Planck's "quantum of action"

Related topics

X-ray, Bragg equation, Continuous (retarded) radiation, characteristic radiation, crystal plane, lattice spacing, minimum wavelength, limiting frequency.

Principle and task

The intensity of X-rays of different frequencies is measured as a function of the anode voltage. From the limiting voltage thus determined, Planck's constant is determined as a function of the frequency.

Equipment

X-ray unit, w. recorder output	09056.97	1
Counter tube, type A, BNC	09025.11	1
Geiger-Müller-Counter	13606.99	1
Screened cable, BNC, l 750 mm	07542.11	1

Problems

1. Determination of the intensity of X-radiation of different frequencies as a function of the anode voltage.
2. Determination of the limiting voltage as a function of the frequency.
3. Determination of Planck's constant *h* from the gradient of the curve obtained in 2.

Set-up and procedure

The experimental set up is as shown in Fig. 1. The anode voltage is directly displayed. The diaphragm (*d* = 2 mm) is inserted in the X-ray outlet aperture. Before commencing to take measurements, both angle indicators in the X-ray experimental chamber must be adjusted.

The LiF crystal is mounted parallel to the pointer and, in order to obtain a definite deflection, is turned slightly to the right. The counter tube is inserted the counter connected and the experimental chamber locked (setting the safety switch). The anode voltage is adjusted to approx. 20–22 kV. The crystal is rotated to 22.5° and the counter tube set to the relative intensity maximum approximately in the range 40 to 50°. The absolute intensity maximum ($K\alpha$ line, $\lambda = 154$ pm $\triangleq$ 22.5°) is sought by making slight changes in the angle of the crystal and the counter tube. The indicators fitted are set to 22.5° and 45° respectively. The background is determined in a preliminary experiment.

The anode voltage must be checked between measurements and readjusted if necessary.

Fig. 1: Experimental set-up for determining the intensity of X-radiation as a function of the anode voltage.

Theory and evaluation

If a beam of electrons of energy E impinges on an anode of a given material the electrons are retarded. Part of the kinetic energy eU is dissipated as radiation, U being the applied anode voltage. The continuous spectrum has a short wavelength limit due to the conversion of the whole of the kinetic energy eU, in a single process, into radiation of frequency f (limiting case):

$$eU = hf = h\frac{c}{\lambda},$$

where c is the velocity of light, λ is the wavelength, f is the frequency, h is Planck's constant, and e is the electron charge.
By giving up their kinetic energy, the electrons can ionise the target atoms by removing electrons from the innermost shell. The exciting atom is converted preferentially to a lower state by an electron from the next or next-but-one shell passing over into the innermost shell and thus emitting the energy difference as an X-ray quantum. If the shell into which the electron passes over is the K-shell the K_α, K_β, etc., lines are obtained accordingly.

If a wave of wavelength λ is incident at an angle θ on a system of parallel crystal-lattice planes of a crystal, then constuctive interference of the reflected wave occurs if the paths of the individual partial waves differ by the wavelength of multiples of it.

Using the nomenclature of Fig. 2, this is the case if

$$n\lambda = 2d\sin\theta;\ n = 1, 2, 3 \ldots, \text{(Bragg equation)}$$

where d is the distance between the lattice planes. At each position of the scattering crystal a certain wavelength is filtered out of the white radiation. The crystal used is LiF with a lattice spacing of

$$d = 2.01 \cdot 10^{-10}\ \text{m}.$$

Thus, a corresponding wavelength of frequency can be allocated to each angular position of the crystal.

The intensity of different wavelengths is measured as a function of the anode voltage.

By extrapolating the curves in Fig. 3 the limiting voltage U_O is determined as a function of the frequency.

From the regression lines to the values plotted in Fig. 4, we obtain the gradient

$$B = 4.12 \pm 0.07 \cdot 10^{-15}\,\frac{\text{J}}{\text{A}}. \qquad \text{(see (1))}$$

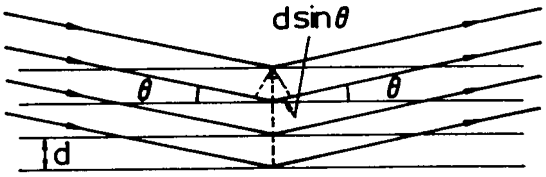
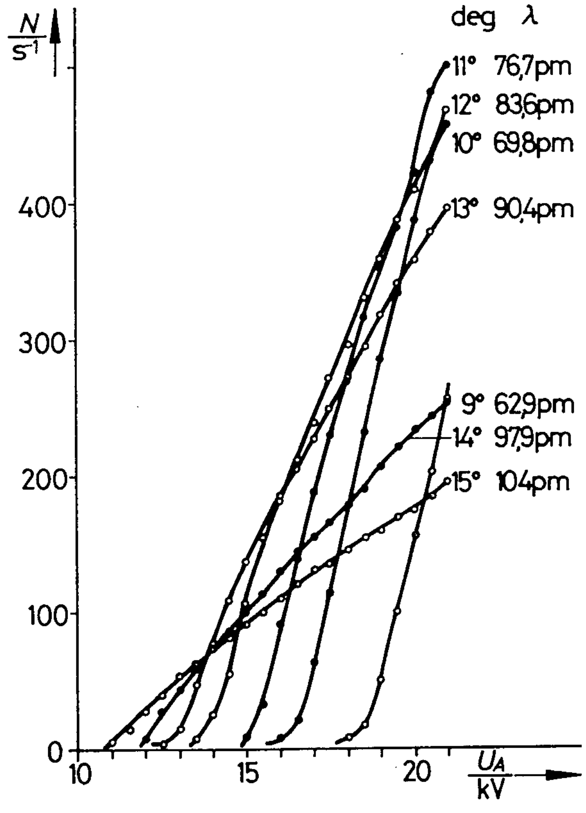


Fig. 2: Interference of waves at lattice planes.

Fig. 3: Intensity of X-radiation of different wavelengths as a function of the anode voltage.



From this, using the electron charge

$$e = 1.60 \cdot 10^{-19}\ \text{As},$$

we obtain Planck's constant

$$h = 6.61 \cdot 10^{-34}\ \text{Js}.$$

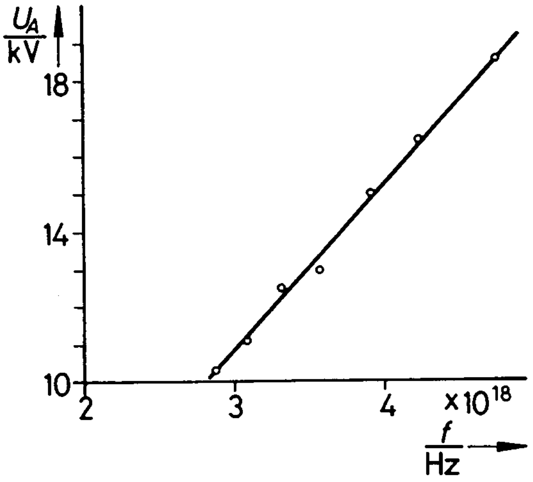


Fig. 4: Interference of waves at lattice planes.