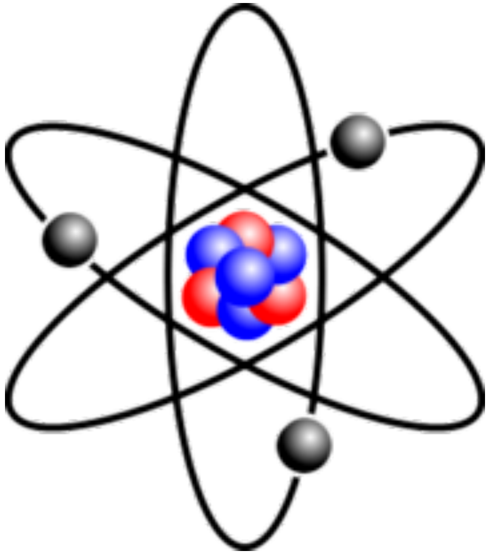
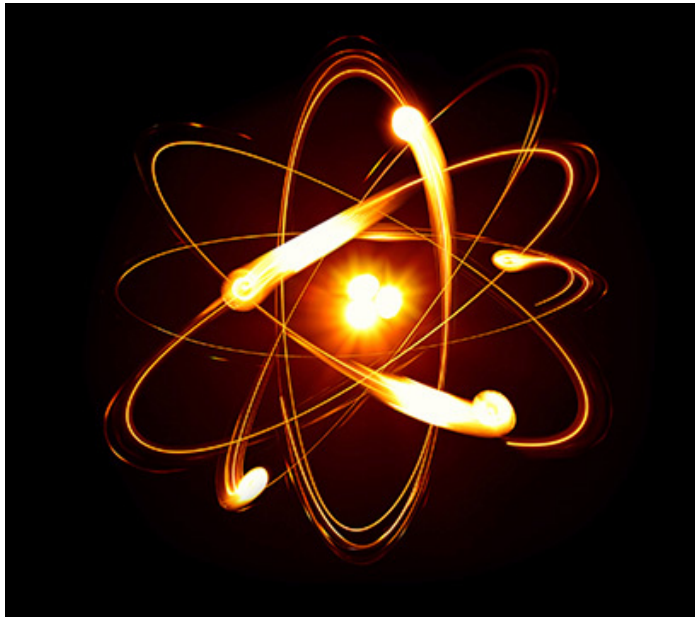
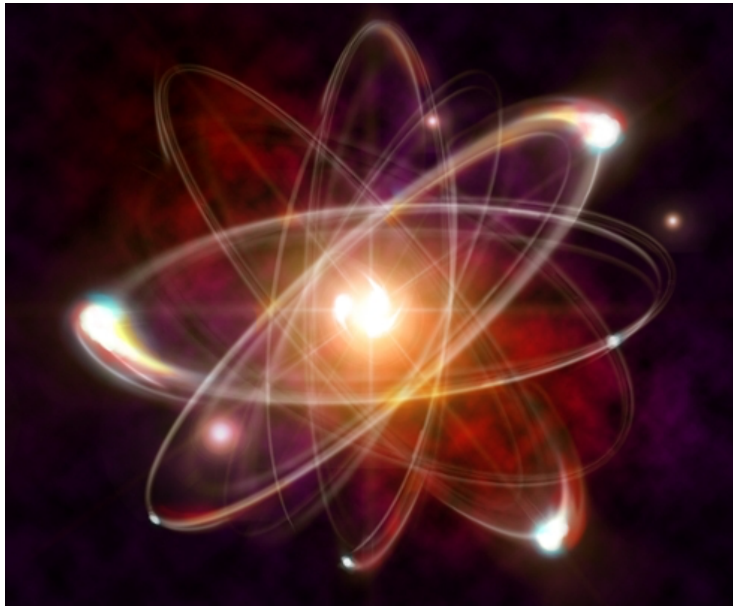
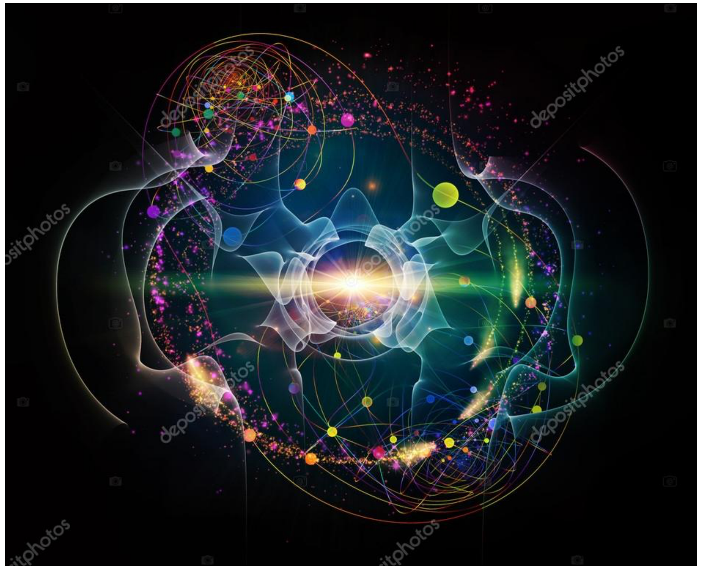
Exercise-1: Which mental picture do you make of an atom?

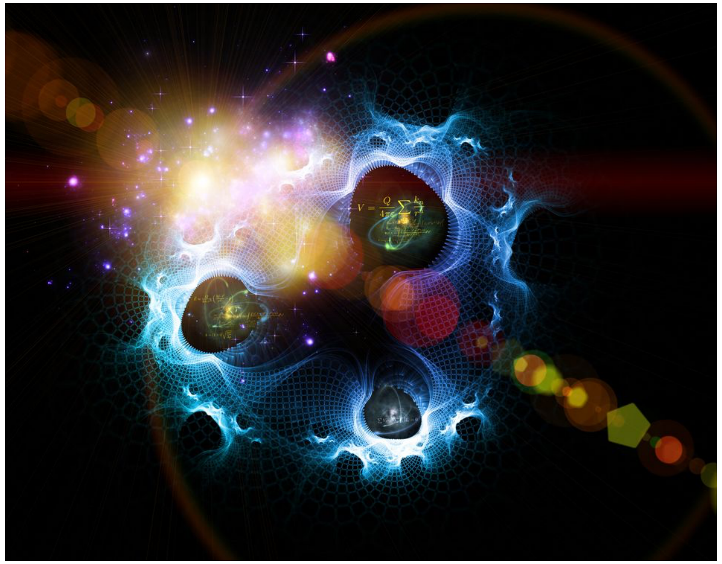
Classic picture of the atom











- Does something “move”?
- How empty is it?
- How does an electron “exist”?

➔ Consider QM interpretations

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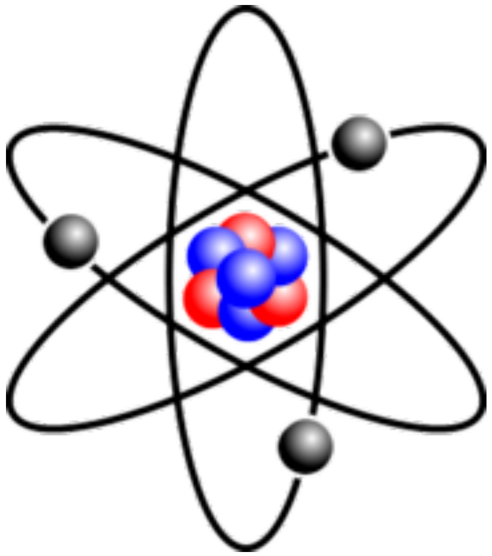

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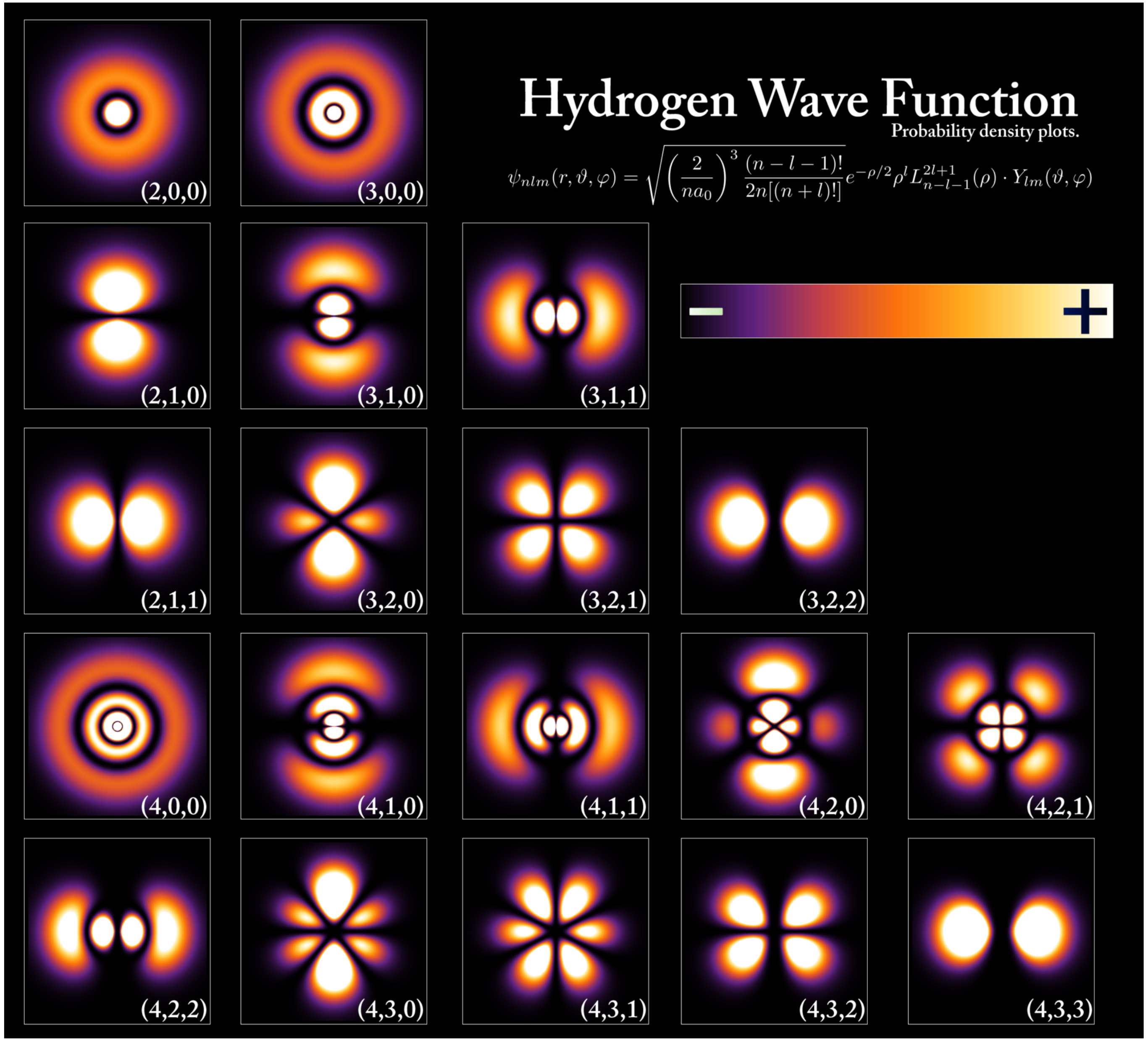


Exercise-2 : The Yukawa Potential

- The electric force is transmitted by a photon with $m = 0$. The wave equation for a static electric field caused by pointlike charge e is: $\nabla^2 V(r) = 0$ (Laplace equation)

  a) Show that the Coulomb potential $V(r) = -e^2 \frac{1}{r}$ fulfills this equation.

  - Note that the potential is ***spherical symmetric,*** ie. use spherical coordinates.

- The nuclear force is transmitted by a pi-meson with $m = m_\pi$. The wave equation for a static nuclear field caused by a pointlike color charge g is: $\nabla^2 U(r) = m^2 U(r)$ (Klein-Gordon equation)

  b) Show that the Yukawa potential $U(r) = -g^2 \frac{e^{-r/R}}{r}$ fulfills this equation for a certain value of R, the ***range*** of the force. What is the relation between R and m_π?

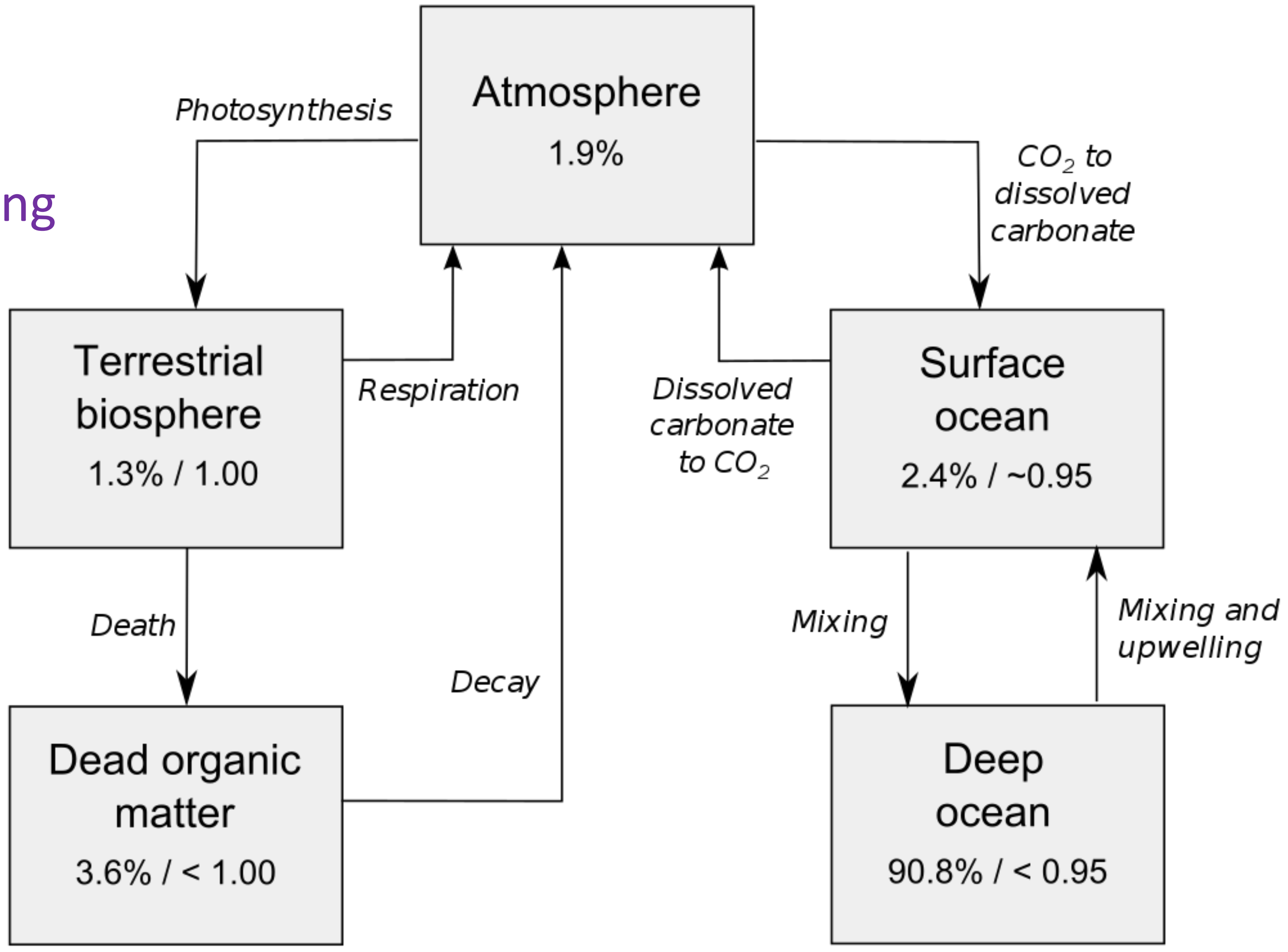
  - Again note that the potential is ***spherical symmetric.***
  - This value is between the electron and proton mass, hence the particle was called a pi-meson or pion.

  c) Calculate the range of the force from Heisenberg's uncertainty relation, using $R = c\Delta t$ and $\Delta E \Delta t \leq \frac{\hbar}{2}$ and $\Delta E = mc^2$.

  d) The weak force is mediated by W (80 GeV) and Z (91 GeV) bosons. What is the estimated range of the weak force?

Exercise-3: How does Radiocarbon dating work?

- Exercise:
  - Explain how the Carbon dating method works



Percentages show the fraction of the total carbon reservoir of each type.
Numbers after slash show ratio of ^{14}C to ^{12}C as fraction of atmospheric ratio.

Exercise-4: Fission vs Fusion

a) Calculate the energy released if 1 gram of U-235 splits into La-148 + Br-87.

b) Calculate the energy released in the fusion process of 0.5 grams of heavy water (D_2O) with 0.5 grams of superheavy water (T_2O), creating He-4 and a neutron. You may neglect the binding energies of the molecules.

c) Compare the energies released per gram of fuel calculated above. Which would you prefer?

Exercise–5: Natural Units

- In particle physics we make often use of natural units
  - Very confusing at first but very convenient when you are used to it ("sloppy")

    Set $c = 2.998\times10^{8}\ m/s \equiv 1$ and $\hbar = 1.055\times10^{-34}\ Js \equiv 1$

    (Just leave them out and put them back at very end of any calculation)
  - Consequence: there is only one basic unit for length, time, mass and energy: GeV

- Exercise: derive the numbers on the conversion table on the next page

Natural Units: conversion table

quantity	symbol in natural units	equivalent symbol in ordinary units
space	x	$x/\hbar c$
time	t	$t/\hbar$
mass	m	mc^2
momentum	p	pc
energy	E	E
positron charge	e	$e\sqrt{\hbar c/\epsilon_0}$

Conversion of basic quantities between natural and ordinary units.

quantity	conversion factor	natural unit	normal unit
mass	$1\text{ kg} = 5.61 \times 10^{26}\text{ GeV}$	GeV	GeV/c^2
length	$1\text{ m} = 5.07 \times 10^{15}\text{ GeV}^{-1}$	GeV^{-1}	$\hbar c/\text{ GeV}$
time	$1\text{ s} = 1.52 \times 10^{24}\text{ GeV}^{-1}$	GeV^{-1}	$\hbar/\text{ GeV}$

Conversion factors from natural units to ordinary units.

Exercise-6 : the quark model; wave function of hadrons

a) Quarks are fermions with spin 1/2. Show that the spin of a meson (2 quarks) can be either a triplet of spin 1 or a singlet of spin 0.

- Hint: use the “Clebsch-Gordon” coefficients in adding quantum numbers. In group theory this is often represented as the product of two doublets leads to the sum of a triplet and a singlet: $2 \otimes 2 = 3 \oplus 1$ or, in terms of quantum numbers: $1/2 \otimes 1/2 = 1 \oplus 0$.

b) Show that for baryon spin states we can write: $1/2 \otimes 1/2 \otimes 1/2 = 3/2 \oplus 1/2 \oplus 1/2$ or equivalently $2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$

c) Let us restrict ourselves to two quark flavours: *u* and *d*. We introduce a new quantum number, called *isospin* in complete analogy with spin, and we refer to the *u*-quark as the isospin +1/2 component and the *d*-quark to the isospin -1/2 component (or *u*= isospin “up” and *d*=isospin “down”).

- What are the possible isospin values for the resulting baryon?

d) *Optional for die-hards!* The Δ++ particle is in the lowest angular momentum state ($L = 0$) and has spin $J_3 = 3/2$ and isospin $I_3 = 3/2$. The overall wavefunction ($L \Rightarrow$space-part, $S \Rightarrow$spin-part, $I \Rightarrow$isospin-part) must be anti-symmetric under exchange of any of the quarks. The symmetry of the space, spin and isospin part has a consequence for the required symmetry of the Color part of the wave function.

- Write down the color part of the wave-function taking into account that the particle is color neutral.