Related topics

Standard deviation; expected value of pulse rate; dead time, diffrent symmetries of distributions.

Principle and task

The aim of this experiment is to show that the number of pulses counted during identical time intervals by a counter tube which bears a fixed distance to a long-lived radiation emitter correspond to a Poisson's distribution. A special characteristic of the Poisson's distribution can be observed in the case of a small number of counts $n < 20$: The distribution is unsymmetrical, i.e. the maximum can be found among smaller numbers of pulses than the mean value. In order to show this unsymmetry the experiment is carried out with a short counting period and a sufficiently large gap between the emitter and the counter tube so that the average number of pulses counted becomes sufficiently small.

Equipment

Counter tube, type A, BNC	09025.11	1
Screened cable, BNC, l 750 mm	07542.11	1
Magnetic base	09053.01	2
Source holder for 09053.01	09053.04	1
Counter tube holder for 09053.01	09053.02	1
Mounting plate R, 32cm×21cm	13001.00	1
Americium-241 source, 370 kBq	09090.11	1
COBRA-interface 2	12100.93	1
Counter tube module	12106.00	1
PC COBRA data cable RS232, 2 m	12100.01	1
Softw. COBRA Statis. radioa. decay	14266.51	1

Poisson's distribution

Set-up and procedure

The experiment is set-up as shown in Fig. 1.

The first step is to adjust the gap between the radioactive source and the counter tube as well as the counting time such that an average pulse rate of about 1…3 results. The following preliminary experiment must be carried out:

— Start the program "Statistics of radioactive decay".
— Select the program option <measure> <parameters>.
— Maintain the presettings <function> "*N*(*t*)"; <background> "substract not"; <number of measurements> "1000!.
— Enter a measuring time of 0.1 s (i.e. the shortest time possible).
— Place the radioactive source about 5 cm away from the admission opening of the counter tube.
— Select the program option <set> <function> and change the presetting of the end value of the Y-axis to "10".
— Activate the pgoram option <measure> <start> (in a very comfortable way simply by means of the <F10> key).

Fig. 1: Experimental set-up: Poisson's distribution and Gaussian distribution of radioactive decay.

— Check the measured value. If necessary, change the gap between the radioactive source and the counter tube in order ob obtain the desired number of counts. To ge low pulse rates, it will be helpuf to put a sheet of paper between the source and the counter tube.

Then a series of measurements (sampling size 1000) is carried out with the aid of experimental equipment and set-up used for the preliminary experiment. The following steps are necessary for this purpose:

— Select the program function <measure> <parameters>.
— If desired, enter a suitable name for the measurement (e.g. "Poisson's distribution").
— Select "statistics" under <display>.
— Hit <ok> to close the window.
— Activate the program function <measure> <Start> (in a very comfortable way simply by means of the <F10> key).
— When the measurement is over, select <file> <save> to store the measurement series on the hard disk.

Repeat the measurement with larger number of counts (reduce the gap; if necessaray, increase the counting period), for instance for an average of 5, 10 and 20 pulses per counting interval.

Theory and evaluation

Figure 2 shows the distribution measured first. It becomes very clear that the distribution is unsymmetrical, i.e. the counts (1) with a maximum frequency are on the left of the middle between the minimum number of counts (10) and the maximum number of counts (10).

Figure 3 shows a comparison of the four measured statistical distributions. With the aid of the program function <graphics> <statistics> <curve> a theoretical curve of the Poisson's distribution was adapted to the measured pulse rate distribution. It becomes clear that the unsymmetry decreases with an increasing number of pulses.

Figure 4 shows the same measured distributions together with an adapted Gaussian curve so that the degree of unsymmetry of the measured distributions can be verified. It can be seen clearly that the approximation of the symmetrical Gaussian distribution is worse than that of the Poisson's distribution as far as the first three measurements are concerned. The maximum of the Guassian distribution is located too far on the right. In the case of the fourth measurement (mean value 20.3) the differences are far less distinctive.

In case of a large sampling size the probability $P(N)$ according to which N pulses are counted is a function of the expected number of pulses which in this case can be replaced by the mean value $\overline{N}$:

$$P(N) = \frac{\overline{N}^N}{N!} \cdot e^{-\overline{N}}$$

This is the so-called Poisson's distribution which regardless of the average number of counts $\overline{N}$ always applies for radioactive decays.

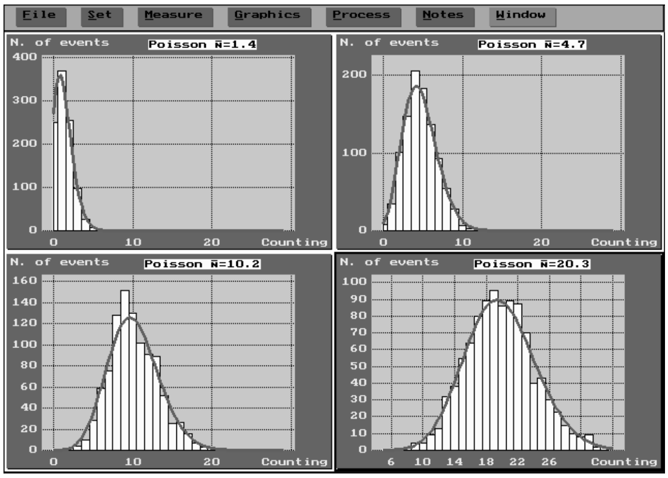

Fig. 3: Comparison of the pulse rate distributions for different average numbers of counts plus the theoretical curves of the Poisson's distributions corresponding to the mean values $\overline{N}$.

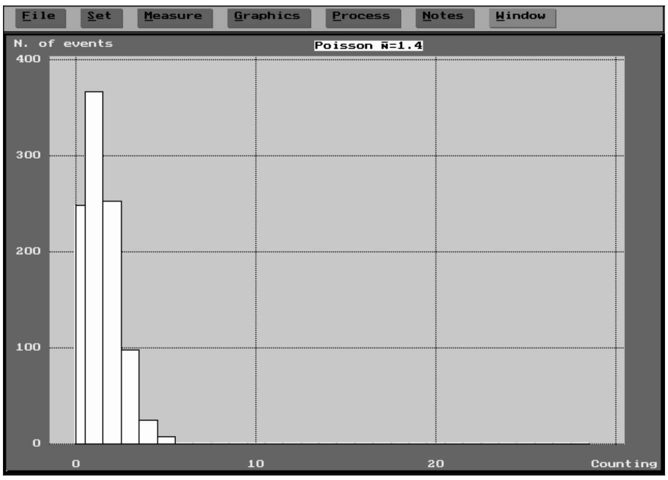

Fig. 2: Distribution of the number of counts with an average number of pulses of 1.4.

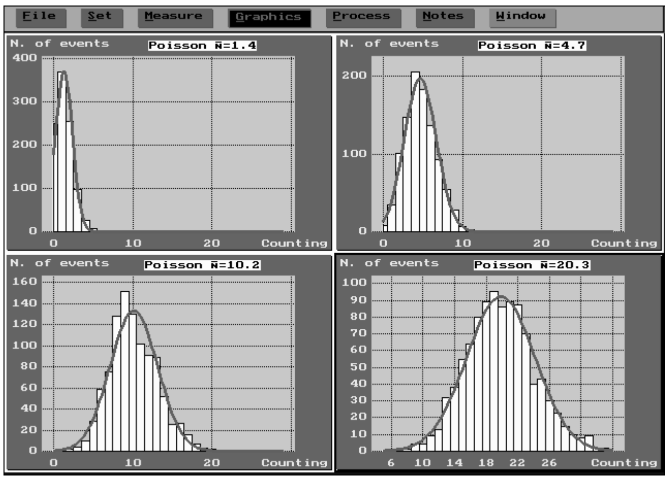

Fig. 4: Like firuge 3 with an adapted Gaussian curve instead of the Poisson's curves.

The adaptation of the theoretical Poisson's distribution carried out by the software can be easily verified. This can be done in a very comfortable way with the aid of a programmable pocket calculator. An even more comfortable way is to use a spread-sheet analysis program which in general can provide the graphical representation, too. The Poisson's distributions shown in figure 5, for instance, were produced by means of the program Excel® by Microsoft®. Furthermore, it is possible to introduce the real measured values into the theoretical curves as well. For this purpose, the COBRA program offers the measured values according to figure 6 after the display type "statistics" has been selected. In order to obtain a representation as shown in figure 5 it is necessary first to produce the representations shown in figure 3 or figure 4. Then the function <windows> must be selected in the main menu line. As a next step the display type <table> must be activated in each of the four windows (e. g. by means of <F5>). The frequencies given in the tables must then be converted into the probability values *P*(*N*) as shown in figure 5. For this purpose the individual frequencies must be devided by the total number of pulses counted.

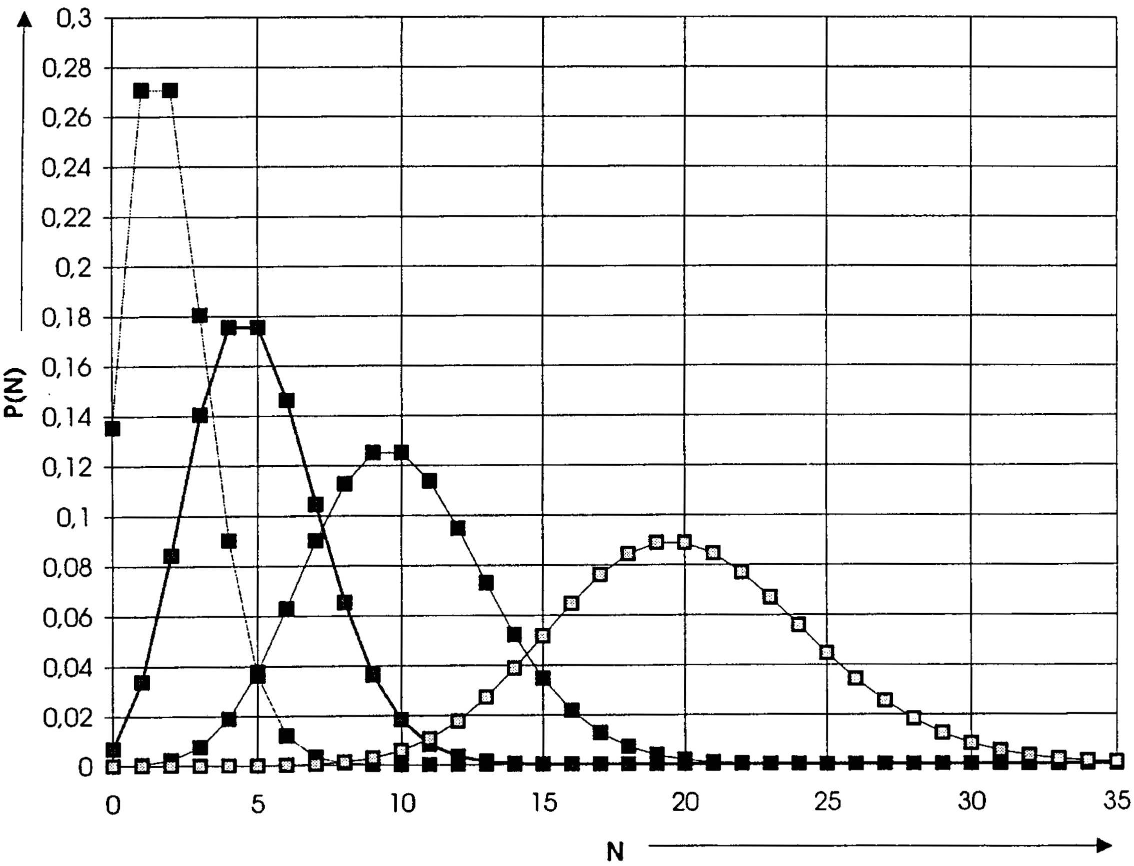

Fig. 5: Poisson's distributions for the average numbers of counts 2, 5, 10 and 20, calculated and represented by means of a spread-sheet analysis program.

Fig. 6: Like figure 3 and 4, representation of the measured values in the form of a table.

File Set Measure Graphics Process Notes Window

Counting	N. of events
0	249
1	367
2	253
3	98
4	25
5	8
6	0
7	0
8	0
9	0
10	0
11	0

Counting	N. of events
0	8
1	35
2	101
3	147
4	205
5	183
6	136
7	93
8	54
9	28
10	6
11	3

Counting	N. of events
4	10
5	28
6	59
7	75
8	128
9	151
10	130
11	102
12	91
13	89
14	52
15	25

Counting	N. of events
14	38
15	55
16	64
17	80
18	89
19	95
20	86
21	89
22	87
23	70
24	40
25	43

Gaussian distribution of radioactive decay

Principle

Not only the Poisson’s distribution, but also the Guassian distribution which is always symmetrical is very suitable to approximate the pulse distribution measured by means of a long-lived radiation emitter and a counter tube arranged with a constant gap between each other. A premise for this is a sufficiently high number of pulses and a large sampling size.
The purpose of the following experiment is to confirm these facts and to show that the statistical pulse distribution can even be be approximated by a Guassian distribution, when (due to the dead time of the counter tube) counting errors occur leading to a distribution which deviates from the Poisson’s distribution.

Set-up and procedure

The experiment is set up as shown in part A.
The second measurement of this experiment requires a sufficiently strong radioactive source with the aid of which pulse rates of more than 3000 pulses/s can be reached. These high pulse rates can be reached by means of a α-emitter since the counter tube responds strongly to these particles. The Am-241 source 09090.11 (370 kBq) which must be arranged very closely to the admission opening of the counter tube is very suitable for this purpose.

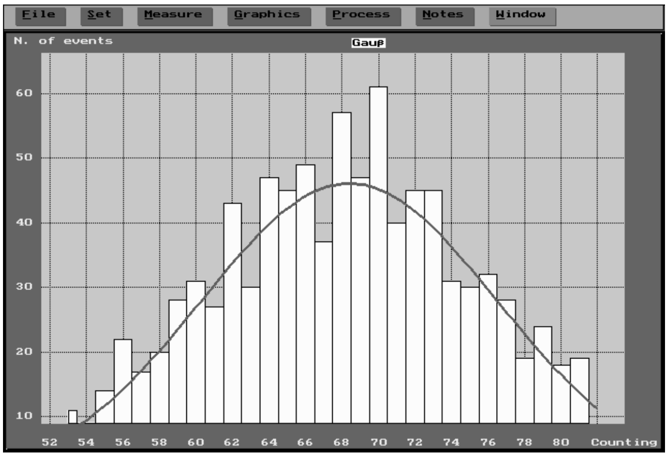

Fig. 7: Pulse rate distribution (average pulse rate approx. 70/s) with an adapted Gaussian curve.

Two different measurements must be carried out. During the first measurement the pulse rate must not exceed 100 pulses/s in order to guarantee that the dead time of the counter tube (about 0.1 ms) has no falsifying effect on the result. The average number of counts per counting procedure should be more than 50…100 pulses so that a Guassian adaptation is possible. The pulse rate for the second measurement should be at least 3000 pulses/s in order to be able to underline the decrease in fluctuation which is due to the influence of the dead time.

The first step is to adjust the gap the between the radioactive source and the counter tube such that an average pulse rate of about 50…100 pulses/s results. The following preliminary experiment must be carried out for this purpose:
— Start the program “Statistics of radioactive decay”.
— Select the program option <measure> <parameters>.
— Maintain the presettings <function> “$N(t)$”; <background> “subtract not”; <number of measurements> “1000”.
— Enter a measuring time of 1 s.
— Place the radioactive source about 5 cm away from the admission opening of the counter tube.
— Hit <ok> to close the window.
— Activate the program option <measure> <start> (in a very comfortable way simply by means of the <F10> key).
— Check the measured values. If necessary, change the gap between the radioactive source and the counter tube in order to obtain an average number fo counts of 50…100.

Then a series of measurements (sampling size 1000) is carried out with the aid of experimental equipment and set-up used for the preliminary experiment. The following steps are necessary for this purpose:
— Select the program function <measure> <parameters>.
— If desired, enter a suitable name for the measurement (e.g. “Gauss”).
— Selet “statistics” under <display>.
— Hit <ok> to close the window.
— Activate the program function <measure> <start> (in a very comfortable way simply by means of the <F10> key).

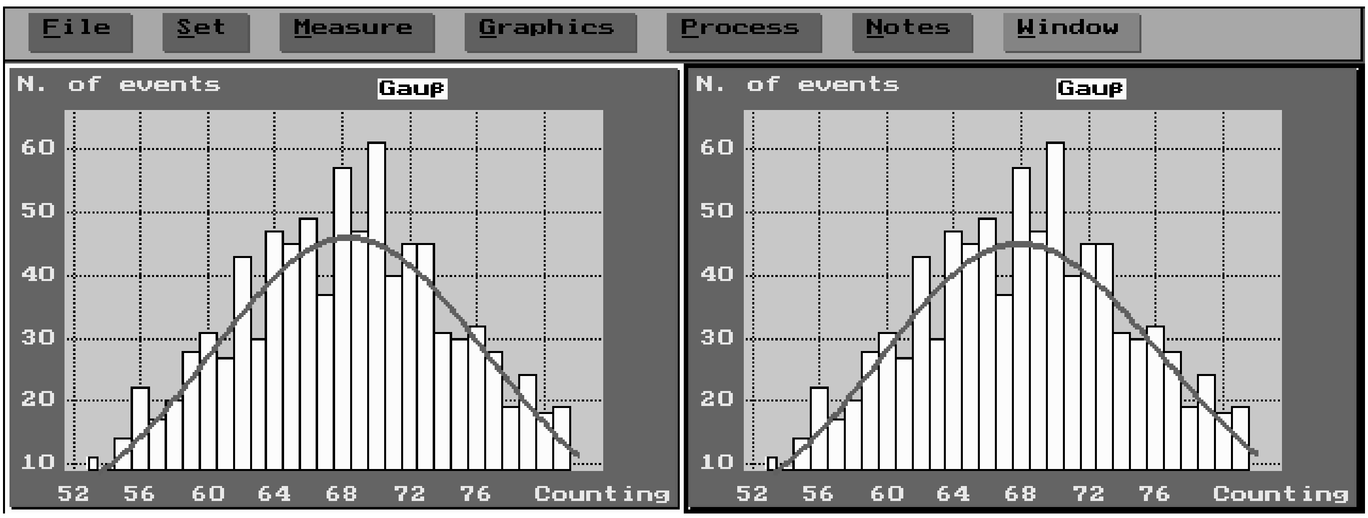

Fig. 8: Distribution like in figure 1; with an adapted Gaussian curve (left window) and a Poisson’s curve (right window).

— When the measurement is over, select <file> <save> to store the measurement series on the hard disk.
— Trace the Guassian curve by means of <graphics> <statistics>.

Figure 7 shows a typical measurment example.

The second measurement must be carried out with the shortest distance possible between the radioactive source and the countertube. **Caution:** If the radioactive source comes into direct contact with the admission opening of the counter tube, the counter tube might be destroyed. The measuring time can be reduced to 0.1 s. It is recommended to increases the end value of the Y-scale, e.g. to 200. Then the second measurement is carried out in the same way as the first one.

Theory and evaluation

Figure 7 shows a puls rate distribution which is sufficiently approximated by the added Gaussian curve. Figure 8 shows that the Poisson’s curve in the right window is nearly identical with the Gaussian curve in the left window. (In order to obtain a representation as shown in figure 8 the measurement of window 1 must be stored on the hard disk and reloaded into window 2. The next steps is to produce the corresponding graphical representation by means of <graphics> <statistics>.)

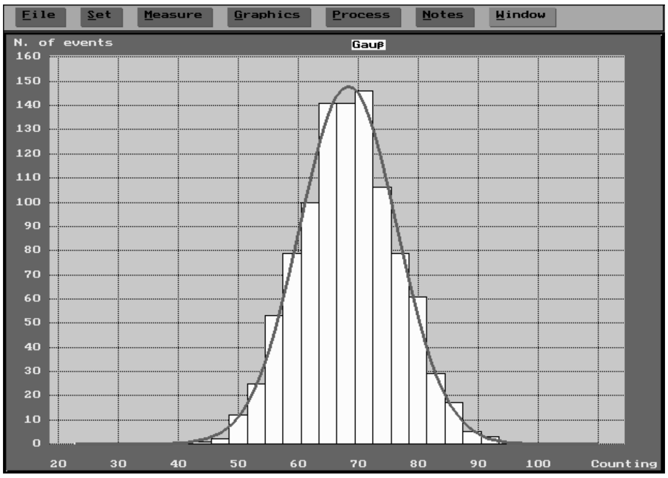

Fig. 9: Like figure 7; here 3 counts are combined in one bar.

However, the strong statistical fluctuations of the frequencies have a disturbing influence on the typical form of the Gaussian distribution. The sampling size of 1000 seems still to be too small to obtain a sufficiently smooth distribution. A special option of the program is to combine several numbers of counts into one single bar of the histrogram (figure 9) so that a nearly ideal distribution results. In order to obtain figure 9 the value 3 must be selected in the <collection> window which can be opened by means of <graphics> <statistics>.

This special option must be used in order to represent the second measurement series (with an extremely high pulse rate), too (see figure 10). It can be seen that a Gaussian curve can be easily adapted to this distribution, too. However, in this case the Poisson’s distribution in the right window does not match the measured distribution since due to the dead time of the counter tube the fluctuation of the number of counts is smaller than the value which is typical of a Poisson’s distribution.

In the case of a large sampling size and a sufficient number of counts the probability $P(N)$ according to which N pulses are counted can not only be described by the unsymmetrical Poisson’s distribution, but also with the aid of the symmetrical Gaussian distribution:

$$P(N) = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-\frac{(N - \overline{N})^2}{2\sigma^2}} \qquad (1)$$

with σ being the standard deviation. The Gaussian curve represented by means of (1) adapts to the symmetrical distributions with various fluctuations which are described by the standard deviation σ. Thus, the Gaussian distribution is of great importance for describing statistically straggling data of the most various kinds. As far as the radioactive decay is concerned the standard deviation is given as the means value $\overline{N}$

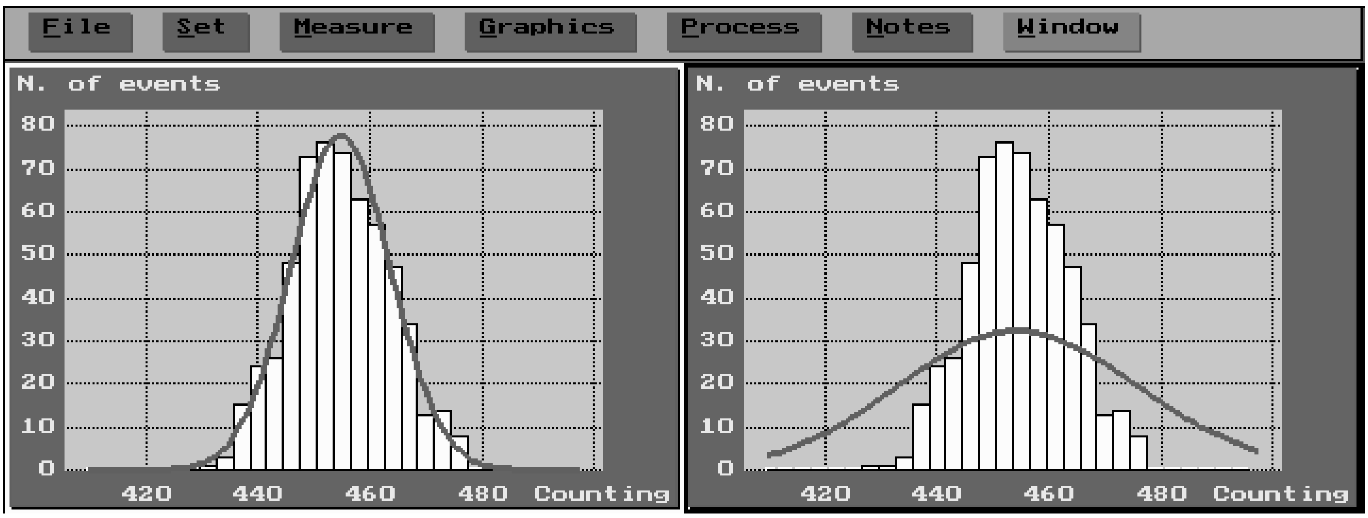

Fig. 10: Pulse distribution for an extremely high pulse rate (4600/s); with an adapted Gaussian curve (left window) and a Poisson’s curve (right window).

in the case of a large sampling size (expected value). $\sigma = \sqrt{\overline{N}}$ so that the general equation (1) is transformed into the following special form:

$$P(N) = \frac{1}{\sqrt{2\pi \cdot \overline{N}}} \; e^{-\frac{(N-\overline{N})^2}{2 \cdot \overline{N}}} \qquad (2)$$

With the aid of several calculation procedures (conversion of *N!* by means of Stirling’s formula and then an expansion into a series) it can be shown that in the case of large numbers of counts the Poisson’s distribution represented by the formula (3) changes into the Gaussian distribution (2).

$$P(N) = \frac{\overline{N}^N}{N!} \, e^{-\overline{N}} \qquad (2)$$

Instead of explaining the mathematical derivation we would like to show how the agreement between the two distributions can be verified by means of the computer and a spread-sheet analysis program linke, for instance, MS-Excel®. In order to obtain the diagrams for the two distributions the corresponding formulas (2) and (3) must be entered. Figure 11 compares the resulting Gaussian distribution (complete line) with the Poisson’s distribution (squares) for a mean value of $\overline{N}$ = 50. It can be clearly seen that the two distributions only differ from each other to a very slight extent.

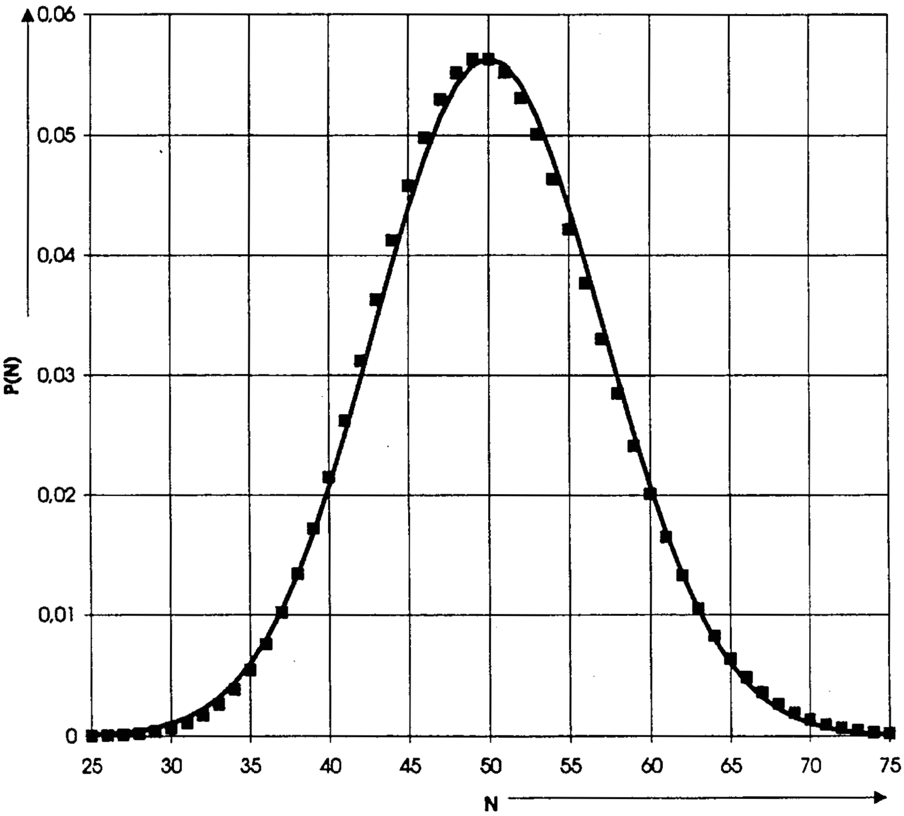

Fig. 11: Gaussian distribution (complete line) and Poisson’s distribution (squares) obtained with the aid of a spread-sheet analyisis program.