Exercise 5. Due Monday Feb 27, 2017. Weight 4 points.

As explained in lecture 4, the exercise is to create a Chebyshev function evaluation of the pdf of detected energy in a scintillator. Assumed is that the scintillator is traversed by minimum-ionizing particles that leave energy according to a Landau-distribution. Assumed is that this distribution can be described by the function

$$pdf_{loss}(E) = N \int_0^{\infty} e^{-x(E - E_p + \ln(x))} \sin(\pi x) dx$$

with E_p = 2.4. This integral is difficult to calculate. It could be approximated with a Chebyshev function calculation.
Also, the scintillator measures noise, according to a Normal distribution with $\sigma = 0.25$ (thus $pdf_{noise}(E_n) = \sqrt{\frac{1}{2\pi\sigma^2}} e^{-E_n^2/2\sigma^2}$).
The detected signal follows the pdf for the noise when there is no particle traversing the scintillator, and the sum of the noise plus energy loss in case of a passing particle. The pdf of this combination is obtained by folding the 2 distributions:

$$pdf_{detect}(E_{det}) = \int_0^{\infty} pdf_{loss}(E)\, pdf_{noise}(E_{det} - E)\, dE = \int_0^{\infty} pdf_{loss}(E) \frac{e^{-(E - E_{det})^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} dE$$

(If you already have a Chebyshev approximation for pdf_{loss} then you can quickly calculate this second pdf of the detected signal).

The probability density distribution for the detected signal in case of 2 particles traversing the detector is again obtained by folding the pdf of the loss of 1 particle (the second particle) and the detected signal of the first particle (i.e. include the noise just once) :

$$pdf_{2part} = \int_0^{E_{det}} [pdf_{detect,1}(E)\ \ pdf_{loss}(E_{det} - E)]\ \ dE$$

The exercise is described in lecture 4. Apart from the pdfs, you have to be able to determine how often a false alarm is triggered (that is, how often the noise exceeds E_noise; the integral of the noise pdf from E_noise to +infinity) and how often a particle leaves less detected energy than a certain threshold. For this, you also need the cumulative pdfs, the integral of the pdf from -infinity to E. This integral is the joint probability that an event generated less energy than E. For the Normal distribution, the cumulative integral is given by the error function (see lecture). For the Landau pdfs, you can calculate the integrals by making a new chebyshev approximation with Chebyshev cpdf = pdf.integral() assuming that pdf is the Chebyshev structure describing your pdf.