Related topics

Bremsstrahlung, characteristic radiation, Bragg scattering, law of absorption, mass absorption coefficient, absorption edge, half-value thickness, photoelectric effect, Compton scattering, pair formation.

Principle and task

Polychromatic X-rays are energy-selected by means of a monocrystal analyzer. The monochromatic radiation serves as the primary radiation source in order to study the absorption behaviour of various metals as a function of the absorber thickness and the primary radiation wavelength.

Equipment

X-ray unit. w. recorder output	09056.97	1
Counter tube, type A, BNC	09025.11	1
Geiger-Müller-Counter	13606.99	1
Absorption set for X-rays	09056.02	1
Screened cable, BNC, l 300 mm	07542.10	1

Problems

1. The intensity attenuation of the primary radiation for aluminum and zinc is to be determined as a function of the material thickness for two different wavelengths. The mass absorption coefficients are to be taken from the corresponding graphic representation.

2. The mass absorption coefficient with a constant foil thickness for aluminum and tin is to be determined as a function of the wavelength and compared with the theoretical values. By graphic representation, it is to be shown that $\mu/\rho = f(\lambda^3)$.

3. Near the absorption edges, the absorption coefficients for copper and nickel are to be determined as a function of the wavelength and plotted graphically. The energies of the K-levels are to be calculated.

4. Prove that $\mu/\rho = f(Z^{3.12})$ applies.

Fig. 1: Experimental set-up for determination of X-ray absorption.

Set-up and procedure

The experiment is set up as shown in Fig. 1. The aperture of $d = 2$ mm is introduced into the outlet of X-rays.

By pressing the "zero key", the counter tube and crystal holder device are brought into starting position. The crystal holders are mounted with the crystal surface set horizontally. The counter tube, with horizontal slit aperture, is mounted in such a way that the mid-notch of the counter tube closes onto the back side of the holder. Finally, the absorption foil holder is carefully placed on the counter tube with slit diaphragm horizontally. The counter tube voltage is 500 V. The counting time should be 50 s or 100 s. In order to keep the relative error to a minimum, always measure up to 1000 pulses. At small pulse rates, the background radiation must be taken into account. That should first be determined carefully at an accelerating voltage of $U_a = 0$ Volt. Due to counter tube dead time τ, high pulse rates N' should be corrected to the true pulse rates N by means of the relation

$$N = \frac{N'}{1 - \tau N'} \; (= \tau = 100 \; \mu sec)$$

For the experiments with the metals Al, Zn, Sn and Ni, chose and accelerating voltage of $U_a = 25$ kV. After a warm-up time of app. 10 minutes, determine the pulse rate of the primary radiation for problem 1. at a glancing angle of $\vartheta = 22.5° \triangleq \lambda = 154$ pm (Cu-K$_\alpha$-line). Then, determine the pulse rate for aluminum and zinc as a function of the metal thickness (combination of 2 metal foils). Repeat the series of measurements for aluminum at a glancing angle of $\vartheta = 15° \triangleq \lambda = 104$ pm. The constancy of the primary radiation intensity should be checked every now and again. The graphic representation of the results is shown in Fig. 2.

Determine the absorption for aluminum ($d = 0.08$ mm) and then for tin ($d = 0.025$ mm) in the interval $\sim 8° < \vartheta < 20°$ in steps of $\Delta\vartheta = 1°-2°$ and plot the results as shown in Fig. 3 and 4.

In order to assure a sufficiently high precision at the evaluation of the experiments with nickel and copper ($d = 0.025$ mm), the number of measuring points near the absorption edges must be increased as well as the measuring interval at larger wavelength (Fig. 5 and 6). The acceleration voltage for copper is exceptionally $U_a = 15$ kV. This means that the primary radiation intensity is only available at $\vartheta > 12°$.

Theory and evaluation

If X-rays with intensity I_0 penetrate matter of layer thickness d, the intensity I behind the layeris determined by

$$I = I_0 e^{\mu(\lambda)d} \qquad (1)$$

The quantity μ (λ) with measuring unit cm^{-1} is called the linear attenuation coefficient and is dependent upon the wavelength λ of the primary radiation.

In order to be able to immediately compare the absorptivity of various materials, it is advantageous to use the so-called half-value thickness $d_{1/2}$. This symbolizes the material thickness which reduces the intensity of primary radiation by one half. In applying the half-value thickness, equation (1) yields

$$d_{1/2} = 0.69 \frac{1}{\mu} \qquad (2)$$

Since the mass attenuation coefficient is proportional to the mass, the mass absorption coefficent μ/ρ (ρ = mass density) is often used with measuring unit cm^2/g.

The intensity attenuation is caused by the following processes:
(1) photoelectric effect
(2) scattering
(3) pair formation

The third pertans to gamma radiation due to the required minimum energy, equal to twice the amount of the electron rest energy

$$2 E_0 = 2 m_0 c^2 = 1.02 \text{ MeV}.$$

Thus, the absorption coefficient for X-rays is comprised of the following components:

$$\mu = \tau + \sigma \qquad (3)$$

τ = photoelectric absorption coefficient
σ = scattering coefficient

For the wavelength range used here, the photoelectric effect is taken into primary account for absorption. Thus: $\tau > \sigma$.

With sufficient precision, the following equation (empirically determined) applies:

$$\left| \frac{\tau}{\rho} \right| = C\lambda^3 - D\lambda^4 \qquad (4)$$

C = $4.86 \cdot 10^{-3} Z^{3.12}$
D = $1.37 \cdot 10^{-6} Z^{5.18}$
Z = atomic number

The above numeric factors of the constants C and D only pertain to wavelengths $\lambda < \lambda_K$ whereas λ_K is understood to be the wavelength corresponding to the absorption edge of the K-energy level. For $\lambda_K < \lambda < \lambda_L$, other numeric factors for C and D apply. If the difference in (4) is calculated using the given C and D values for various wavelengths and Z-values, the value $|\tau/\rho|$ is primarily determined by the quantity $D \lambda^4$. This means, however, that the following equation applies to the mass attenuation coefficient:

$$\mu/\rho \approx \tau/\rho = f(\lambda^3, Z^{3.12}) \qquad (5)$$

Accordingly, it becomes evident that the absorption increases drastically with the wavelength of the primary radiation as well as with the atomic number of the absorbing element.

Since absorption is an entirely atomic characteristic, it follows that a molecular absorption is ompiled from the addition of the absorption coefficients of the elements.

In order to produce a monochromatic primary radiation necessary for the absorption analysis from the copper X-ray spectrum, a monocrystal is used as monochromator. The wavelength λ is determined by means of the Bragg equation

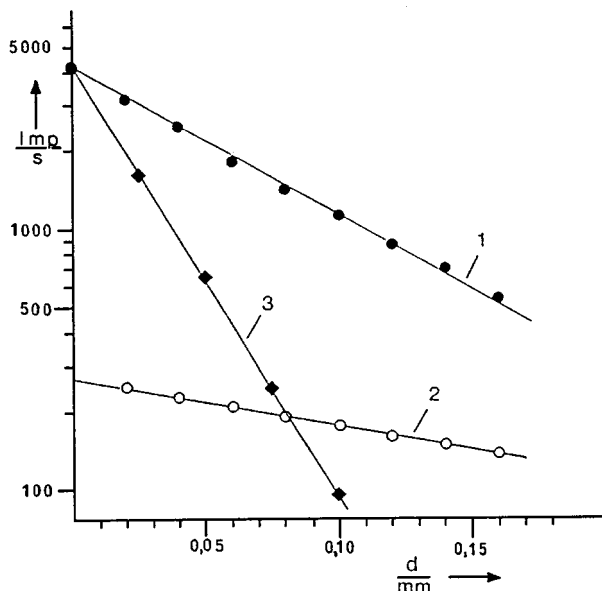$$2 d \sin \vartheta = n \lambda \qquad (6)$$

ϑ = glancing angle
n (1, 2, 3…) = scattering order
d = 201.4 pm = lattice constant
of the LiF-crystal

In Fig. 2, the measured pulse rates are recorded for varying absorbe thickness on a semi-logarithmic scale. The curves 1 (λ =154 pm) and 2 (λ =104 pm) apply to aluminum (Z=13, ρ =2.7 g/cm^3) and to two different primary radiation wavelengths. Curve 3 (λ =154 pm) applies to zinc (Z=30, ρ =7.14 g/cm^3).

Fig. 2: Semi-logarithmic representation of the pulse rates as a function of the absorber thickness; $U_a = 25$ kV
Curve 1: Al (Z = 13); $\lambda = 154$ pm
Curve 2: Al (Z = 13); $\lambda = 104$ pm
Curve 3: Zn (Z = 30); $\lambda = 154$ pm

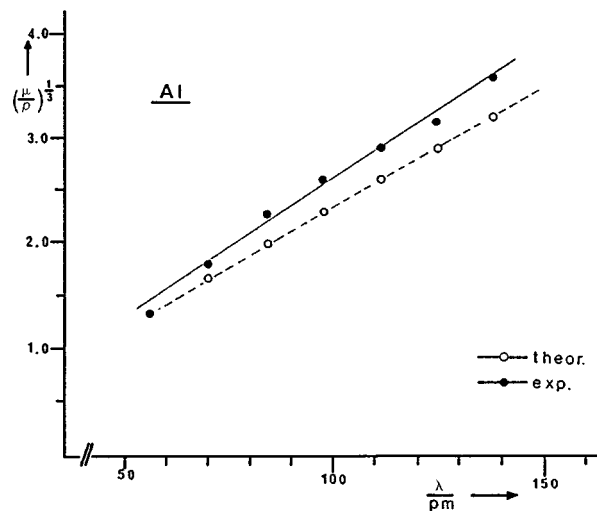Fig. 3: $\sqrt[3]{\mu/\rho}$ of aluminum as a function of the primary radiation wavelength; $Ua = 25$ kV.

Fig. 3 shows that the experimentally determined wavelength dependency of aluminum absorption corresponds well with the values calculated according to (4). In Fig. 4, the corresponding result for tin (Z = 50, $\rho = 7.28$ g/cm^3) is shown in comparison with the literature values.

If the wavelength of the X-rays is reduced so that their energy is equal to one of the energies of the atomic levels of the absorber, a sudden increase in absorption is observed. This situation for copper is shown in fig. 5 (Z = 29, $\rho = 8.96$ g/cm^3). Thus, according to (5), the quantity $\sqrt[3]{\mu/\rho}$ falls linearly up to a critical wavelenth λ_K then rises suddenly for $\lambda < \lambda_K$ and decreases linearly once again. This experiment yields $\lambda_K = 137.5$ pm. Using this value and the equation

$$E_K = \frac{h \cdot c}{e \cdot \lambda_K} \qquad (7)$$

$h = 6.626 \cdot 10^{-34}$ Js Planck's quantum of action
$c = 2.998 \cdot 10^8$ ms^{-1} velocity of light
$e = 1.602 \cdot 10^{-19}$ C elementary charge,

it is possible to obtain the following value for the energy of the copper K-level:
Cu – E_K = 9.02 keV   (Literature value: 8.98 keV).

For comparison, the absorption calculated according to (4) for the range $\lambda < \lambda_K$ has additionally been plotted. The agreement between calculation and experiment for small wavelengths worsens for larger wavelengths due to the gain of short-waved photons. These photons are produced by 2nd order of diffraction scattering. Since the numerical values of C and D are not available for the range $\lambda_K < \lambda < \lambda_L$, a corresponding comparative curve is not given.

The absorption curve for nickel (Z = 28, $\rho = 8.99$ g/cm^3) is shown in Fig. 6. Since the atomic number of nickel is smaller than that of copper,

Ni – E_K < Cu – E_K also applies here as well as

Ni – λ_K > Cu – λ_K. This is actually true in the case of

From Fig. 2, it is apparent that absorption increases with both the wavelength of the primary radiation as well as with the atomic number. The results of Fig. 2 found by means of (1) to (5) are listed in Table 1.

Table 1: Wavelength dependency of absorption

	μ [cm^{-1}] exp.	$d_{1/2}$ [cm] exp.	μ/ρ [cm^{2}g^{-1}] exp. calc./lit.		$\left(\frac{\mu_1/\rho}{\mu_2\rho}\right)$	$\left(\frac{\lambda_1}{\lambda_2}\right)^3$
Al (Z = 13; $\rho = 2.7$ gcm^{-3} $\lambda_1 = 154$ pm	126	$5.5 \cdot 10^{-3}$	46.7	45.2		
					$\frac{46.7}{14.3} = 3.26$	3.24
$\lambda_2 = 104$ pm	38.5	$1.8 \cdot 10^{-2}$	14.3	14.0		
Zn (Z = 30; $\rho = 7.14$ gm^{-3})						
$\lambda_1 = 154$ pm	375	$1.8 \cdot 10^{-3}$	52	59	–	–

For aluminum, the experimentally determined mass absorption coefficients correspond well with those coefficients calculated from (4). The wavelength dependency according to (5) is also apparent. A Z-dependency check of the mass absorption coefficents for aluminum and zinc according to (5) is not possible here since the primary radiation wavelength lies within the K-absorption edge of zinc. The validity of (5) is only true outside of the edges.

Ni – λ_K = 152.5 pm. According to (7), with the value of Ni – λ_K
Fig. 4: The mass absorption coefficient and $\sqrt[3]{\mu/\rho}$ of tin as a function of the primary radiation wavelength; U_a = 25 kV.
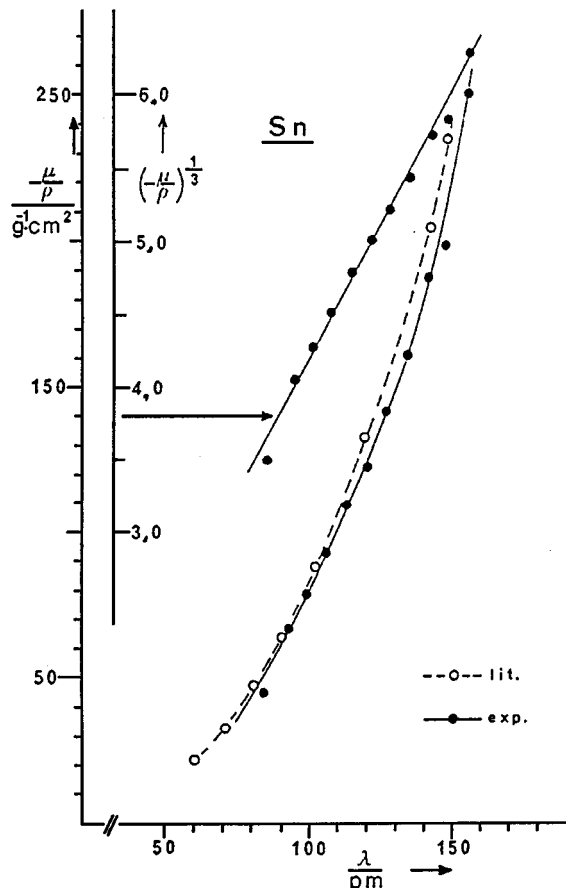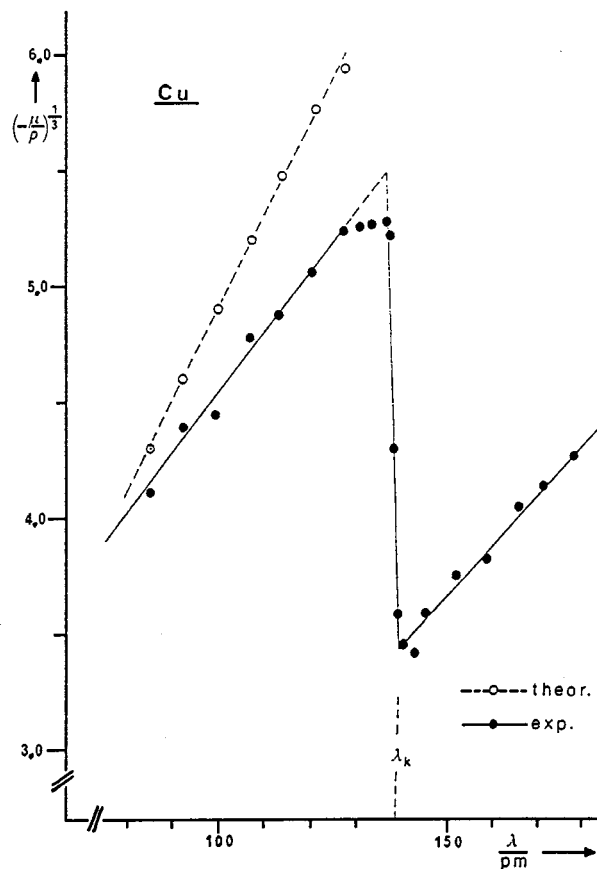

Fig. 5: Absorption edge of copper
U_a = 15 kV; λ_K = 137.5 pm

one obtains:
Ni – Ek = 8.13 keV (Literature value: 8.33 keV)

Nickel filters are used to monochromatize the radiationof copper X-ray tubes. In this case, only the characteristic Kα-copper radiation

$E_{K\alpha}$ = E_K – $E_{L2,3}$ = (8.98 – 0.95) keV ≈ 8.03 keV is allowed to pass, while the K_β-line

$E_{K\beta}$ = E_K – $E_{M2,3}$ = (8.98 – 0.074) keV ≈ 8.9 keV is absorbed. (See also experiment 5.4.3). The reduction in the absorption curve at λ ≈ 100 pm is the result of the 2nd order of diffraction scattering.
A short calculation may be worthwhile here to helb emphasize this fact.

The beginning of the X-ray spectrum at high energy is determined by the acceleration voltage of the X-ray tube. (See also Experiment 5.4.3) According to (7), an acceleration voltage of 25 kV is in compliance with a wavelength of λ_c = 49.6 pm. If this value is inserted in the Bragg equation for the 2nd order of diffraction (n = 2), the scattering is expected to be under glancing angle ϑ = 14.3°.
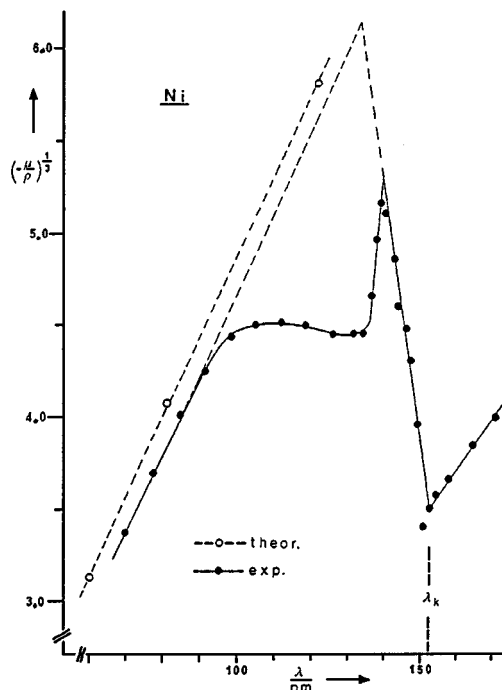

Fig. 6: Absorption edge of copper
U_a = 25 kV; λ_K = 152.5 pm

Under this glancing angle, the X-rays with wavelength λ = 99.2 pm are emitted from the 1st order of diffraction so that the primary radiation colliding with the absorber under a lancing angle of $\vartheta > 14.3°$ ($\lambda > 99.2$ pm) contains a percentage of photons with shorter wavelengths. In consequence the absorber appears to be more transparent thanit is. This disturbing effect does not appear as drastically in Fig. 5. Due to the conscious choice of a reduced primary voltage of U_a = 15 kV, the bremsspectrum is only activated under larger glancing angles ϑ.

Furthermore, the intensity of the bremsspectrum at U_a = 15 kV is less than at U_a = 25 kV.

Finally, Fig. 7 shows the absorption dependence on the atomic number Z. Keep in mind that the quantities C and D in (4) have different values for the ranges – $\lambda < \lambda_K$ and $\lambda > \lambda_K$. Thus, the Z-dependence of the absorption coefficients can only be cmpared within equivalent absorption ranges. For the line $\lambda < \lambda_K$, the values for Ci (Z = 17) and Ca (Z = 20) calculated according to (4) are used additionally to support the measured values.

Fig. 7: $^{3.12}\sqrt{\mu/\rho} = f(z)$

Liteature
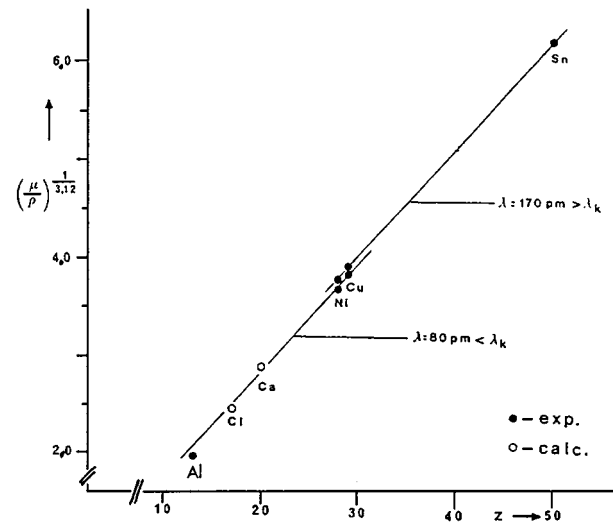
1. Energy level values:
   Handbook of chemistry and physics; CRC-Press, Inc.; Florida
2. Mass absorption coefficient calculation:
   I. A. Victoreen; J. Appl. Phys.;
   Vol. 20; p. 1141; 1949.