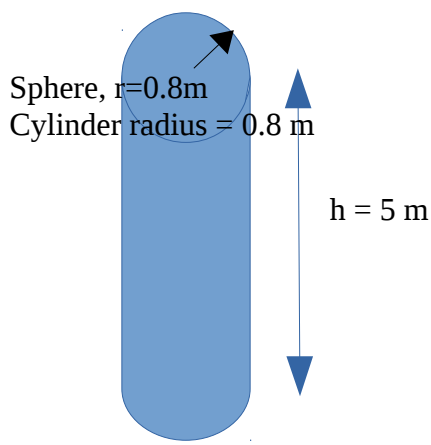
Exercise 8. Due Monday March 28. 5 points

In this exercise, we will use Monte Carlo techniques to calculate the equilibrium temperature that will be reached by a satellite orbiting the earth. The satellite loses heat via radiation according to the law of Stefan-Boltzmann: $dP/dA = \epsilon \sigma T^4$ with dP/dA the power per unit area that is radiated away, ε the emissivity (1 for a perfect black body, 0 for a perfectly reflecting white body), T the temperature in Kelvin, and $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$. The cylinder receives radiation from the sun in visible light; the emissivity of the satellite equals 0.2 in visible light. It also receives reflected sunlight from the part of the earth where it is day, the earth reflects 0.4 of the incident sunlight back (in a solid angle of 2 pi; you can multiply the flux from the sun with the cosines of the angles normal to the earth surface and with the albedo of 0.4 to see how much is reflected in the direction of the satellite). Furthermore the earth emits infrared light with an emissivity of 1 and a black-body temperature of 290 K (you may assume that homogeneous). For the temperature of the sun one can take 5700 K; the radius of the sun is 0.7 million km and the distance to the sun 150 million km (giving a flux at earth of $(r/R)^2 \sigma T^4 = 1303\, W/m^2$ ,close to the true value).

The satellite has the shape of a cylinder with height 5m and radius 0.8m, and at the top a half-sphere with a radius of 0.8 m, the bottom is flat.
The satellite orbits the earth in the equatorial plane; this plane makes an angle of 23 degrees with the planetary orbit of the earth around the sun. (radius earth = 6700 km).




Calculate the temperature that the satellite will get on average, when in thermal equilibrium. You can draw random parts of the surface of the satellite and integrate the infalling radiation from the earth and the sun; for the sun it is best to take a constant flux from the direction of the sun and determine the angle of that direction with the surface element, or analytically integrate over the solid angle seen from the surface element dA towards the sun; drawing a random direction will require too many trials for a good averaging. For the earth, one can use priority sampling or just randomly draw from 2pi solid angle and see whether it hits or misses. Also, you can draw random positions along the orbit of the satellite. Assume that the top of the satellite is pointing up, (declination 0 degree, in the direction of Polaris).

The averaged, summed, infalling radiation is also radiated away in all directions; the integral over the incoming radiation times the surface area of the detector equals $A\sigma T^4$. With A the total area of the satellite and T the temperature that the satellite reaches in equilibrium.