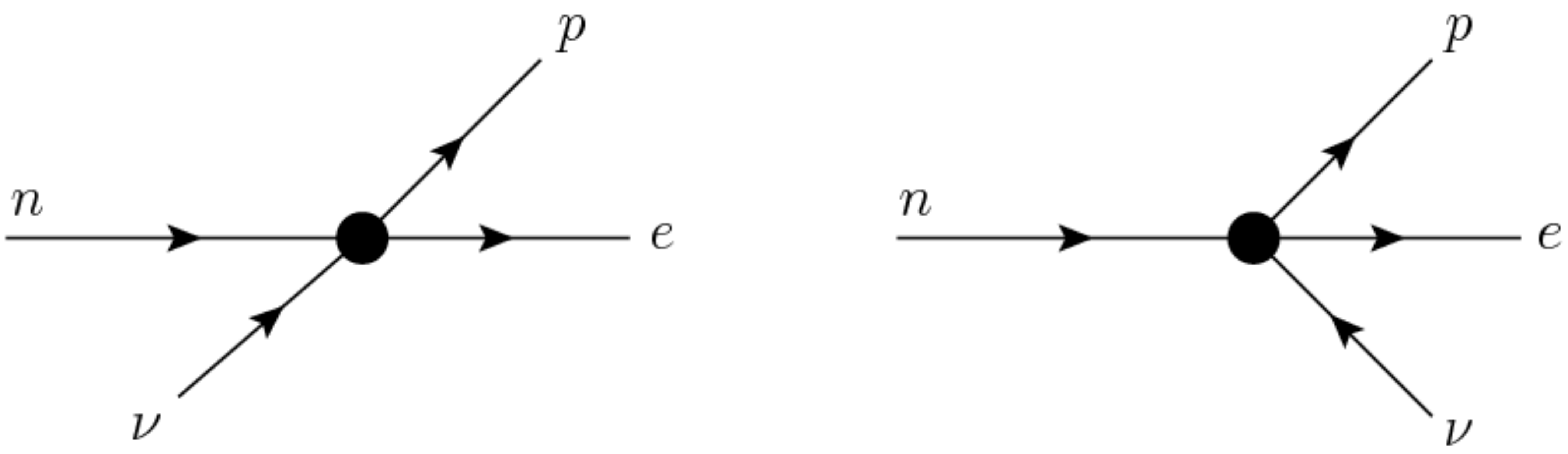
Part 2

The Weak Interaction

Historical: Fermi 4-point interaction

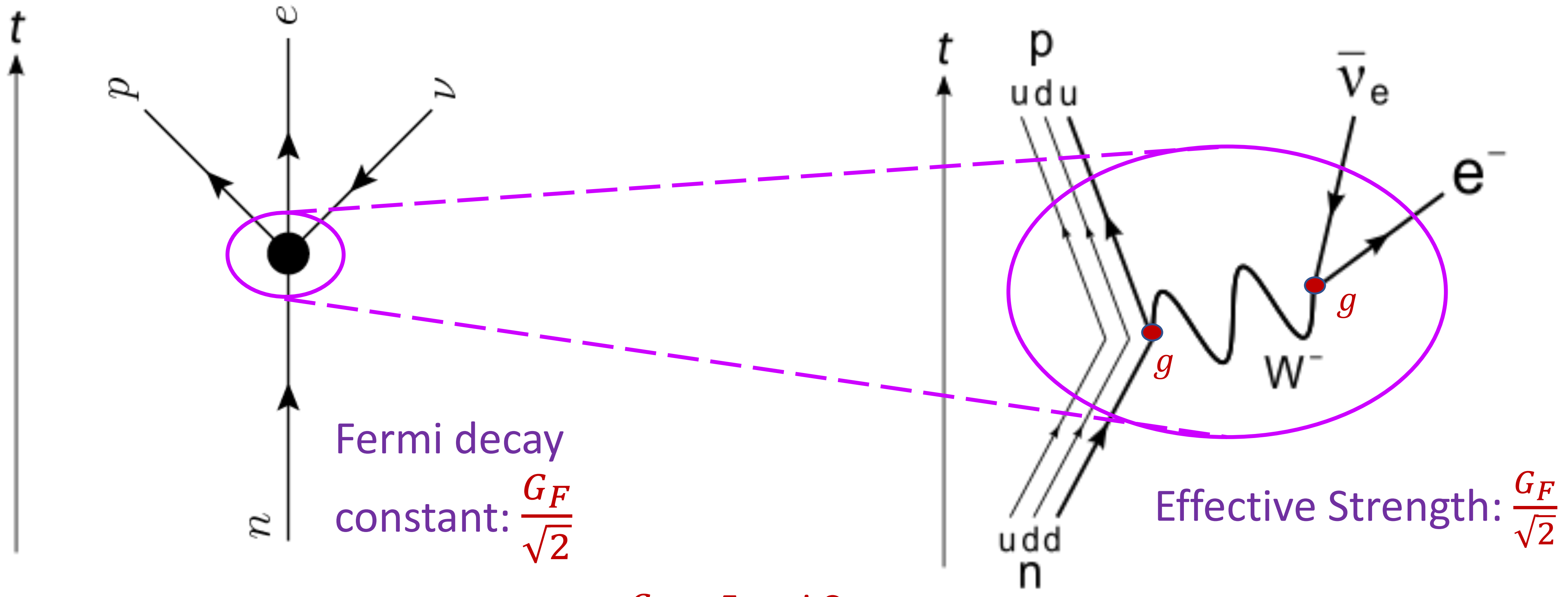
- Becquerel discovered radioactivity base on the weak force
  - $n \to p + e + \overline{\nu}$
- The original model for the weak interaction is from Fermi
  - For obvious reasons it is called the 4-point ‘contact’ interaction (1933).
  - The strength of the interaction was given by Fermi’s constant $G_F = 10^{-5}$

$$n + \nu \to p + e \qquad\qquad n \to p + e + \overline{\nu}$$



Quantum zoom-in on Fermi 4-point interaction

- Fermi 4-point interaction
- Very short range W exchange

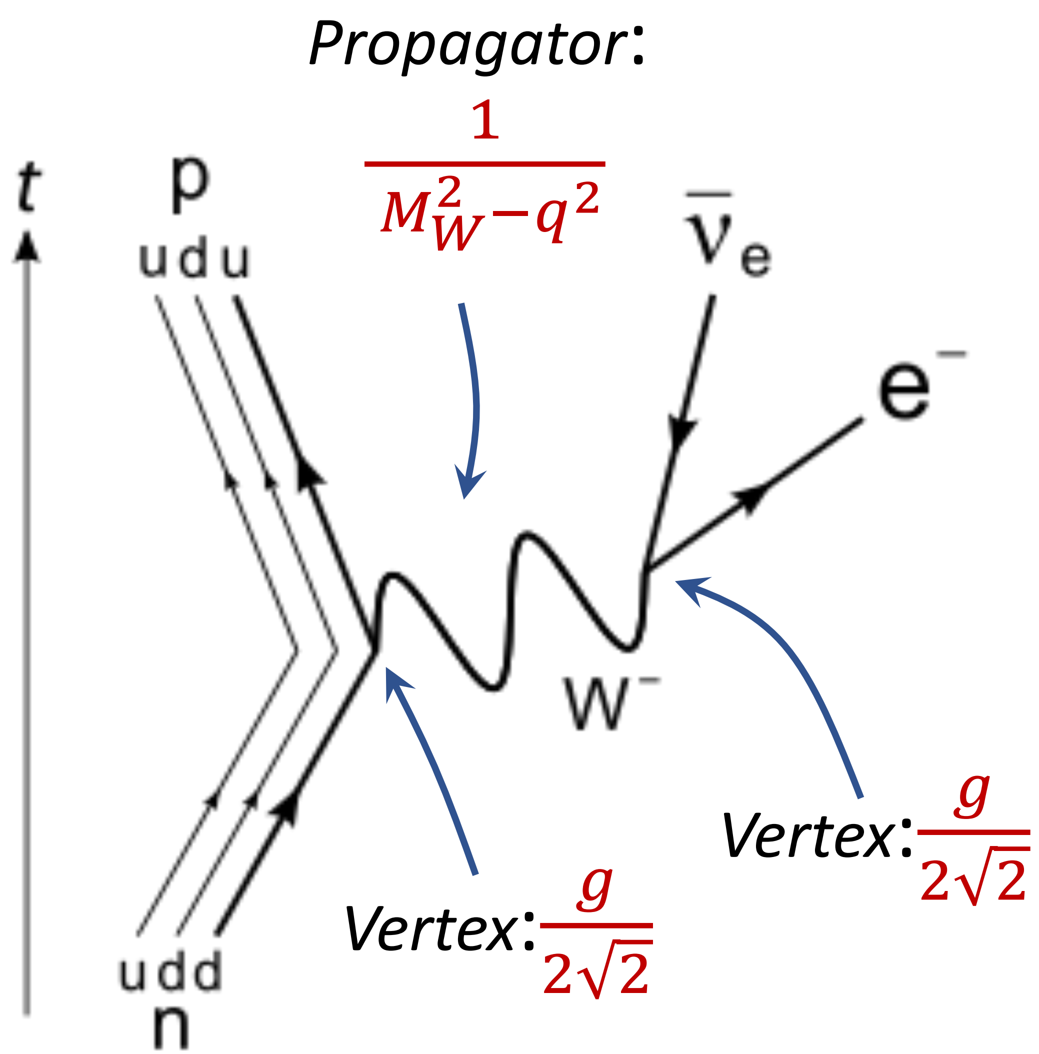


G_F = Fermi Constant

- Similar to electrodynamics the weak interaction has a propagator
  - W spin-1 boson
- At quark level it involves a $d \rightarrow u$ transition

The Weak Interaction

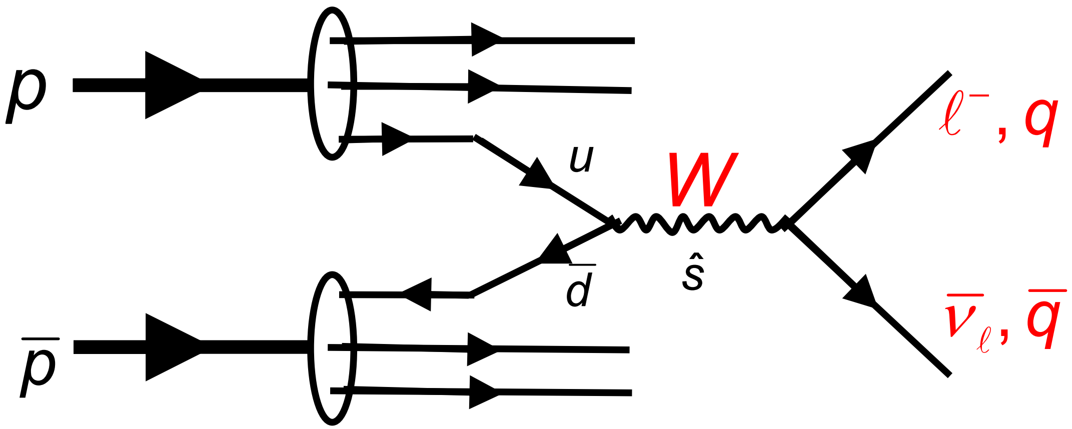
- Feynman rules



- W-boson has electric charge
  - The process is called a *charged current*
- q^2 is the momentum transfer from the hadronic to leptonic vertex
  - Here due to mass difference $m_n - m_p$
- Effect of propagator is only noticeable when $q^2 \sim M_W^2$
  - If q^2 is large: ***resonance***
    - In that case weak force is strong
  - If q^2 is small: then effectively a 4-point Fermi coupling
    - Propagator $\sim 1/M_W^2$ ➔ weak force
    - $G_F = \frac{g^2}{8M_W^2}$; $\alpha_W = \frac{g^2}{2\pi} = \frac{1}{30} > \alpha_{QED}$
- The weak coupling constant is large!
  - Interaction is weak due to large W mass

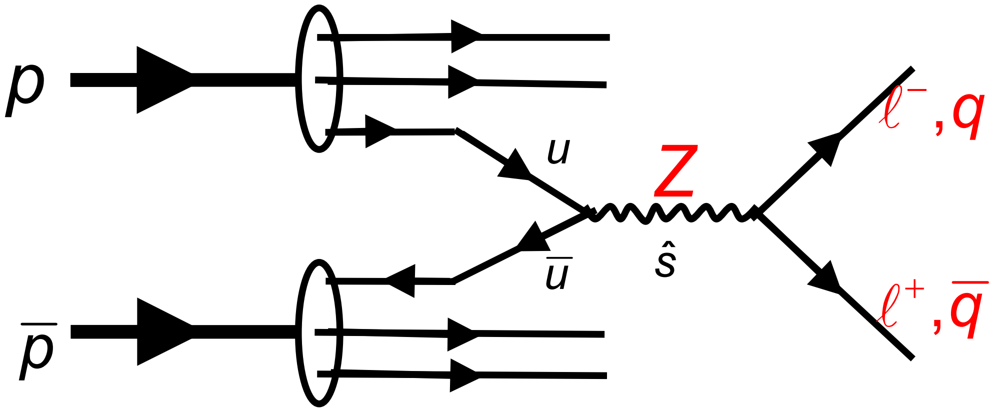
W and Z bosons

- Glashow, Salam, Weinberg (GSW model, 1968) predict three weak mediators
  - W^+ , W^- , Z - bosons
- Discovery of the W and Z bosons at CERN, 1983, UA1 and UA2 experiments
  - Look at high energy proton-proton collisions
  - $M_W = 80\ GeV/c^2$, $M_Z = 91\ GeV/c^2$ as predicted by GSW model



$$p\bar{p} \rightarrow W \rightarrow \ell \bar{\nu}_\ell + X$$

Flavour *changing charged* currents: $u\bar{d}$

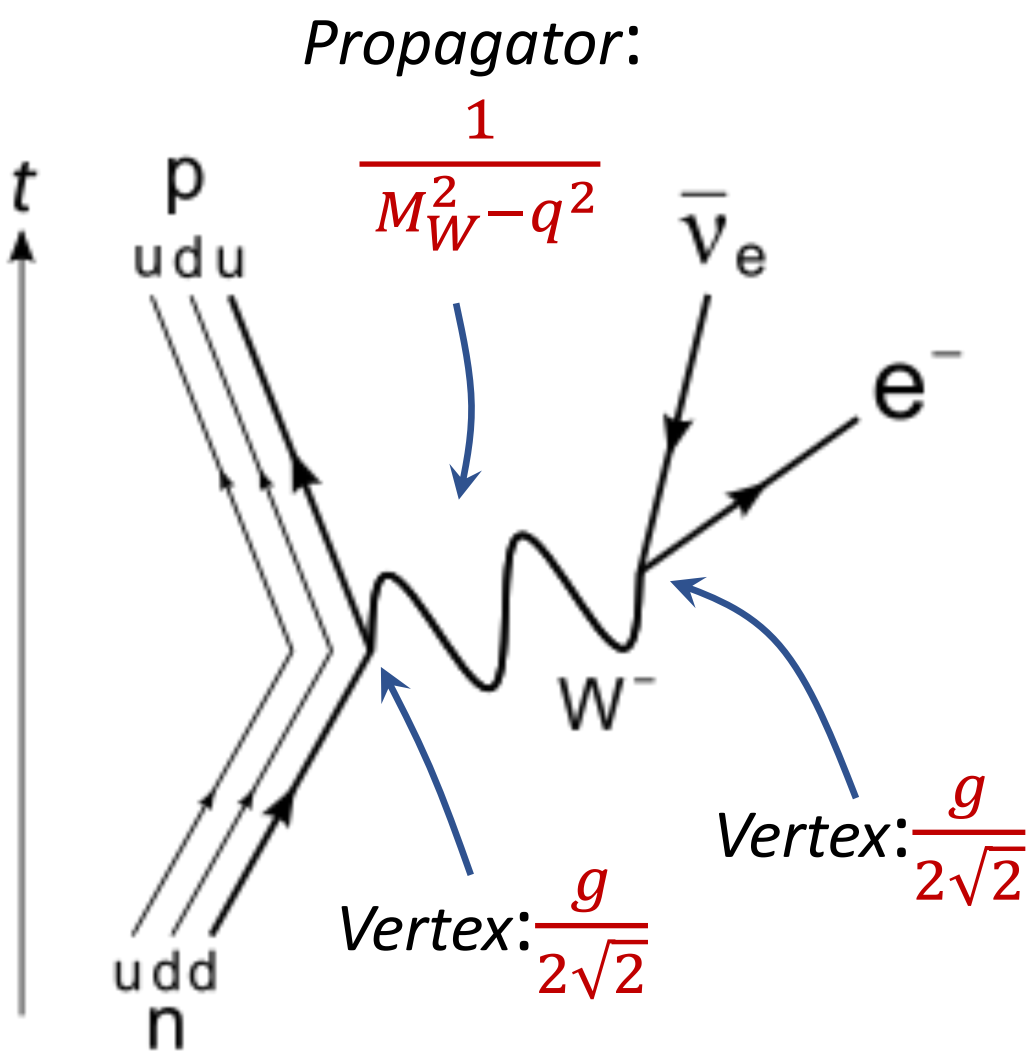


$$p\bar{p} \rightarrow Z \rightarrow f\bar{f} + X$$

Flavour *conserving neutral* currents: $u\bar{u}$

Virtuality

- How can beta decay $n \to p + e + \bar{\nu}$ work, if the W has a mass of 80 GeV??



- Exchange if “virtual” particles, also called “off-shell”: $E^2 - \vec{p}^2 \neq m^2$
- Heisenberg:

  E is undetermined as $\Delta E \Delta t \geq \hbar/2$ with $\Delta E = \sqrt{\vec{q}^2 + m^2}$

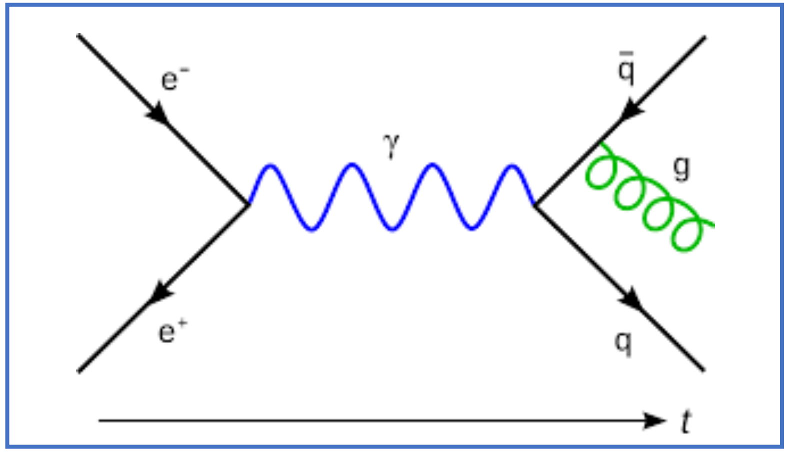
  For small q → range force R

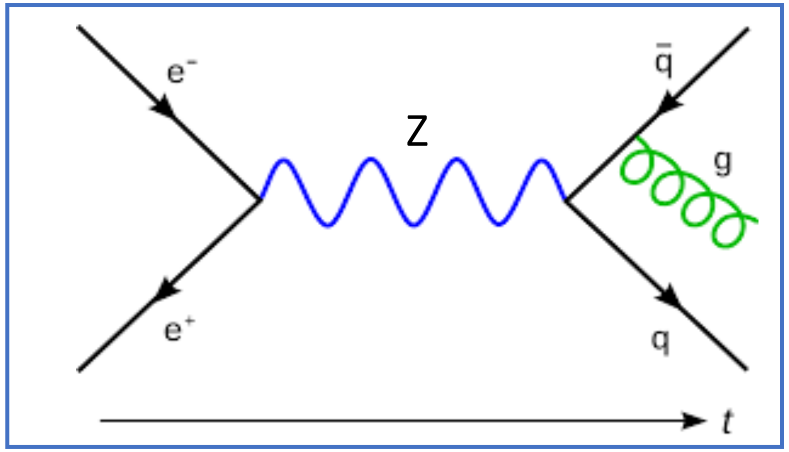
  $R \sim c\Delta t \approx \frac{\hbar}{2mc} \sim 1.2\times10^{-18}\, m$

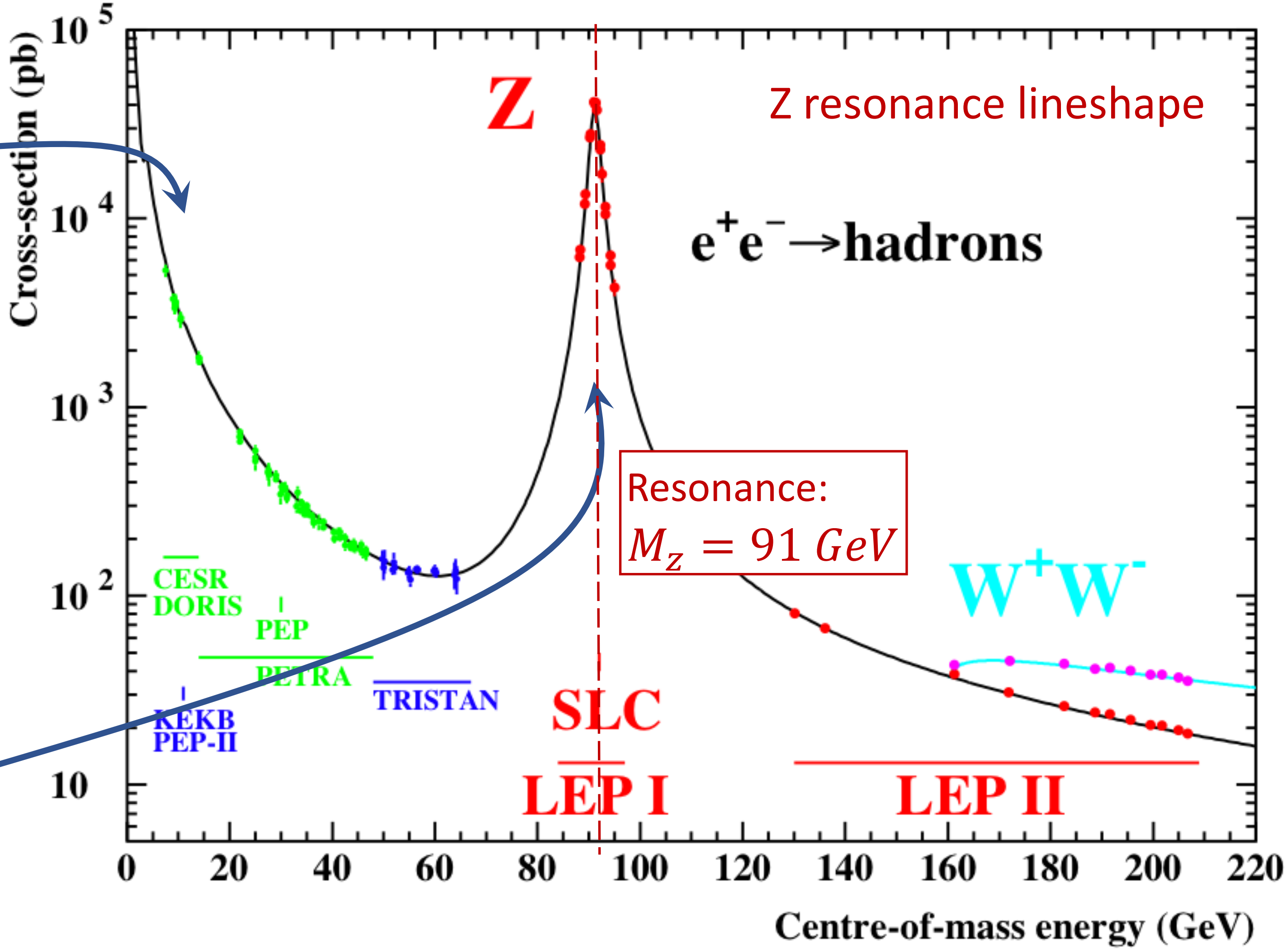
- Notice the resonance behaviour for $q^2 \approx M_W^2$

Precision studies with Z bosons: “Electroweak Force”

- The LEP collider did many precision studies of the weak interaction with e^+e^- annihilation collisions at collision energy around 91 GeV/c^2.

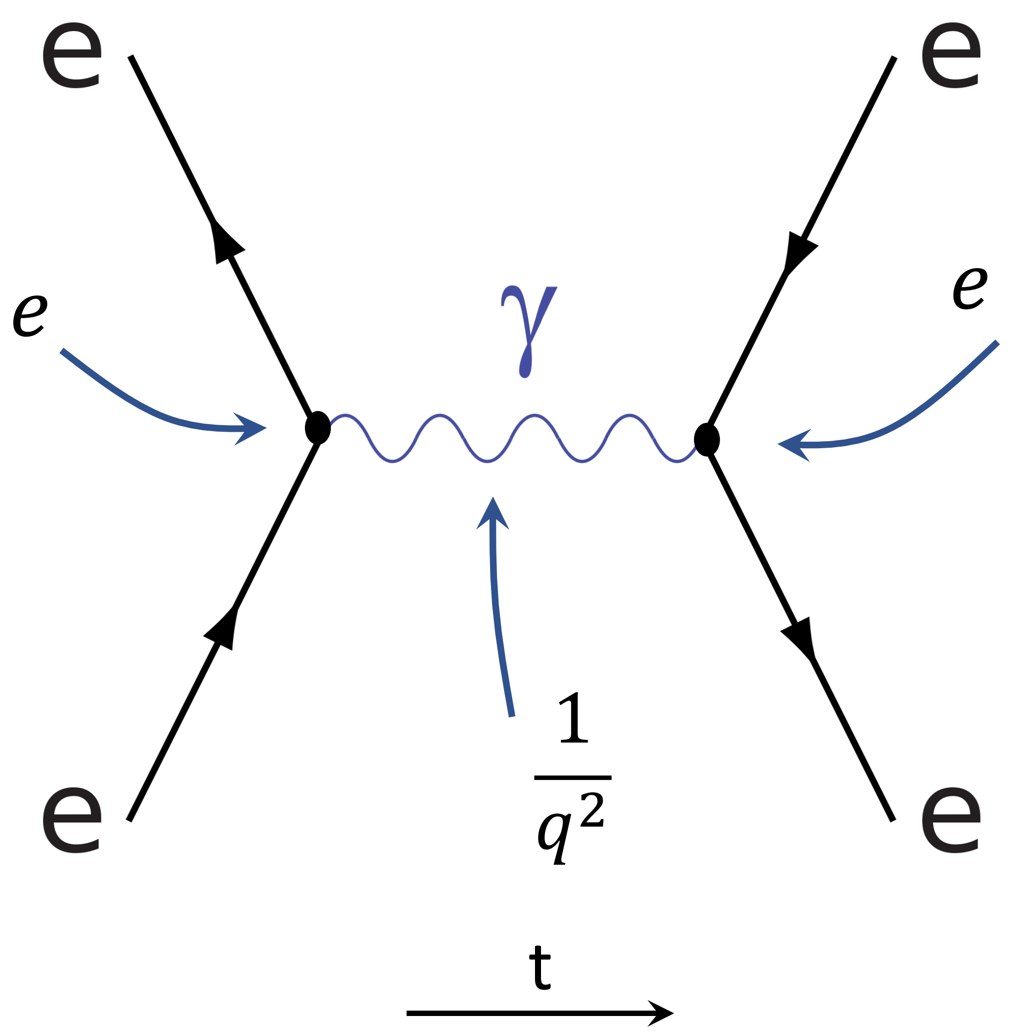




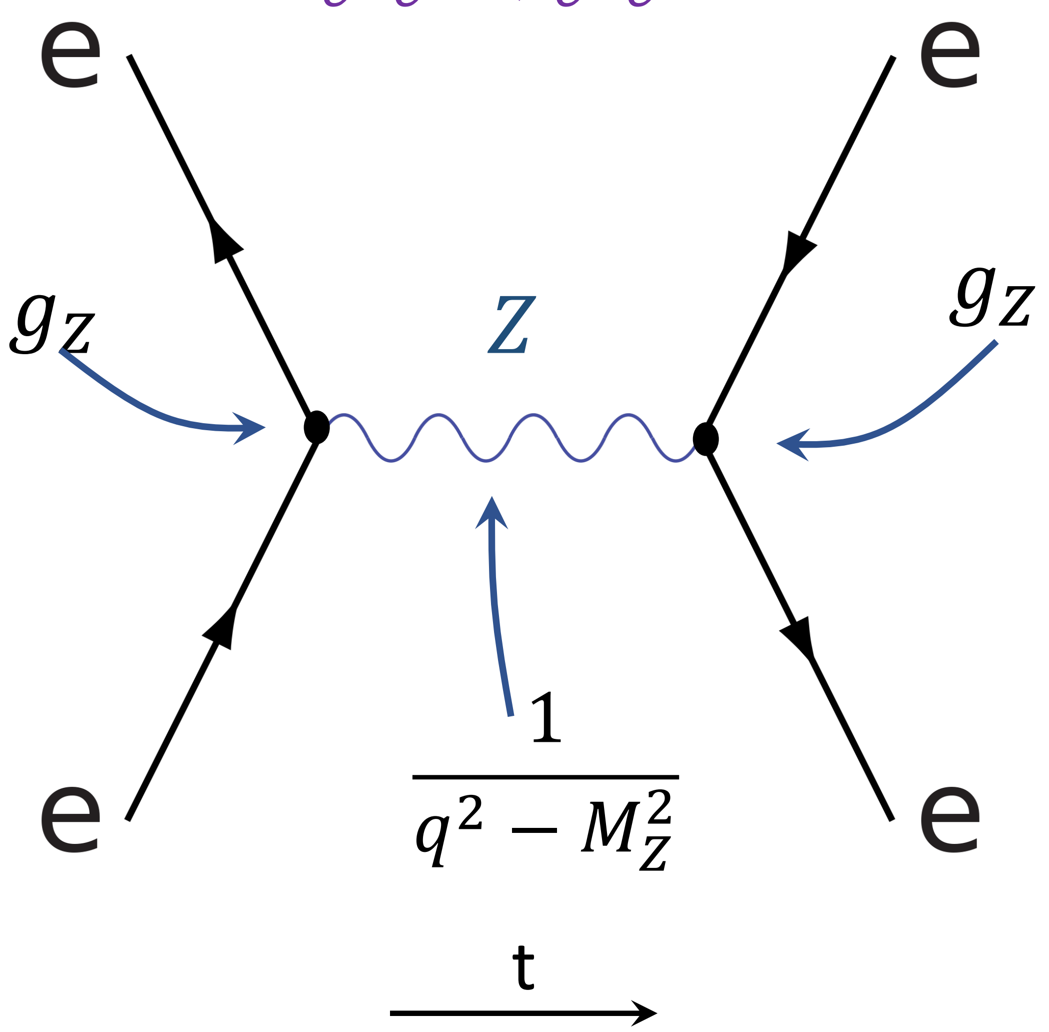


Similarity of Electromagnetic (γ) and Weak (Z) force

- Electromagnetic:
  $e^+e^- \rightarrow e^+e^-$

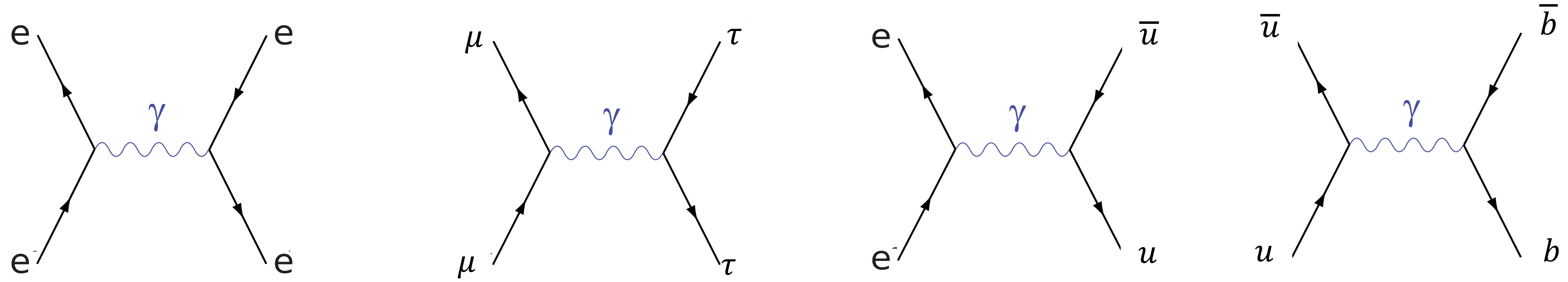


- Weak neutral current:
  $e^+e^- \rightarrow e^+e^-$

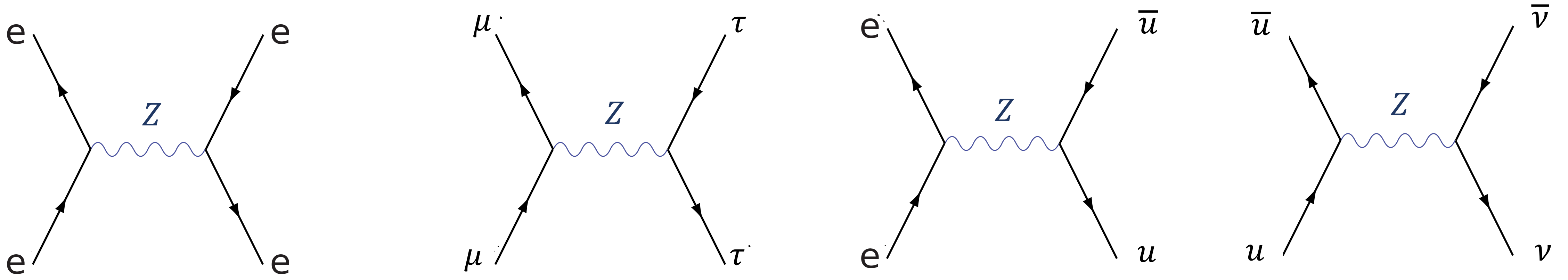


Examples of processes: “neural currents”

- Electromagnetic: couples to electric charge



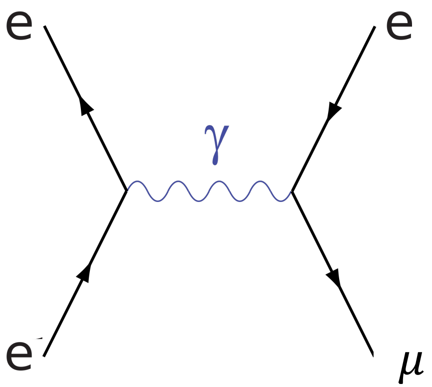
- Weak “Neutral Currents”: couples to weak charge

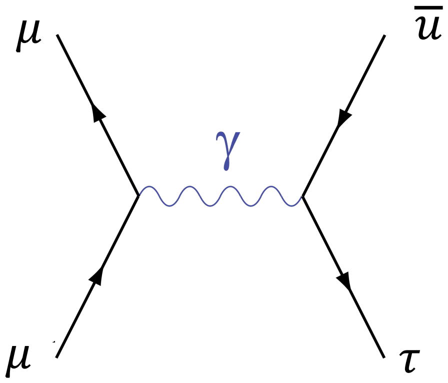


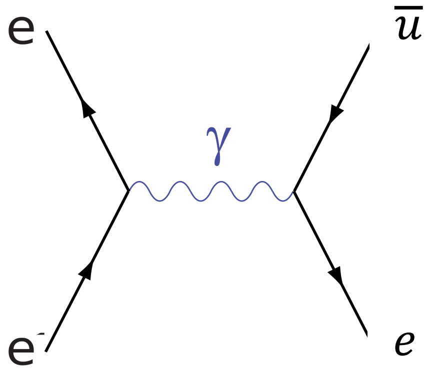
Z boson also couples to neutrinos: neutrinos have weak charge

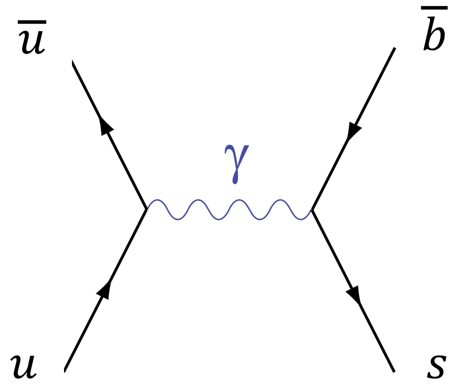
Examples of processes that ***do not exist***

- Electromagnetic:

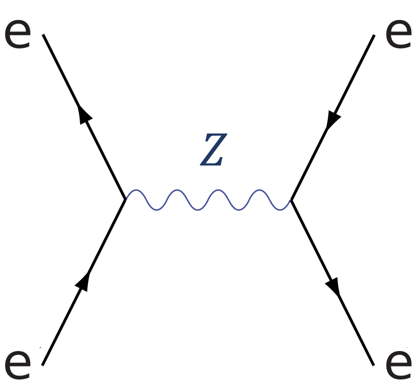


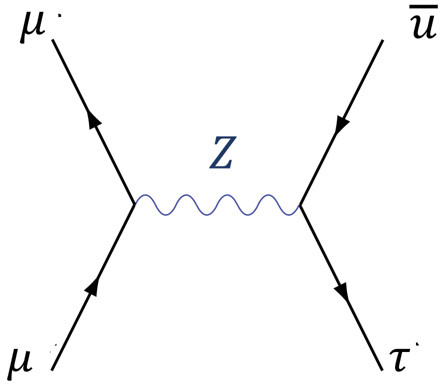


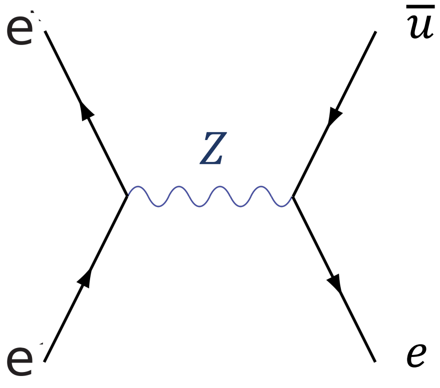


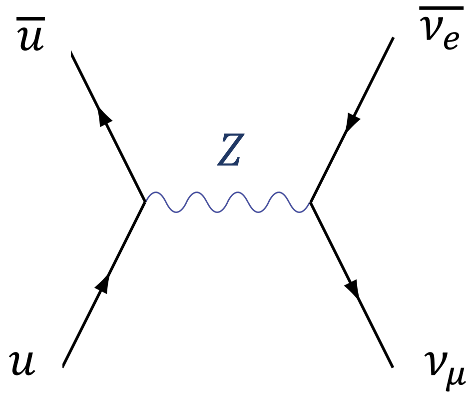


- Weak “Neutral Currents”:





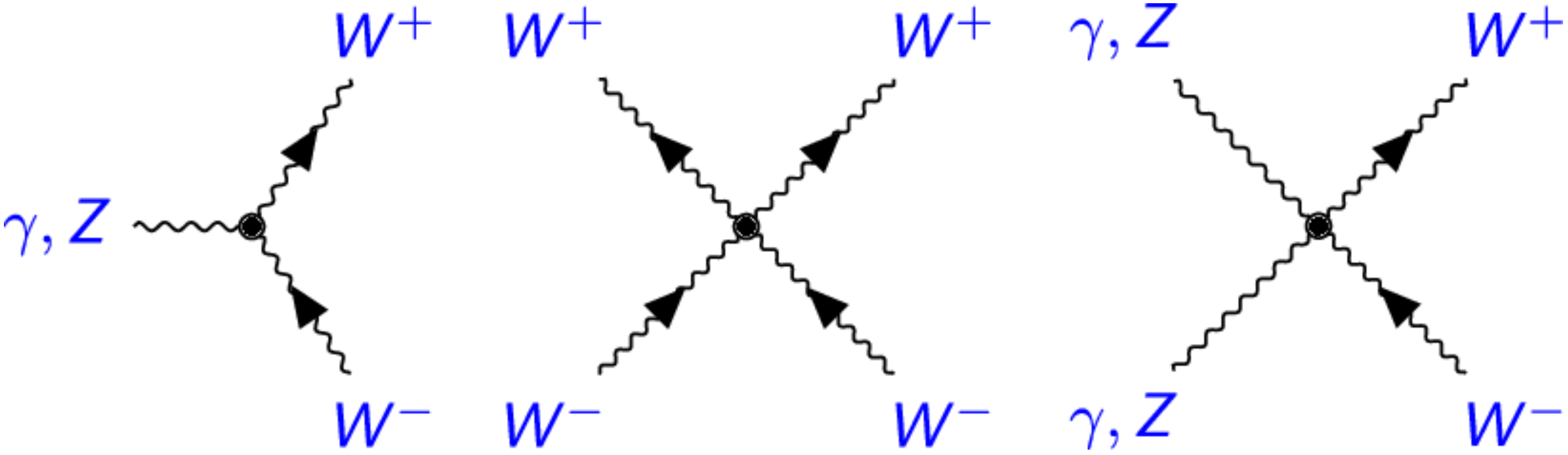




γ and Z only couple to particles within one generation and only to quark or lepton pairs

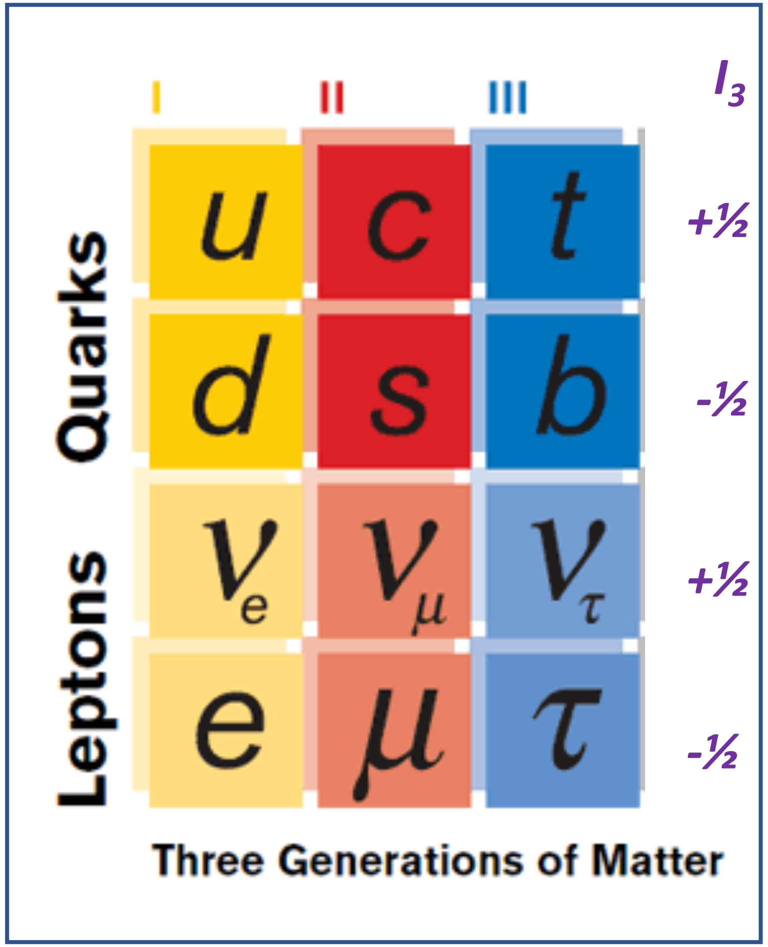
Self coupling

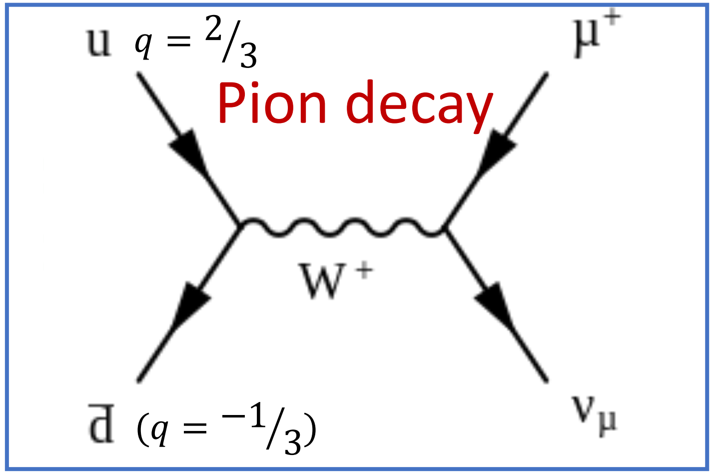
- In electromagnetism a photon has no electric charge
- In the weak interaction the force carriers have charge themselves
  - The weak charge is called “weak isospin”
  - The following diagrams are possible

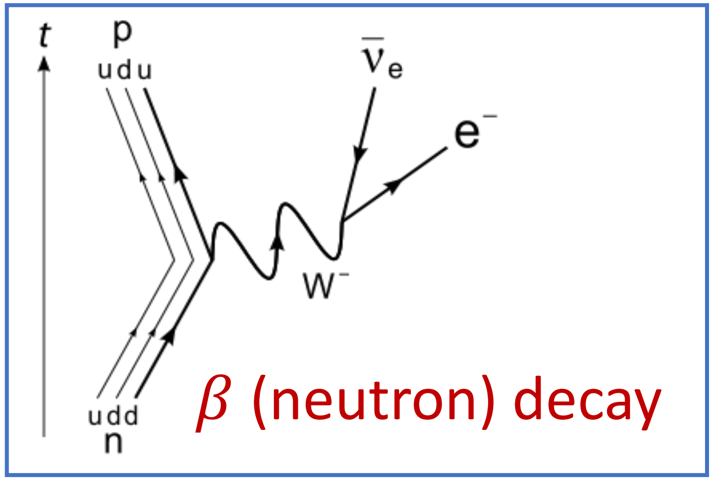


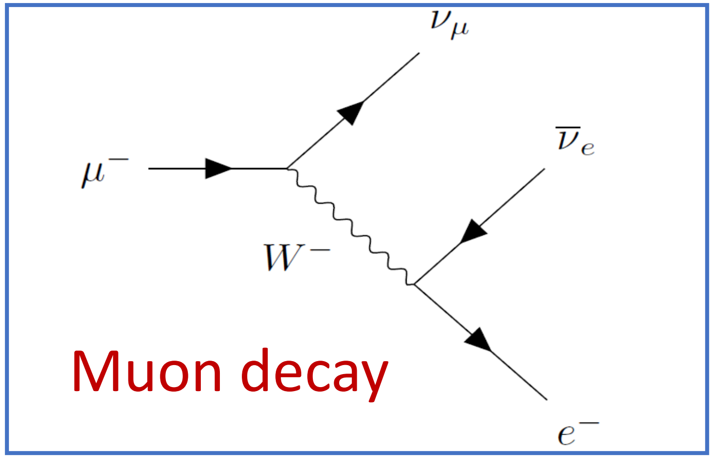
The W weak force: “charged currents”

- The W-boson carries electric charge
- The W connects the weak isospin (I_3) ‘up’ and ‘down’ type particles of a generation
  - W^+ and W^- do the opposite
- 2nd and 3rd generation muon and tau can decay because neutrino’s are light!







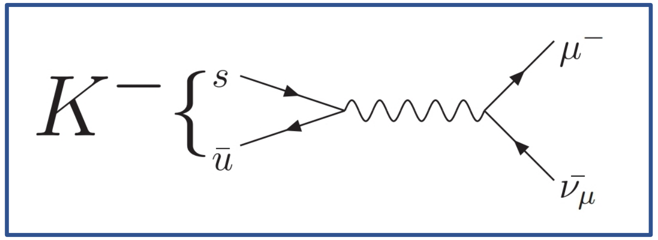


- How does a kaon decay?
  - $K^- = s\bar{d}$
- The c-quark is heavier than the s quark:
  - $s \to c\ e\ \overline{\nu_e}$ is not possible

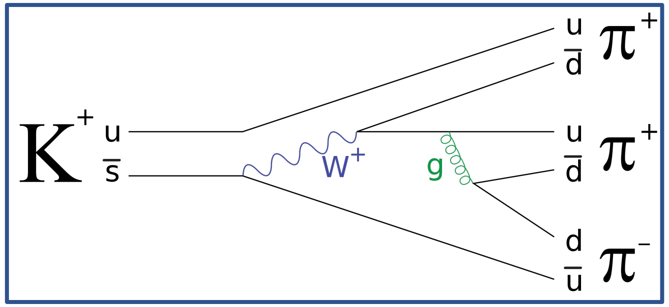
Generations

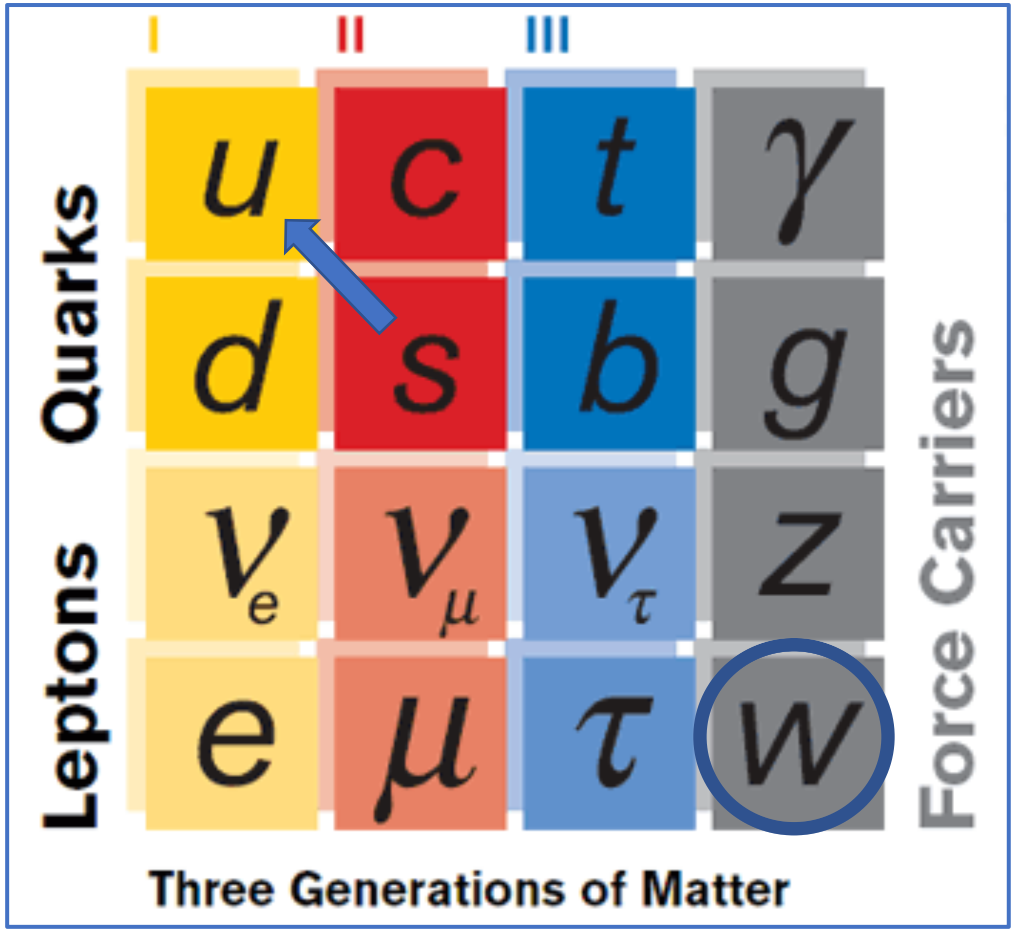
- Does the W ***only*** couple between particles of one generation?
  - Leptons: yes
  - Quarks: no!
    - Couplings between generations are possible for the W!

- Kaon decays
  - ”Leptonic”: $K^+ \rightarrow \mu^+ \nu_\mu$



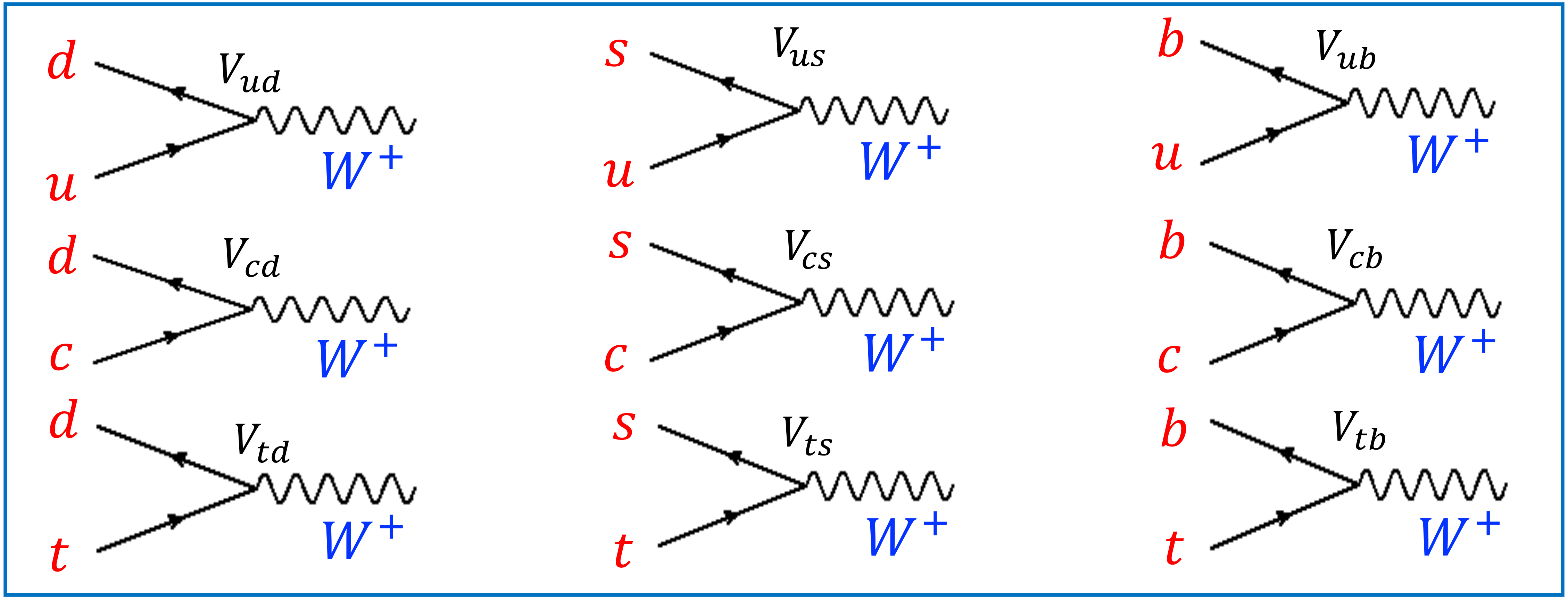
  - ”Hadronic”: $K^+ \rightarrow \pi^+ \pi^+ \pi^-$



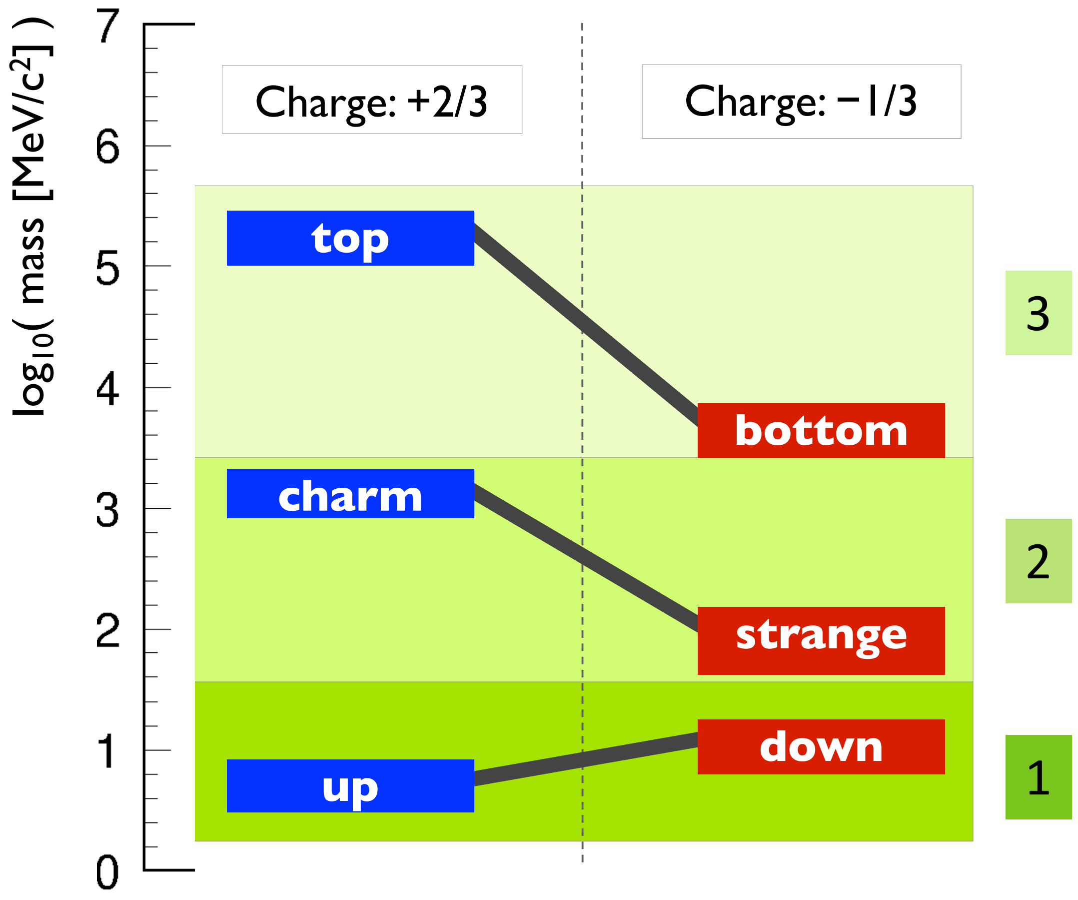


The strange world of W-quark couplings

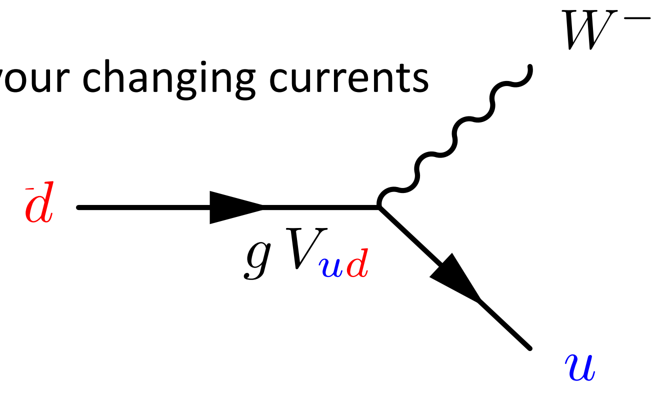
- All possible combinations of "up"-type to "down"-type quark couplings are possible
  - They are not all equally strong!



Flavour Changing Quark Interactions

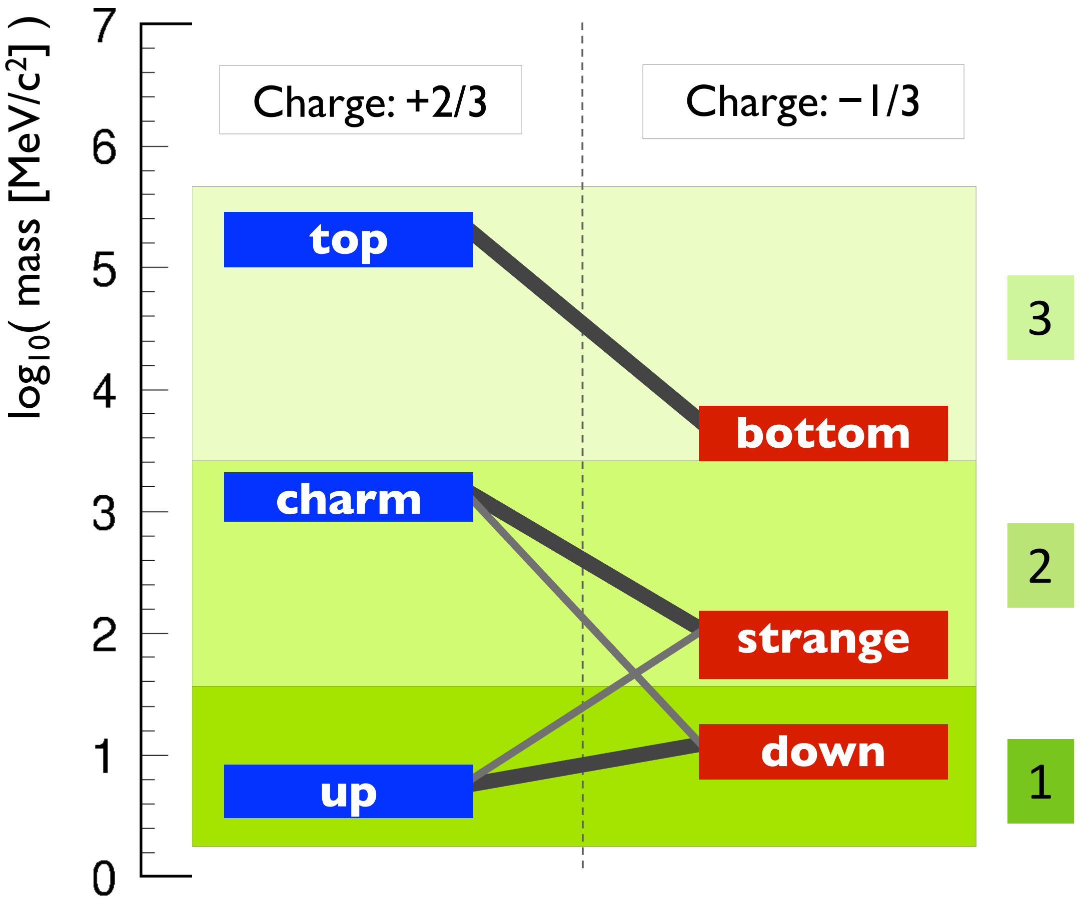


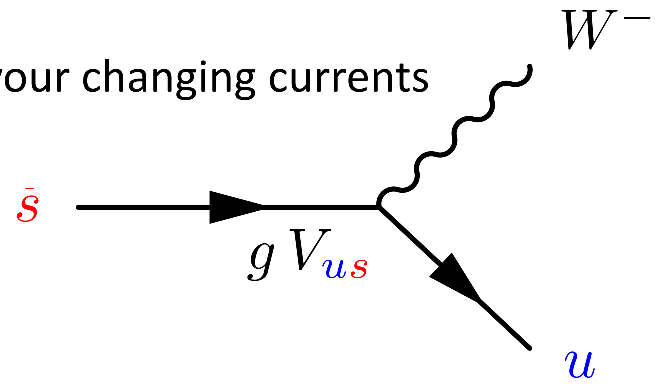
Flavour changing currents



$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & & \\ & V_{cs} & \\ & & V_{tb} \end{pmatrix}$$

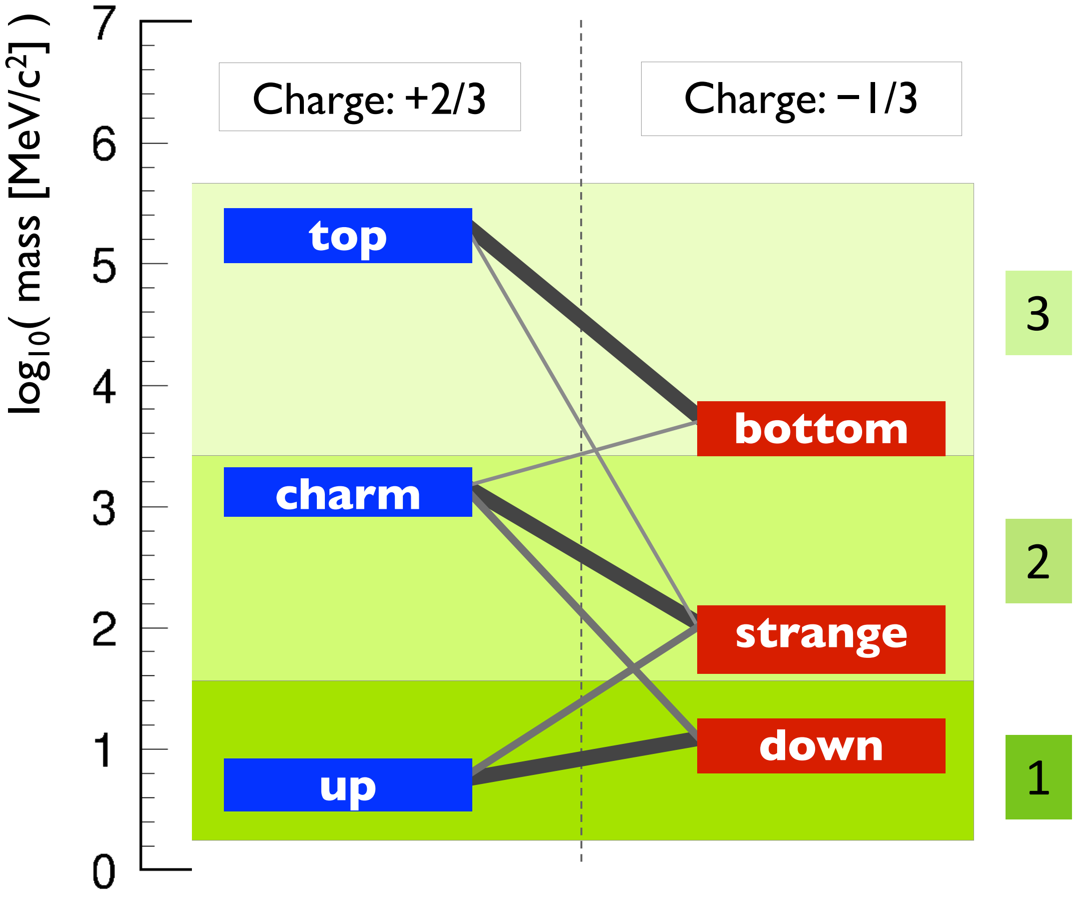
Flavour Changing Quark Interactions



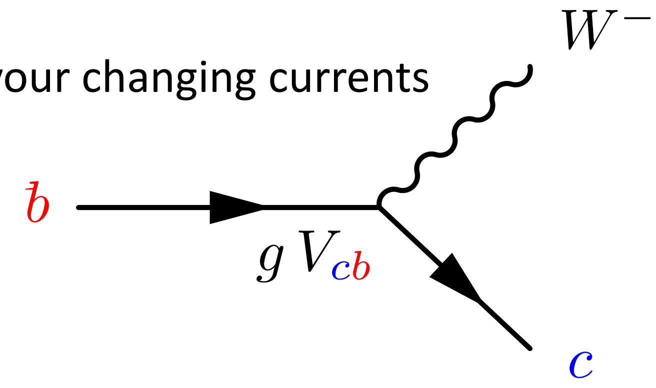


$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & \\ V_{cd} & V_{cs} & \\ & & V_{tb} \end{pmatrix}$$

Flavour Changing Quark Interactions

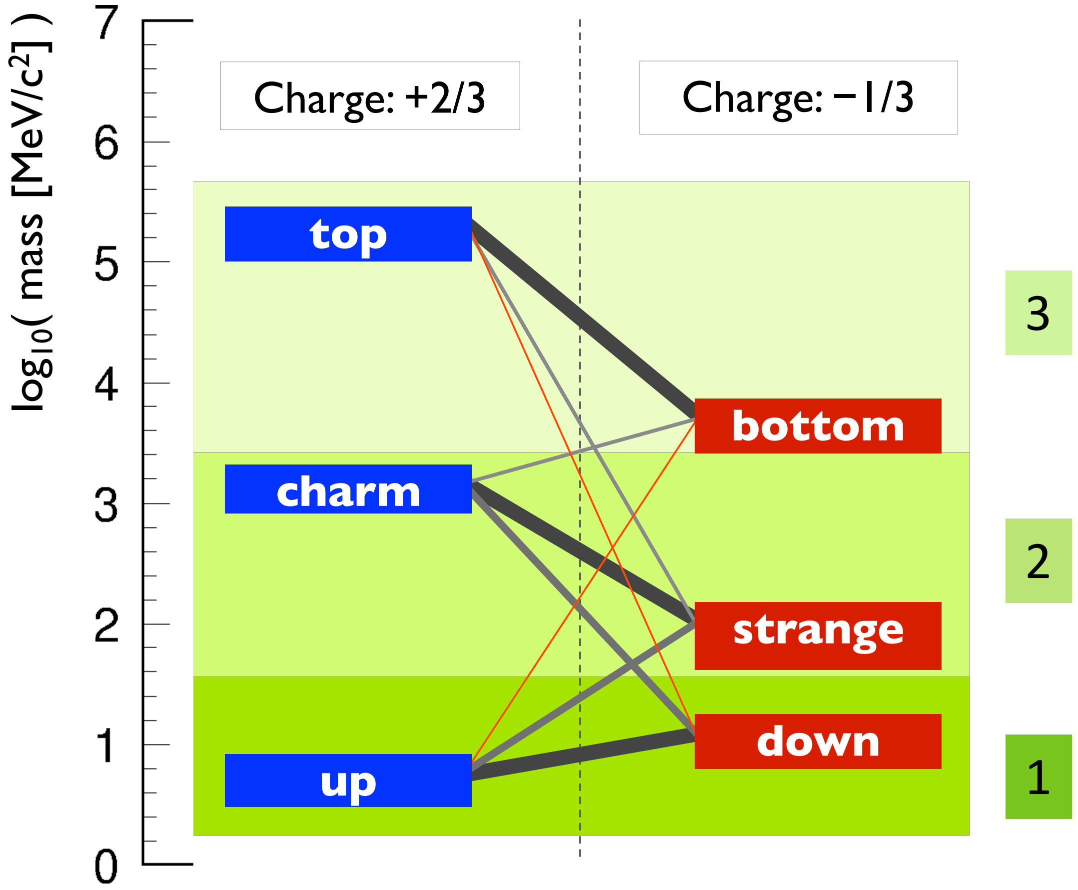


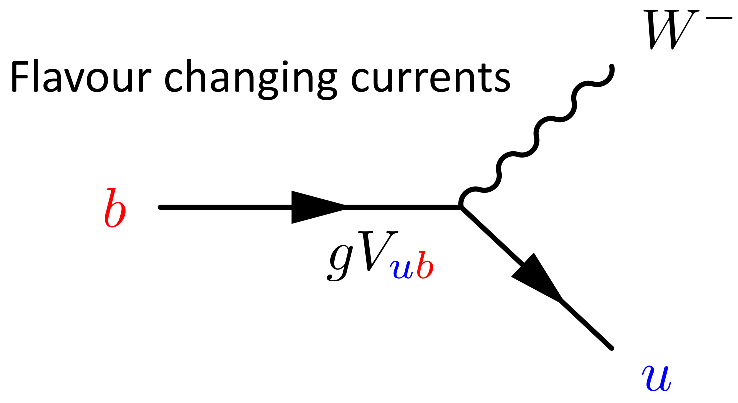
Flavour changing currents



$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & \\ V_{cd} & V_{cs} & V_{cb} \\ & V_{ts} & V_{tb} \end{pmatrix}$$

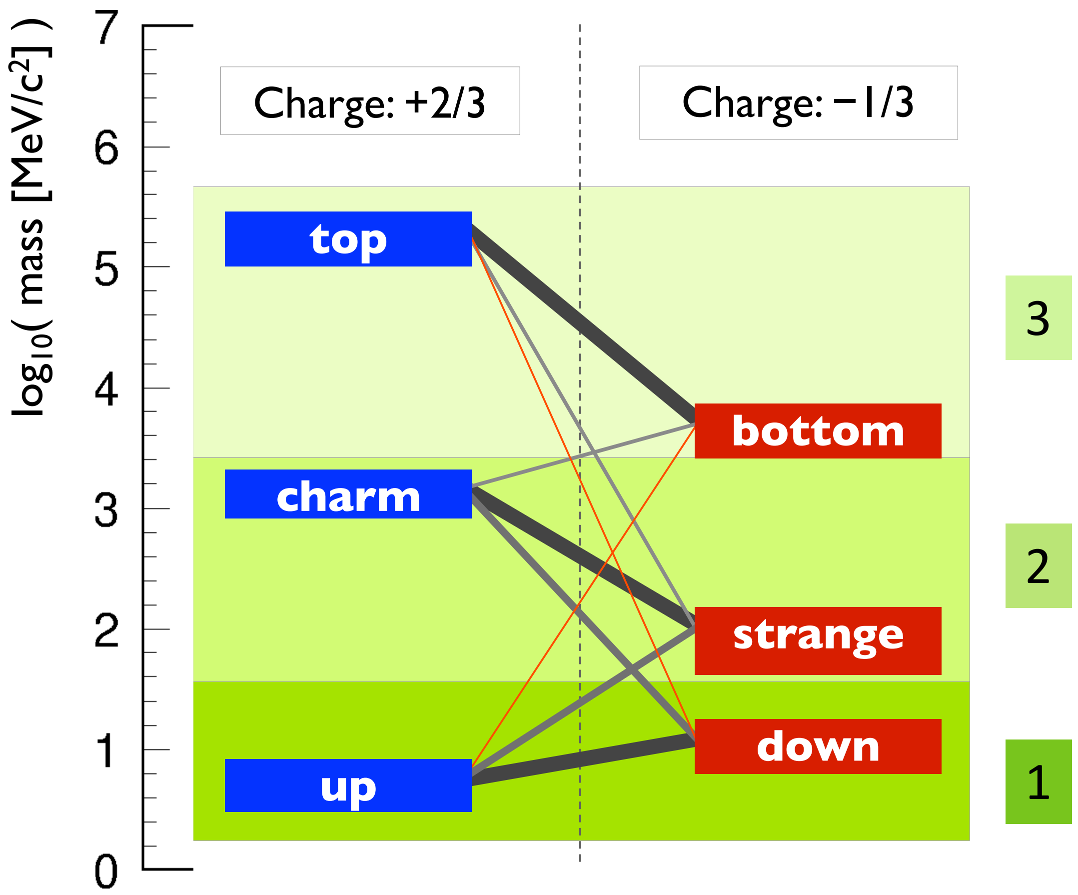
Flavour Changing Quark Interactions

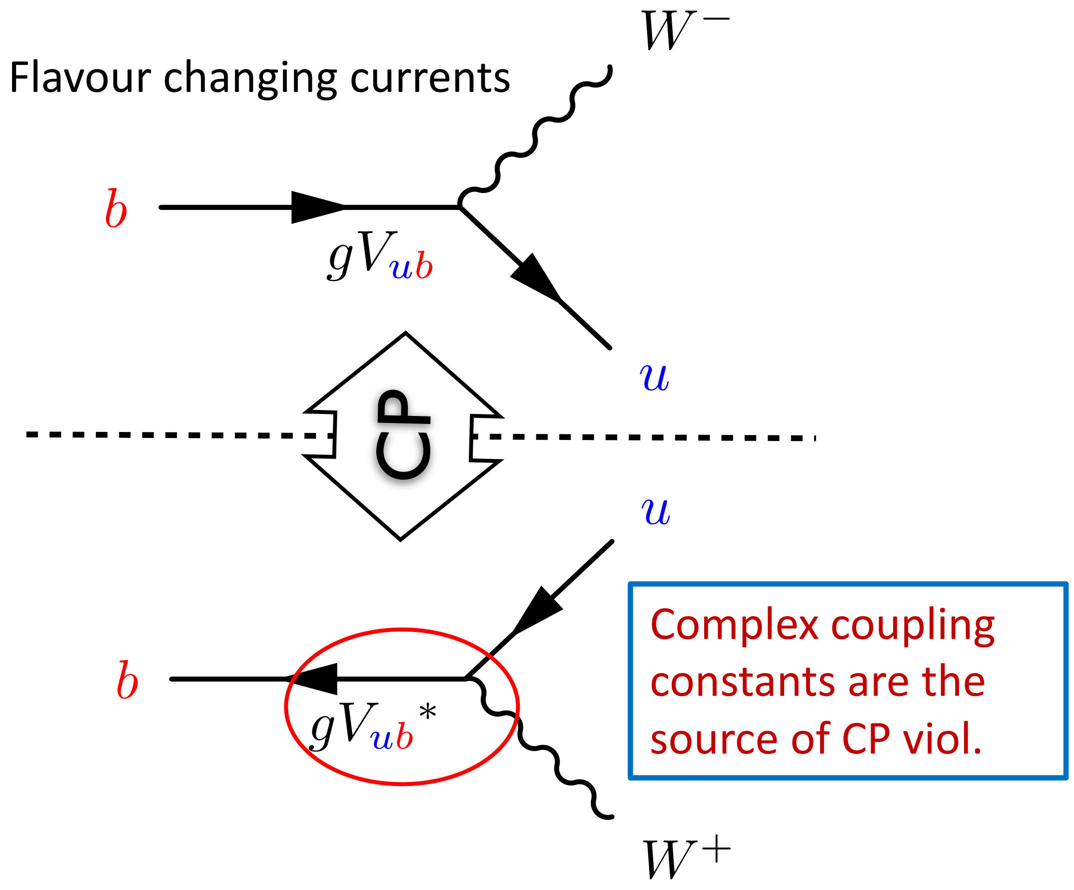




$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Flavour Changing Quark Interactions – CP Violation





- Particles and antiparticles have complex conjugated coupling constants
  - This leads to CP violation
  - Matter dominated universe

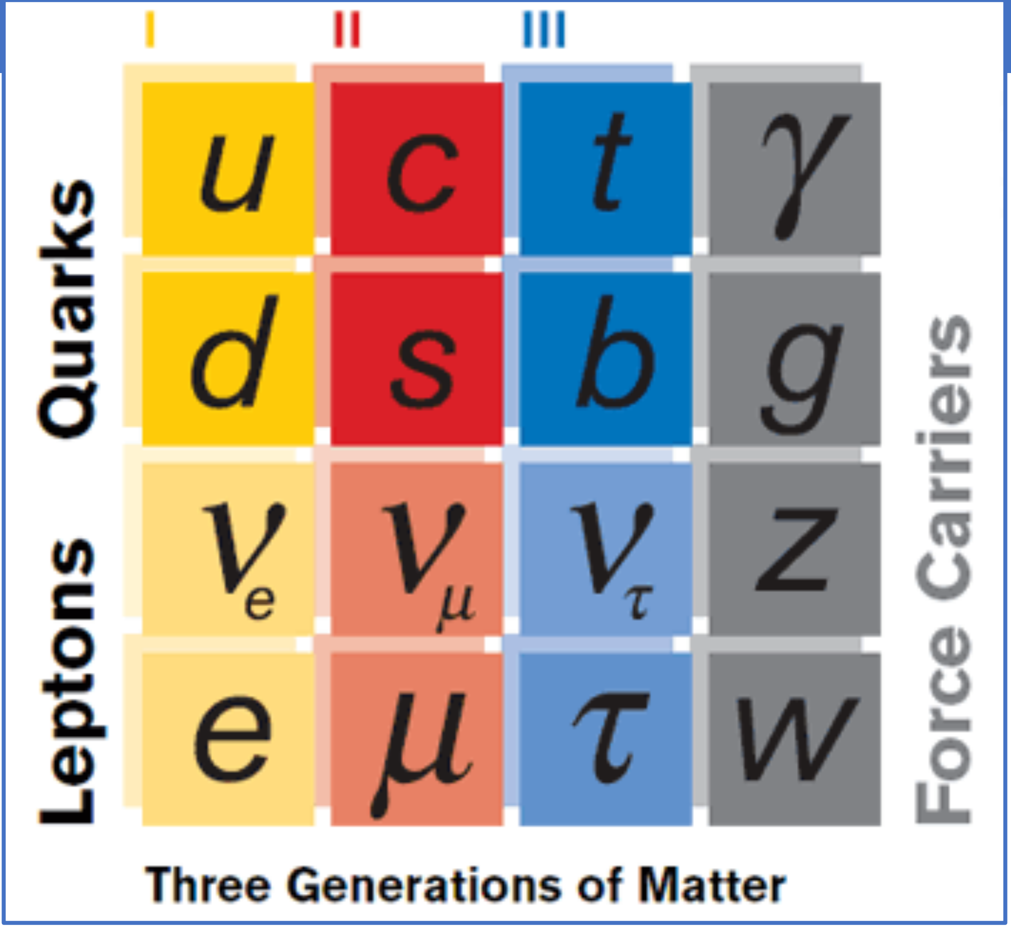
$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Why a matrix notation?

- Model:
  - Charged weak current does ***not*** couple to
    $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$
  - but instead to
    $\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} t \\ b' \end{pmatrix}$
  - Where

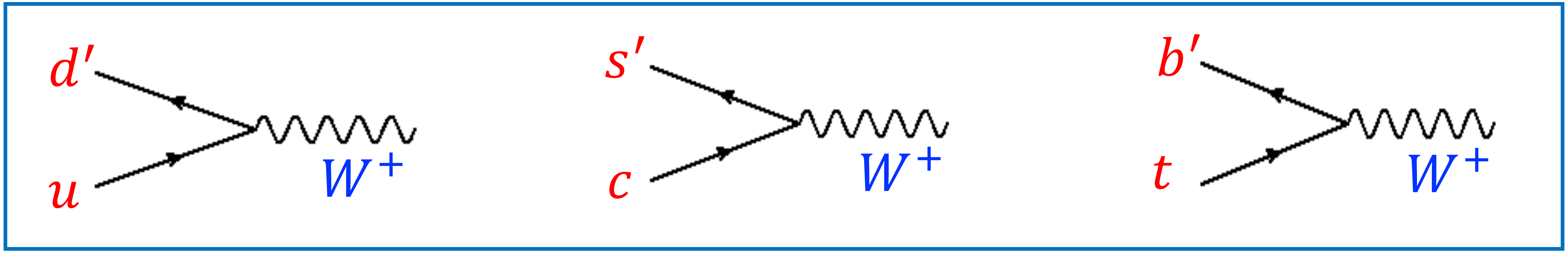
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM matrix:
Cabibbo,
Kobayashi
Maskawa

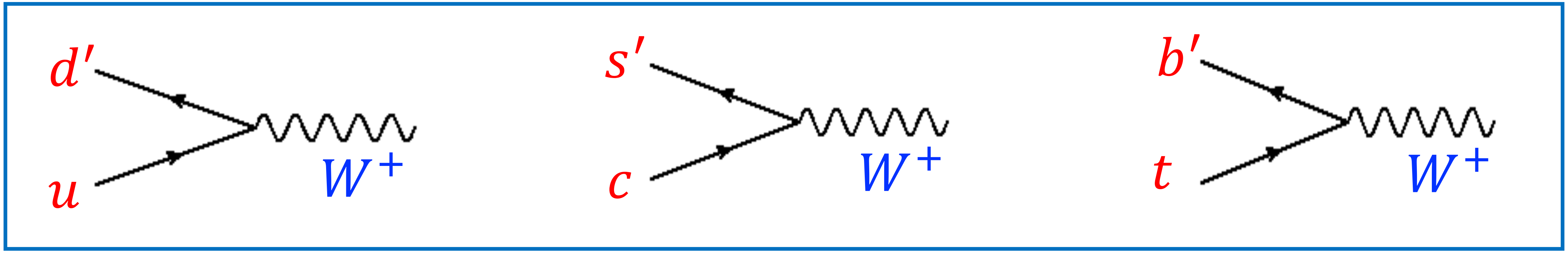


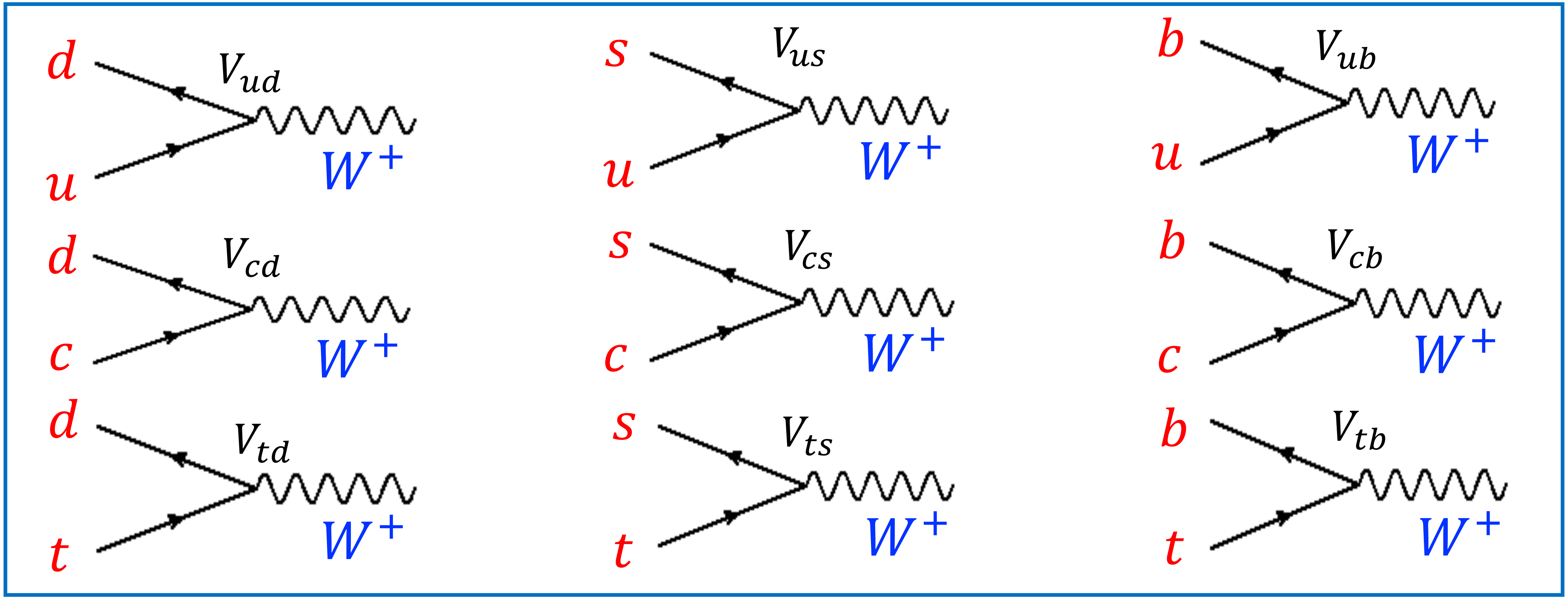
A story of eigenstates

- *Mass* eigenstates $|q\rangle$ are the eigenstate solutions of the free Hamiltonian
- *Weak* or *flavour* eigenstates $|q'\rangle$ are the eigenstate solutions of the weak interaction Hamiltonian.
  - They are unitary linear combination, or “rotation” of mass eigenstates.
- The the weak interaction can be written as:



Flavour eigenstates and Mass eigenstates





The CKM matrix V_{CKM} - 3 vs 2 Generations

$$V_{CKM}: \quad \begin{array}{c|ccc} & d & s & b \\ u & V_{ud} & V_{us} & V_{ub} \\ c & V_{cd} & V_{cs} & V_{cb} \\ t & V_{td} & V_{ts} & V_{tb} \end{array}$$

- Wolfenstein parametrization: V_{CKM} =

$$\begin{pmatrix} 1 - {}^{1}/_{2}\,\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - {}^{1}/_{2}\,\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

➔ 1 CP violating phase

$$V_{CKM}: \quad \begin{array}{c|cc} & d & s \\ u & V_{ud} & V_{us} \\ c & V_{cd} & V_{cs} \end{array}$$

V_{CKM} =

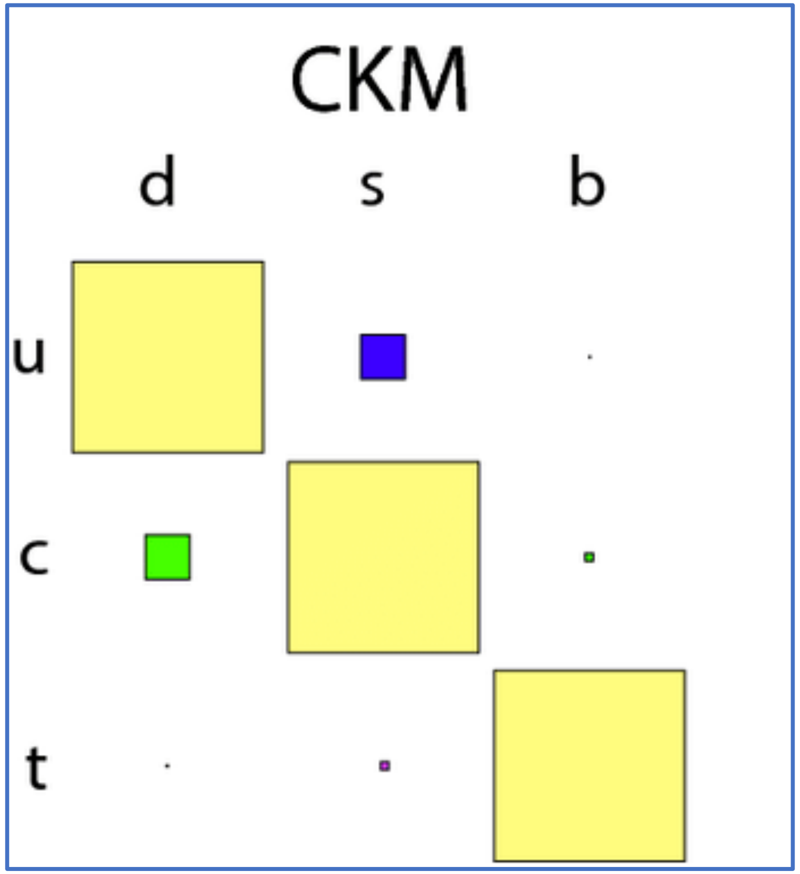
$$\begin{pmatrix} 1 - {}^{1}/_{2}\,\lambda^2 & \lambda \\ -\lambda & 1 - {}^{1}/_{2}\,\lambda^2 \end{pmatrix}$$

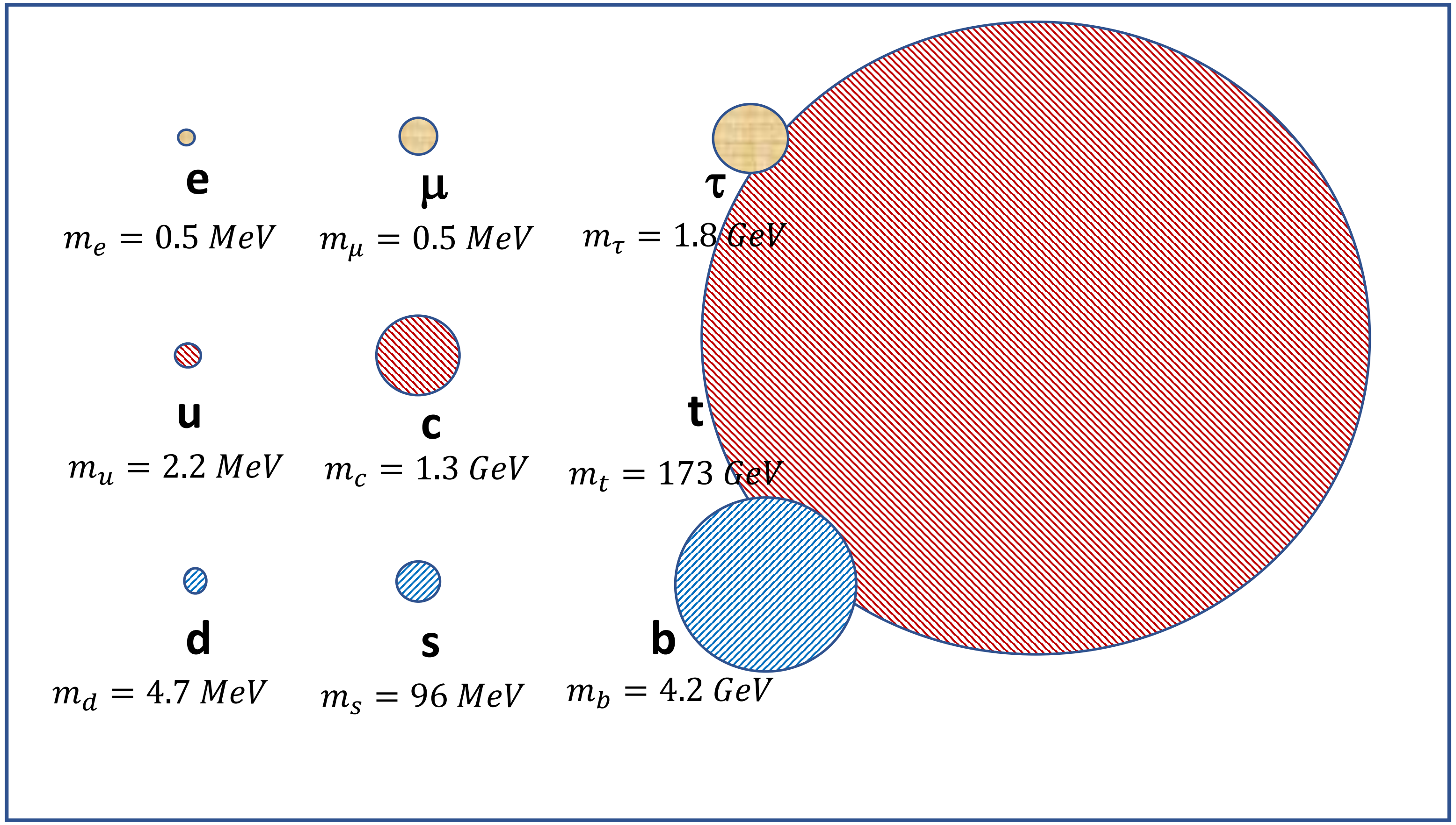
➔ No CP violation

- *3 generations is the minimal particle content to generate CP violation* (In Standard Model).

The Flavour Puzzle

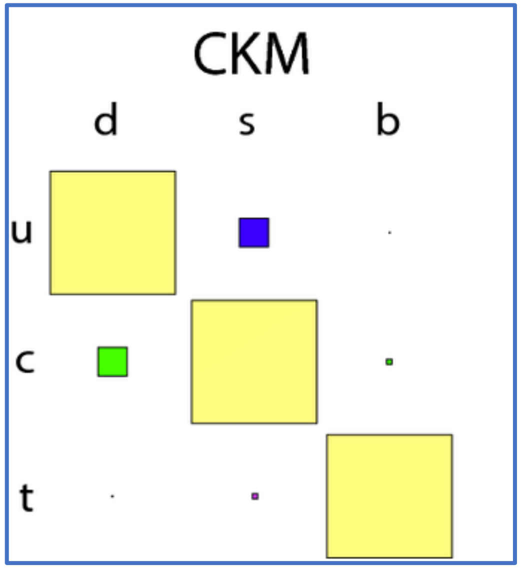
- Why 3?
- Why are the couplings what they are?
- Is there a relation with the masses of the quarks?

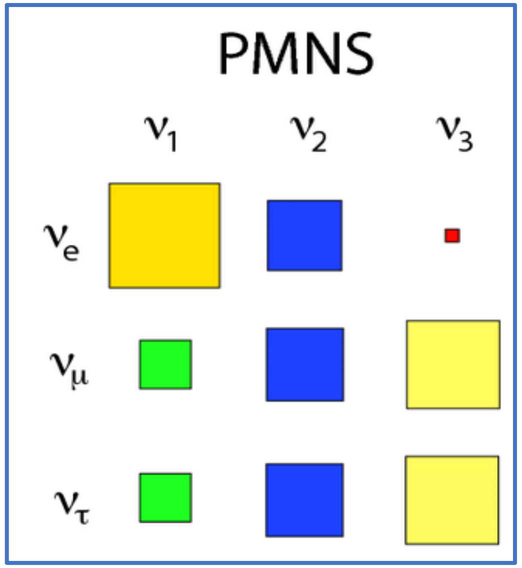




Flavour in the leptons!

- It turns out neutrino's have mass, too!
  - The mass is very tiny
- The generation mixing also occurs for neutrino's
  - Slightly different nomenclature:
  - Quarks (CKM): $\begin{pmatrix} u' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$
  - Leptons (PMNS): $\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} \\ U_{2e} & U_{2\mu} & U_{c\tau} \\ U_{3e} & U_{3\mu} & U_{3\tau} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$
    - U_{PMNS} : Pontecorvo, Maki, Nakagawa, Sakata mixing matrix
- (Difficult) Question: why is lepton mixing not seen in decays?
  - It is only seen in neutrino oscillations



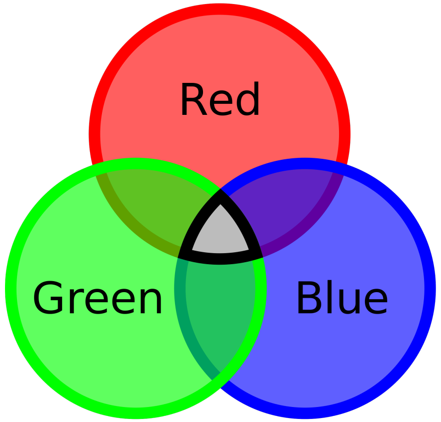


Griffiths section 2.3

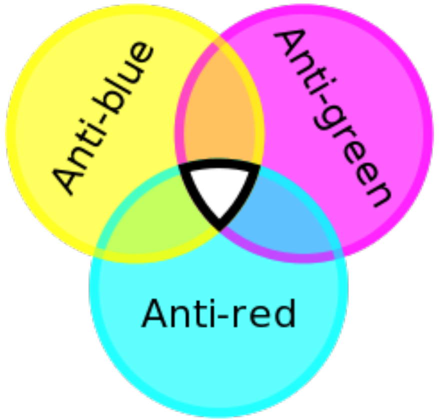
The Strong Interaction
Quantum Chromodynamics (QCD)

Color singlets

- Quarks are “locked-up” in hadrons
  - Technical term: confinement
- Quarks carry color charge: r , g , b
- All physical objects are color neutral or color *singlets: confinement!*
  - Baryons: rgb or $\overline{r}\,\overline{g}\,\overline{b}$
  - Mesons: $r\,\overline{r}$ or $b\overline{b}$ or $g\,\overline{g}$
- The color force, transmitted by gluons, is very strong
  - Requires a lot of energy to separate a color charge from a color neutral object
  - Trying to do so will produce another color neutral object

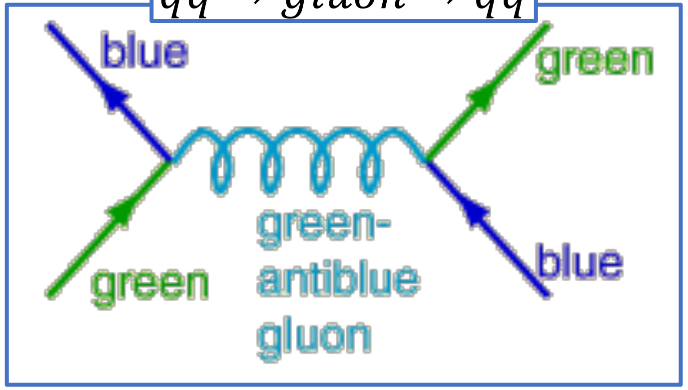


baryon



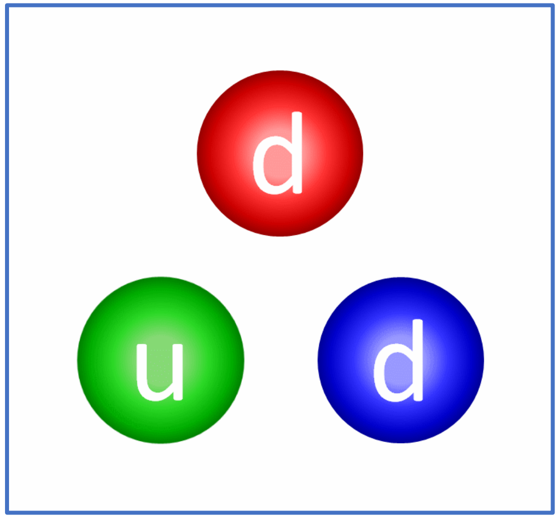
anti-baryon

$q\overline{q} \rightarrow gluon \rightarrow q\overline{q}$



Gluon color: $g\,\overline{b}$
There are 8 gluons
+ 1 colorless singlet

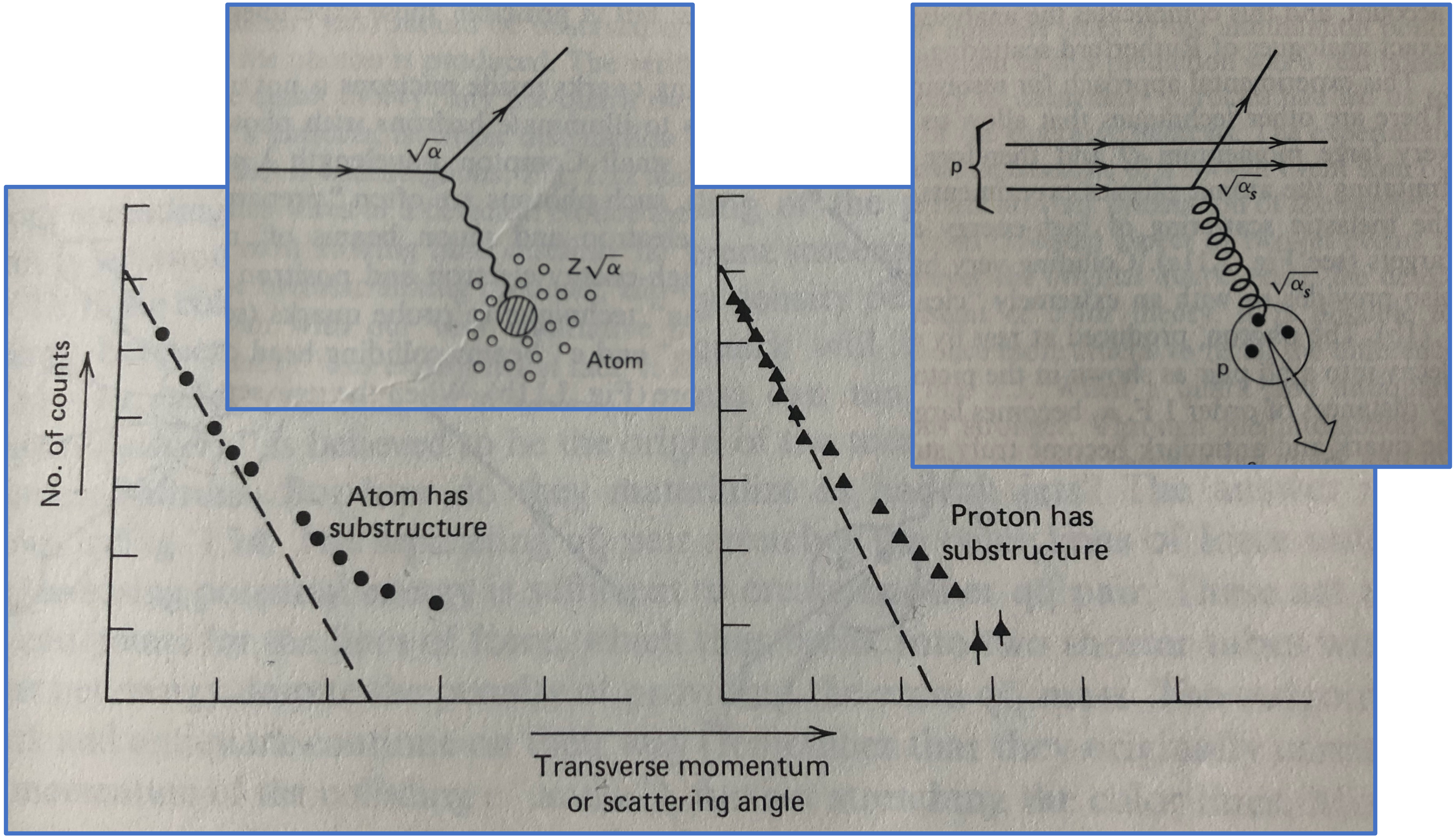
- Animation of a color neutral neutron:



- Are quarks and gluons real or a bookkeeping devise?

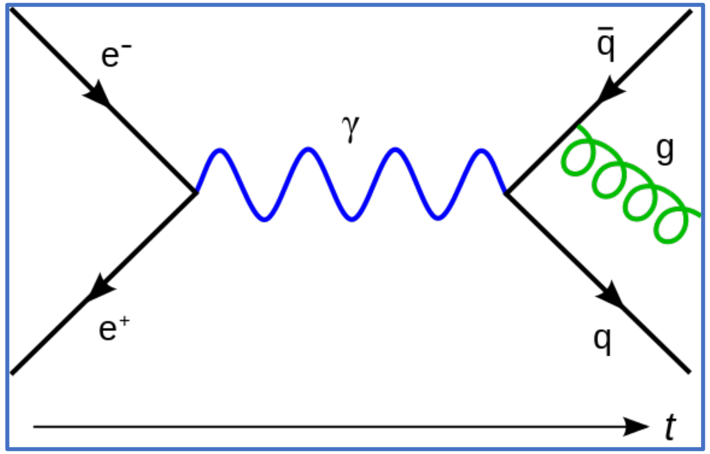
Proton Substructure: discovery of "partons" or "quarks"(1968)

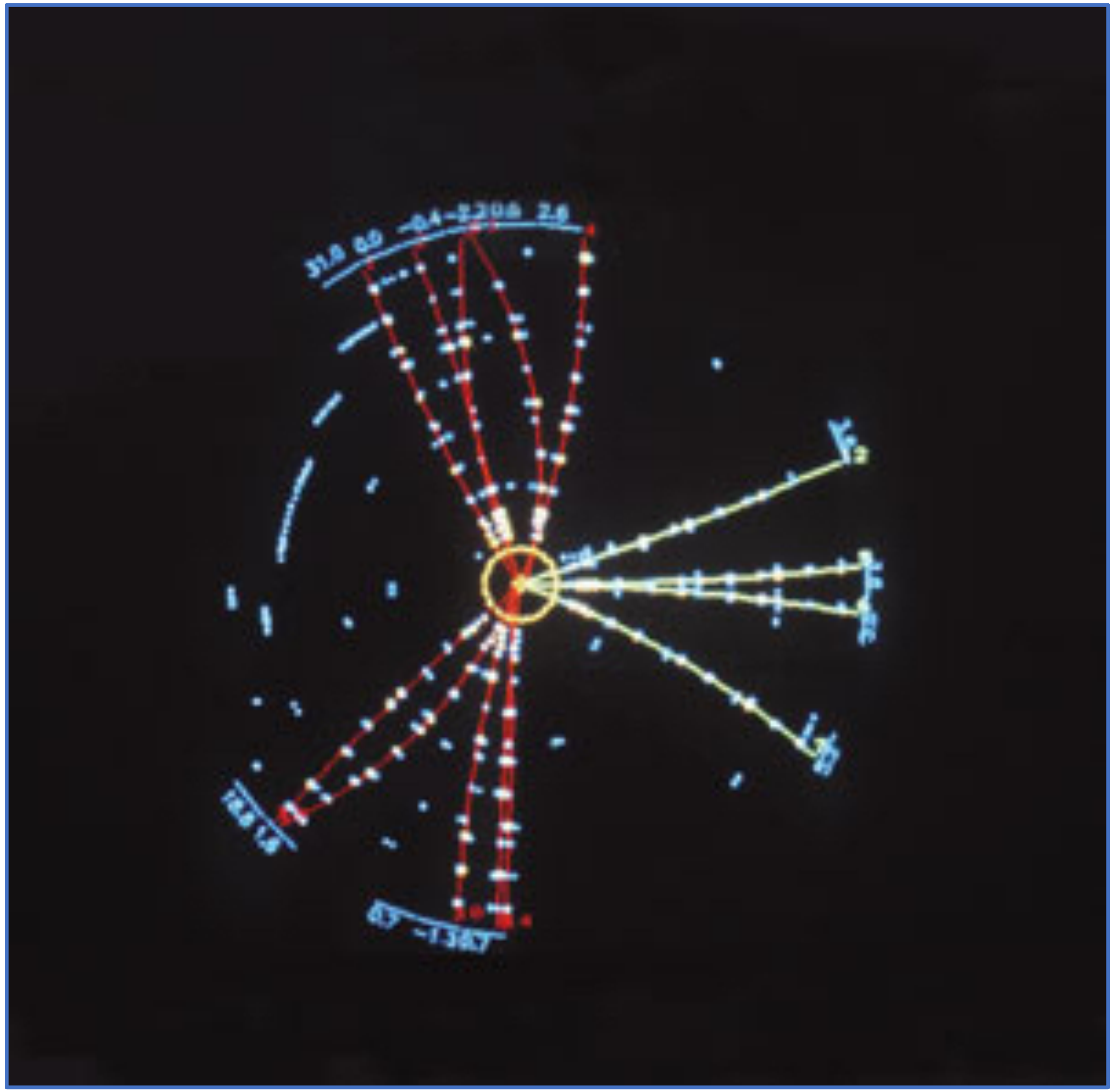
- Similarly to Rutherford scattering ("substructure of the nucleus") deep inelastic scattering of electrons on protons show the proton substructure



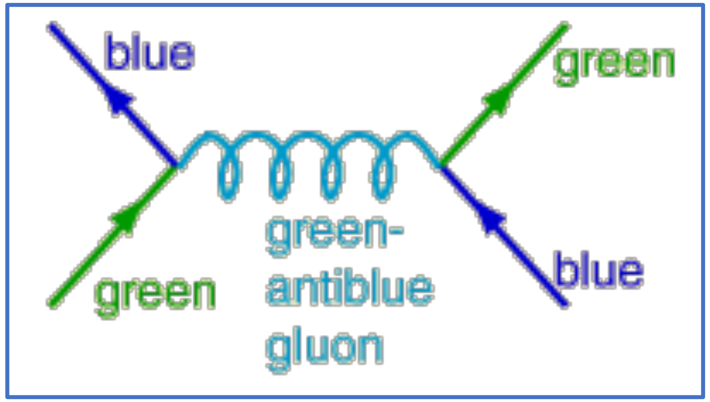
Discovery of the Gluon (1979)

- The gluon was discovered in e^+e^- collisions at DESY in Hamburg
  - "3-jet events"

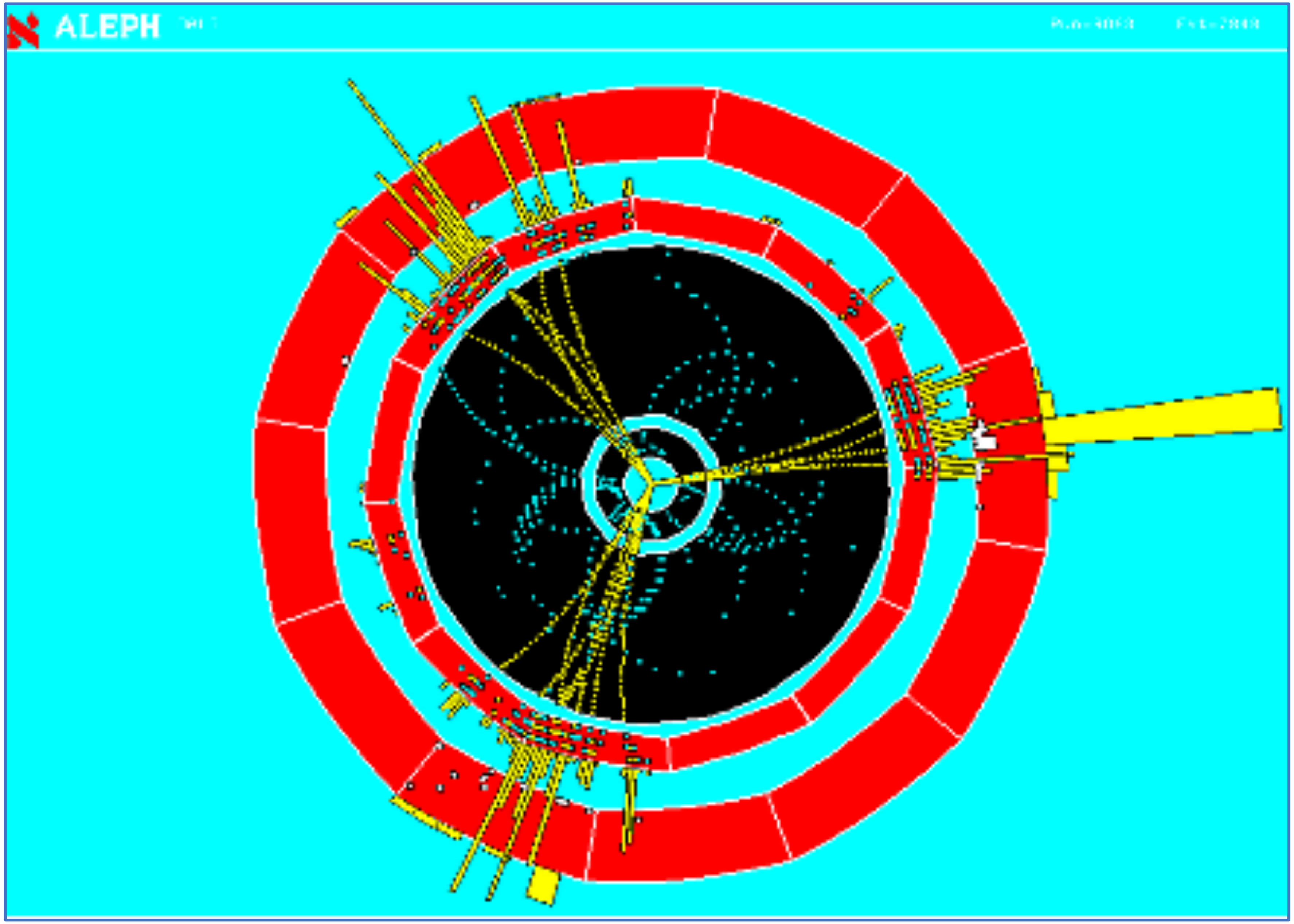




- Feynman diagram for quark-quark interaction
  - At LHC

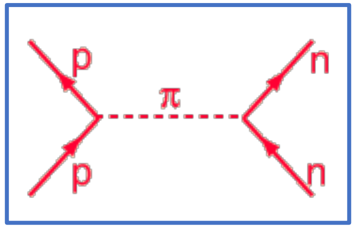


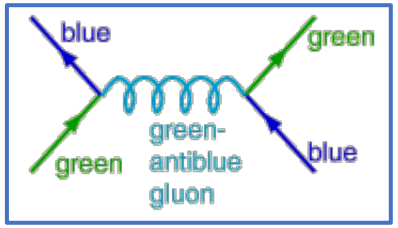
3-Jet event at LEP - Delphi (1989 – 2000)

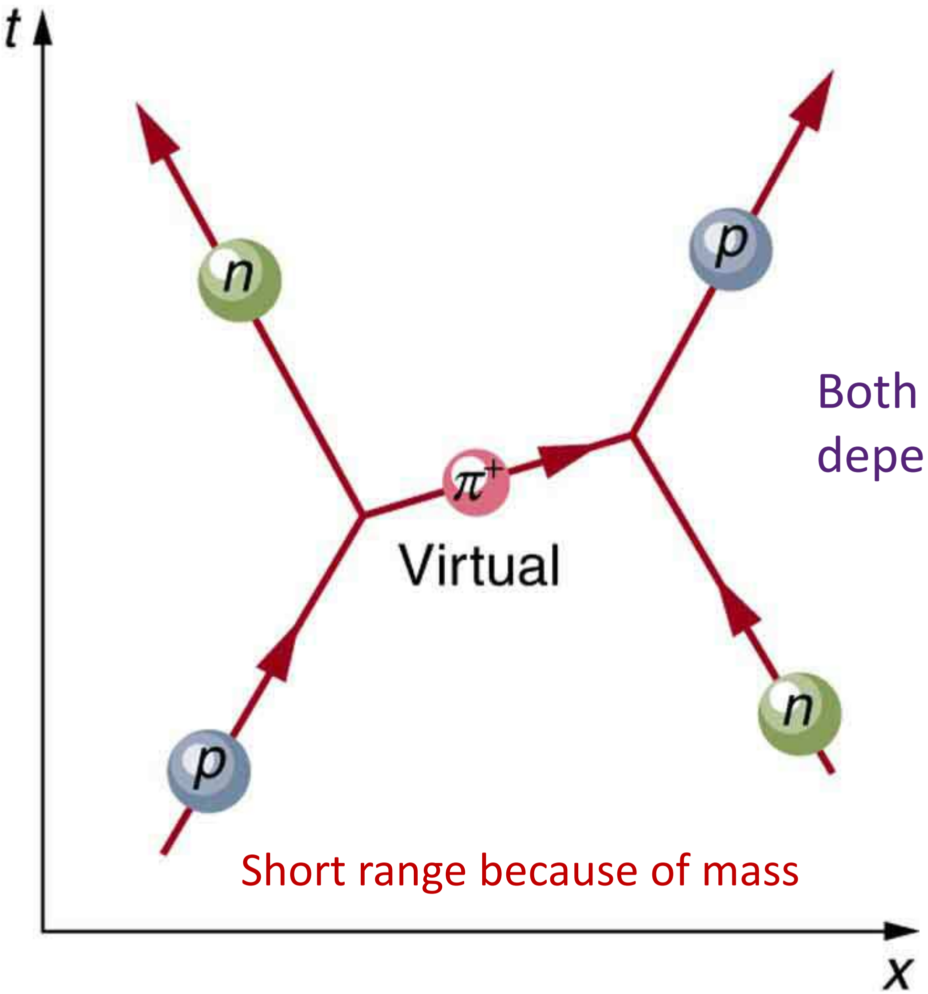


Yukawa’s pion exchange vs gluon exchange

- Yukawa: Nuclear force is carried by *massive* gluon
- QCD: Strong force is mediated by *massless* gluon

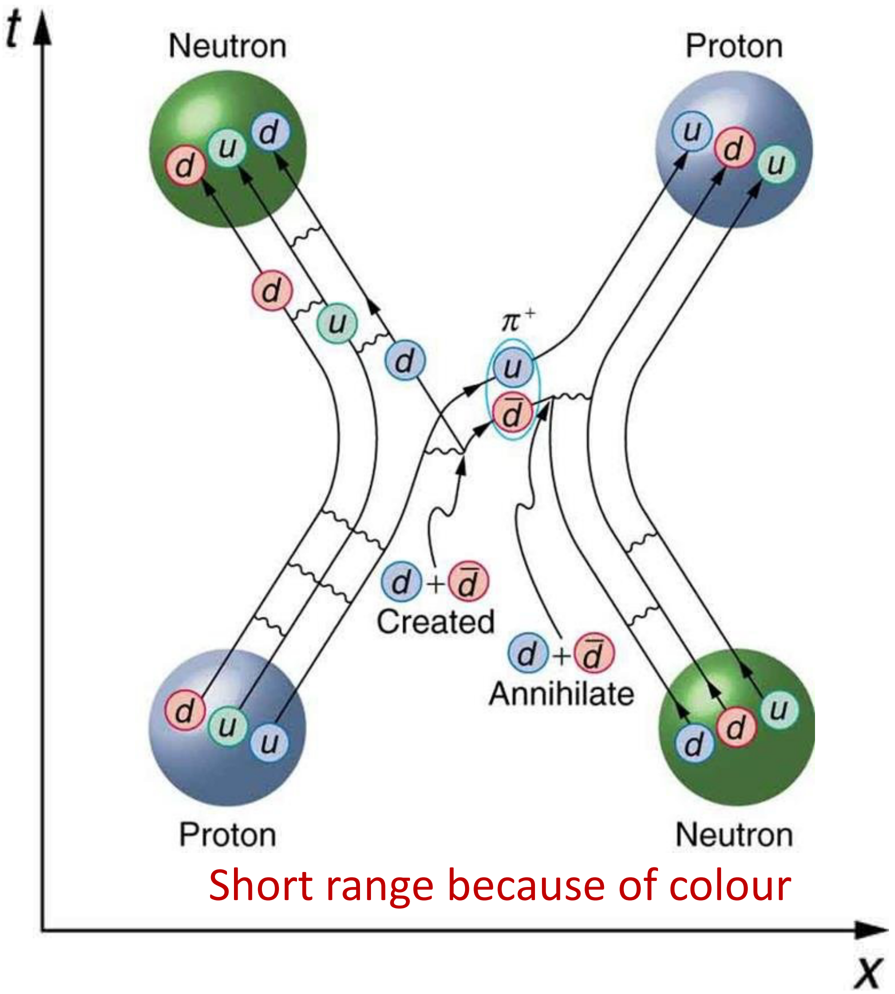






Short range because of mass

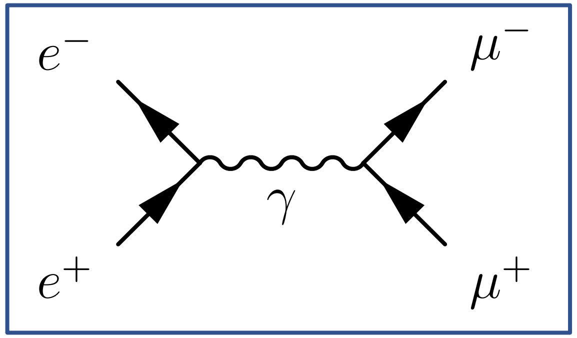
Both are “correct” depending on “zoom”

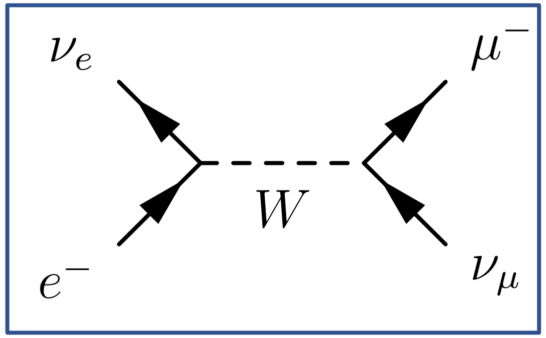


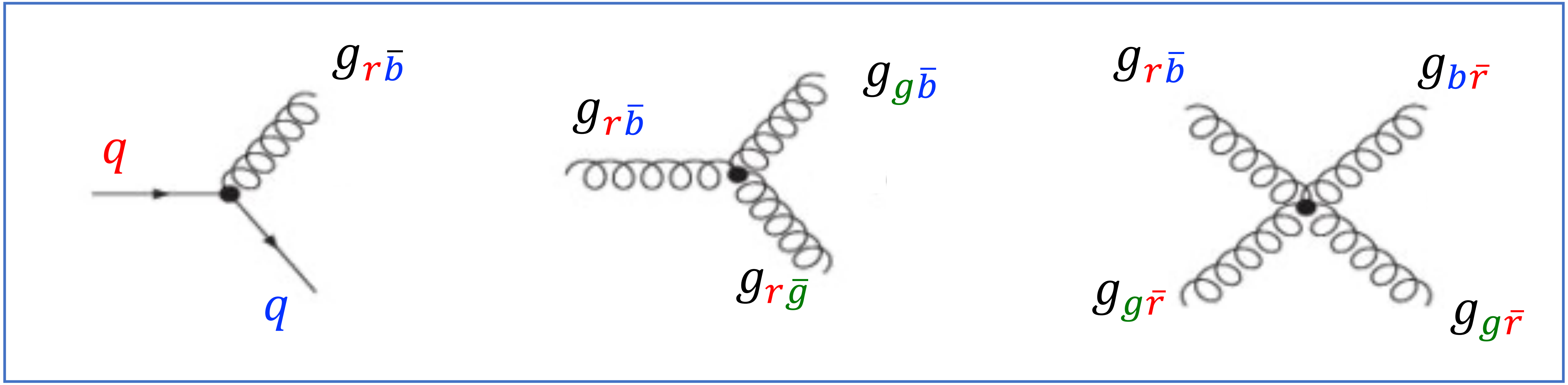
Short range because of colour

Fundamental Vertices of the strong interaction

- QED: photon couples to charge
- Weak: W couples to isospin
- Strong gluon couples to color
  - 3 colors and anti-colors
  - Gluon carries charge itself!
    - 8 different gluons (9 – 1 symmetric)





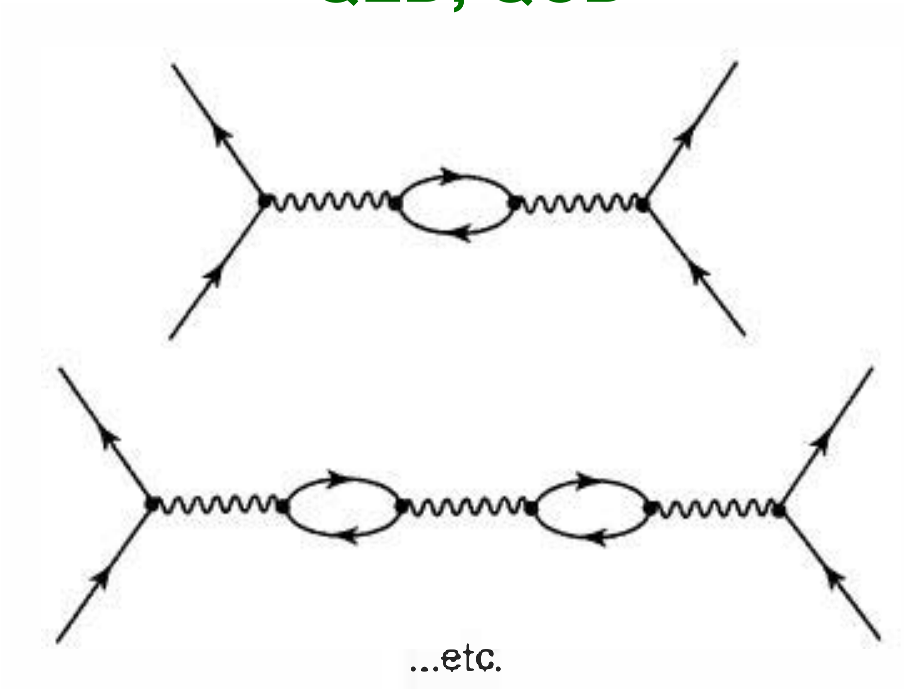


- Leptons carry no color: leptons do not feel strong (nuclear) interaction

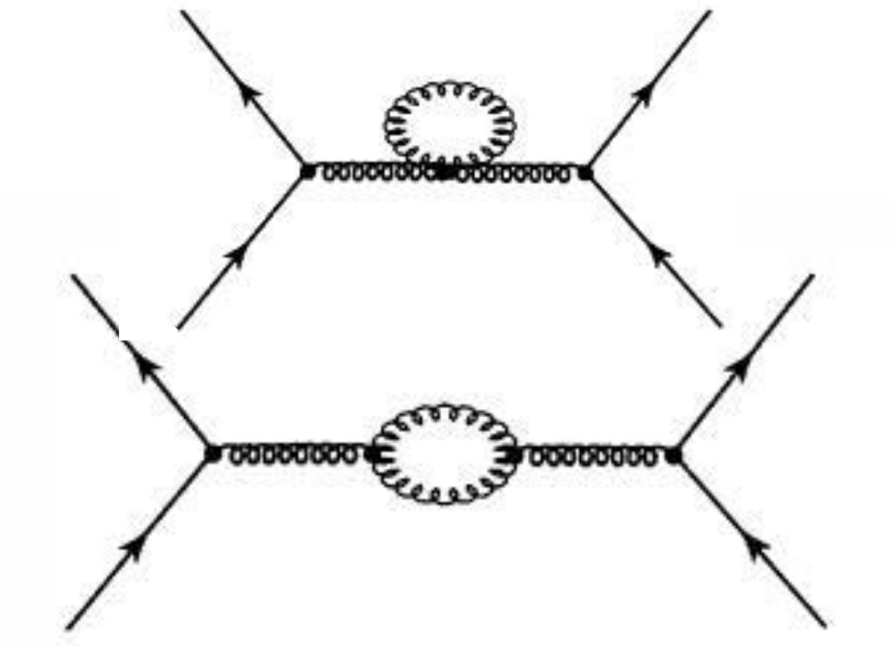
Abelian and non-Abelian

- Photons have no electric charge:
  - Photons *do not* interact with each other ("Abelian" in group theory)
- Gluons carry color charge:
  - Gluons *do* interact with each other ("non-Abelian" in group theory)

QED, QCD



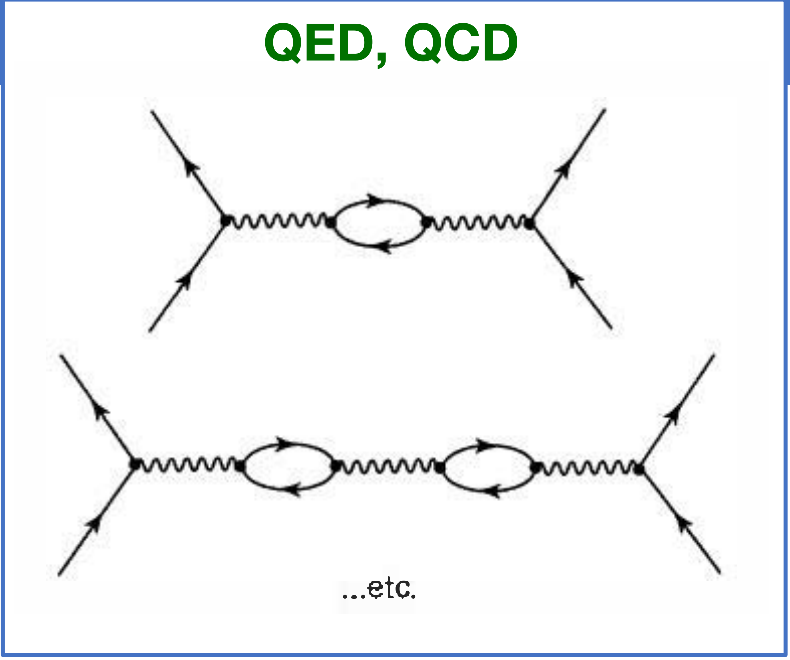
QCD



- Vacuum behaves like dielectric: by QED and even more so by QCD

Screening

- Charge screening effect:
  - Around a charge $q(+)$ dipoles align
  - Halo of negative charge
  - Effective charge (at distance) reduced



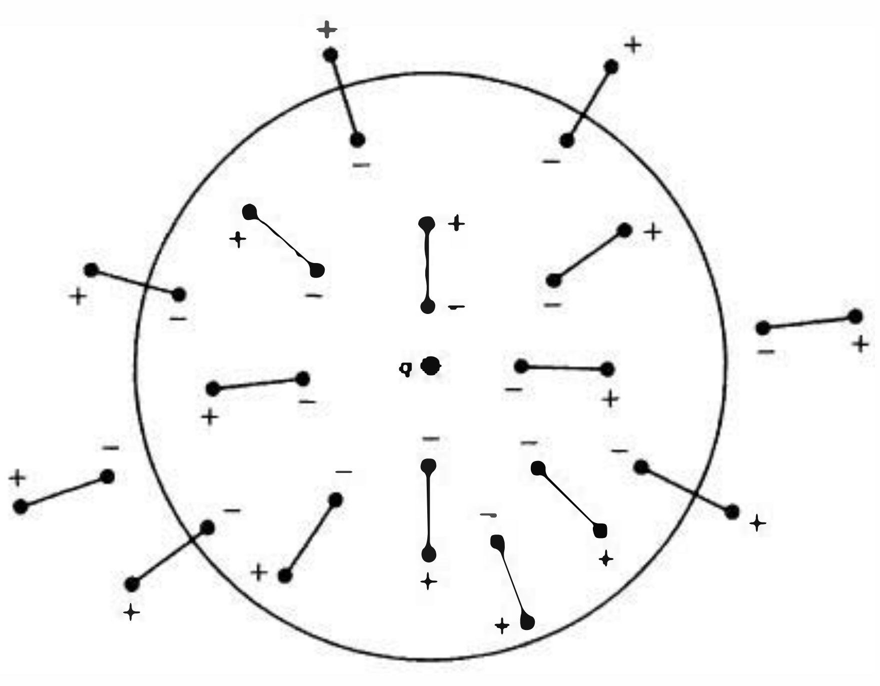
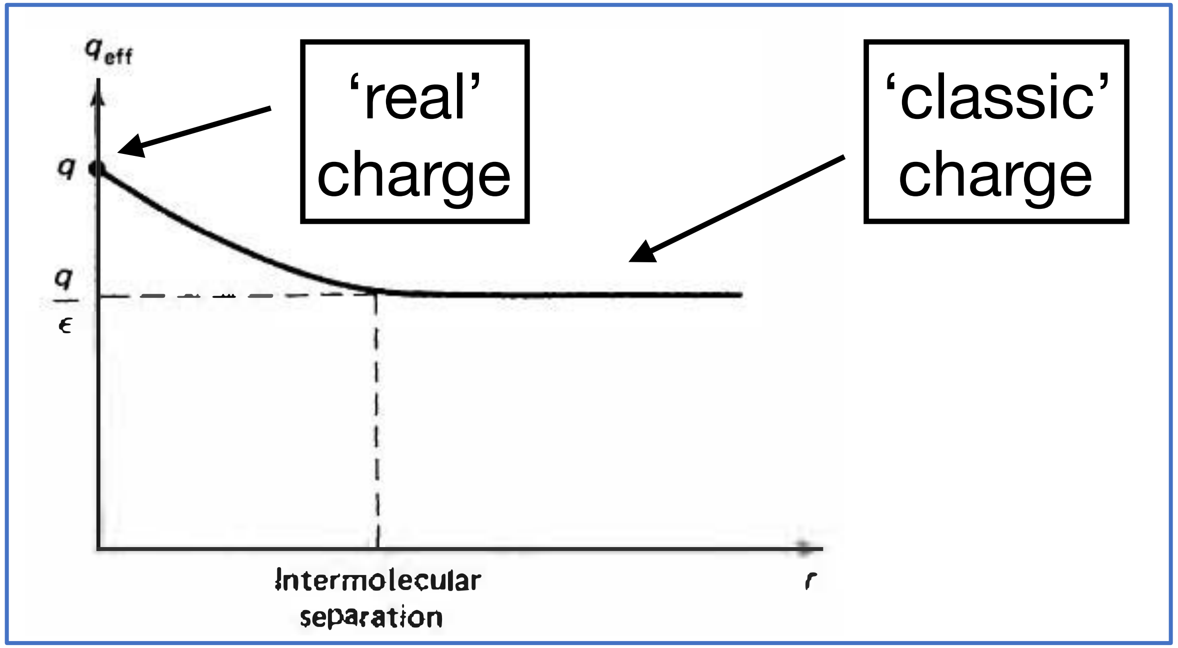


Fig. 2.1 Screening of a charge q by a dielectric medium.



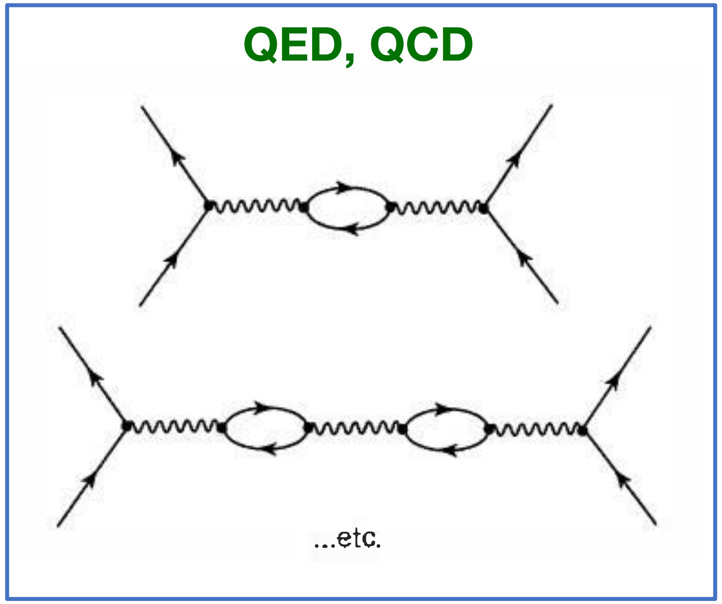
Anti-screening and asymptotic freedom

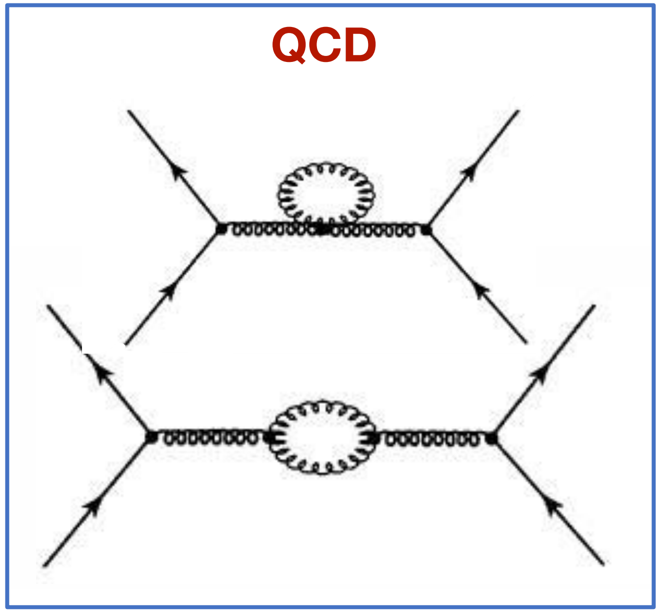
- Anti-screening:
  - gluon self coupling loops have opposite effect
  - Crucial parameter:

    $a \equiv 2f - 11c$

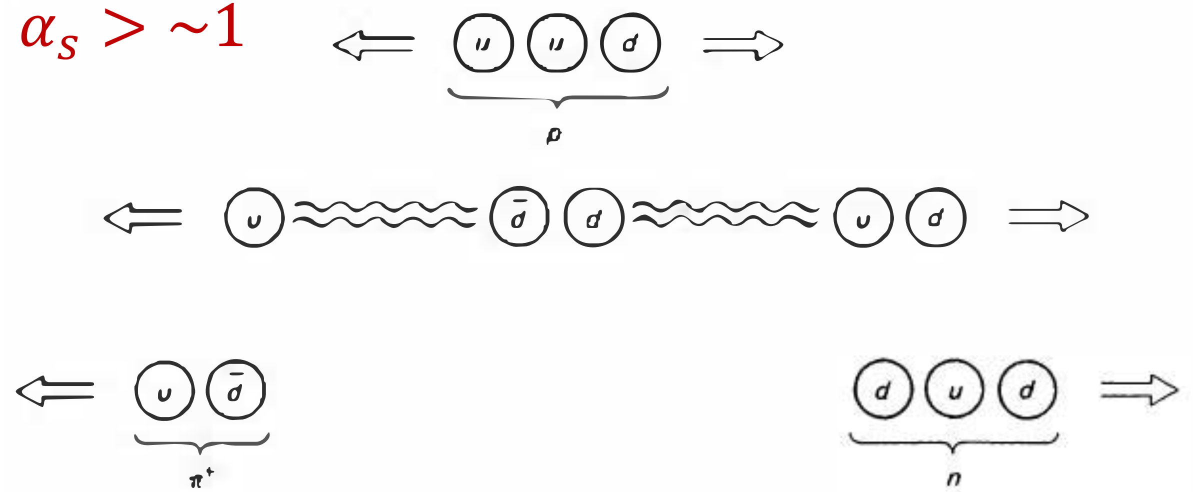
    where: f = number of flavours

    c = number of colors
  - In case $a > 1$ (QED) coupling *increases* for short distance:
  - In case $a < 1$ (QCD) coupling *decreases* for short distance





- *Confinement*: at large distance:
  - $\alpha_s > \sim 1$



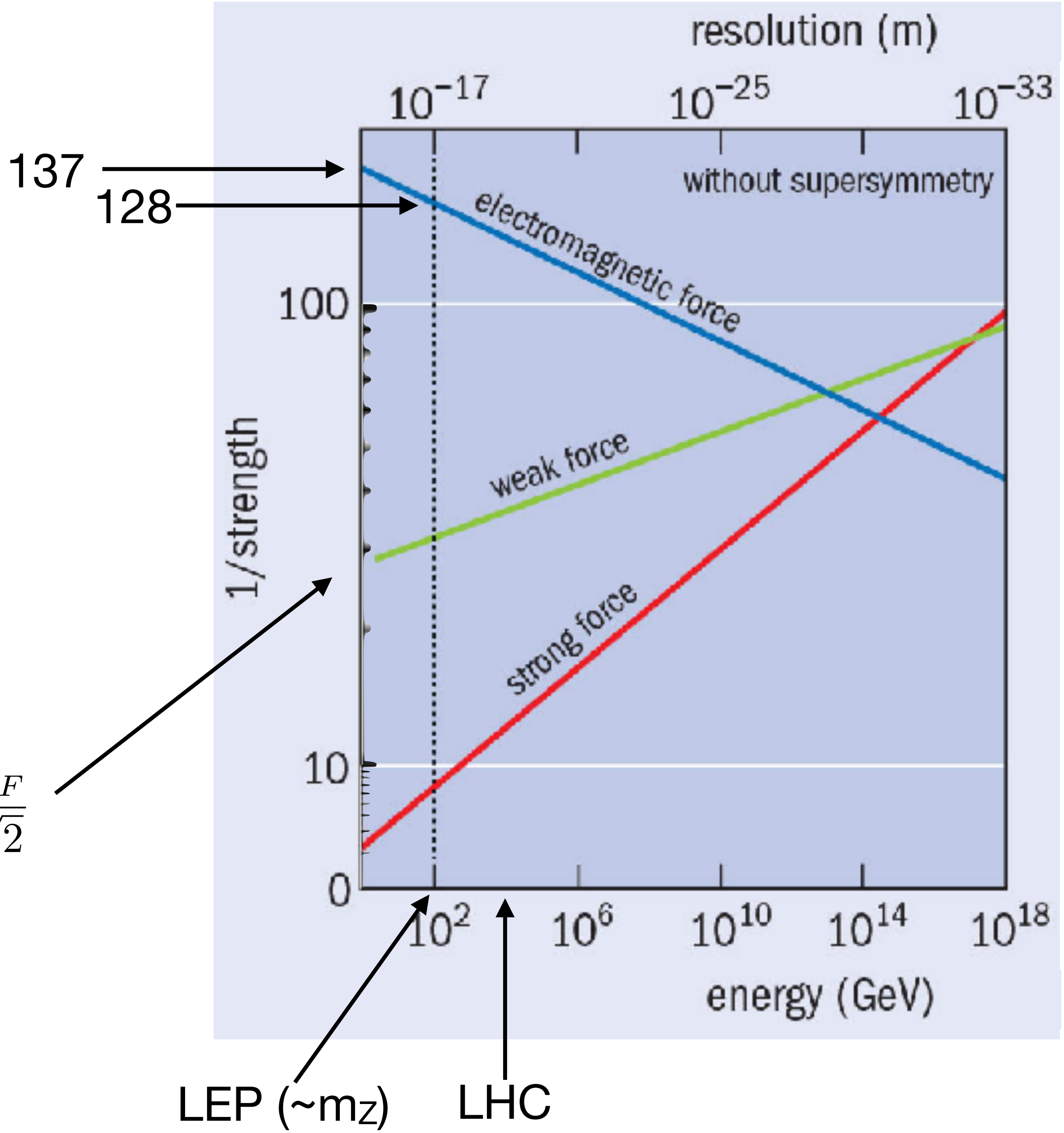
- *Asymptotic freedom:* at short distance:
  - Coupling $\alpha_s \sim 0$ at very small distances
  - Quarks and gluons become "free"

➔Quark gluon plasma

Running coupling constants and unification?

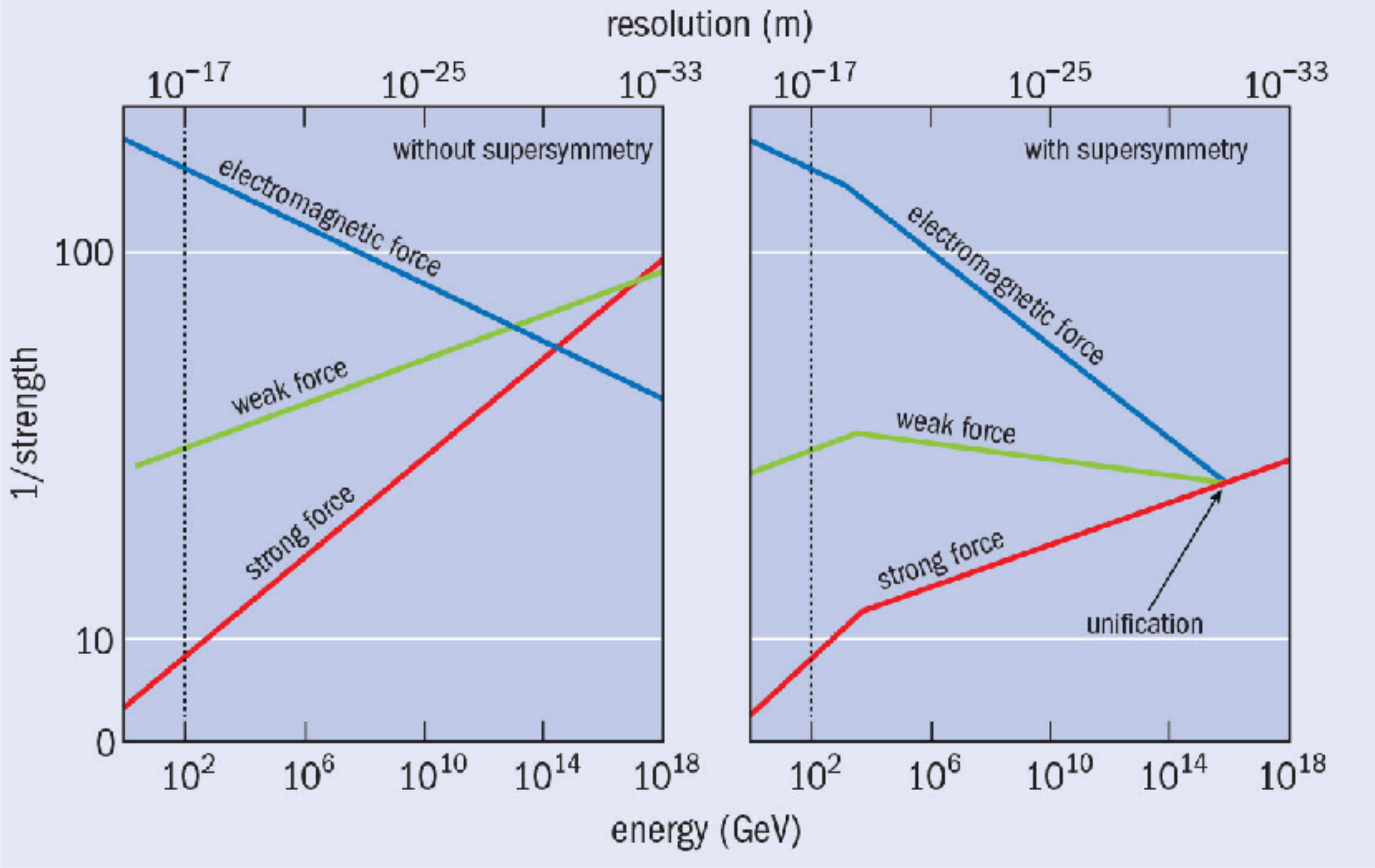
- Running couplings: 'beta'-function
- Higher energy means shorter distance
- Lines do not cross in one point

$$\frac{g_W^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$



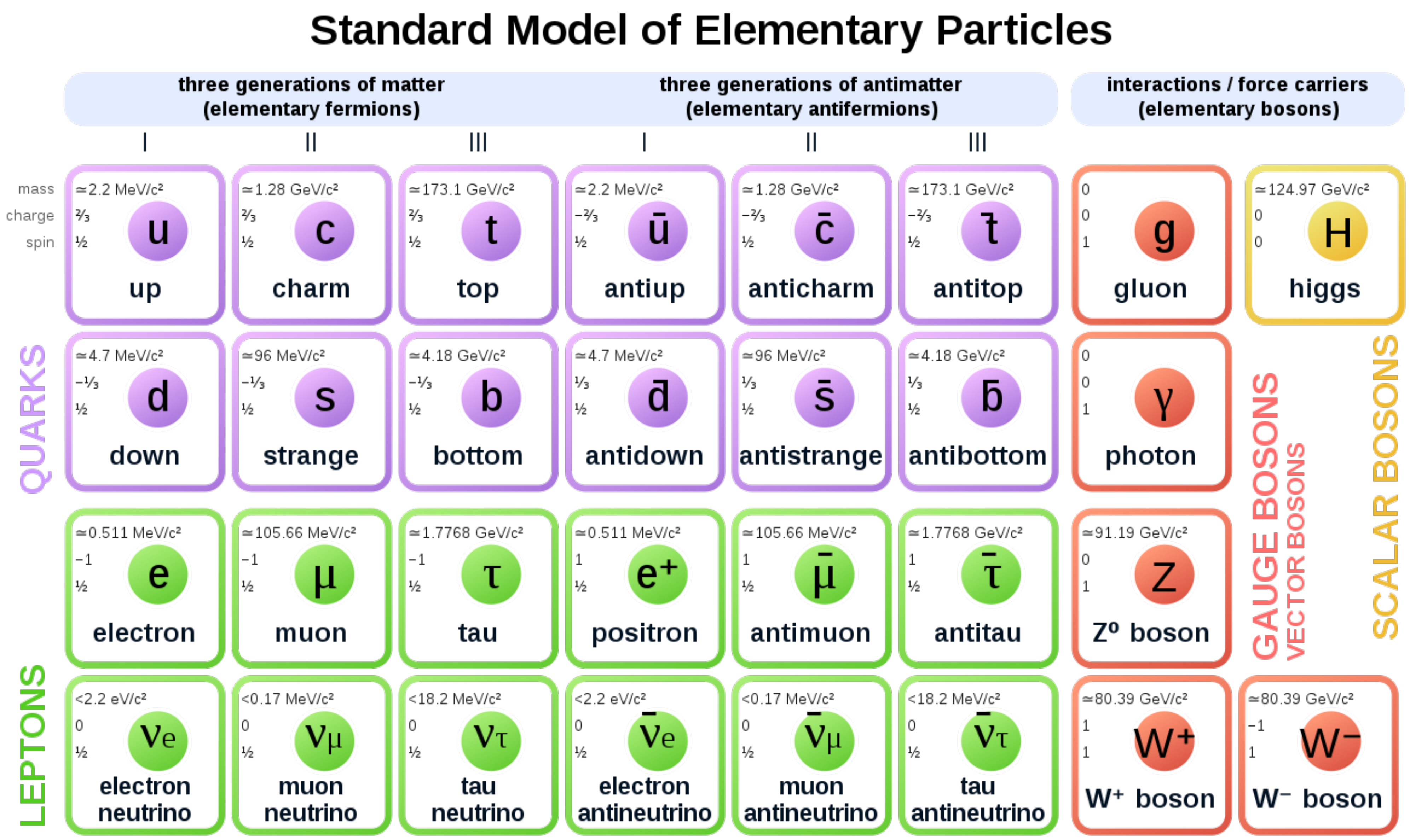
Running coupling constants and unification?

- Unification with SUSY?
- 10^{15} GeV
  - Grand Unified Theory
- 10^{19} GeV:
  - Planck scale → quantum gravity?



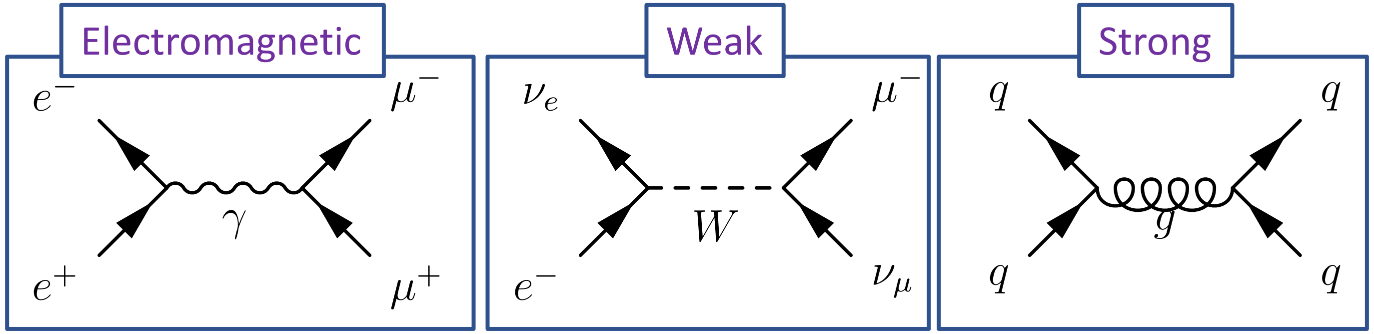
- 10^{15} GeV —> Grand Unified Theory?
- Planck scale (~10^{19} GeV) —> Gravity! —> Theory of Everything?

Standard Model of particles and forces



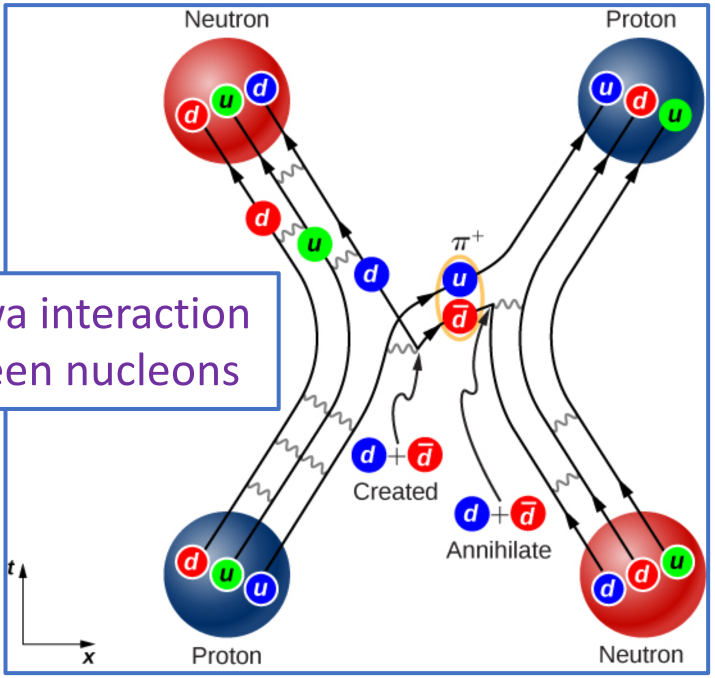
Summary Forces: Intermediate Vector Bosons

- SM: forces are transmitted by vector mesons, particles with spin-1:
  - Electromagnetism:
    - Long range : photon γ , $M_\gamma = 0$
    - Photon carries no E.M. charge
  - Weak interaction:
    - Very short range: W and Z bosons, $M_W = 80\ GeV$, $M_Z = 91\ GeV$
    - Weak isospin charge. W and Z have I-charge.
  - Strong interaction:
    - Very short range: gluon g , $M_g = 0$
      - Gluons carry color → confinement → very short range
    - Pairs of quarks can transmit strong force: Yukawa's mesons
      - Short range



- Gravitation: transmitted by spin-2 particle?
  - Graviton
    - Long range: $M_g = 0$

Yukawa interaction between nucleons



Prepare(!): Relativistic and 4-vector notation

See Griffiths chapter 3

- Lorentz transformation along x^1axis using $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$ is:

  $$x^{0\prime} = \gamma(x^0 - \beta x^1)$$
  $$x^{1\prime} = \gamma(x^1 - \beta x^0)$$
  $$x^{2\prime} = x^2$$
  $$x^{3\prime} = x^3$$

- Coordinate four-vector notation: $x^\mu = (x^0, x^1, x^2, x^3)$
  - Time: $x^0 = ct$, Space: $\vec{x} = (x^1, x^2, x^3)$
  - General $Contravariant$ vector $A^\mu = (A^0, \vec{A})$ transforms like x^μ
- Lorentz transformations leave the "length" invariant $A \cdot A = |A|^2 = {A^0}^2 - |\vec{A}|^2$ which can be regarded as the product of a *contra*variant vector with a *co*variant vector: $A_\mu = (A^0, -\vec{A})$
- Metric tensor: $g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$
- Scalar product: $A \cdot B = A^\mu B_\mu = g_{\mu\nu} A^\mu B^\nu$, where a sum is always implicit over *contra*variant and *co*variant indices.
- The scalar product is always Lorentz invariant.

Exercise – 7 : 4-Vector notation

a) Start with the expression for a Lorentz transformation along the x^1 axis. Write down the *inverse* transformation (i.e. express (x^0, x^1) in $(x^{0\prime}, x^{1\prime})$)

b) Use the chain rule to express the derivatives $\partial/\partial x^{0\prime}$ and $\partial/\partial x^{1\prime}$ in $\partial/\partial x^0$ and $\partial/\partial x^1$

c) Use the result to show that $(\partial/\partial x^0, -\partial/\partial x^1)$ transforms in the same way as (x^0, x^1)

d) In other words the derivative four-vectors transform inversely to the coordinate four-vectors:

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right) \text{ and } \partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

Note the difference w.r.t. the minus sign!

Exercise – 8: Kinematics: Z-boson production

- The Z-boson particle is a carrier of the weak force. It has a mass of 91.1 GeV. It can be produced experimentally by annihilation of an electron and a positron. The mass of an electron, as well as that of a positron, is 0.511 MeV.
  a) Draw the Feynman interaction diagram for this process.
  b) Assume that an electron and a positron are accelerated in opposite directions and collide head-on to produce a Z-boson in the lab frame. Calculate the minimal beam energy required for the electron and the positron in order to produce a Z-boson.
  c) Assume that a beam of positron particles is shot on a target containing electrons. Calculate the beam energy required for the positron beam in order to produce Z-bosons.
  d) This experiment was carried out in the 1990's. Which method do you think was used? Why?

"Quick" Exercises 9, 10, 11

9. [Griffiths exercise 2.2] "Crossing lightsabers"
    - Draw the lowest-order Feynman diagram representing Delbruck scattering: $\gamma + \gamma \rightarrow \gamma + \gamma$
    - This has no classical analogue. Explain why.
10. [Griffiths exercise 2.4]
    - Determine the invariant mass of the virtual photon in each of the lowest-order Feynman diagrams for Bhabha scattering
11. [Griffiths exercise 2.7]
    - Examine the processes in Griffiths exercise 2.7 and state which one is possible or impossible, and why / with which interaction.

    Hint: draw the corresponding Feynman diagrams if needed.

Exercise – 12: Penguins

- One of the flagship analyses of the LHCb experiment is the decay $B_s^0 \to J/\psi\phi$. It is used for the analysis of CP violation.
  - Draw the lowest order (tree) Feynman diagram of the process. ($B_s^0 = (\bar{b}s), J/\psi = (c\bar{c}), \phi = (s\ \bar{s})$) . Hint: it consists of an 'internal' $W \to cs$ transition. Why would this diagram be called 'colour suppressed'?
  - I addition to the tree diagram, there is a famous 'loop' contribution, called a *penguin* diagram. Here, the b transforms into an s due to the emission and re-absorption of a W and a quark, while this quark radiates off a gluon that turns into a $c\bar{c}$. Draw this diagram. Try to twist and turn the diagram such that it may look as a penguin?
  - Ask for the funny story behind this name
  - Which flavour can this internal quark have? Which option has the largest probability?
  - The precision of LHCb is such that these penguin contributions are becoming a nuisance. Based in the vertices and CKM elements, can you guess how much weaker the penguin diagram is with respect to the tree?