Exercise 1. Determination of stability of golden mean approximation.

Powers of the golden mean, $\phi = \frac{\sqrt{5}-1}{2}$,

can be calculated using the following recursion relation:

$$\phi^{n+2} = \phi^{n} - \phi^{n+1}.$$

However, this relation is numerically unstable.

Determine, by comparing the result of this recursive relation for higher powers of phi to the direct calculation (obtained by $\phi^n = e^{n \ln \phi}$), after how many terms the difference between those two calculations is larger than 0.1% and after how many terms it is larger than 50%. Give the values of n and ϕ^n, calculated with the recursion relation and with the direct method, both for single precision (4 bytes) and double precision (8 bytes).

The direct calculation (using *pow(phi, n)* or *exp(n log(phi))*) gives an error close to the machine accuracy, which may be verified by dividing the result of the direct calculation n times by the calculation of ϕ. The latter operation (n successive divisions or multiplications) will result in a fractional error that is approximately equal to n times the machine accuracy, since the relative error grows linearly in each step (Why?). This accumulated error, although larger than the error in n random multiplications, is still much smaller than the error obtained by the recursion relation.

Do you understand the results?

This exercise has the lowest weight of 1 point. Mail the source code and your answer to henkjan@nikhef.nl before Thursday Feb 9, 0:00; the results will be discussed next lecture.