Related topics
X-ray tube, bremsstrahlung, characteristic radiation, energy levels, crystal structures, lattice constant, interference, Bragg equation, order of diffraction.

Principle and task
X-ray spectra are analyzed by means of different monocrystals and plotted by a recorder. The energy of the characteristic lines is determined through the glancing angle position of their various orders of diffraction.

Equipment
X-ray unit, w. recorder output 09056.97 1
Potassium Bromide Crystal, mounted 09056.01 1
Counter tube, type A, BNC 09025.11 1
Pulse rate meter 13622.93 1
xyt recorder 11416.97 1
Screened cable, BNC, l 750 mm 07542.11 1
Connecting cord, 1000 mm, red 07363.01 1
Connecting cord, 1000 mm, blue 07363.04 1
Connecting cord, 2000 mm, red 07365.01 1
Connecting cord, 2000 mm, blue 07365.04 1

Problems
1. The intensity of the X-rays emitted by the copper anode at maximal anode voltage is to be recorded by means of a LiF-monocrystal as a function of the Bragg angle (glancing angle).
2. Using the various anode voltages, the measurement must be repeated according to step 1.
3. The measurement from step 1 is to be repeated, this time with the KBr-monocrystal as the analyzer.
4. The energy values of the characteristic copper lines are to be calculated.

Set-up and procedure
The experiment is set up as shown in Fig. 1. The aperture of $d = 2$ mm is introduced into the outlet of X-rays.

By pressing the “zero key”, the counter tube and crystal holder device are brought into starting position. The crystal holders are mounted with the crystal surface set horizontally. The counter tube, with horizontal slit aperture, is mounted in such a way that the mid-notch of the counter tube closes onto the back side of the holder.

Typical adjustment settings of the peripheral equipment are:
- Pulse rate meter: counter tube voltage 500 V
  Sensitivity $10^5$ Imp/min
  Time constant 0.5 or 1.5 s
- x, y-recorder: x-axis (β-axis) 1 V/cm additionally variable
  y-axis (intensity) 0.1 V/cm, additionally variable

Fig. 1: Experimental set-up for the analysis of X-rays.
The output of the pulse rate meter is connected to the y-input of the recorder. The angle-proportional direct-current voltage (0.1 V/degree) of the X-ray unit lies on the x-input. The plotting of the spectra is performed at a slow velocity of rotation (switch settings “V1” and “Auto”); crystal and counter tube must rotate in synchronization.

First, a spectrum with maximal anode voltage is recorded by means of the LiF-crystal. This measurement is then repeated using various anode voltages. In order to obtain a more comprehensive comparability of the results, the spectra should be plotted staggerly in y-direction.

Analysis by means of the KBr-monocrystal then follows (Fig. 6).

Note:
The counter tube should never be exposed to primary radiation for any longer period of time.

Theory and evaluation

When electrons of high energy impinge on the metallic anode of the X-ray tube, X-rays with continuous energy distribution (the so-called bremsstrahlung) are produced. (See also experiment 5.4.2). Lines specific to the anode materials, the so-called characteristic X-ray lines whose energies are not dependent upon the anode voltage, are then superimposed on this continuum. The anode atoms can be ionized through electron impact in the K-shell, for example. The vacancy is then filled by an electron from a higher energy level. The energy released through this deenergization process can be transformed into an element-specific X-ray. Fig. 2 shows the energy-level scheme of a Cu-atom. Characteristic X-rays produced from either the L → K or the M → K transitions are called Ka and Kb lines respectively. M1 → K and L1 → K transitions do not take place due to quantum mechanical selection rules.

Accordingly, characteristic lines for Cu with the following energies are to be expected (Fig. 2):

\[ E_{K_a} = E_K - \frac{1}{2} (E_{L_2} + E_{L_3}) = 8.038 \text{ keV} \]  
\[ E_{K_b} = E_K - E_{M_{2,3}} = 8.905 \text{ keV} \]

The mean value Ka is used due to the inseparability of lines Ka1 and Ka2.

Analysis of polychromatic X-rays is made possible through the use of a monocrystal. When X-rays with wavelength λ impinge on the monocrystal under glancing angle θ, constructive interference after scattering only appears when the paths of the partial waves on the lattice planes n differ by one or more wavelengths (Fig. 3).

This situation is explained by the Bragg equation:

\[ 2d \sin \theta = n \lambda \quad (2) \]

\(d = \text{interplanar crystal spacing}; n = \text{order of diffraction}\)

If d is assumed to be known, it is possible to calculate the energy of the X-rays by means of the glancing angle θ supplied by the spectra and by drawing upon the following relation:

\[ E = h \cdot f = \frac{hc}{\lambda} \quad (3) \]

From combining (3) and (2) follows:

\[ E = \frac{n \cdot h \cdot c}{2 \cdot d \cdot \sin \theta} \quad (4) \]
Fig. 4 shows that well-defined lines are superimposed upon the bremspectrum continuum. The angle position of these lines remains unaltered at variation of the anode voltage (Fig. 5). This result, as well as the evaluation (See table) according to (4), indicate that these lines are characteristic copper X-ray lines in which the line at $\theta = 43.9^\circ$ is allocated to the 2$^{nd}$ order of diffraction.

In the KBr-monocrystal (instead of the LiF-crystal) is used to analyze the copper X-ray spectrum, the Bragg scatterings are allowed an order of diffraction of up to 3 ($n = 3$) (Fig. 6). These structures, additional in comparison with Fig. 4, are caused by the greater lattice constant of the KBr-crystal.

The energy values of the characteristic copper X-ray lines listed in the table correspond well with the literature values. These values were calculated through the use of (4) and the following constants:

- Planck constant: $h = 6.6256 \cdot 10^{-34}$ Js
- Velocity of light: $C = 2.9979 \cdot 10^8$ m/s
- Lattice constant: LiF (100) $d = 2.014 \cdot 10^{-10}$ m, KBr (100) $d = 3.295 \cdot 10^{-10}$ m
- and the equivalent $1\text{ eV} = 1.6021 \cdot 10^{-19}$ J

Table of Results

<table>
<thead>
<tr>
<th>Line</th>
<th>$\delta$/degree</th>
<th>$E_{\text{exp.}}$/keV</th>
<th>$E_{\text{exp.}}/E_{\text{el.}}$</th>
<th>$K_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiF-Analyzer (Fig. 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n = 1$</td>
<td>20.4</td>
<td>8.830</td>
<td>0.8%</td>
<td>$K_{\beta}$</td>
</tr>
<tr>
<td></td>
<td>22.7</td>
<td>7.976</td>
<td>0.8%</td>
<td>$K_{\delta}$</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>43.9</td>
<td>8.878</td>
<td>0.3%</td>
<td>$K_{\beta}$</td>
</tr>
<tr>
<td>KBr-Analyzer (Fig. 6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n = 1$</td>
<td>12.4</td>
<td>8.761</td>
<td>1.6%</td>
<td>$K_{\beta}$</td>
</tr>
<tr>
<td></td>
<td>13.7</td>
<td>7.994</td>
<td>1.2%</td>
<td>$K_{\delta}$</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>25.1</td>
<td>8.870</td>
<td>0.4%</td>
<td>$K_{\beta}$</td>
</tr>
<tr>
<td></td>
<td>28.0</td>
<td>8.015</td>
<td>0.3%</td>
<td>$K_{\delta}$</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>39.5</td>
<td>8.873</td>
<td>0.4%</td>
<td>$K_{\beta}$</td>
</tr>
<tr>
<td></td>
<td>44.7</td>
<td>8.024</td>
<td>0.2%</td>
<td>$K_{\delta}$</td>
</tr>
</tbody>
</table>

A variation of the evaluation is possible by using the calculated characteristic copper X-ray lines from one spectrum in order to produce the corresponding crystal lattice constant from the other spectrum.
Fig. 5: X-ray intensity of copper at various anode voltages LiF (100)-monocrystal as Bragg analyser.

Anode voltage $U = 25$ kV.

Fig. 6: X-ray intensity of copper as a function of the glancing angle KBr (100)-monocrystal as Bragg analyser.

Anode voltage $U = 25$ kV.
The bremspectrum in Fig. 6 indicates an important intensity drop in the direction of smaller glancing angles at 8.2° and 16.5°. This drop falls together with the expected theoretical bromide K-absorption edge ($E_K = 13.474$ keV) in the 1st and 2nd order of diffraction. The K-absorption edges of potassium, lithium, and fluorine cannot be observed since the bremspectrum intensity in these energy ranges is too low. (For K- and L-absorption edges, see experiment 5.4.5).

From Fig. 5, it is clearly evident that the high-energy limit of the bremspectrum is (in accordance with the Duane-Hunt displacement law) dependent upon the anode voltage. (See also experiment 5.4.3).

**Note:**
The atomic energy values were taken from the “Handbook of Chemistry and Physics”, CRC Press, Inc., Florida.