Half-life and radioactive equilibrium

Related concepts

Parent substance, daughter substance, rate of decay, disintegration or decay constant, counting rate, mean life, disintegration product.

Principle and task

The half-life of a Ba-137 m daughter substance eluted (washed) out of a Ca-137 isotope generator is measured directly and is also determined from the increase in activity after elution.

Equipment

Isotope generator Cs-137, 400 kBq	09047.60	1
Pulse rate meter	13622.93	1
Digital multimeter	07134.00	1
Counter tube, type A, BNC	09025.11	1
Stopwatch, digital, 1/100 sec.	03071.01	1
Aluminium, sheet, 1×20×200mm, 5 pcs	31074.00	1
Tripod base -PASS-	02002.55	1
Support rod -PASS-, square, l 250 mm	02025.55	1
Right angle clamp -PASS-	02040.55	2
Universal clamp	37715.00	2
Glass beaker, short, 250 ml	36013.00	2
Test tube, 160×16 mm, 100 pcs	37656.10	1
Screened cable, BNC, l 750 mm	07542.11	1
Connecting cord, 500 mm, red	07361.01	1
Connecting cord, 500 mm, blue	07361.04	1

Problems

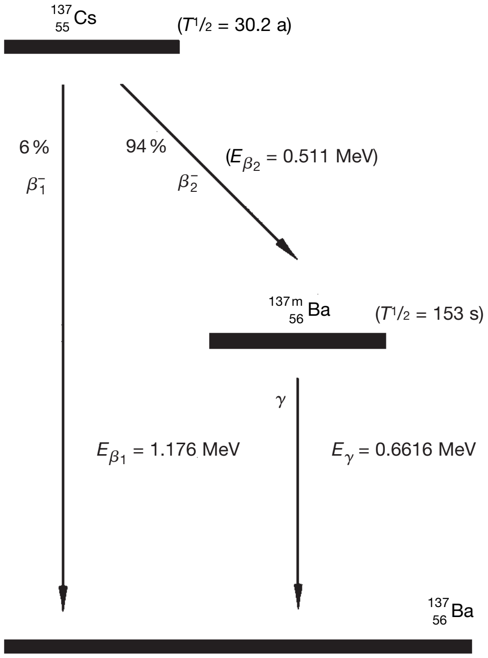
1. To record the counting rate as a function of the counter tube voltage (counter tube characteristic) when the isotope generator activity is constant (radioactive equilibrium).
2. To measure the activity of the isotope generator as a function of time immediately after elution.
3. To measure the activity of a freshly eluted solution of Ba-137 m as a function of time.

Set-up and procedure

1. Set up the experiment as shown in Fig. 1. Fix up the counter tube immediately in front of the isotope generator in order to record the counter tube characteristic. Set the counter tube voltage on the counting ratemeter and measure it with the multimeter at the sockets provided. The multimeter will then be used to display the counter rate.

Fig.1: Experimental set-up for determining the half-life of Ba-137 m.

Fig. 2: Term diagram of caesium-137.



2. The counter tube voltage selected is in the range of the plateau (450 V DC/500 V). In accordance with the instructions the isotope generator is eluted into a glass beaker which sould then be placed as far away from the counter tube as possible. Make a U-shaped cap from a strip of aluminium sheet and put it over the tube: it will absorb the electrons in the betadecay phase which would otherwise iterfere with the experiment. To measure the increase in activity it is advisable to read off the impulse rate every 30 seconds after elution (counting ratemeter time constant = 10 seconds). Activity and impulse count are sufficiently proportional at low impulse rates.

3. To measure the half-life of the Ba-137 m isotope, first elute the isotope generator in a test tube, then place it as far away from the rest of the equipment as possible. The counter tube (without the aluminium cap) can now be set up immediately in front of the bottom end of the test tube.

Theory and evaluation

Caesium-137 has a half-life of 30 years; it undergoes beta-decay into barium-137, 92 % of the daughter products passing through the intermediate (virtual) state Ba-137 m (Fig. 2). Ba-137 m has a half-life of 156 seconds, emitting a gamma quantum and being transformed into stable Ba-137.

The decay chain is thus

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3$$

where

N_1 = number of Cs-137 atoms

N_2 = number of Ba-137 m atoms

N_3 = number of Ba-137 atoms

λ_1 = disintegration constant of Cs-137

λ_2 = disintegration constant of Ba-137 m.

The rates of disintegration are:

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \quad \text{and}$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

Solving this system of two differential equations gives for $N_2(t)$, with the initial condition that $N_2(t = 0) = 0$:

$$N_2(t) = N_1(0)\,\frac{\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

The activity of the daughter substance is thus

$$A_2(t) = \lambda_2 N_2 = A_1(0)\,\frac{\lambda_2}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

As the half-life of the parent substance $T_{1/2}(1)$ is much greater than that of the daughter substance $T_{1/2}(2)$ in this case, then $\lambda_1 << \lambda_2$. If we now neglect λ_1 in relation to λ_2, we obtain

$$A_2(t) = A_1\left(1 - e^{-\lambda_2 t}\right),$$

with A_1 constant.

When t is very long,

$$A_2(t) - A_1 \quad \text{and}$$

$$\lambda_2 N_2 = \lambda_1 N_1,$$

i.e. the substances are in equilibrium (dynamic equilibrium, steady state).

If this equilibrium is disturbed by the removal of the daughter substance, the system will try to restore equilibrium by forming more of it. For the incease in the daughter substance, we have:

$$N_2(t) = \frac{\lambda_1}{\lambda_2} N_1\left(1 - e^{-\lambda_2 t}\right),$$

with N_1 constant.

Here, the formation constant of the daughter substance is equal to its disintegration constant λ_2.

The activity increases in accordance with

$$A_2(t) = A_1\left(1 - e^{-\lambda_2 t}\right),$$

with A_1 constant.

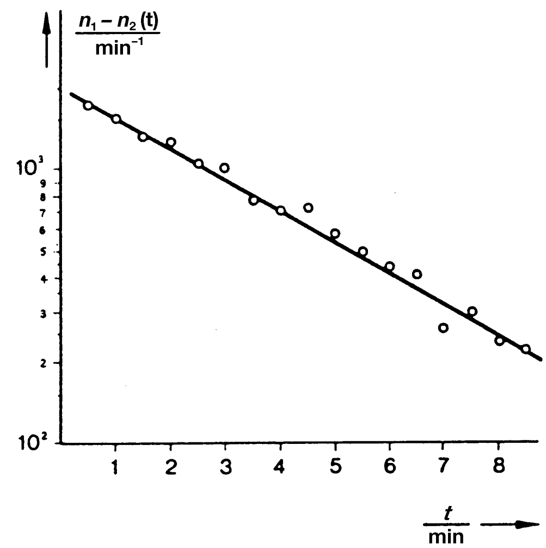
If the equilibrium activity

$$A_2(t = \infty) = A_1$$

is measured prior to elution, then λ_2 can be calculated from the measured values $A - A(t)$ because

$$A_1 - A_2(t) = A_1\, e^{-\lambda_2 t} \qquad (1)$$

Fig. 3: Difference between the counting rates before (n_1) and after (n_2) elution, as a function of time.



Using the measured values from Fig. 3, and by matching the curve for the expression

$$y = a\,e^{bx} \text{ with equation} \qquad (1)$$

we obtain the value

$$\lambda_2 = 0.270 \pm 0.008 \text{ min}^{-1}$$

for the formation constant.

The related half-life

$$T_{1/2} = \frac{\ln 2}{\lambda_2}$$

is thus

$$T_{1/2} = 2.57 \pm 0.08 \text{ min.}$$

The daughter substance eluted at time $t = 0$ decays in accordance with

$$N_2^{\prime}(t) = N_2^{\prime}(t = 0) \cdot e^{-\lambda_2 t};$$

i. e. activity decreases in accordance with

$$A_2^{\prime}(t) = A_2^{\prime}(t = 0) \cdot e^{-\lambda_2 t} \qquad (2)$$

Using the measured values in Fig. 4, and by matching the curve for the expression

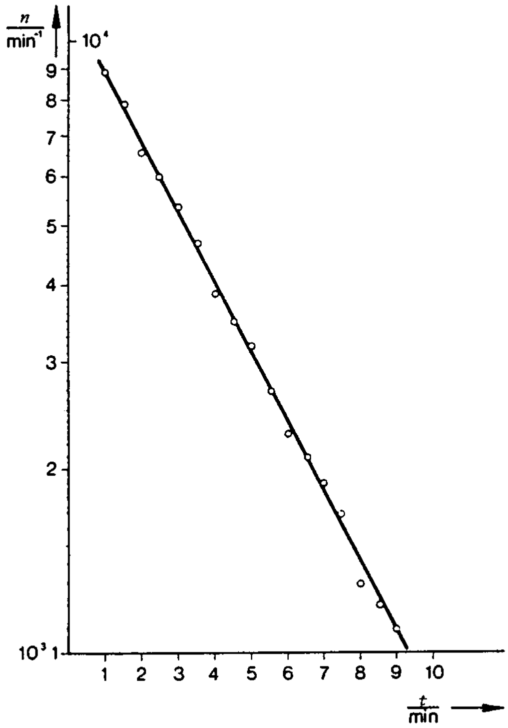
$$y = a\,e^{bx} \text{ with equation (2),}$$

we obtain the values

$$\lambda_2 = 0.264 \pm 0.003 \text{ min}^{-1}$$

for the disintegration constant.

Fig. 4: Counting rate of the eluted daughter substance as a function of time.



The related half-life

$$T_{1/2} = \frac{\ln 2}{\lambda_2}$$

is thus

$$T_{1/2} = 2.62 \pm 0.03 \text{ min.}$$

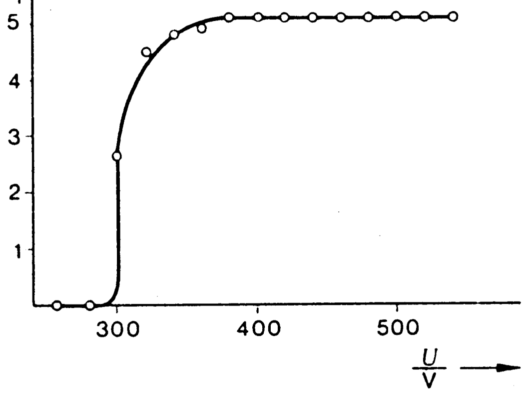
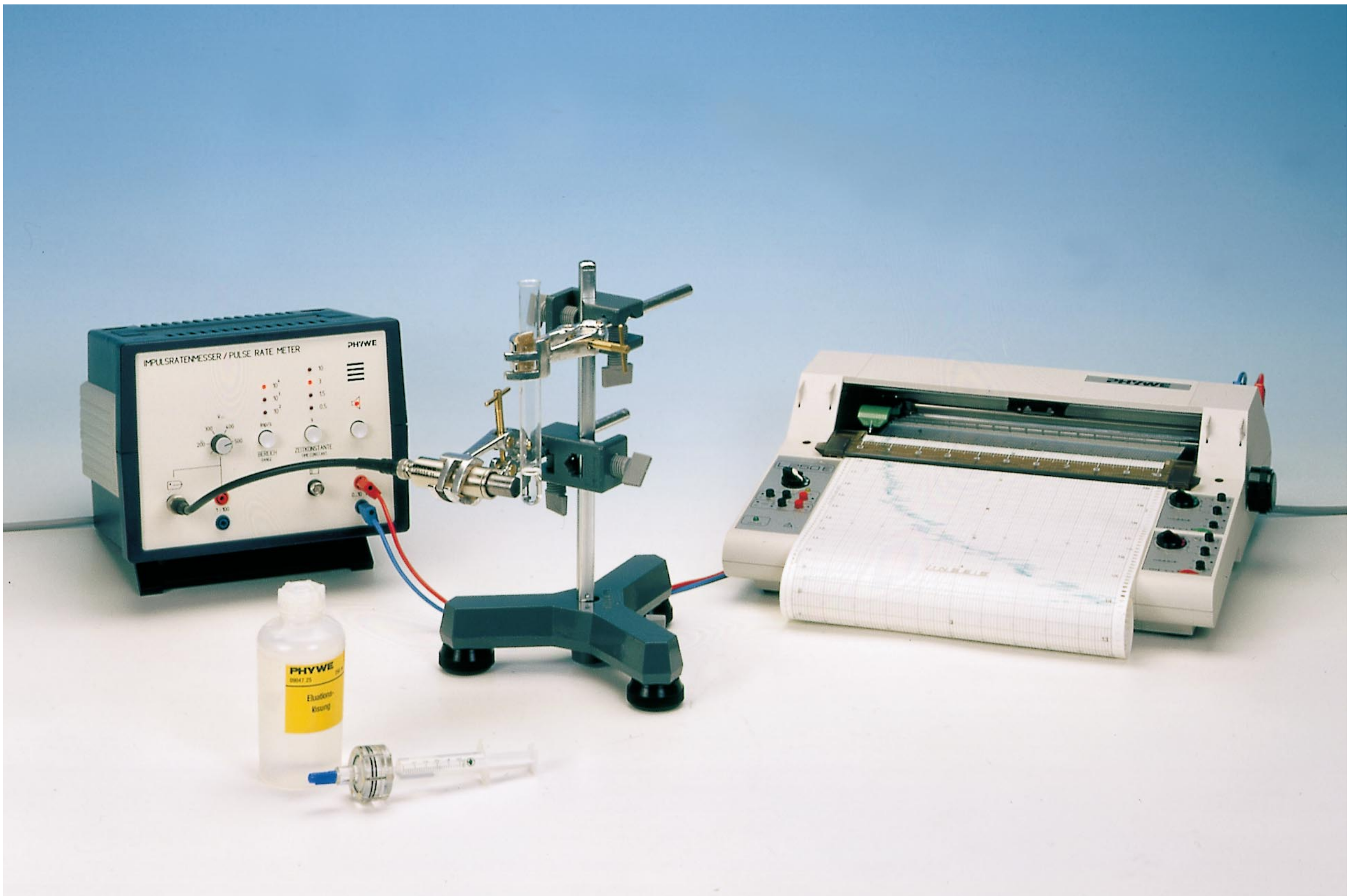


Fig. 5: Counting rate at constant activity, as a function of the counter tube voltage.

Fig. 6: Experimental set-up for determining the half life of Ba-137 m with yt-recorder.



Notes

The counting rate *n* recorded by the counting equipment is essentially proportional to the activity *A* being measured.
The values in Fig. 2 were taken from the “Handbook of Chemistry and Physics”, CRC Press, Florida, USA, 6oth edition 1980.

Annexe

A more elegant way for determination of the experimental values with the help of a yt-recorder (11414.95) is shown in Fig. 6.