Related topics

X-rays, Bragg equation, absorption, transmission, Compton effect, Compton wavelength, rest energy, conservation of momentum and energy conservation principle, relativistic electrons, Einstein's mass/energy relation.

Principle and task

X-rays strike a scatterer and are then scattered according to Compton. By means of a counter tube, the portion scattered under 90° is recorded. By positioning an absorber in front of as well as above a scattering body, the Compton wavelength can be determined from a pre-recorded and measured transmission curve due to the varying intensity attenuation of the X-rays.

Equipment

X-ray unit. w. recorder output	09056.97	1
Compton attachment	09052.01	1
Counter tube, type A, BNC	09025.11	1
Geiger-Müller-Counter	13606.99	1
Screened cable, BNC, l 750 mm	07542.11	1

Problems

1. The transmission cuve of an aluminium absorber as a function of the wavelength is determined by means of Bragg scattering and plotted graphically.
2. The measurement of problem 1. is to be repeated, this time for a limited wavelength interval with maximum angular resolution.
3. By using a scatterer, the intensity of the X-rays scattered under 90° is determined; then, the intensity attenuation is determined for an aluminium absorber in two different positions.
4. The Compton wavelength is determined from the different transmissions yielded from problem 3. and compared with the theoretical value.

Set-up and procedure

The experiment is set up as shown in Fig. 1. The aperature of d = 2 mm is introduced into the outlet X-raxs. By pressing the "zero key", the counter tube and crystal holder device are brought into starting position. The crystal holders are mounted with the crystal surface set horizontally. The counter tube, with horizontal slit aperture, is mounted in such a way that the mid-notch of the counter tube output of the X-ray machine is connected to the corresponding input of the digital counter. The counter tube voltage is app. 500 V.
First the zero pulse rate N_0[Imp/sec] is determined at a counter tube anode voltage of U_a = 0 V.
Beginning at a glancing angle of ϑ = 10° and at maximal anode voltage, the X-ray pulse rate reflected by the crystal N_1 (ϑ) is determined in steps of 1° up to $\vartheta \approx$ 18°, using the digital counter.

Fig. 1: Experimental set-up with Compton scattering device.

This is done at a synchronized rotation of the crystal and counter tube in an angular relationship of 1:2. Due to the necessary exactitude, however, a number of pulses ≥8000 should be measured. If the measured number of pulses is I, the relative measuring error is given by the ratio:

$$\frac{\Delta I}{I} = \frac{\sqrt{I}}{I} = \frac{1}{\sqrt{I}}$$

The aluminium absorber is then inserted between the X-ray outlet and the crystal. Using this arrangement, the measurements are repeated to determine the pulse rate N_2 (ϑ).
At high pulse rates N, not all incoming photons are recorded due to the dead time τ = 100 µs of the counter tube.
Determination of the true pulse rates N' follows with the help of the relation:

$$N' = \frac{N}{1 - \tau N} \quad (1)$$

By means of the Bragg equation

$$2d \sin \vartheta = \lambda \quad \text{lattice constant } (d = 2.014 \cdot 10^{-12} \text{ m}), \quad (2)$$

the wavelength length λ is calculated as a function of the glancing angle ϑ.
From the ratio of the corrected pulse rates, the transmission values are calculated as a function of the wavelength and plotted graphically (Fig. 4).
Subsequently, in order to determine with adequate precision the wavelength change due to scattering from the transmission curve, the measurement is repeated in the interval of $10.0° \leq \vartheta \leq 11.2°$ in steps of $\Delta\vartheta = 0.2°$ (Fig. 5).
Before the Compton attachment – complete with counter and screening tube – is installed, remove the crystal and the counter tube slit diaphragm, turn the counter tube holder to its 90° final position, and insert the aperture of d = 5 mm. Then determine the following pulse rates at maximum anode voltage (Fig. 2):

N_3: with plexiglass scatterer but without A1-absorber
N_4: with scatterer and absorber in position 1
N_5: with scatterer and absorber in position 2

If necessary, dead time and background radiation must also be taken into account.

Note: The counter tube should never be exposed to primary radiation for any longer period of time.

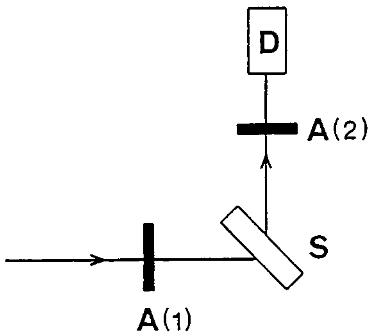
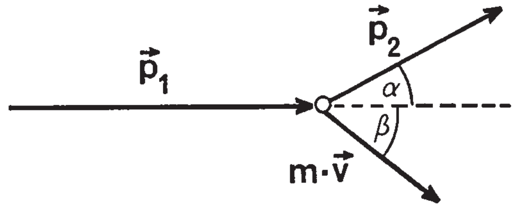


Fig. 2: Schematic representation of the 90° Compton scattering attachment
S = Scatterer
A = A1-absorber in positions 1 and 2
D = Detector.

Fig. 3: Scattering geometry of the Compton effect.



Theory and evaluation

A schematic representation of the scattering geometry of the Compton effect is shown in Fig. 3. The incident photon with an energy loss under scattering angle α is veered away from its original direction, while the formerly idle, free electron is emitted from the collision point with an energy gain under angle β with respect to the direction of incidence of the photon.

The energy conservation principle yields

$$hf_1 + m_0c^2 = hf_2 + mc^2 \quad (3)$$

h = Planck's quantum of action
f_1/f_2 = photon frequency before/after collision
m/m_0 = electron mass/electron rest mass
c = velocity of light

From the momentum conservation principle follows:

$$\vec{P}_1 = \vec{P}_2 + m\vec{v} \quad (4)$$

$\vec{P}_1/\vec{P}_2$ = photon impulse before/after collision
$\vec{v}$ = electron velocity

By using the cosine formula, Fig. 3 yields:

$$m^2v^2 = P_1^2 + P_2^2 - 2\, p_1p_2 \cos \alpha \quad (5)$$

If photon impulse $p = \frac{h}{\lambda}$ is inserted in (5), the result is:

$$m^2v^2 = \frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2} - 2\,\frac{h^2}{\lambda_1\lambda_2} \cos \alpha \quad (6)$$

Squaring (3) yields the necessary formula:

$$c^2\,(m - m_0)^2 = \frac{1}{c^2}\left[(hf_1)^2 + (hf_2)^2 - 2\,h^2f_1f_2\right] \quad (7)$$

$$= \frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2} - \frac{2h^2}{\lambda_1\lambda_2}$$

Subtracting (7) and (6) produces:

$$m^2v^2 - c^2\,(m-m_0)^2 = \frac{2h^2}{\lambda_1\lambda_2}\,(1-\cos \alpha) \quad (8)$$

$$= \frac{4h^2}{\lambda_1\lambda_2}\,\sin^2 \frac{\alpha}{2}$$

Taking into account the relativistic electron mass

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (9)$$

yields from (8)

$$c^2\,(m^2-m_0^2)-c^2\,(m-m_0)^2 = 2\,c^2 m_0\,(m-m_0) \quad (10)$$

$$= \frac{4h^2}{\lambda_1\lambda_2}\sin^2\frac{\alpha}{2}$$

Eliminating electron mass m by means of (3) yields:

$$m_0\,c\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right) = \frac{2h}{\lambda_1\lambda_2}\sin^2\frac{\alpha}{2} \quad (11)$$

By rearraging (11), the desired wavelength change of the photon as a function of scattering angle α is finally attained.

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{2h}{m_0 c}\sin^2\frac{\alpha}{2} \quad (12)$$

The wavelength change and energy transfer attain maximum values for central collision (180° back-scattering).
The change in wavelength for 90° scattering is called the Compton wavelength.
The following holds true for this:

$$\lambda_c = \frac{h}{m_0 c} = 2.426 \cdot 10^{-12}\ \text{m} \quad (13)$$

$h = 6.626 * 10^{-34}$ Js
$m_0 = 9.109 * 10^{-31}$ kg
$c = 2.998 * 10^{8}$ ms^{-1}

A photon with Compton wavelength λ_c contains the energy.

$$E_c = h f_c = \frac{h \cdot c}{\lambda_c} = m_0\,c^2 \quad (14)$$

i.e., E_c is the electron rest energy. Fig. 4 shows the transmission T as a function of the wavelength λ. There after, T decreases almost linearly with greater wavelenghts (smaller photon energy). The later ascent is caused by the 2nd order of Bragg scattering.
By allowing the X-rays to collide with a scatterer, it is possible to determine the pulse rate N_3 scattered in a 90° direction and then the pulse rates N_4 (absorber in front of the scatterer) and N_5 (absorber behind the scatterer). It can be seen that

$$T_1 = \frac{N_4}{N_3} > T_2 = \frac{N_5}{N_3}$$

The means that the wavelength of the scattered radiation is greater than the wavelength of the primary radiation.
For the scattering rates, the following values have been found:

I_3 = 8329 Imp/39.5 s ; $\Delta I_3/I_3 \sim \pm 1.1\,\%$
N_3 = 210.9 Imp/s ; N'_3 = 215 Imp/s
I_4 = 8125 Imp/236.2 s ; $\Delta I_4/I_4 \sim \pm 1.1\,\%$
N_4 = 34.4 Imp/s ; N'_4 = 34.2 Imp/s
I_5 = 7275 Imp/261.5 s ; $\Delta I_5/I_5 \sim \pm 1.2\,\%$
N_5 = 27.8 Imp/s ; N'_5 = 27.6 Imp/s

(N' value = dead time and background radiation corrected)

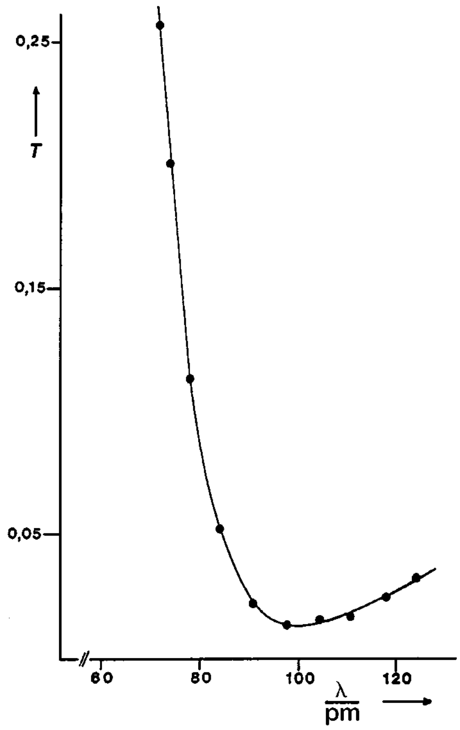
$$T_1 = \frac{N'_4}{N'_3} = \frac{34.2\ \text{Imp/s}}{215\ \text{Imp/s}} = 0.159 \quad ; \quad \frac{\Delta T_1}{T_1} \sim \pm 2.2\,\%$$

$$T_2 = \frac{N'_5}{N'_3} = \frac{27.6\ \text{Imp/s}}{215\ \text{Imp/s}} = 0.128 \quad ; \quad \frac{\Delta T_2}{T_2} \sim \pm 2.3\,\%$$

The T-values recorded in Fig. 5 yield within their error limits the following:

$$\Delta\lambda = \lambda_c = (2.35 \pm 0.25) * 10^{-12}\ \text{m}$$

Fig. 4: Transmission of the A1-absorber as a function of the wavelength.



This value corresponds acceptably well with the theoretical value of the Compton wavelength.

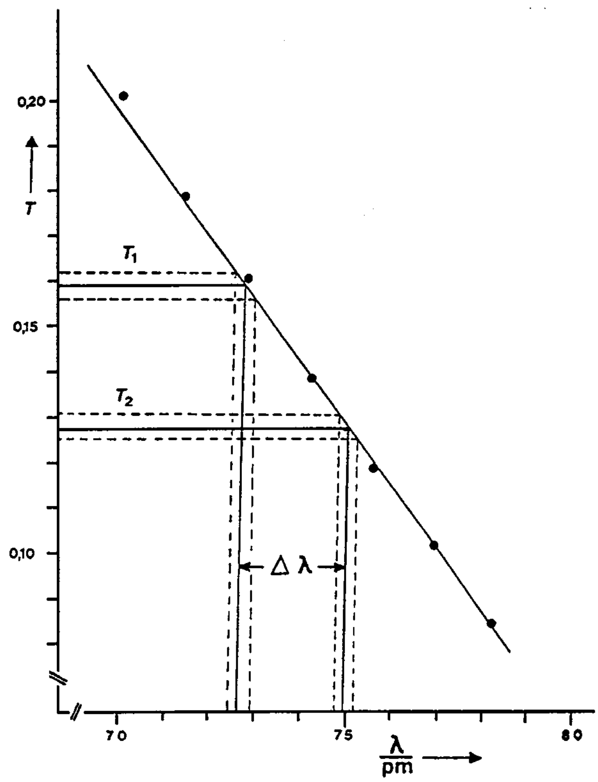


Fig. 5: Transmission of the A1-absorber as a function of the wavelength with higher angular resolution.