Exercise 3. Due Friday, Feb. 17, 2017 (12:00). weight 2 points.

Assume, that the values of the function $f(x)=\ln(|x+\pi|)$ are known for the values of $x=-10+0.25i$, with 81 integers *i* from 0 to 80. Interpolate these function values for all values of $x=-10+0.01j$, from x=-10 to x=+10, using the natural cubic spline, a (N-1)-order polynomial (i.e. a N-point interpolation function), and the power series and Pade approximant for N coefficients.

For this, assume that at x=0 the first N-1 derivatives of *f*(*x*) are known. The power series can be written as $\ln(\pi)-\sum_{i=1}^{N-1}\frac{1}{i(-\pi)^i}x^i$ for values of $x>-\pi$ (Check). For $x<-\pi$, one can replace the *x* in the power series by $x'=-2\pi-x$ or alternatively create a new power series, that takes into account that the absolute value of $x+\pi$ equals -($x+\pi$) for $x<-\pi$, which is more work (you need 2 functions and 2 Pade approximants in that case). The coefficients of the power series can be used to calculate the function value at all values of *x* and also to calculate the Pade approximant at all values of *x*.

Determine the average deviation from the true function result to the interpolated result, give the RMS value of the deviations of the interpolated results for each of these interpolation schemes (i.e. calculate $\sqrt{\sum_j (interp-calc)^2/2001}$) for the 2001 values of x for the 2 interpolation schemes, the power series, and the Pade approximation. Do this for N=4,7,10,13 and 16 (so interpolation and approximation with 4,7,10,13 and 16 coefficients).

Note, that the power series and Pade approximants extrapolate results over the full range in x, while the interpolated results need 81 function determinations and interpolate between grid points around the value of x. Therefore, it is surprising how accurate the Pade approximation turns out to be.