Related topics
Radioactive radiation, beta-decay, conservation of parity, anti-neutrino, gamma quanta, half-value thickness, absorption coefficient, term diagram, pair formation, Compton effect, photoelectric effect, conservation of angular momentum, forbidden transition, weak interaction, dead time.

Principle and task
The inverse square law of distance is demonstrated with the gamma radiation from a $^{60}$Co preparation, the half-value thickness and absorption coefficient of various materials determined with the narrow beam system and the corresponding mass attenuation coefficient calculated.

Equipment
Radiative source, Co-60, 3.7 MBq 09097.50 1
Vernier caliper 03010.00 1
Screened cable, BNC, 1750 mm 07542.11 1
Lead brick with hole 09021.00 2
Counter tube, type A, BNC 09025.11 1
Geiger-Müller Counter 13606.99 1
Absorption material, lead 09029.01 1
Absorption material, Plexiglas 09029.04 1
Absorption material, iron 09029.02 1
Absorption material, concrete 09029.05 1
Absorption material, aluminium 09029.03 1

Problems
1. To measure the impulse counting rate as a function of the distance between the source and the counter tube.
2. To determine the half-value thickness $d_{1/2}$ and the absorption coefficient $\mu$ of a number of materials by measuring the impulse counting rate as a function of the thickness of the irradiated material. Lead, iron, aluminium, concrete and Plexiglas are used as absorbers.
3. To calculate the mass attenuation coefficient from the measured values.

Set-up and procedure
1. Secure the counter tube and the $^{60}$Co-source in the holes of the lead brick. It is advisable to take three counts of the impulses $N'\,(r)$ (each count 1 minute) for each of about 10 different distances $r$ between specimen and counter tube, and to calculate the counting rate from their mean value

$$\frac{N'(r)}{1\,min} = \bar{N}'(r).$$

Care should be taken to ensure that the preparation is about 2 mm behind the front edge of the radiation pencil and that the counter tube has an effective length of 40 mm. If we start out from the distance $r'$ between the front edge of the radiation pencil and that of the counter tube, we get $r = r' + 2.2$ cm

Fig. 1: Experimental set-up for measuring the half-value thickness of different materials.
Fig. 2: Measuring absorption with a broad and a narrow beam of radiation

a) Narrow beam system

b) Broad beam system

from a point at the centre of the counter tube. Then determine the background radiation $N_0$ without the preparation and subtract it from the measured counting rates $N'(r)$:

$$
\hat{N}(r) = \hat{N}'(r) - N_0
$$

2. Set up the experiment as shown in Fig. 1. Mask out a narrow beam from the gamma radiation emitted by the specimen, with the aid of two lead blocks with holes in them. This arrangement, known as the narrow beam system, ensures that most of the scattered radiation due to the absorber is suppressed.

After determining the background radiation count $N_0$ without the preparation, count the impulses with the preparation, first without absorber and then with the various thicknesses of absorber; count for three minutes in each case and calculate the impulse counting rates

$$
\frac{N'(d)}{1 \text{ min}} = \hat{N}'(d)
$$

Subtract the background correction from the impulse counting rates so as to obtain

$$
\hat{N}(d) = \hat{N}'(d) - N_0
$$

Theory and evaluation

The cobalt isotope $^{60}\text{Co}$ has a half-life of 5.26 years; it undergoes beta-decay to yield the stable nickel isotope $^{60}\text{Ni}$—see Fig. 3.

As with most beta emitters, disintegration leads at first to daughter nuclei in an excited state, which change to the ground state with the emission of gamma quanta. Whereas the energy levels of the beta electrons can assume any value up to the maximum because of the antineutrinos involved, the gamma quanta which participate in the same transition process have uniform energy, with the result that the gamma spectrum consists of two discrete, sharp lines (Fig. 3).

The impulse counting rate $\hat{N}(r)$ per area $A$ around a point-source decreases in inverse proportion to the square of the distance provided the gamma quanta can spread out in straight lines and are not deflected from their track by interactions.

$$
\frac{r_2}{r_1} = 2, \quad A_2 = 4 \cdot A_1, \quad A_2 = \left(\frac{r_2}{r_1}\right)^2 \cdot A_1
$$
The reason for this is that, as shown by Fig. 4, the area of a sphere round the source through with a beam of rays passes, increases as the square of the distance $r$. In vacuum (in air), therefore

$$\frac{N(r)}{A} = \frac{N(0)}{A} \cdot \frac{1}{4\pi} r^2$$

If we plot the counting rate $\dot{N}(r)$ versus the distance $r$ on a log-log scale, we obtain a straight line of slope $-2$.

From the regression lines from the measured values in Fig. 5, applying the exponential expression

$$\dot{N}(r) = a \cdot r^b,$$

we obtain the value

$$b = -2.07 \pm 0.01$$

for the exponent.

This thus proves the applicability of the inverse square law.

The attenuation of the gamma rays when they pass through an absorber of thickness $d$ is expressed by the exponential law

$$\dot{N}(d) = \dot{N}(0) \cdot e^{-\mu d},$$

where $\dot{N}(0)$ is the impulse counting rate after absorption in the absorber, and $\dot{N}(0)$ is the impulse counting rate when no absorption takes place: $\mu$ is the absorption coefficient of the absorber material and depends on the energy of the gamma quantum.

The absorption of the gamma rays is brought about by three independent effects – the Compton effect, the photoelectric effect and pair formation.

The relative contributions of these three effects to total absorption depends primarily on the energy of the quanta and on the atomic number of the absorber.
We can see from the $\mu/E$ curves in Fig. 7 that lead is particularly suitable as an absorber of gamma rays of low or high energy.

The attenuation of gamma rays therefore takes place predominantly in the electron shell of the absorber atoms. The absorption coefficient $\mu$ should therefore be proportional to the number of electrons in the shell per unit volume, or approximately proportional to the density $\rho$ of the material.

The mass attenuation coefficient $\mu/\rho$ is therefore roughly the same for the different materials.

The half-value thickness $d_{1/2}$ of a material is defined as the thickness at which the impulse counting rate is reduced by half, and can be calculated from the absorption coefficient in accordance with

$$d_{1/2} = \frac{\ln 2}{\mu}.$$

From the regression lines from the measured values in Fig. 7 we obtain the following values for $\mu = b$ and for $d_{1/2}$ and $\mu/\rho$, with the relevant standard errors, using the exponential expression

$$\hat{N} = ae^{bd}.$$

### Lead: ($\rho = 11.34$ g cm$^{-3}$)

- $\mu = 0.62$ cm$^{-1}$,
- $s_\mu = 0.009$ cm$^{-1}$
- $d_{1/2} = 1.12$ cm,
- $s_{d_{1/2}} = 0.02$ cm
- $\frac{\mu}{\rho} = 0.055$ cm$^2$g$^{-1}$;
- $s_{\mu/\rho} = 0.001$ cm$^2$g$^{-1}$

### Aluminium: ($\rho = 2.69$ g cm$^{-3}$)

- $\mu = 0.15$ cm$^{-1}$,
- $s_\mu = 0.01$ cm$^{-1}$
- $d_{1/2} = 4.6$ cm,
- $s_{d_{1/2}} = 0.3$ cm
- $\frac{\mu}{\rho} = 0.056$ cm$^2$g$^{-1}$;
- $s_{\mu/\rho} = 0.004$ cm$^2$g$^{-1}$

### Iron: ($\rho = 7.86$ g cm$^{-3}$)

- $\mu = 0.394$ cm$^{-1}$,
- $s_\mu = 0.006$ cm$^{-1}$
- $d_{1/2} = 1.76$ cm,
- $s_{d_{1/2}} = 0.03$ cm
- $\frac{\mu}{\rho} = 0.050$ cm$^2$g$^{-1}$;
- $s_{\mu/\rho} = 0.001$ cm$^2$g$^{-1}$

### Concrete: ($\rho = 2.35$ g cm$^{-3}$)

- $\mu = 0.124$ cm$^{-1}$,
- $s_\mu = 0.009$ cm$^{-1}$
- $d_{1/2} = 5.6$ cm,
- $s_{d_{1/2}} = 0.4$ cm
- $\frac{\mu}{\rho} = 0.053$ cm$^2$g$^{-1}$;
- $s_{\mu/\rho} = 0.004$ cm$^2$g$^{-1}$

### Plexiglas: ($\rho = 1.119$ g cm$^{-3}$)

- $\mu = 0.078$ cm$^{-1}$,
- $s_\mu = 0.004$ cm$^{-1}$
- $d_{1/2} = 8.9$ cm,
- $s_{d_{1/2}} = 0.5$ cm
- $\frac{\mu}{\rho} = 0.066$ cm$^2$g$^{-1}$;
- $s_{\mu/\rho} = 0.003$ cm$^2$g$^{-1}$

**Note**

As the preparation is completely enclosed in metal the beta rays are largely absorbed, so that the source can be regarded purely as a gamma emitter.

Water also can be examined as an absorber provided that suitable Plexiglas containers are available.

The relevant safety regulations governing radiation must be observed.