Exercise – 20 : Charge Current

- Show that the definition $W_\mu^\pm = \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}$ leads to the charged current:

  $\mathcal{L} = -W_\mu^+ J^{\mu^+} - W_\mu^- J^{\mu^-}$ with $J^{\mu^+} = \frac{g}{\sqrt{2}} \overline{\Psi} \gamma_\mu \tau^+ \Psi$ and $J^{\mu^-} = \frac{g}{\sqrt{2}} \overline{\Psi} \gamma_\mu \tau^- \Psi$

Exercise – 21 : Symmetry breaking

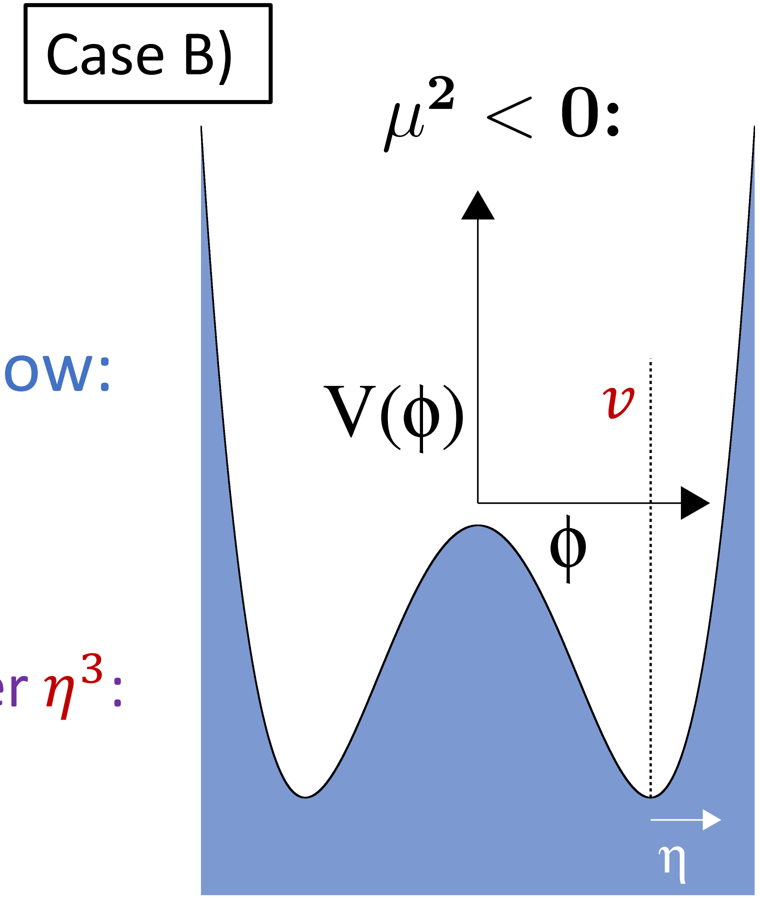
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

- Redefine coordinates: $\eta \equiv \phi - v$
- Exercise: re-write the Lagrangian in ϕ and v to show:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4$$

  - Ignore the constant term $\frac{1}{4}\lambda v^4$ and neglect higher order η^3:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2$$



  - This describes a new scalar field η with a mass $\frac{1}{2}m_\eta^2 = \lambda v^2 \Rightarrow m_\eta = \sqrt{2\lambda v^2}$ $(= \sqrt{-2\mu^2})$
  - Price to pay: Lagrangian is no longer symmetric under $\eta \to -\eta$ in the new field.

Exercise – 22 : Mass of the proton

Besides giving mass to the weak vector bosons, it was briefly flashed that the same Higgs mechanism is responsible for giving mass to the fermion masses in the Standard Model, through ad-hoc Yukawa couplings. The mass of a 'naked' quark can be estimated through models of soft QCD, where it enters as a parameter for e.g. the binding energy of a meson. For up and down, they are found to be roughly 2 resp. 5 MeV/c .

a) What fraction of the proton mass is due to the Higgs mass of the constituent quarks?

b) Can you find out where the other part of the proton mass comes from?

Exercise – 23 : Parity

a) Find the eigenvalue of the parity operator P, for the function $y(x) = 10x^5 + 3x^3$

b) (Optional for die-hards only – not required)
Find an expression for the parity of the Y_{lm} functions. Show that the parity is $(-1)^l$. This means that if a state has orbital angular momentum, there is an additional factor of $(-1)^l$ to the Parity eigenvalue!

c) [Griffiths 4.37 a)] Explain why the decay $\eta \to \pi\pi$ is forbidden for both strong and electromagnetic interactions.

Exercise – 24 : Helicity vs Chirality

a) Write out the chirality operator γ^5 in the Dirac-Pauli representation.

b) The helicity operator is defined as $\lambda = \vec{\sigma} \cdot \hat{p}$. Show that helicity operator and the chirality operator have the same effect on a spinor solution, i.e.

$$\gamma^5 \psi = \gamma^5 \begin{pmatrix} \chi^{(s)} \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi^{(s)} \end{pmatrix} \approx \lambda \begin{pmatrix} \chi^{(s)} \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi^{(s)} \end{pmatrix} = \lambda \psi$$

with: $\chi^{(1)} = \binom{1}{0}$; $\chi^{(2)} = \binom{0}{1}$

in the relativistic limit where $E \gg m$

c) Show explicitly that for a Dirac spinor:

$\bar{\psi} \gamma^\mu \psi = \overline{\psi_L} \gamma^\mu \psi_L + \overline{\psi_R} \gamma^\mu \psi_R$ making use of $\psi = \psi_L + \psi_R$ and the projection operators: $\psi_L = \frac{1}{2}(1 - \gamma^5)$ and $\psi_R = \frac{1}{2}(1 + \gamma^5)$

d) Explain why the weak interaction is called left-handed.

Wolfgang Pauli

"I cannot believe God is a weak left-hander."

Exercise – 25 : Bilinear Covariants

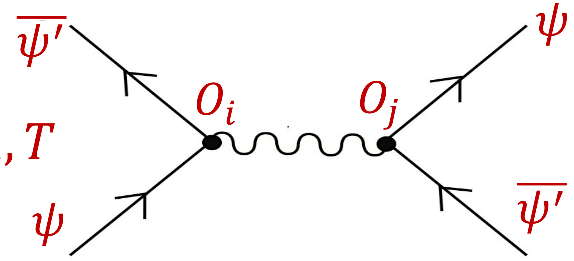
- What is the reason that gamma matrices *cannot be* Lorentz 4-vectors?
- The space-time dependence is included by combining the wave function $\psi(t,x)$ with the gamma matrices γ^μ. Show explicitly that each of the following so-called *bilinear covariants* no longer carry Dirac spinor indices:

  $S{:}\,\bar{\psi}\psi,\;\; V{:}\,\bar{\psi}\gamma^\mu\psi,\;\; T{:}\,\bar{\psi}\sigma^{\mu\nu}\psi,\;\; A{:}\,\bar{\psi}\gamma^5\gamma^\mu,\;\; P{:}\,\bar{\psi}\gamma^5\psi$ where $\sigma^{\mu\nu} \equiv \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$
- How many Lorentz indices does each of these combinations have?
- Can you think of additional independent forms that contract two Dirac spinors into an object without Dirac indices? These bilinear combinations are generally called currents as they are the most general form in which currents can occur.
- Explain the names of these currents:

  S = Scalar, V = Vector, T = Tensor, A = Axial vector, P = Pseudo-scalar
- The most general type of interaction will be of the form represents :

$$\mathcal{M} = G \sum_{i,j}^{S,P,V,A,T} C_{ij}(\bar{\psi}O_i\psi')(\bar{\psi}O_j\psi')$$

where Operator O_i represents S, P, V, A, T



Exercise – 26 : Symmetries

a) What do you think is the difference between an exact and a broken symmetry?

b) Can you explain the name *spontaneous* symmetry breaking means?

c) Which symmetry is involved in the gauge theories below? Which of these gauge symmetries are exact? Why/Why not?

   i. U1(Q) symmetry
   ii. SU2(u-d-flavour) symmetry
   iii. SU3(u-d-s-flavour) symmetry
   iv. SU6(u-d-s-c-b-t) symmetry
   v. SU3(colour) symmetry
   vi. SU5(Grand unified) symmetry
   vii. SuperSymmetry