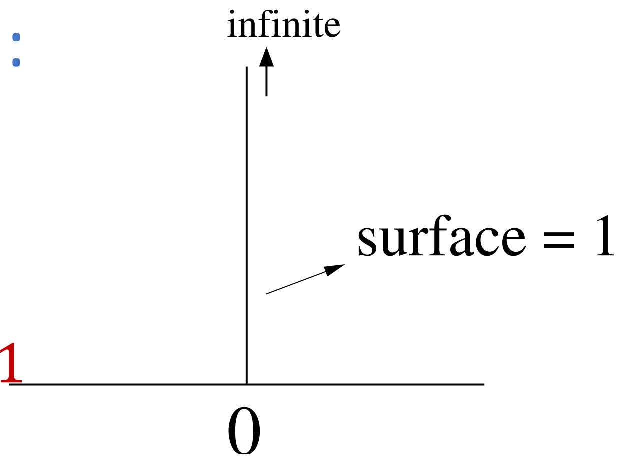
Exercise – 27: Dirac delta function (1)

See Griffiths Appendix A

- Consider a function defined by the following prescription:

$$\delta(x) = \lim_{\Delta \to 0} \begin{cases} 1/\Delta & \text{for } |x| < \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$



- The integral of this function is normalized: $\int_{-\infty}^{\infty} \delta(x)\,\mathrm{d}x = 1$

- And for a function $f(x)$ we have: $\int_{-\infty}^{\infty} f(x)\delta(x)\,\mathrm{d}x = f(0)$

- Exercise:
  a) Prove that: $\delta(kx) = \frac{1}{|k|}\delta(x)$
  b) Prove that: $\delta\big(g(x)\big) = \sum_{i=1}^{n} \frac{1}{|g'(x_i)|}\delta(x - x_i)$, where $g(x_i) = 0$ are the zero-points
     - Hint: make a Taylor expansion of g around the 0-points.

Exercise – 27: Dirac delta function (2)

- The delta function has many forms. One of them is: $\delta(x) = \lim_{\alpha \to \infty} \frac{1}{\pi} \frac{\sin^2 \alpha x}{\alpha x^2}$

c) Make this plausible by sketching the function $\sin^2(\alpha x) / (\pi \alpha x^2)$ for two relevant values of α

- Remember the Fourier transform:

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(k)\, e^{ikx}\, \mathrm{d}k$$

$$g(k) = \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, \mathrm{d}x$$

d) Use this to show that another (important!) representation of the Dirac delta-function is given by:

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx}\, \mathrm{d}k$$

🡐 We will use this later in the lecture!

Exercise – 28: Kinematics relation

- Show explicitly that by inverting the equation:

$$m_1 = \sqrt{p^2 + m_2^2} + \sqrt{p^2 + m_3^2}$$

it follows that:

$$p = \frac{1}{2m_1}\sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2m_2^2 - 2m_1^2m_3^2 - 2m_2^2m_3^2}$$

Exercise – 29: Pion Decay

Griffiths exercise 6.6

Calculate the lifetime of the neutral pion π^0

The neutral pion decays mainly via: $\pi^0 \rightarrow \gamma\gamma$. Assume that the amplitude has dimensions $[\text{mass}] \times [\text{velocity}]$.

a) Motivate the reason that the amplitude should be proportional to the coupling constant α. Try to sketch a diagram of the decay.

b) Use Fermi's golden rule for two-body decays to estimate the lifetime of the pion.

c) Compare it with the experimental value. What do you think?

Exercise – 30: Two-to-Two Scattering

Griffiths exercise 6.15

Consider the process: $A + B \rightarrow A + B$ in the ABC theory

a) Draw the (two) lowest-order Feynman diagrams, and calculate the amplitudes
b) Find the differential cross-section in the CM frame, assuming $m_A = m_B = m\,, m_C = 0$, in terms of the (incoming) energy E and the scattering angle θ.
c) Assuming next that B is much heavier than A, calculate the differential cross-section in the lab frame.
d) For case c), find the total cross-section.