Exercise 3

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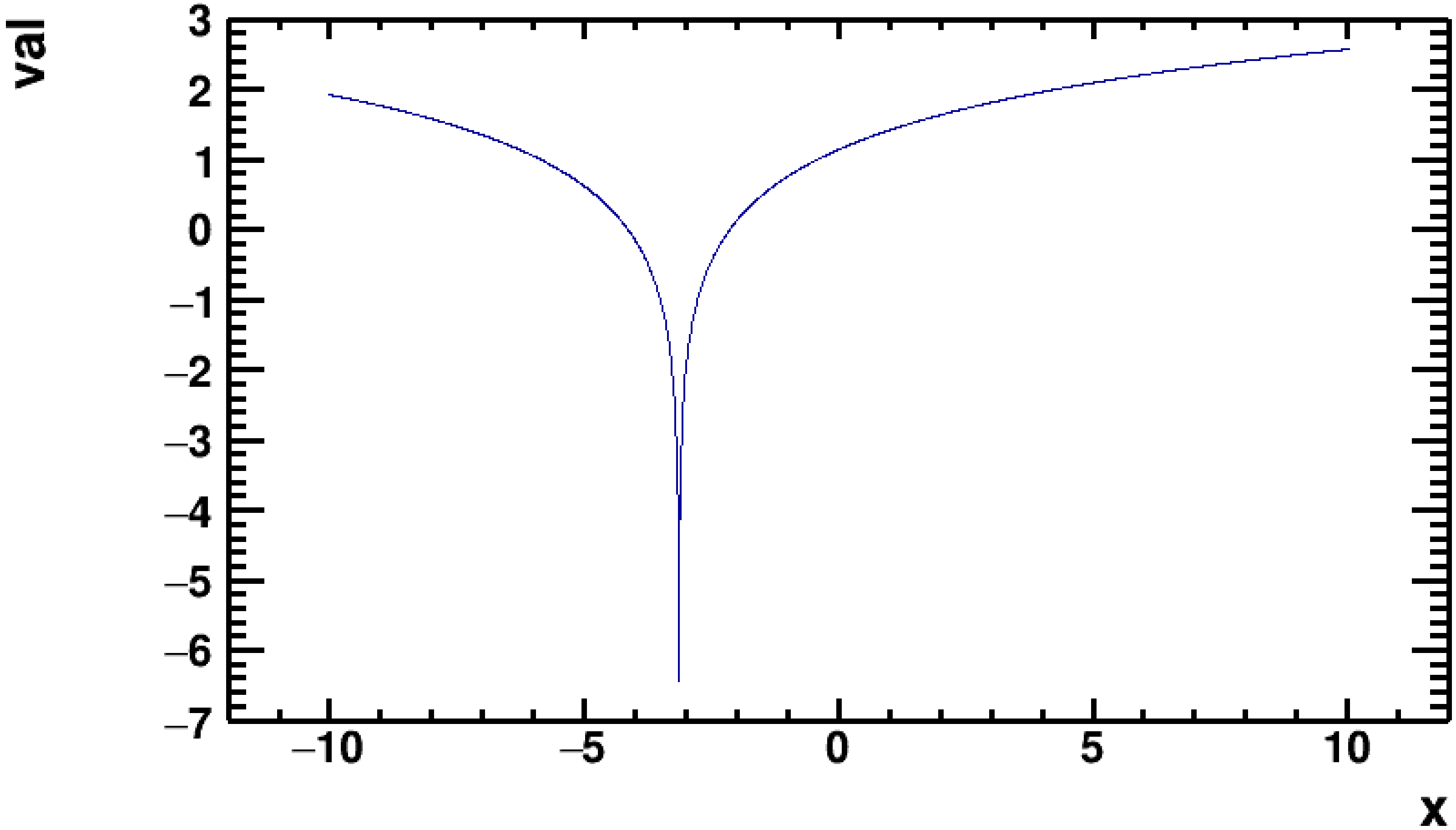
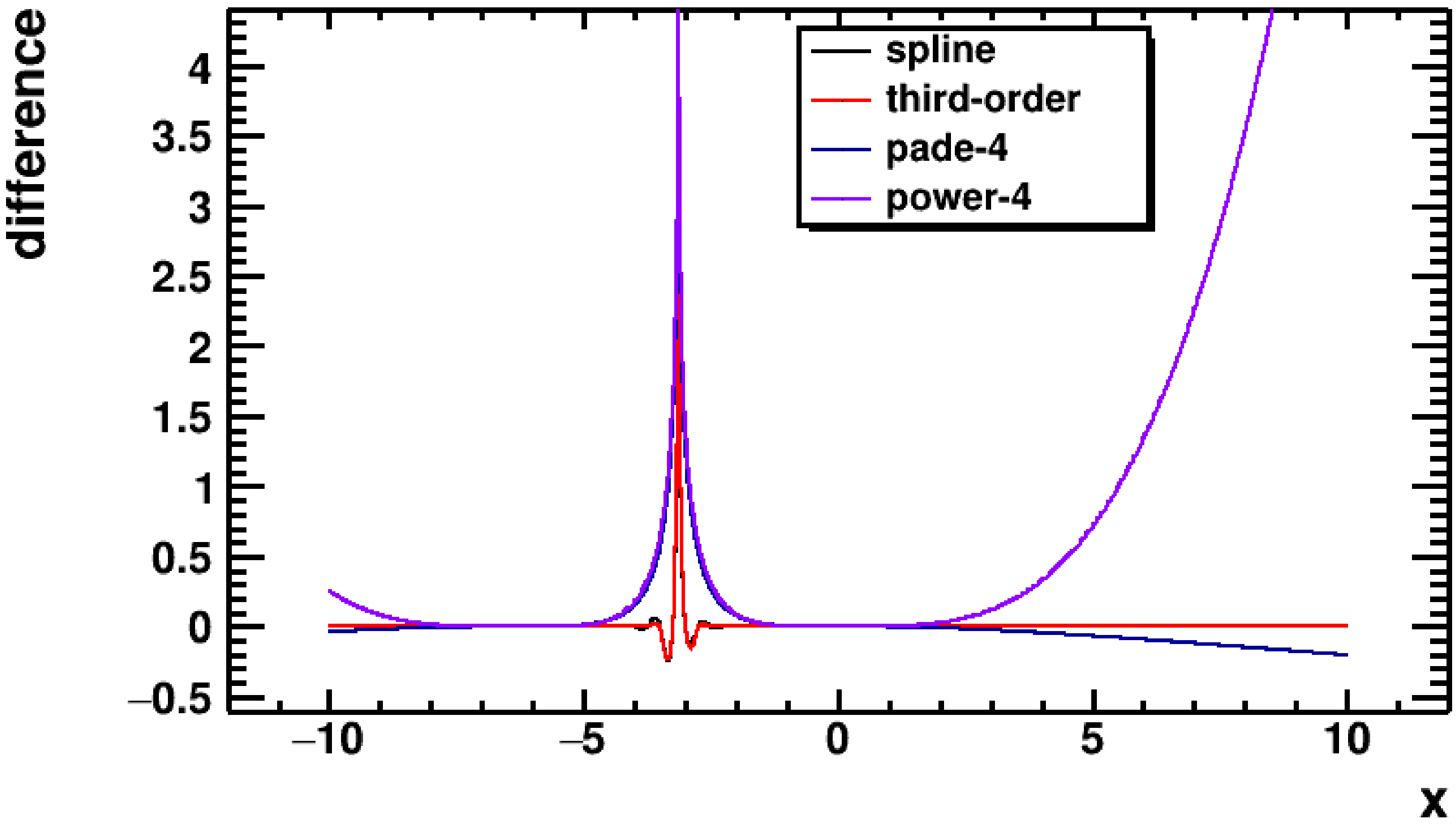
- Interpolation routines.
  - Interpolation schemes switch on grid point: else the function values are not continuous
  - Spline: continuous derivatives (cubic: to third order)
  - Power series: Taylor expansion; uses derivatives to extrapolate from x=0
  - Pade series: Rational expansion with same derivatives as power series
- Function: ln|x+pi| , pole at -pi
  - For power series and expansion, calculate the series with x>-pi (if x < -pi replace with y = -2x+pi; in this manner you can use the same coefficients.
- Code: interpolation.cpp

Exercise 3

- Function values at [-10,10] in steps of 0.01

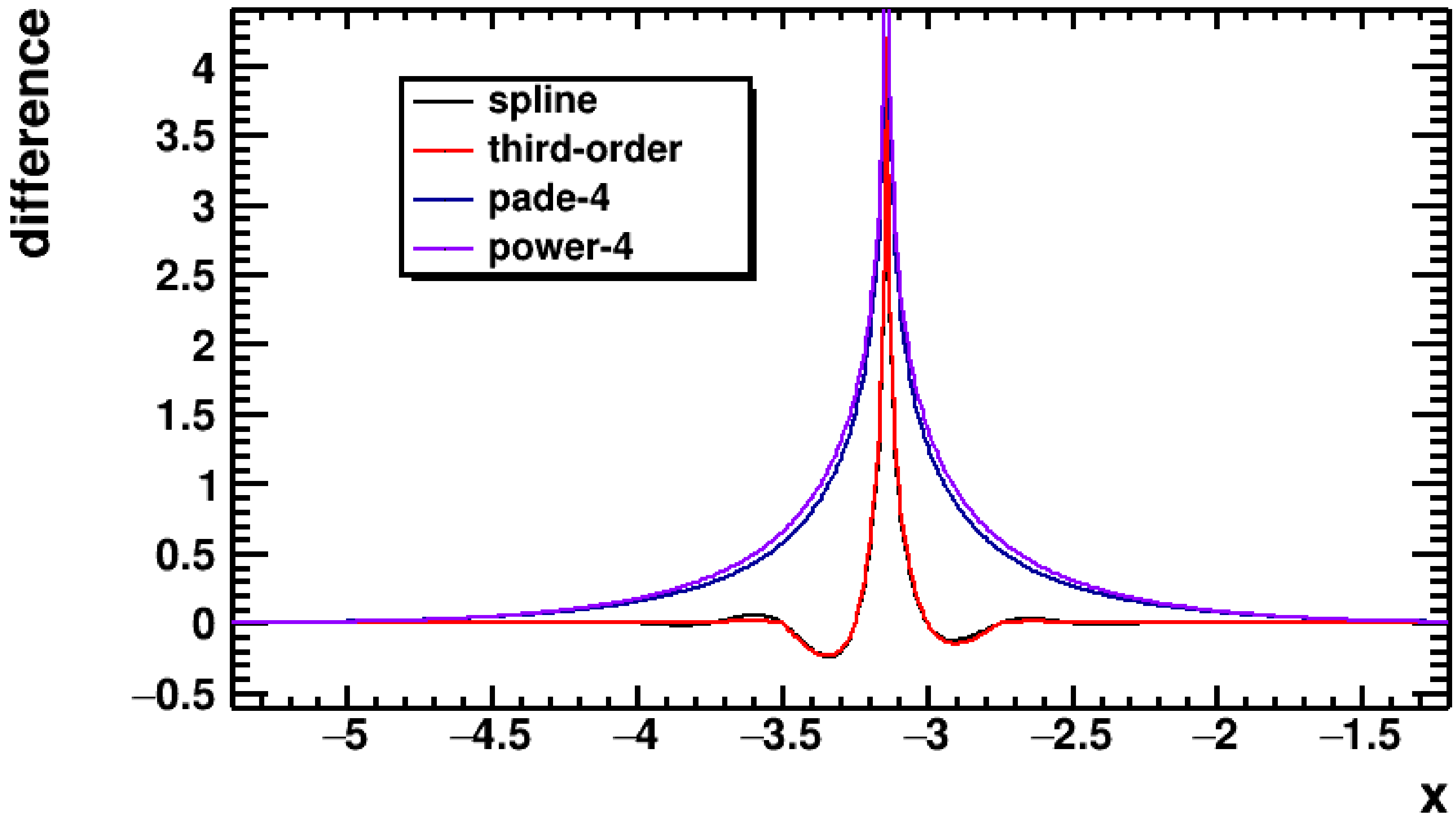


Interpolation schemes, N=4



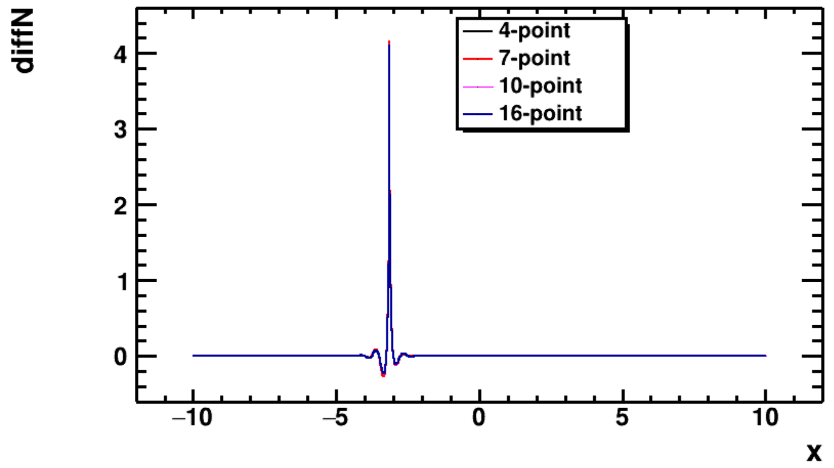
The difference with the true function is plotted for all interpolated results (2001 steps). Spline and 3rd-order polynomial interpolation differ slightly. Third-order power series and pade approximation differ at high x.

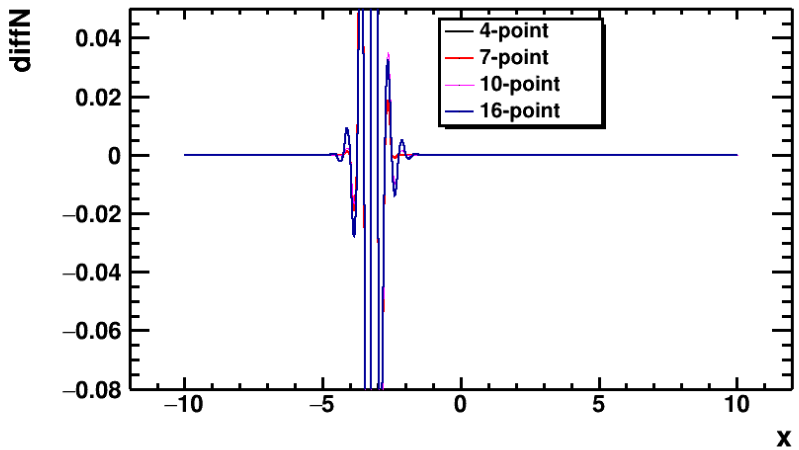
Interpolation schemes, N=4



Zoom around the pole. 3^Rd-order interpolation is very good when the pole is not in between the used grid points (so flat from -3.5 to -4) whereas spline has to be continuous (stiffer) and wiggles around a bit.

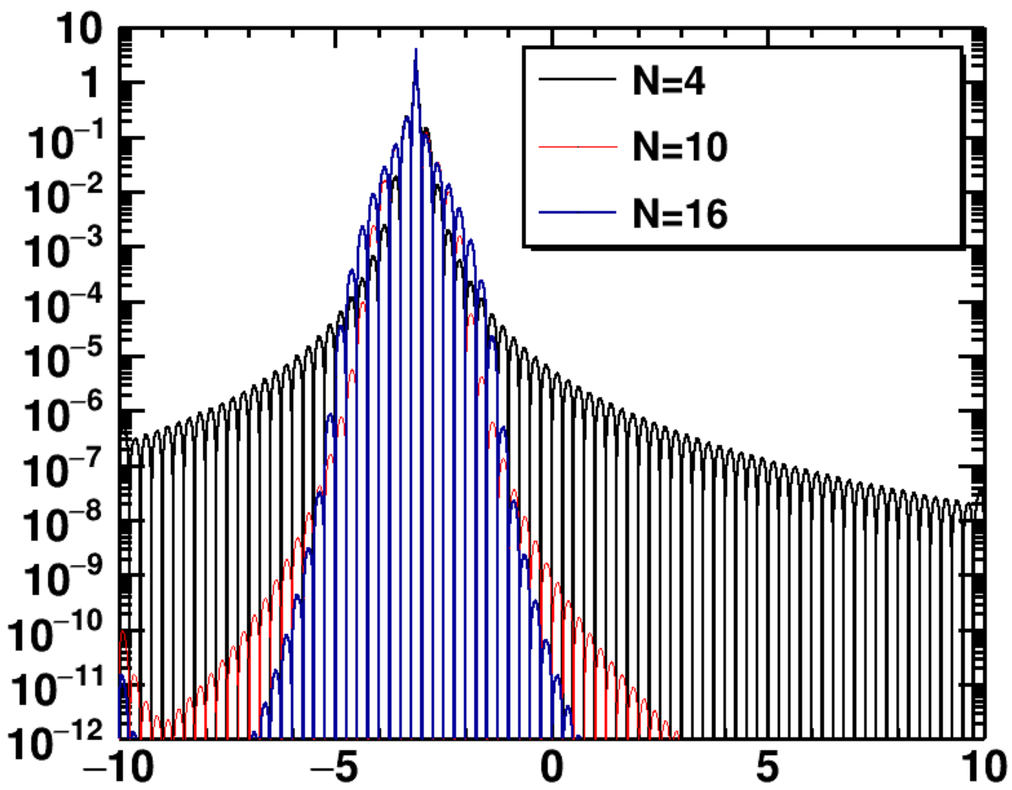
Polynomial interpolation



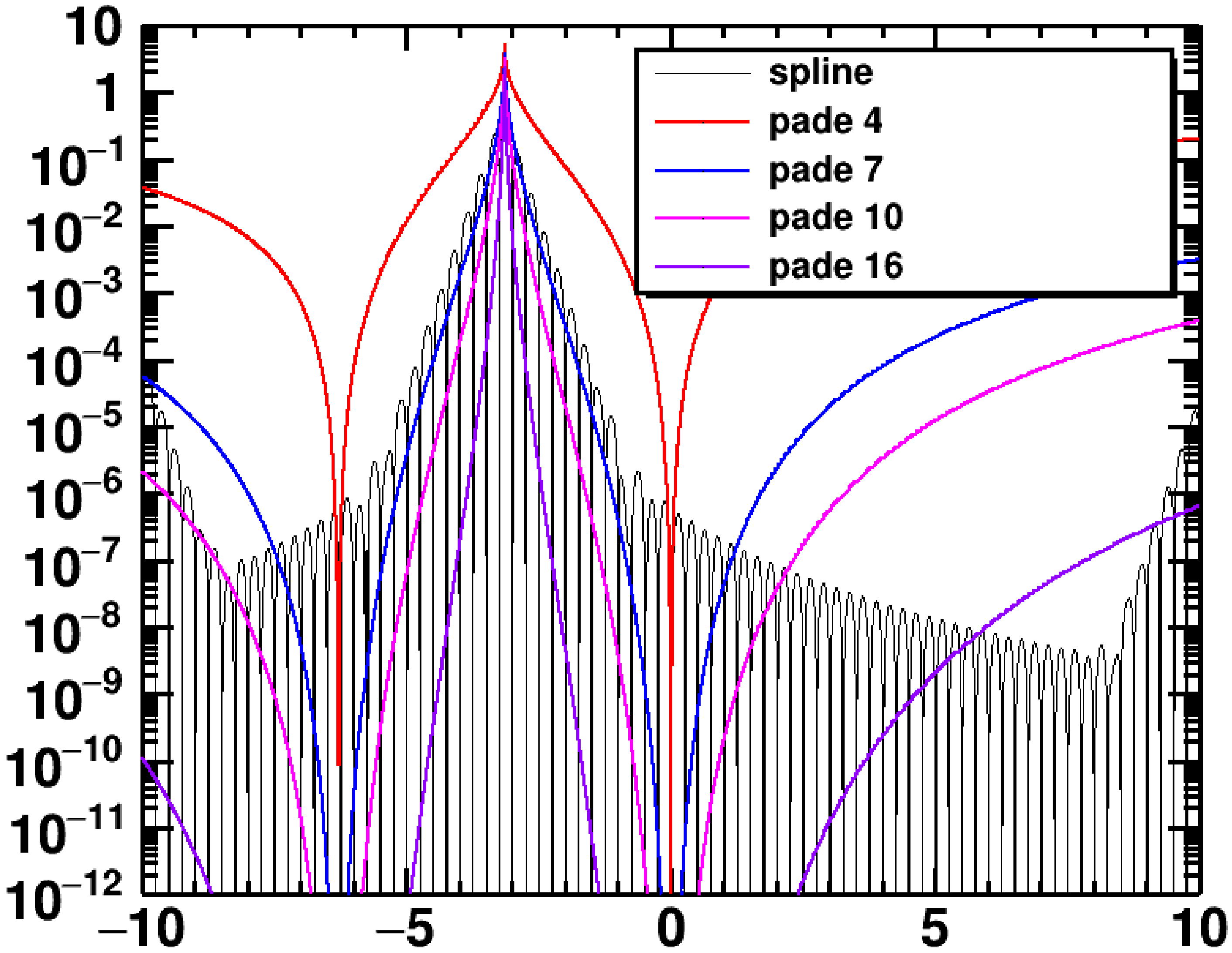


Top: difference of interpolation results for N=4,7,10,16.
Top right: zoom around the pole

Right: plot of the absolute value of the difference. At all grid points the difference is zero, and in between grid points it is maximal. The lowest-order interpolation gives the worst results in the tails away from the pole, but the best results close to the pole.
Also, close to the boundary of the interval (+/-10) the difference for high N rises, due to the fact that you need to use N gridpoints that are not symmetrically placed around the interpolated result.



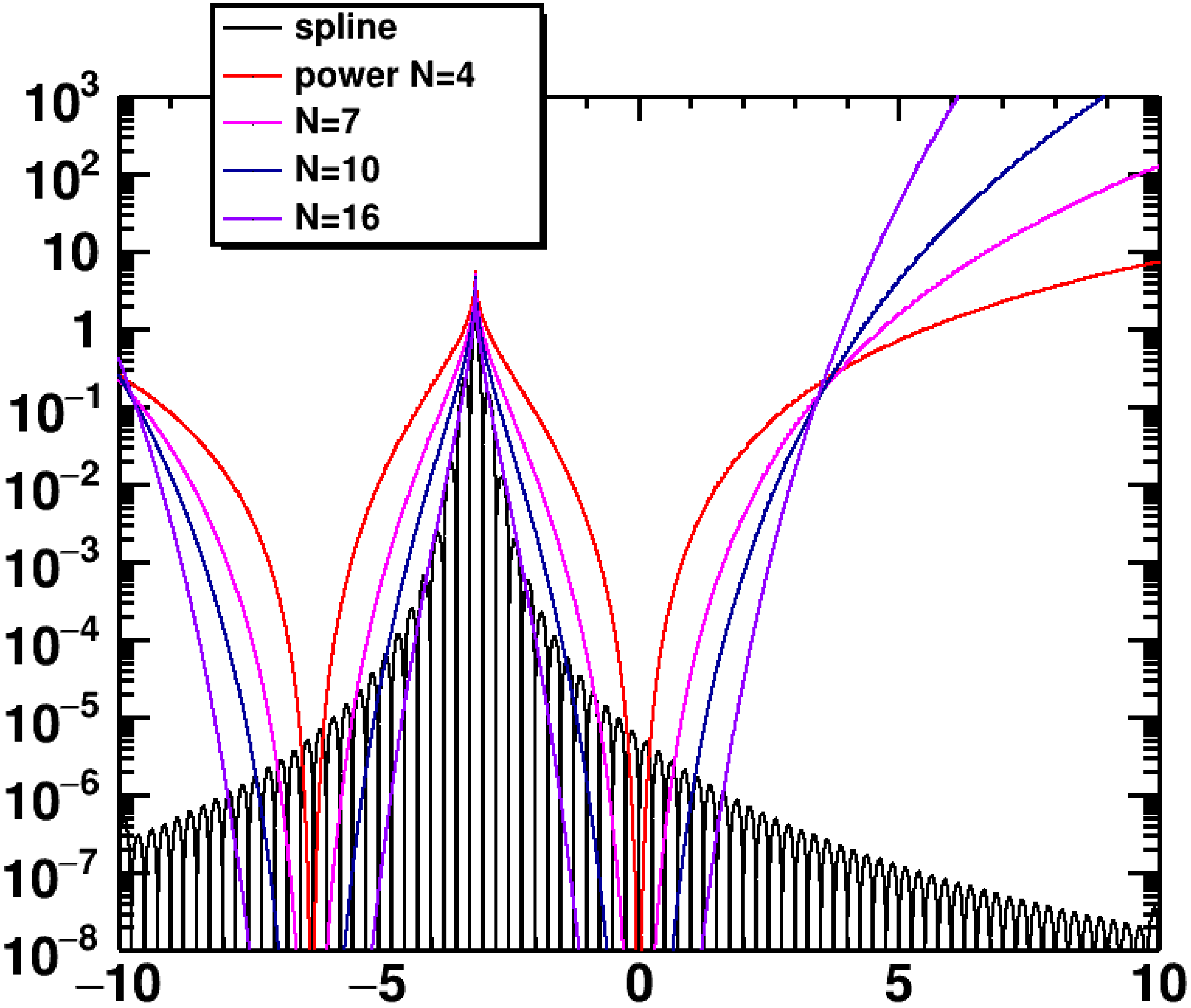
Pade extrapolation



Pade results for N=4..16.

Each higher order is giving a better interpolation. Already for N=7, the Pade result is better than spline/polynomial interpolation in an interval +/- 5 around the pole, which is remarkable since the grid point difference is 20 times smaller!

Power series extrapolation



Power series results for N=4..16.

The power series deviates a lot at higher values of x. Indeed, the coefficient for the 15-th derivative is multiplied with 10^15 at x=10, so a small deviation results in a large offset.

Whereas the rational Pade extrapolation is doing better at higher N, the power series is only doing better at low values of x. The 16-point Pade series beats all interpolation schemes everywhere (for these grid points up to infinity) the power series is just marginally better than spline in the region of the pole for N=16

Results

- Results for the average RMS as asked in the exercise:
  - Although Pade does not have guidance from the grid points, it does better than all interpolation schemes already at N=7.

N-point	Cubic spline	Polynomial interpolation	Pade extrapolation	Power series extrapolation
4	0.143426	0.147625	0.329451	1.94232
7		0.145481	0.141727	24.4738
10		0.142948	0.110082	447.385
13		0.143019	0.073422	9582.11
16		0.141847	0.064702	224472