Duane-Hunt displacement law and Planck's "quantum of action"

LEP 5.4.03

Related topics

X-ray tube, bremsstrahlung, characteristic X-rays, energy levels, crystal structures, lattice constant, interference, Bragg equation.

Principle and task

By means of an x,y-recorder, X-ray spectra are recorded as a function of the anode voltage. From the short-wave length limit of the bremsspectrum, the Duane-Hunt displacement law and Planck's "quantum of action" are determined.

Equipment

X-ray unit, w. recorder output	09056.97	1
Counter tube, type A, BNC	09025.11	1
Pulse rate meter	13622.93	1
xyt recorder	11416.97	1
Screened cable, BNC, l 750 mm	07542.11	1
Connecting cord, 1000 mm, red	07363.01	2
Connecting cord, 1000 mm, blue	07363.04	2

Problems

1. The intensity of the X-rays emitted by the copper anode at various anode voltages is to be drawn as a function of the Bragg angle by an x,y-recorder.
2. The short wavelength limit (or maximum energy) of the bremsspectrum is to be determined for the spectra of 1.
3. The Duane-Hunt displacement law and Planck's "quantum of action" are to be verified by these measurements.

Set-up and procedure

The experiment is set up as shown in Fig. 1. The aperture of *d* = 2 mm is introduced into the outlet of X-rays.

By pressing the "zero key", the counter tube and crystal holder device are brought into starting position. The crystal holders are mounted with the crystal surface set horizontally. The counter tube, with horizontal slit aperture, is mounted in such a way that the mid-notch of the counter tube closes onto the back side of the holder.

Typical settings of the peripheral equipment are:

Pulse rate meter:	Couter tube voltage	500 V
	Sensitivity	10^5 imp/min
	Time constant	0.5 or 1.5 s
x,y-recorder:	x-axis (ϑ-axis)	1 V/cm, additionally variable
	y-axis (intensity)	0.1 V/cm, additionally variable

The output of the pulse rate meter is connected to the y-input of the recorder. The angle-proportional direct-current voltage (0.1 V/degree) of the X-ray unit lies on the x-input. The plotting of the spectra is performed at a slow velocity of rotation (positions "V_1" and "Auto"), crystal and counter tube must rotate in synchronization.

First, at maximum anode voltage, a general spectrum (Fig. 2) is drawn. After, that, the bremsspectra for various anode voltages are recorded up to the K_β-line. (Fig. 3)

Fig. 1: Experimental set-up for energy analysis of X-rays.

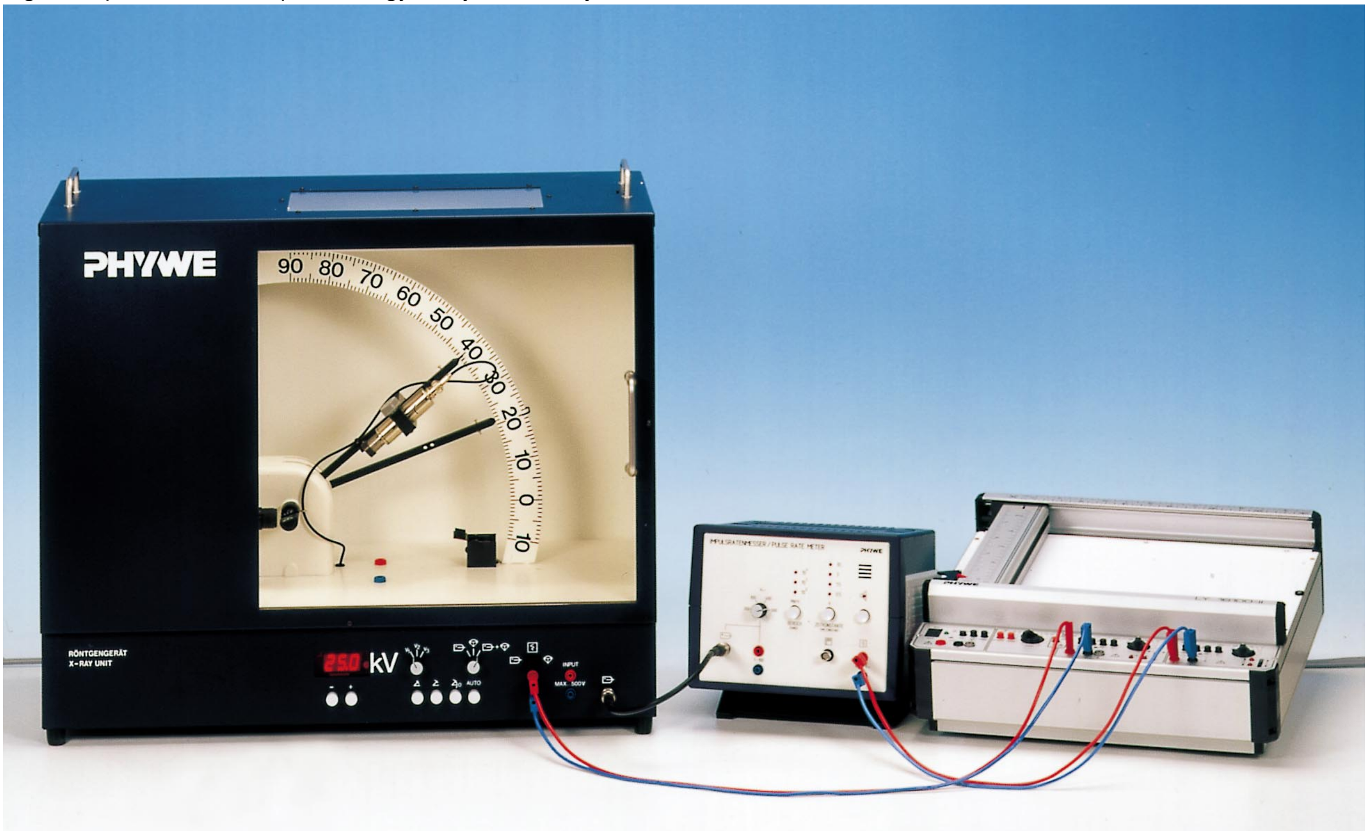
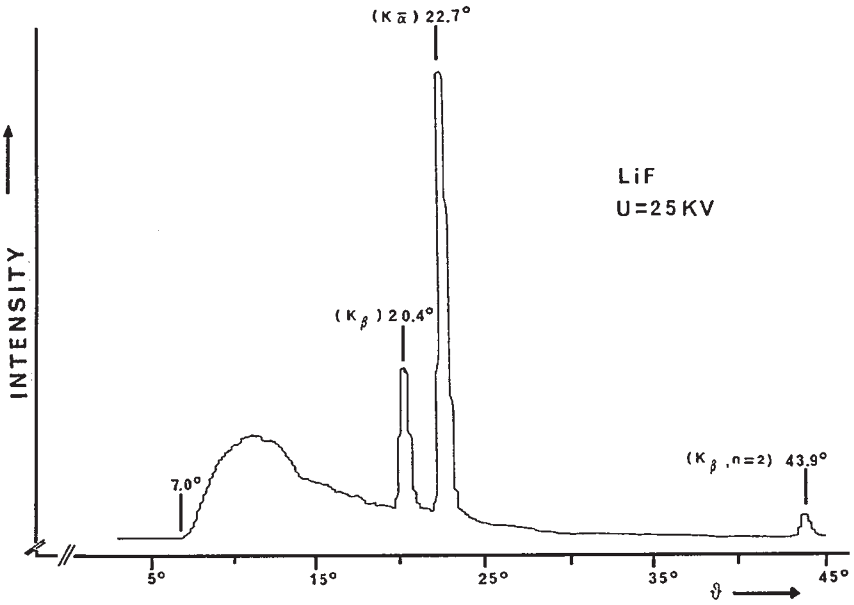


Fig. 2: Copper X-ray spectrum with LiF-analyzer.



In order to obtain exact angle determination of the bremsspectra, the sensitivity of the x-channel must be increased accordingly and calibrated carefully; likewise, the zero-line of the y-axis must be displaced for each new measurement.

Note:

The counter tube should never be exposed to primary radiation for any longer period of time.

Theory and evaluation

A positve voltage, lying on the anode of the x-ray tube, accelerates the electrons emitted from the cathode with a rather low energy distribution.

In reaching the anode, the electrons have the kinetic energy:

$$E_{kin} = eU \quad (e = \text{elementary charge}) \qquad (1)$$

At arrival, a part of the electrons will be progressively slowed down, thus converting their kinetic energy into electro-magnetic radiation with continuous energy distribution. This bremsspectrum has a short wavelength limit which has been determined in that the entire kinetic energy of some electrons is converted into radiation in just on step. In 1915, Duane and Hunt empirically found that the product of anode voltage and the shortest wavelength λ_{min} is constant, and that the following formula holds:

$$U \cdot \lambda_{min} \sim 1.25 \cdot 10^{-6}\ \text{V} \cdot \text{m} \qquad (2)$$

This relationship can easily be derived from Einstein's energy equation, according to which:

$$E_{kin} = eU = hf_{max} = h \cdot \frac{c}{\lambda_{min}} \qquad (3)$$

where $h = 6.6256 \cdot 10^{-34}$ Js — Planck's constant
$c = 2.9979 \cdot 10^{8}\ \text{ms}^{-1}$ — velocity of light
$e = 1.6021 \cdot 10^{-19}$ As — elementary charge

The shortest wavelength is calculated to be:

$$\lambda_{min} = 1.2398 \cdot 10^{-6} \frac{1}{U}\ \text{V} \cdot \text{m}$$

The analysis of the polychromatic X-rays is carried out by the use of a monocrystal. If the X-rays impinge under a glancing angle ϑ, constuctive interference will only appear in reflection if the paths of the partial waves reflected on the lattice planes differ by one or more wavelengths. This situation is described by the Bragg equation.

This situation is explained by the Bragg equation:

$$2d \sin \vartheta = n \cdot \lambda \qquad (4)$$

(LiF-lattice constant d = 201.4 pm, n = order of diffraction)

The short wavelength limit of the bremsspectrum is determined by the appertaining glancing angle ϑ. In conjunction with (4), λ_{min} can be calculated.

In Fig. 5, λ_{min} is represented as a function of the reciprocal value of the anode voltage.
The slope of the resulting line is:

$$m = \frac{\lambda_{min}}{1/U} = (1.233 \pm 0.007) \cdot 10^{-6}\ \text{V} \cdot \text{m}$$

This formula is acceptably consistent with the Duane-Hunt displacement law.

By using the same measured curves, it is also possible to determine Planck's "quantum of action".

Fig. 3: Bremsspectra as functions of the anode voltage.

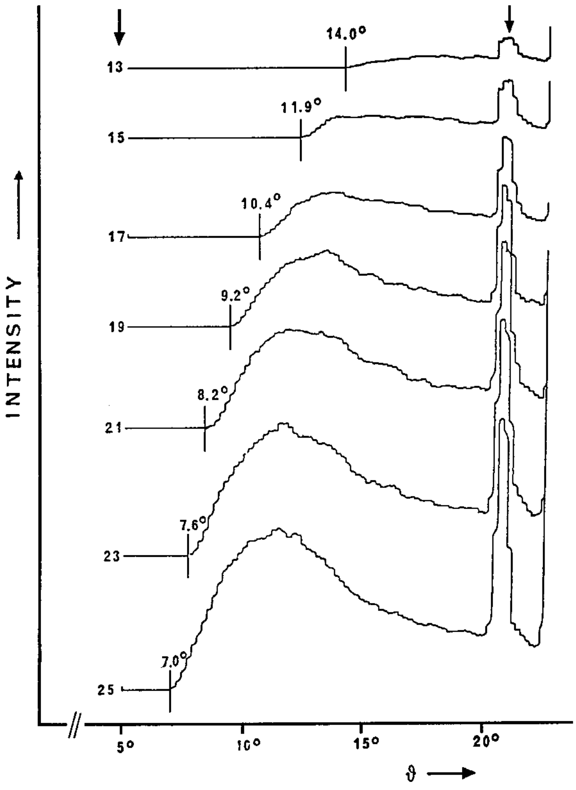
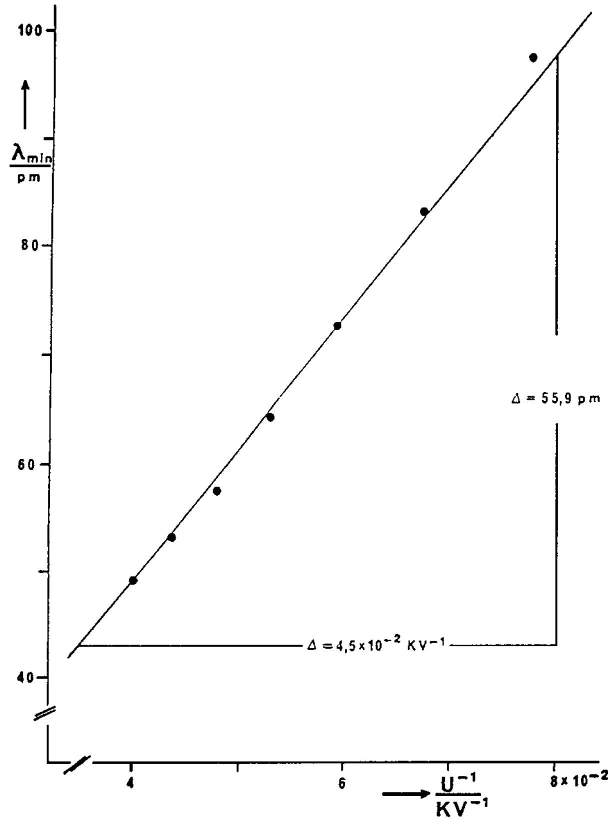


Fig. 5: Duane-Hunt displacement law. λ_{min} as a function of reciprocal anode voltage.



From (3) and (4) follows:

$$U = \frac{h \cdot c}{2 \cdot e \cdot d \cdot \sin\vartheta} \qquad (5)$$

In Fig. 6, the anode voltage is drawn as a function of ($\sin \vartheta^{-1}$). Using the slope m of the resulting line, it is possble to attain:

$$h = m \cdot \frac{2 \cdot e \cdot d}{c} = (6.59 \pm 0.04) \cdot 10^{-34}\,\text{Js}; \; \frac{\Delta h}{h} = \pm 0.5\%$$

In conjunction with (5) and glancing angle values from Fig. 2, the energies of the characteristic copper X-ray lines can be calculated to be:

n = 1	E-$K_{\bar{\alpha}}$	=	7.98 KeV
	K-K_{β}	=	8.83 KeV
n = 2	E-K_{β}	=	8.88 KeV

These values are in good approximation with the literature values. (See also experiment 5.4.1)

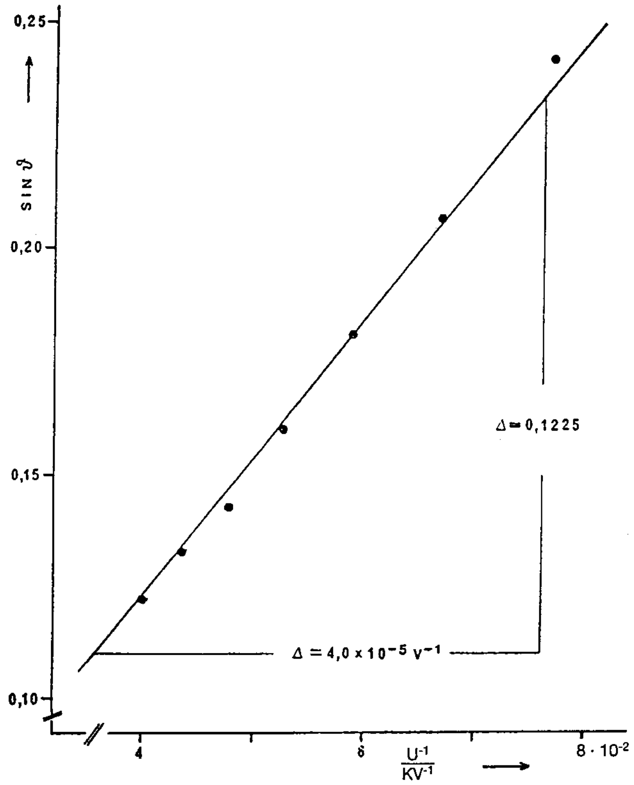


Fig. 6: Planck's "quantum of action". $\sin \vartheta_{min}$ as a function of reciprocal anode voltage.

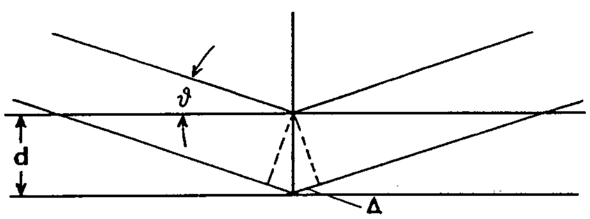


Fig. 4: Bragg scattering on the lattice planes.